Compositeness Talk 10/25/10 Dean Robinson

References
- Overviews Peccei "Gauge Theories of the Eighties" Lect. Notes Phys (181) 355 (1983)
  Harari Phys Repts 104 159 (1984)
- Anomaly matching 't Hooft "Recent devel. in gauge theories" (Les Houches) 1979
- Complementarity DRS Nucl. Phys B 173 208 (1980)

- **Motivation**
  - Well known open problems by 1980
    - → Hierarchy problem (HP)
    - → EWSB
    - → Fermion mass splittings ($m_e \sim m_p/4000$)
    - → Neutrino masses
    - → GUT?
    - → Flavor existence
    - → Strong CP problem.
- Knee-jerk response: What if basic constituents aren't so basic? ⇒ Compositeness. This was a very popular idea 70's → mid 80's. Compositeness of Higgs, quarks, leptons and even gauge bosons (!) investigated
- Skip to end of story:

  Advantages of idea
  - → Solves HP, EWSB can be via a scalar condensate (eg technicolor)
  - → Natural ~~fermion masses~~ mass hierarchies
  - → Repeating copies of representations if ∃ chiral sym breaking.
  - → We already have a composite theory: QCD!

  Problems
  - → Often UV completions are horrendous. Need to avoid FCNCs and precision EW constraints. $f_{IR} \leq f_{UV}$ general feature (Appelquist)
  - → Probably need several stages of confinement strong $\xrightarrow{\text{confine}}$ weak $\xrightarrow{\text{run}}$ strong.
  - → Death by options!

Our focus today: constraints on chiral bound states "interesting composites"
dynamical scheme to generate chiral sym breaking pattern and masses.

General setup

- We consider a confining × flavor group $G_c \times G_F$, with fermionic content $\{T\}$. $\{T\}$ are all chiral, called preons. Note chiral ⟺ complex representation ⟺ no masses.

- At some scale Λ, G_c coupling becomes strong.
  → Confinement!
  → Well-below Λ, new EFT, whose dof are G_c singlets.

- Immediate questions
  1) What are the physical bound states? Triquarks, pentaquarks, etc? Lots of options!
  2) What is the mass spectrum? Are there any massless dofs?
  3) Is G_F broken by confinement?

Confinement as an $\boxed{\text{algebraic}}$ exercise in terms of singlets! Need $\boxed{\text{dynamical}}$ info

't Hooft Anomaly Matching

- Answer to 2) and 3) partially given/controlled by 't Hooft anomaly matching constraint.

- Sketch of idea. Consider preon theory

	G_c (asym. free)	G_F (weakly gauged)
Preons	r_c	r_F
Spectators	1	r^s_F

← Left handed, by convention

- By construction,

$$A_{r_c}(G_c^3) = A_{r_c}(G_c \text{ inst.}) = 0$$

(Schwinger–Dyson)

$$A_{r_F}(G_F) = -A_{r_F^s}(G_F)$$

- At some scale, confinement occurs! Assume G_F is <u>unbroken</u>. New EFT

	G_c	G_F
Bound states	1	r_F'
Spectators	1	r_F^s

- Spectators do not form bound states: they are G_c singlets!
- We must have

$$A_{r_F'}(G_F) = - A_{r_F^s}(G_F)$$

Hence

$$\boxed{A_{r_F'}(G_F) = A_{r_F}(G_F)}$$

Flavor Anomalies of bound states and preons must match

- Note that

$$\text{massless dofs} \Longleftrightarrow \text{chiral} \Longleftrightarrow \text{non-trivial anomalies}$$
$$\text{massive dofs} \Longleftrightarrow \text{real (reducible)} \Longleftrightarrow \text{trivial anomalies.}$$

Hence 't-Hooft anomaly matching is a ~~state~~ constraint on the <u>chiral/massless</u> bound states!

- Immediate contrapositive:

If $A_{r_F'}(G_F) \neq A_{r_F}(G_F)$, then G_F must be spontaneously broken!

- Generalization. Suppose $G_F \to G_F'$ under confinement. This occurs usually if $\langle \Psi\Psi \rangle \neq$

Then

$$A_{r_F'}(G_F') = A_{r_F}\underbrace{(G_F' \subset G_F)}_{\text{anomaly of decomposed rep}^{\text{ns}}.}$$

Flavor anomalies of unbroken symmetry must match

Example: Toy QCD

	G_c	G_F		
	$SU(N)$	$SU(n)_L$	$SU(n)_R$	$U(1)_V$
Ψ_L	$\square$	$\square$	1	1
Ψ_R^c	$\bar{\square}$	1	$\bar{\square}$	-1

- Ignore $U(1)_A$. This is an accidental global symmetry. We don't care if it is anomalous
- Convention: $A(\square) = 1, \quad C(\square) = 1$ (Normalization)
  $Tr[T^a_\square, \{T^b_\square, T^c_\square\}] = A[\square] d^{abc}$ $\quad Tr[T^a_\square, T^b_\square] = C[\square]\delta^{ab}$
- Clearly G_c anomalies all vanish.

$$A_{\Psi_L}[SU(n)^3_L] = N = -A_{\Psi_R^c}[SU(\bar{N})_R]$$

$$A_{\Psi_L}[SU(n)^2_L U(1)_V] = N, \quad A_{\Psi_R^c}[SU(n)^2_R U(1)_V] = -N. \qquad -①$$

$$A[U(1)^3_V] = 0$$

- Generic bound state

$$\chi = (\Psi_L)^{m_L} (\Psi_R^c)^{\bar{m}_R}$$

Here notation is

$$(\psi^+)^p = \psi^{-p}$$

Allows us to keep track of instanton anomalies etc. We are cheating slightly: should be $(\Psi_L)^{m_L}(\Psi_L^c)^{\bar{m}_L}$ etc. Only important thing is $m_L - \bar{m}_L$, though.

- Constraints on $m_L, \bar{m}_R$.

G_c singlet : $m_L + (N-1)\bar{m}_R = kN, \quad k \in \mathbb{Z}$ $\qquad -②$

Fermion : $(m_L + \bar{m}_R) \bmod 2 = 1$ $\qquad -③$

- Also, we want left-handed bound states, in order to compare anomalies. Four possible cases

2	$m_L > 0$	$\bar m_R > 0$	$\Rightarrow$	m_L odd (even), $\bar m_R$ even (odd)
1	$m_L > 0$	$\bar m_R < 0$	$\Rightarrow$	m_L odd, $\bar m_R$ even (ψ_R^* is right hande
1	$m_L < 0$	$\bar m_R > 0$	$\Rightarrow$	m_L even, $\bar m_R$ odd
	$m_L < 0$	$\bar m_R < 0$		Forbidden.

This is all equivalent to

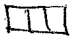
$$\left[1+\mathrm{sgn}(m_L)\right](m_L \bmod 2) + \left[1+\mathrm{sgn}\,\bar m_R\right](\bar m_R \bmod 2) = 2 \quad -④$$

which solves fermionic constraint (spin-statistics!) ③.

- Bound state spectrum of left handed baryons is now, up to multiplicities.

$SU(N)$	$SU(n)_L$	$SU(n)_R$	$U(1)_V$
1	$(\Box)^{m_L}$	$(\bar\Box)^{\bar m_R}$	$m_L - \bar m_R$

- ~~So for~~ Now let's suppose $N = n = 3$, toy qcd, and suppose we only have triquark baryons: $|m_L| + |\bar m_R| = 3$.

~~Spectrum~~ m_L	$\bar m_R$	$SU(n)_L$	$SU(n)_R$	$U(1)_V$
3	0		1	3
			1	3
			1	3
due to ③ $\to$ ~~m~~ 2	1			-3
				-3

etc.

Instanton anomaly matching with ① for $SU(2)_L$

$$\cancel{3}\left[\ell_1 \frac{(n+2)(n+3)}{2} + \ell_2(n^2-3) + \ell_3 \frac{(n-2)(n-3)}{2} + \ell_4\, n(n-2) + \ell_5 \cancel{3} n(n+2) + \ell_6\, n(n+1) + \ell_7\, n(n-1)\right] = \cancel{3}^1$$

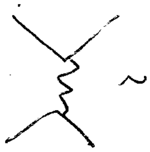
⑥

- For $n=3$, no solution (take mod 3 of both sides)!

  ⇒ chiral symmetry breaking of $SU(3)_L \times SU(2)_R$. In actual QCD it is broken completely.

Complementarity and Tumbling

- A scheme to determine pattern of chiral SSB and mass spectra: Tumbling.
- Assume: 1) ~~At~~ At confinement scale Λ, $\exists$ a single scalar condensate with non-trivia vev. This is the MAC, determined crudely via potential


$\sim$ quadratic Casimirs

2) MAC induces SSB. Decompose $G_c \times G_F$ rep$\hat{s}$; this is a hybridization of confinement and SSB, but we never actually compose representations!

3) New symmetry $G_c' \times G_F'$. Under decomposition real rep$\hat{s}$ ~~get become~~ have mass $\sim \Lambda$, chiral rep$\hat{s}$ are massless
4) Persistent mass condition: Once a rep$\hat{}$ becomes massive, remains massive at sc
5) New chiral rep$\hat{s}$ form new MAC at Λ_2. Repeat.
We say $G_c \times G_F$ 'tumbles' to $G_c' \times G_F'$
6) Tumbling stops when chiral rep$\hat{s}$ are all $G_c^{(n)}$ singlets.

$$\begin{tikzcd}
\{T\}\; G_c \times G_F \arrow[r, "MAC", "\Lambda_1"'] & G_c' \times G_F'\; \{T'\} \arrow[d, "run"] & \\
& \{T'\}\; G_c' \times G_F' \arrow[r, "MAC", "\Lambda_2"'] & G_c'' \times G_F''\; \{T''\} \arrow[d] \\
& & \vdots
\end{tikzcd}$$

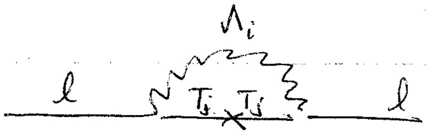
real rep$\hat{s}$ mass Λ_1

'chiral ~~chiral rep$\hat{s}$~~

End up with massive rep$\hat{s}$ + chiral rep$\hat{s}$ $\{T^\wedge\}$ This is the Higgs picture.

- $\boxed{\text{We are guaranteed that anomalies of Higgs picture and preons match!}}$

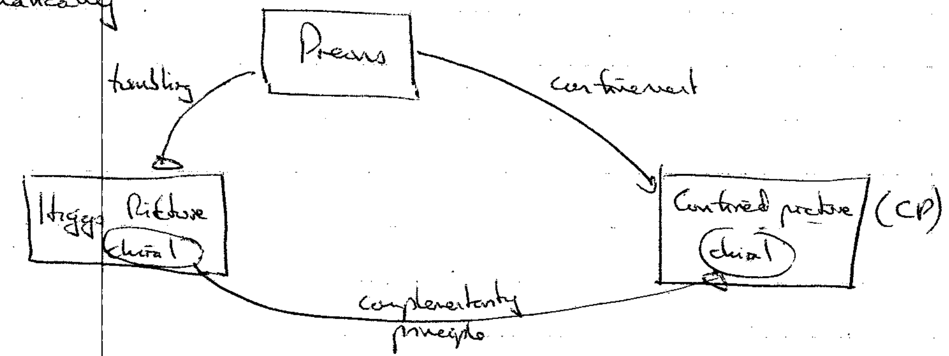
- We can also have ~~2Mas~~ secondary mass generation (2MG): a real rep^c does not contain MAC element, so masses are loop generated (via irrelevant operators)



$$m_l \sim \frac{\langle \bar{T}_j T_j \rangle}{\Lambda_i^2} \sim \frac{\Lambda_j^3}{\Lambda_i^2} \quad j > i.$$

- So far we haven't "confined" anything! How does this relate to confinement? Higgs picture allows us to encode dynamical info, via <u>complementarity principle</u>

- Schematically



∃ a 1-1 map between Higgs picture chiral fields and confined picture chiral fields s.t. chiral CP fields satisfy anomaly matching. Map is a "vev projection" i.e. if ϕ is ~~the~~ a MAC, then

$$\text{Higgs} \quad (\cdots)\langle\phi\rangle \longrightarrow (\cdots)\phi \quad \text{confined}$$

<u>Example</u>: $SU(5)\times U(1)$ without sym breaking.

Consider.

	G_c	G_F
	$SU(5)$	$U(1)$
ψ	$\square$	3
χ	$\bar{\boxminus}$	-1

We have $A_{\psi\oplus\chi}(SU(5)^3) = 0$ and $A_{\psi\oplus\chi}(SU(5)^2U(1)) = 3 - (5 \cdot 2)$
$= 0$

$A(U(1)^3) = 27.5 - 10 = 125.$

Higgs picture:

MAC $\phi \sim \psi\chi^* \sim \bar{\square}_{\mathbf{2}}$ This breaks $SU(5) \to \overset{\sim G_c'}{SU(4)}$ and breaks $U(1)$.

We have decomposition $SU(5) \to SU(4) \times U(1)_{\alpha}$

$$\square_5 \longrightarrow (\square)_{\alpha} \oplus 1_{-4\alpha}$$

$$\bar{\boxminus} \to \boxminus_{-2\alpha} \oplus \bar{\square}_{3\alpha} \qquad \alpha \in \mathbb{R}/\{0\}.$$

~~Let us suppose~~

Now, explicitly

$$\phi_i = \varepsilon_{ijklm}\psi^j\chi^k$$

Let us suppose only $\langle\phi_5\rangle \neq 0$. Ie $\langle\phi_5\rangle \sim \Lambda^3$. Then since

$$\bar{\square}_2 \to \bar{\square}_{2-\alpha} \oplus 1_{2+4\alpha}$$

Then for $\alpha = -\frac{1}{2}$, can identify $U(1)' = U(1)_{-\frac{1}{2}} \otimes U(1)$ and $\langle\phi_5\rangle \sim 1_0$ of $U(1)'$

Hence $U(1)'$ is unbroken!

We now have new decomposed theory

		$SU(4)$	$U(1)'$
	ψ^i	$\square$	$\frac{5}{2}$
$i=1,..,4$	ψ^5	1	5
	χ^{ij5}	$\boxminus$	0
	χ^{ijk}	$\bar{\square}$	$-\frac{5}{2}$

chiral !

really reducible ⇒ Dirac fermion.

real irrep ⇒ Majorana fermion

- ψ^5 is only chiral ~~x~~ dof remaining. Since it is an $SU(4)$ singlet, tumbling stops.

NB $\mathcal{A}_{\psi^5}[U(1)'] = 5^3 = 125 = \mathcal{A}_{\psi\oplus\chi}[U(1)^3]$ as promised.

Confined Picture:

Left handed baryonic SU(5) singlets

	SU(5)	U(1)
$\psi^2\chi$	$\Box\otimes\Box\otimes\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}\supset 1$	5
$\psi(\chi^\dagger)^2$	$\Box\otimes\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\otimes\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\supset 1$	5
$(\psi)^5$	$\Box^5\supset 1$	15
χ^5	$\left(\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}\right)^5\supset 1_5$	-5

lots more – arbitrarily large no. of bound states

- It is unclear which are chiral baryons, except that anomalies must match.

- Map: In Higgs picture

$$\psi_5 \propto \psi^i\langle\phi_i\rangle \qquad \text{(projection)}$$
$$= \psi^i\,\epsilon_{ijklm}\langle\psi^j\chi^{klm}\rangle$$

$$\to \psi^i\,\epsilon_{ijklm}\,\psi^j\chi^{klm}$$

$$\Rightarrow \psi^2\chi\,!$$

Hence we identify $\psi^2\chi$ as chiral dof.

Moreover

$$\boxed{A_{\psi^2\chi}(U(1)) = 125}$$

Complementarity worked!

- Tumbling + complementarity $\Rightarrow$ natural fermion mass spectra, with control of chiral bound states.

Conclusion:

- 't Hooft anomaly matching is a powerful necessary condition for a ~~[illegible]~~ set of bound states to be chiral.
- Complementarity is a crude, but effective, dynamical picture of confinement. Allows determination of mass spectra and chiral bound states.