INTERPRETATION of SEIBERG DUALITY

Plan:

- SUSY Reminder (SEIBERG duality & Moduli space generalities).
- ρ-Meson in QCD & Hidden Local Symmetry (HLS).
- Magnetic dual as HLS of the electric theory.

References

- first & foremost: Flip's notes from an c.2010 BSM talk.
- [hep-th/1010.4105]
- [hep-th/1202.2863]

Consider SUSY QCD with N_f-flavors: $[N_f > N_c]$

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$
Q	$\square$	$\square$	$\mathbb{1}$
$\bar{Q}$	$\bar{\square}$	$\mathbb{1}$	$\bar{\square}$

If $N_f < 3N_c$ the β-function is negative,: the theory is asymptotically free & confines in the IR.

We expect the low energy ~~th~~ EFT to be a theory of mesons & baryons. No ~~gau~~ $SU(N_c)$ left, e.g. χPT in QCD.

WRONG!!!

Thus ~~two~~ theory flows to a <u>MAGNETIC GAUGE THEORY</u>:

	$SU(N_f-N_c)$	$SU(N_f)_L$	$SU(N_f)_R$
$\tilde{q}$	$\square$	$\square$	$\mathbb{1}$
$\bar{q}$	$\bar{\square}$	$\mathbb{1}$	$\bar{\square}$
M	$\mathbb{1}$	$\bar{\square}$	$\bar{\square}$

with a non-vanishing superpotential:

$$W = \bar{q} M q$$

<u>Fancier stuff</u>: Is the ~~magn~~ magnetic th. asymptotically free?

$$N_f < 3\tilde{N}_c \mapsto N_f < 3(N_f - N_c) \mapsto N_f > \tfrac{3}{2} N_c$$

so there are three region in Seiberg:

- $N_f > 3N_c \to$ the electric th. is ~~was~~ IR-free, no need for a magnetic th.
- $\tfrac{3}{2}N_c < N_f < 3N_c \to$ electric th. ~~&~~ magnetic th. both strongly coupled in the IR but they both flow to the same IR-fixed point: <u>CONFORMAL</u> <u>WINDOW</u>.
- $N_f < \tfrac{3}{2}N_c \to$ electric th. strongly coupled but magnetic th. IR-free: <u>COOL</u>! <u>SEIBERG</u> is <u>USEFUL</u>!

<u>MODULI SPACE</u> suppose I have $(\partial_\mu\phi)^2 - m^2\phi^2$

what's the minimum? $\phi = 0$

but $(\partial_\mu\phi)^2$ ~~is~~ has a "moduli space" $\phi = v$ for all v ~~minimize the~~ are possible minima

<u>SUSY</u> we should minimize the D-term:

$$D^a = g\left(\phi^* T^a \phi - \bar{\phi} T^a \bar{\phi}^*\right)$$

where ϕ is the scalar component of q & $\bar{q}$ or ~~Q & Q~~ Q & $\bar{Q}$.

it can be shown (Terning § 3.4) that

$N_f > N_c$: $\quad \langle \phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_{N_c} \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$; $\langle \bar{\phi} \rangle = \begin{pmatrix} \bar{v}_1 & & & 0 & \cdots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & \bar{v}_{N_c} & 0 & \cdots & 0 \end{pmatrix}$

[<u>remember</u> $\phi \equiv q$ is a ~~$SU(N)$~~ $\square$

	$SU(N_c)$	$SU(N_f)$
q	$\square$	$\square$

$\sim N_c \times N_f$ matrix]

In a generic point of the ~~moduly~~ moduli space:

1) $SU(N_c)$ is completely broken.

2) $SU(N_f)_L \times SU(N_f)_R \to SU(N_f - N_c)_L \times SU(N_f - N_c)_R$

<u>XPT</u> Big picture:

- Theory of pions is valid ~~for~~ ~~Q~~ roughly QCD below Λ_{QCD}.
- No gauge group, pure non-linear σ-model.
- I have a $SU(2)_L \times SU(2)_R$ global broken to $SU(2)_V$ & the pions are the goldstone bosons of the broken $SU(2)_A$.

IMPORTANT χPT is a theory of MASSLESS degrees of freedom!

consider $U(x) = e^{i T^a \frac{\pi^a}{f_\pi}}$ & $U(x) \to g_L U g_R^\dagger$

π's are goldston fields, so $\langle \pi \rangle = 0 \Rightarrow \langle U \rangle = \mathbb{1}$
which in fact breaks $SU(2)_L \times SU(2)_R \to SU(2)_V$.

~~We ta~~

Build $\mathcal{L}$: $\mathcal{L} = \frac{1}{4} f_\pi^2 \,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U\right]$

& expand in the π's!

Alternative way. U is a generic $SU(2)$ matrix

I can write $U = \xi_L \xi_R^\dagger$ ξ_L & $\xi_R \in SU(2)$

& $\xi_L \to g_L \xi_L$ & $\xi_R \to g_R \xi_R$.

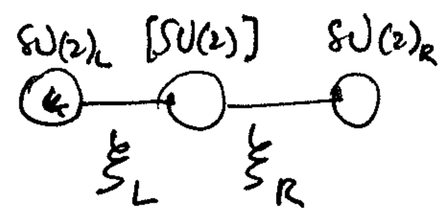
but there is a redundancy:

$\xi_L \to \xi_L h$ $\xi_R \to \xi_R h$ $h \in SU(2)$

~~basi~~ this is a quiver ~~g~~ gauge th:

	$SU(2)_L$	$SU(2)_R$	$[SU(2)]$
ξ_L	$\square$	$\mathbb{1}$	$\square$
ξ_R	$\mathbb{1}$	$\square$	$\square$

:



$$\frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu U \partial^\mu U\right] \to -\frac{f_\pi^2}{4} \mathrm{Tr}\left[\left(\xi_L^\dagger \partial_\mu \xi_L - \xi_R^\dagger \partial_\mu \xi_R\right)^2\right]$$

~~in order~~ we could make the global $[SU(2)]$ a local symmetry by promoting $\partial_\mu \to D_\mu = \partial_\mu - i g \rho_\mu^a T^a$

the ρ_μ^a can be identified with the three rhos:

$$\rho_\mu^a \equiv \rho_\mu^{0,\pm} .$$

REMARKS:

- We can add another gauge / $SU(2)_L \times SU(2)_R$ inv. term to the Lagrangian: $a \mathrm{Tr}\left[\left(\xi_L^\dagger D_\mu \xi_L + \xi_R^\dagger D_\mu \xi_R\right)^2\right]$

for ~~certain values of~~ $a=2$ this model is phenomenologically pretty remarkable:

- $M_\rho^2 = 2 g_{\rho\pi\pi}^2 f_\pi^2 \quad (\sim g^2 v^2)$
- $g_{\rho\gamma} = 2 g_{\rho\pi\pi} f_\pi^2$

so it's not total bullshit!

Let's look at the ~~symm~~ SB pattern:

π's / U $\quad \langle U \rangle = \mathbb{1}$

$SU(2)_L \times SU(2)_R \to SU(2)_V$

ξ_L / ξ_R moduli space:

$\langle \xi_L \rangle = \mathbb{1} \quad \langle \xi_R \rangle$ generic

$\langle \xi_L \rangle$: $SU(2)_L \times [SU(2)] \times SU(2)_R$

$\downarrow$

$[SU(2)_V] \times SU(2)_R$

$\langle \xi_R \rangle$: $[SU(2)_V] \times SU(2)_R \to SU(2)_V$

- In the SB pattern there is a mixing between the $SU(2)_L$ & the redundant local $[SU(2)]$.

<u>GENERAL RESULT</u> (without derivation)

A $\mathcal{G}/H$ non-linear σ-model is gauge equivalent to a $\mathcal{G}$ flavor & H as a broken gauge group.

χPT: $\mathcal{G} = SU(2)_L \times SU(2)$; $H = SU(2)$

In SQCD (electric) (remember the VEVS)

$$SU_L(N_f)_L \times SU_R(N_f)_R \longrightarrow SU(N_f - N_c)_L \times SU(N_f - N_c)_R$$

so in the far IR such a theory is

- equiv. with a $SU(N_f)_L \times SU(N_f)_R$ flavor group with a broken $\underbrace{SU(N_f - N_c)_L \times SU(N_f - N_c)_R}_{\text{resembles the magnetic thy.}}$

MAGNETIC DUAL

In order for analogs of the rho mesons to appear we need to move away from the origin of the moduli space ($SU(N_f-N_c)$ needs to be broken).

Consider the following direction

$$q = \begin{pmatrix} \chi \\ \varphi \end{pmatrix} = v \begin{pmatrix} \mathbb{1}_{(N_f-N_c)\times(N_f-N_c)} \\ 0 \end{pmatrix}$$

& both $\langle \tilde{q} \rangle = \langle M \rangle = 0$,

It's convenient to decompose them:

$$\tilde{q} = \left(\tilde{\chi}_{(N_f-N_c)\times(N_f-N_c)} , \tilde{\varphi}_{(N_f-N_c)\times N_c} \right) ; \quad M \equiv \begin{pmatrix} X_{(N_f-N_c)\times(N_f-N_c)} & Y \\ \tilde{Y} & Z_{N_c\times N_c} \end{pmatrix}$$

so the $\boxed{W: \tilde{q} M q = \tilde{\chi} X \chi + \tilde{\chi} Y \varphi + \tilde{\varphi} \tilde{Y} \chi + \tilde{\varphi} Z \varphi}$

as q gets a VEV $(\tilde{\chi}, X, \tilde{\varphi} \tilde{Y})$ become massive.

Also $[SU(N_f-N_c)]$ is completely broken & all but 1 $\chi_{(N_f-N_c)\times(N_f-N_c)}$'s are eaten by gauge fields.

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f-N_c)_L \times SU(N_c)_L \times SU(N_f)_R$$

so there are $2N_fN_c - 2N_c^2 + 1$ goldstones: $(\varphi^i_s, \chi^c_i = \delta^c_i)$

The low energy theory is a $SU(N_f)_L \to SU(N_f-N_c) \times SU(N_c)$ non-linear σ-model which has no gauge group & only the π's ~~&~~ & the massless ~~the~~ mesons Y, Z.

The only χ's which is not eaten by the gauge fields is the ~~one who along~~ field describing the excitation along the direction in the moduli space we are moving along.

This approach would be equivalent to writing down the standard chiral Lagrangian with just U.

Let's look at the fields

π^c_1: c → transforms under $SU(N_f-N_c)_L$; 1 → transforms under $SU(N_c)_L$

the π's are somewhat the ~~goldst~~ φ's.

The index c ~~comes~~ descends from a gauge index at higher energy.

~~In fact~~ : $\cancel{\langle \chi \rangle : SU(N_f)_L \times [SU(N_f - N_c)] \to [SU(N_f-N_c)_V] \times SU(N_c)_L}$

φ initially does not transform under $SU(N_f - N_c)_L \subset SU(N_f)_L$

$$\left(\begin{array}{c|c} M & \\ \hline & \end{array}\right)\begin{pmatrix}\chi \\ \varphi\end{pmatrix} = \begin{pmatrix} M\chi \\ \varphi\end{pmatrix}$$

but it does under $[SU(N_f - N_c)]$:

$$\begin{pmatrix}\chi \\ \varphi\end{pmatrix} g^T = \begin{pmatrix}\chi g^T \\ \varphi g^T\end{pmatrix}$$

but $\langle \chi \rangle : SU(N_f)_L \times [SU(N_f - N_c)] \to [SU(N_f - N_c)_V] \times SU(N_c)_L$

or in other words:

$$\left(\begin{array}{c|c} M & \\ \hline & \end{array}\right)\begin{pmatrix}\mathbb{1} \\ 0\end{pmatrix} g^T = \begin{pmatrix} M g^T \\ 0\end{pmatrix} \equiv \begin{pmatrix}\mathbb{1} \\ 0\end{pmatrix}$$

if $g^T = M^{-1}$

non-linear σ-model

π's which transform under both $SU(N_f-N_c)_L$ & $SU(N_c)_L$

	$SU(N_f-N_c)_L$	$SU(N_c)_L$
π	$\square$	$\square$

$\xi_L\ \xi_R$

	$SU(N_f-N_c)_L$	$SU(N_c)_L$	$[SU(N_f-N_c)]$
ξ_L	$\square$	$\mathbb{1}$	$\square$
ξ_R	$\mathbb{1}$	$\square$	$\square$

If we ~~embede~~ embed $SU(N_f-N_c)_L \times SU(N_c)_L$ in $SU(N_f)_L$:

$$\left(\begin{array}{c|c} N_f-N_c & \\ \hline & N_c \end{array}\right)$$

$\xi_L \equiv \chi$; $\xi_R \equiv \varphi$ & the redundant $[SU(N_f-N_c)]$ is the magnetic gauge group.

FINAL REMARKS: • there is a lot more than I ~~said~~ understand, e.g. in SQCD VMD comes for free!

• Since in SQCD there is a limit in which $[SU(N_f-N_c)]$ is restored. ($v \to 0$) there are Higgs fields

associated with the SB & that
get eaten by the ρ's (~~eg.~~ i.e. χ's)

- ~~There are rema~~ We only looked at one specific direction in moduli space, so how general is this association is? It seems to be much deeper. There are sentence I don't understand like:
"The property of SUSY theories that allows us to reconcile this difference is that their potentials have enhanced ~~flavor~~ complex flavor symmetries..."

[hep-th/1202.2863]