Qualitative plots of bound-state wave functions

- The wave function of the **ground state** has, in the classically **allowed region** with **E > V**, the **smallest possible "curvature":**

$$\Phi''(x)/\Phi(x) = -\frac{2m(E-V)}{\hbar^2}$$

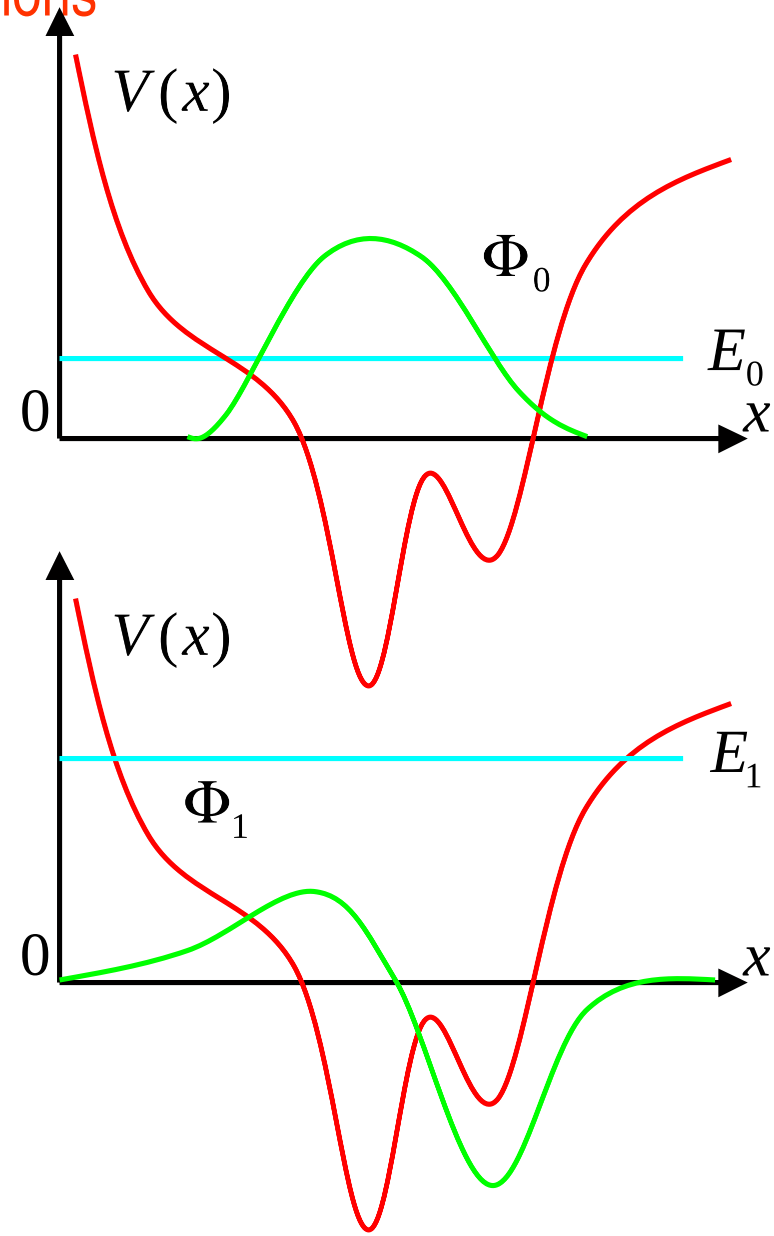
- The wave function of the **ground state** has, in the classically **forbidden region** with **E < V**, the **fastest exponential falloff:**

$$\Phi''(x)/\Phi(x) = \frac{2m(V-E)}{\hbar^2}$$

- For continuous potentials, the wave function is continuous in its first 2 derivatives.

- The stationary state **n** has **n** nodes:

$$\Phi(x_{node}) = 0$$



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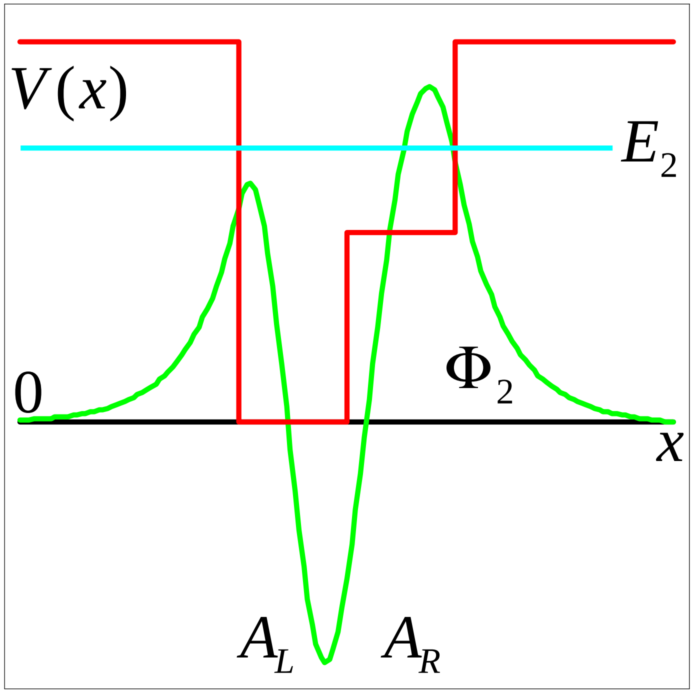
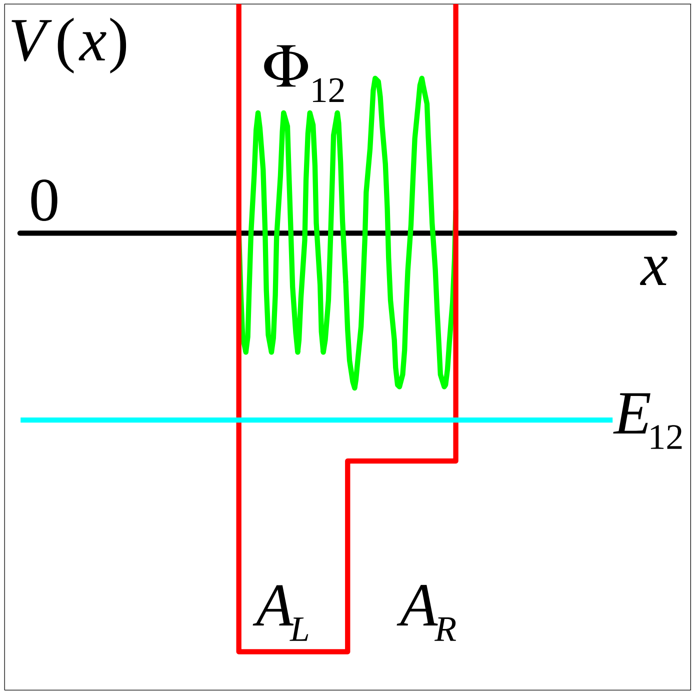
The amplitude of the wave function





Smoothness at center:

$$\left.\begin{aligned}\Phi(c) &= A_L \sin(k_L c+\varphi_L) = A_R \sin(k_R c+\varphi_R)\\ \Phi'(c) &= k_L A_L \cos(k_L c+\varphi_L) = k_R A_R \cos(k_R c+\varphi_R)\end{aligned}\right\} \frac{A_R^2}{A_L^2} = \frac{\Phi(c)^2 + \frac{\Phi'(c)^2}{k_R^2}}{\Phi(c)^2 + \frac{\Phi'(c)^2}{k_L^2}}$$

Correspondence principle:
The probability to find a particle with small velocity is larger since it spends more time.

Regions of smaller **k** have larger maximum values of the amplitude than adjacent regions of larger **k**.

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Symmetric potentials

$$-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\frac{\partial}{\partial x}\Phi(x)+V(x)\Phi(x)=E\,\Phi(x)$$

$$V(x)=V(-x)$$

If $f(x)$ is a stationary solution of the Schrödinger equation for energy E, then so is $f(-x)$

One can use this function to find a symmetric and an anti symmetric wave function, except if **f(x)** is already symmetric or anti-symmetric.

$$\Phi(x)=\begin{cases}\Phi_s \propto f(x)+f(-x)\\ \Phi_a \propto f(x)-f(-x)\end{cases}$$

But for each E there is only one solution, and thus **f(x)** has to be **symmetric** or **anti-symmetric.**

Odd parity: $\Phi(x)=-\Phi(-x) \quad \rightarrow \quad \Phi(0)=0$

Even parity: $\Phi(x)=\ \ \Phi(-x) \quad \rightarrow \quad \Phi'(0)=0$

When searching for wave functions to a symmetric potential, it is helpful that it suffices to start the search with either

$$\Phi(0)=0 \quad \text{or} \quad \Phi'(0)=0$$

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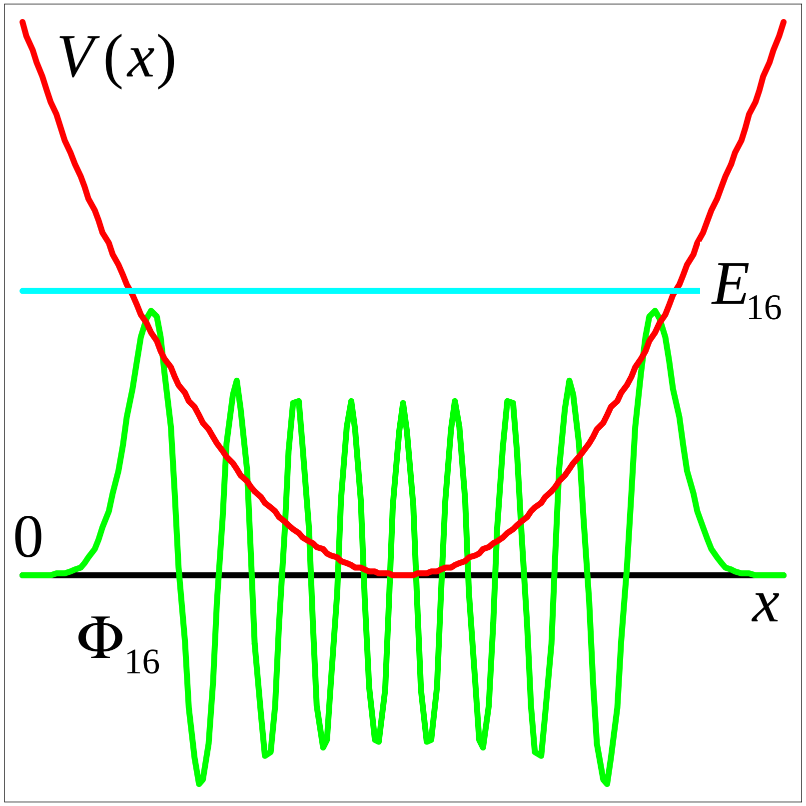
Summary for the qualitative determination of wave functions

1. Odd or even symmetry in the case of a symmetric potential.

2. The correct number **n** of nodes for the **n**th energy level above the ground state.

3. The correct relative wavelengths (shorter or longer) for different values of the potential at different places inside the potential well, shorter where the well is deeper and shorter for states of higher energy.



4. The correct relative maximum values of amplitudes at adjacent points of different potential inside the well. Amplitudes get larger where the well gets shallower.

5. The correct relative rate of decrease (more or less gradual) of the wave function with distance outside the well. The decrease is more gradual for a shallower potential and for a state of higher energy.

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4) Solutions of Schrödinger's equation in one dimension

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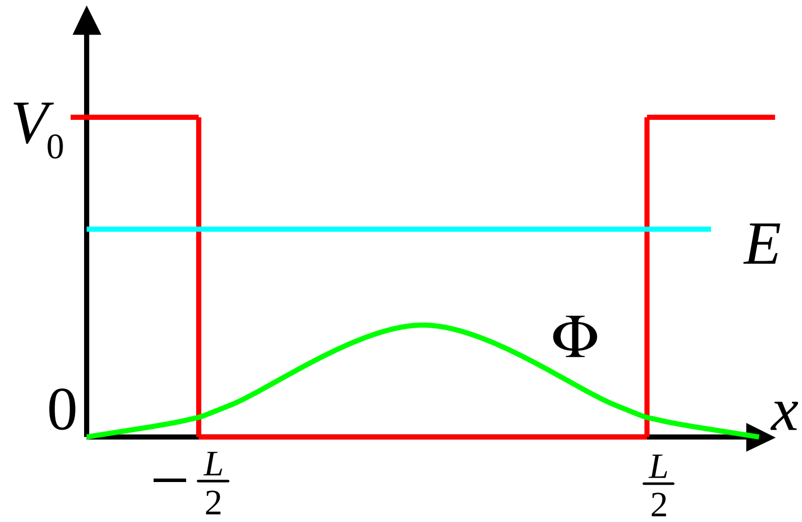
The square well

Used as approximation for a neutron in a nucleus
or for states of an electron in a linear molecule.



Stationary solutions:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Phi(x)+V(x)\Phi(x)=E\,\Phi(x)$$

Right: $\frac{\partial^2}{\partial x^2}\Phi_R(x)=\alpha^2\,\Phi_R(x)\,,\quad \alpha=\sqrt{\frac{2m}{\hbar^2}(V_0-E)}$

$$\Phi_R=Ae^{-\alpha x}+\cancel{\tilde{A}e^{\alpha x}}\,,\quad \Phi_R'=-A\alpha e^{-\alpha x}$$

Center: $\frac{\partial^2}{\partial x^2}\Phi(x)=-k^2\,\Phi(x)\,,\quad k=\sqrt{\frac{2m}{\hbar^2}E}$

$$\begin{aligned}\Phi_C &= B\sin(kx+\varphi)\\ \Phi_C' &= Bk\cos(kx+\varphi)\end{aligned}\quad,\qquad \varphi=\begin{cases}0 & \text{for } \Phi(-x)=-\Phi(x)\\ \frac{\pi}{2} & \text{for } \Phi(-x)=\Phi(x)\end{cases}$$

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