BSM: Riccardo - $1/N$

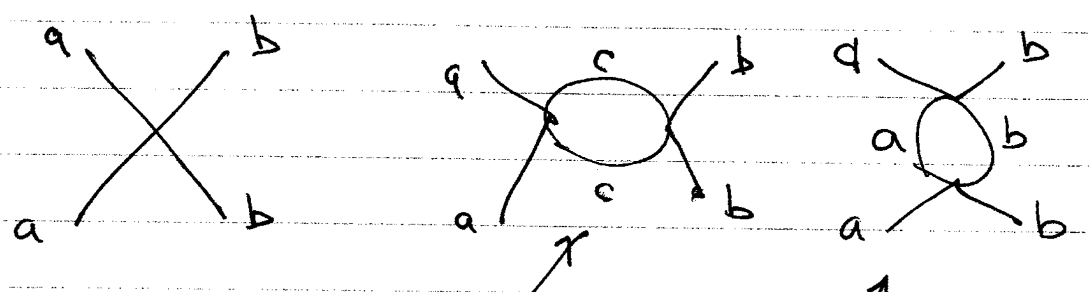
4 MARCH

EXACT SOLUTIONS of QFT — PERTURBATION IN $1/N$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 - \frac{1}{2}\mu_0^2 \phi^a\phi^a - \frac{1}{8}\lambda(\phi^a\phi^a)^2$$

$SO(N)$ invariant

eg:

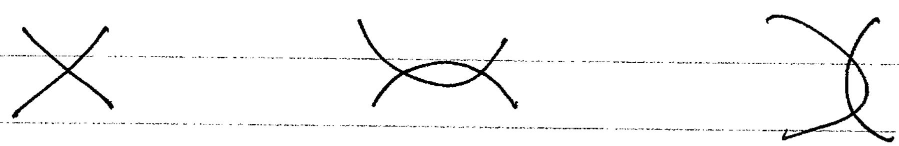


λN (sum c)

internal line labels are fixed

λ_0 $N\lambda_0^2$ λ_0^2

"Large N": fix $\boxed{g_0 = \lambda_0 N}$



g_0/N g_0^2/N g_0^2/N^2

SAME ORDER IN $1/N$

Introduce auxiliary field

$$\mathcal{L} \to \mathcal{L} + \frac{1}{2}\frac{N}{g_0}\left(\sigma - \frac{1}{2}\frac{g_0}{N}\vec{\phi}\cdot\vec{\phi}\right)^2$$

$\Uparrow$

eom: $\left(\sigma - \frac{1}{2}\frac{g_0}{N}\vec{\phi}\cdot\vec{\phi} = 0\right)$

"integrated in" a "heavy field"
(ignored kinetic term, b/c heavy)

$$\mathcal{L} = \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{2}\mu_0^2|\vec{\phi}|^2 + \frac{1}{2}\frac{N}{g_0}\sigma^2 - \underline{\underline{\frac{1}{2}\sigma\vec{\phi}\cdot\vec{\phi}}}$$

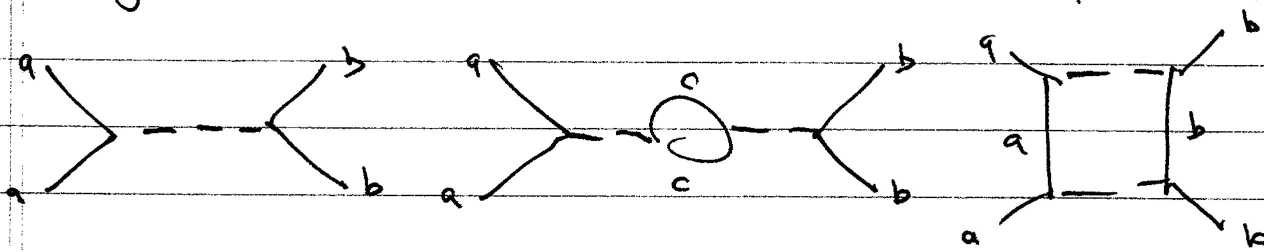
NO MORE QUARTIC

σ PROPAGATOR: $\frac{i g_0}{N}$

only appearance of N!

$\Delta_\sigma^2 \sim \frac{1}{N^2}$

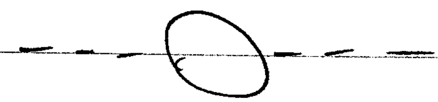
Diagrams now look like:



$\Rightarrow$ NOW ALL LOOPS HAVE A SUM, GIVE FACTOR OF $\boxed{N}$

So now throw out subleading in $1/N$

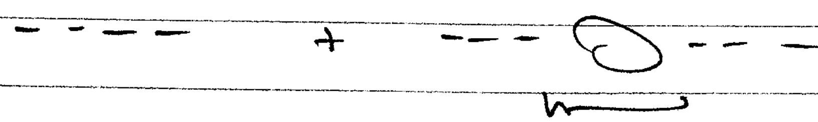
for now, only look @ internal ϕ
external σ

eg 

$t \sim N$

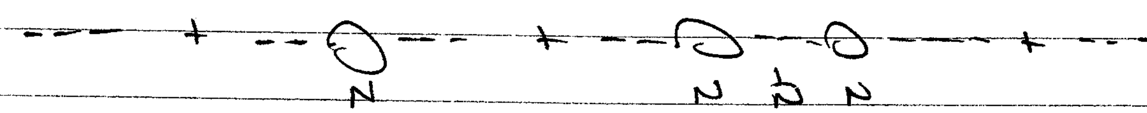
[easy to replace external ϕs]

↑ OUR LEADING DIAGRAMS ARE



WANT TO MAKE THIS AN EFFECTIVE VERTEX

EACH DIAGRAM GOES LIKE N, eg



$$e^{S_{eff}} = \int D\phi \, e^{-S(\phi,\sigma)}$$

$$\hookrightarrow S_{eff} = N \, S(\phi,\sigma)$$

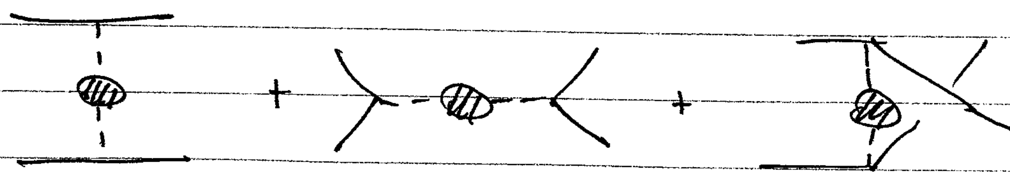
So for a graph of σ:

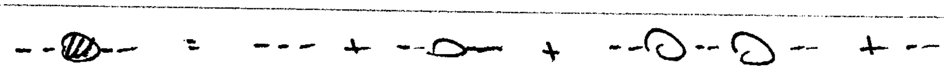
$$N^{V-I-E} = N^{-E-\overset{\text{loops of } \sigma}{L}+1}$$

of σ

smallest $1/N$ power: no loops fewest external lines.

So eg. 4-point in ϕ^4:



Where: 

EXACT σ PROPAGATOR

So the trick: force non perturbativity into something easy to re-sum.

Peskin: $\tilde{N}$ solution of NLEM

↳ do functional integral over σ (quadratic)

<u>our main focus</u>: $\mathbb{CP}^{N-1}$ model in $d=2$

$\mathbb{C}$ projective space

theory of <u>$N \times N$ traceless</u>, <u>hermitian</u> matrices
(Adjoint of $SU(N)$)

POLYNOMIAL

$$\mathcal{L} = \frac{1}{2} \operatorname{Tr}\left((\partial\phi)^2\right) - \lambda \operatorname{Tr} P(\phi)$$

chosen s.t. minima have:

$$\phi = g_0^{-1}\left[N^{1/2} z z^{+} - \underbrace{N^{-1/2} I}_{\text{N-1 EQUAL EIGENVALS}}\right]$$

1 DIFF EIGENVAL

$z^+ z = 1$

outer product

nonlinear limit: $\lambda \to \infty$
forced to live in vacuum state

phase of z not dynamical $\to$ gauge dof
$$\vec{z} \to e^{i\delta} \vec{z}$$

So our $\mathcal{L}$ must have a $U(1)$ gauge sym.

$$\mathcal{L} = \frac{N}{g_0^2} \left((\partial_\mu \vec{z}^{\dagger})(\partial_\mu \vec{z}) - j_\mu j^\mu \right) \qquad \leftarrow \boxed{z^{\dagger} z = 1}$$

rescale

$$j_\mu = \frac{1}{2i} \left[\vec{z}^{\dagger} \partial_\mu \vec{z} - \partial_\mu \vec{z}^{\dagger} \vec{z} \right]$$

$$\mathcal{L} = (\partial z^{\dagger})(\partial z) - g_0^2 N^{-1} j_\mu j^\mu$$

w/ $\boxed{z^{\dagger} z = N / g_0^2}$

$$\mathcal{L} \to \mathcal{L} + g_0^2 N^{-1}(j_\mu + g_0^{-2} N A_\mu)^2$$

AUXILIARY FIELD

EOM SETS THIS TERM TO ZERO

$$= (\partial_\mu - iA_\mu) z^\dagger \cdot (\partial_\mu + iA_\mu) z$$] now quadratic in z.

local sym is $A_\mu \to A_\mu - \partial_\mu \theta$

$z \to e^{i\theta} z$

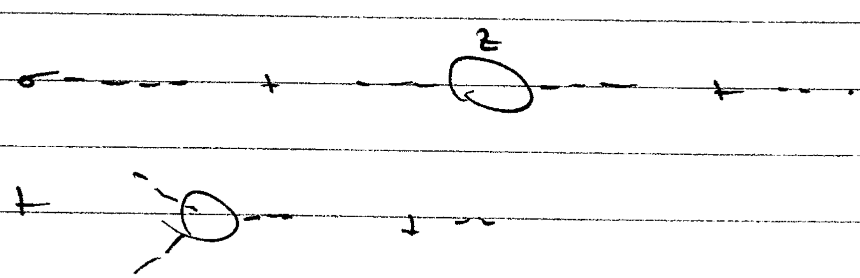
[describing theory w/ more redundancy]

introduce: $\Delta \mathcal{L} = -\sigma\left[z^\dagger z - g_0^2 N\right]$

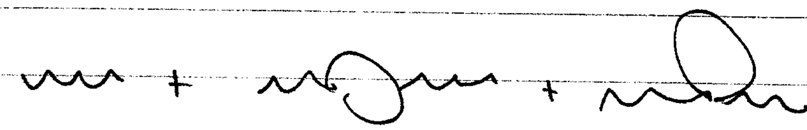
; now z is <u>free</u> (unconstrained)

<u>Next</u>: integrate out z: quadratic & unconstrained

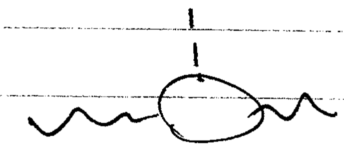
want an eff action of σ & A_μ



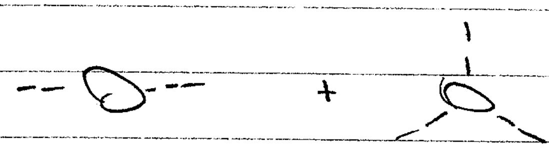
A_μ terms:



mixed terms:



$V(\sigma) =$



coleman-weinb.

$$= -N\left[\frac{\sigma}{g_0^2} + \frac{\sigma}{4\pi}\left(\ln\frac{\sigma}{\Lambda^2} - 1\right)\right]$$

$\langle\sigma\rangle \neq 0$

gives mass for z $(-\sigma z^\dagger z)$

det.

$$Z_I = \int D\sigma DA\; e^{-N\,\mathrm{Tr}\log\left(-(\partial + iA)^2 - \frac{\sigma g_0^2}{N}\right) + i\int d^2x\,\sigma}$$

in the large N limit : stationary phase approx

↳ hope $A_\mu = 0$, so expand here

$\sigma = \sigma_0$, const.

↳ gives same $V(\sigma)$ as before.

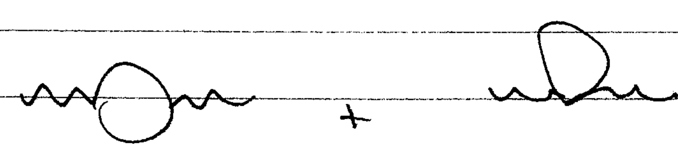
(exp small value of σ)

↳ $\frac{g^2\langle\sigma\rangle}{N} = \mu^2 e^{-4\pi/g_R}$

gives mass for z

expansion of det gives kinetic term for A_μ

eg:



→ $\partial_{[\mu}A_{\nu]}$ term

Interp: A_μ ~ vector meson

z's ARE CONFINED

(All these diagrams have external z states)