GOLDSTONE FERMION DARK MATTER

JHEP 1109:035,2011 `[arXiv:1106.2162]`
& work in progress

Flip Tanedo

Cornell University

In collaboration with B. Bellazzini, M. Cliche, C. Csáki, J. Hubisz, J. Shao

UC Davis Particle Theory Seminar, 22 Oct 2012

The WIMP Miracle

Contains factors of M_{Pl}, s_0, $\cdots$

$$\Omega_{\text{DM}} h^2 \approx 0.1 \left(\frac{x_{\text{f}}}{20}\right)\left(\frac{g_*}{80}\right)^{-\frac{1}{2}}\left(\frac{\langle \sigma v\rangle_0}{3\times 10^{-26}\ \text{cm}^3/\text{s}}\right)^{-1}$$

$$\sim \left\langle \frac{\alpha^2 v}{(100\ \text{GeV})^2}\right\rangle$$

Logarithmic miracle: Within orders of magnitude!

Abundance vs direct detection

$$\boxed{\sigma_{\text{ann.}} \sim 0.1\ \text{pb}}$$

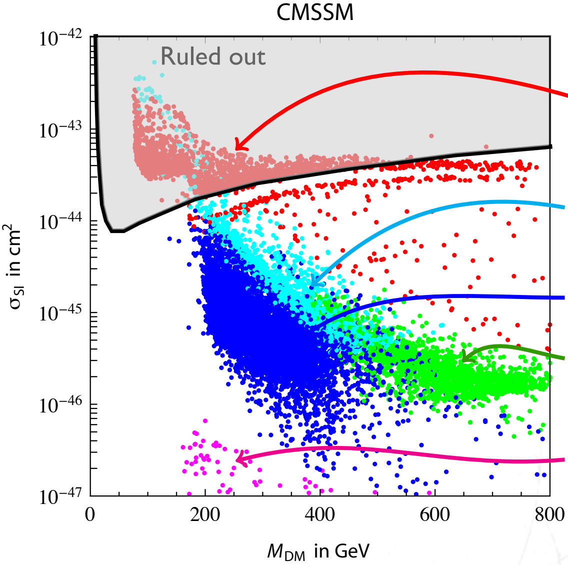
$$\boxed{\sigma_{\text{SI}} \sim 7.0 \times 10^{-9}\ \text{pb}}$$

50 GeV WIMP

Typical strategy: pick parameters such that σ_{SI} is suppressed, then use tricks to enhance $\sigma_{\text{ann.}}$.

- Tune the neutralino composition ($\widetilde{B}$ vs. $\widetilde{W}$, $\widetilde{H}$)
- Coannihilations (accidental slepton degeneracy)
- Resonant annihilation

Abundance vs direct detection



Well tempered neutralino

h^0 resonance

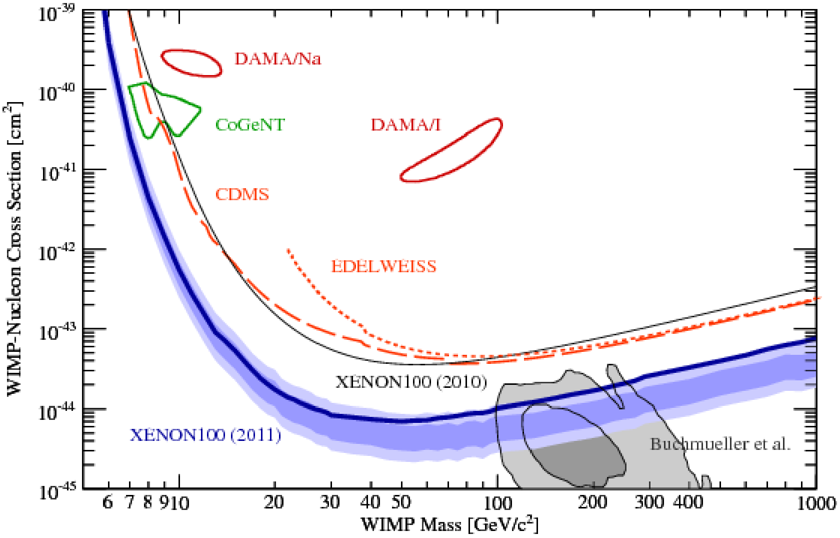
slepton co-annihilations

A^0 resonance

stop co-annihilations

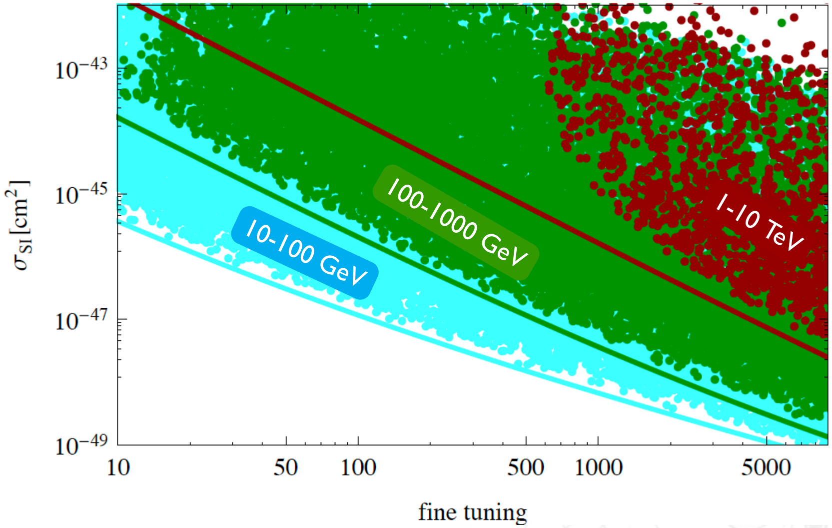
Farina, Kadastik, Raidal, Pappadopulo, Pata, Strumia [1104.3572]

The timid amoeba



1104.2549

MSSM Dark Matter and EWSB Tuning



Perelstein and Shakya [1107.5048]

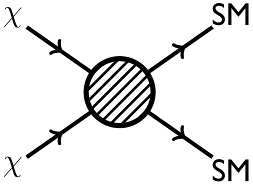
Abundance vs direct detection

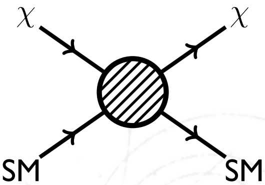
$$\boxed{\sigma_{\text{ann.}} \sim 0.1\ \text{pb}}$$

$$\boxed{\sigma_{\text{SI}} \sim 7.0 \times 10^{-9}\ \text{pb}}$$

50 GeV WIMP

Assumed that these come from the same effective operator:





Can we separate into two different sectors?

One way to do this is with a Goldstone supermultiplet.

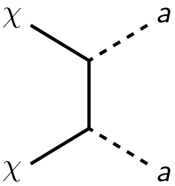
Abundance vs direct detection

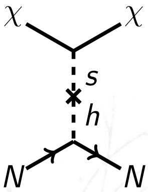
$$\boxed{\sigma_{\text{ann.}} \sim 0.1 \text{ pb}}$$

$$\boxed{\sigma_{\text{SI}} \sim 7.0 \times 10^{-9} \text{ pb}}$$

50 GeV WIMP

Assumed that these come from the same effective operator:





Can we separate into two different sectors? (Higgs portal)

One way to do this is with a Goldstone supermultiplet.

Motivation: natural WIMP

Typical MSSM WIMP: σ_{SI} too large

Want to naturally suppress direct detection while maintaining 'miracle' of successful abundance.

If LSP is part of a *Goldstone multiplet*, $(s + ia, \chi)$, additional suppression from derivative coupling.

- Like a weak scale axino, but unrelated to CP
- Like singlino DM, but global symmetry broken in SUSY limit

Goldstone Fermion Dark Matter

Parameterized class of models with a hidden sector and a spontaneously broken U(1)

Motivation: a natural WIMP

Annihilation: *p*-wave decay to Goldstones

$$\frac{1}{f}\overline{\chi}\gamma^{\mu}\gamma^{5}\chi\partial_{\mu}a \quad \Rightarrow \quad \langle\sigma v\rangle \approx \left(\frac{m_\chi^2}{f^4}\right)\left(\frac{T_f}{m_\chi}\right) \approx \boxed{\text{1 pb}}$$

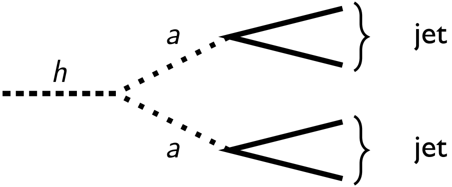
Direct detection: CP-even Goldstone mixing with Higgs

$$\frac{m_\chi v}{f^2} \sim 0.01 \quad \Rightarrow \quad \sigma_{\text{SI}} = \left(\frac{m_\chi v}{f^2}\right)^2 \sigma_{\text{SI}}^{\text{MSSM}} \approx \boxed{\mathcal{O}(10^{-45}\ \text{cm}^2)}$$

‘Historical’ Motivation: Buried Higgs

Idea: Light Higgs buried in QCD background

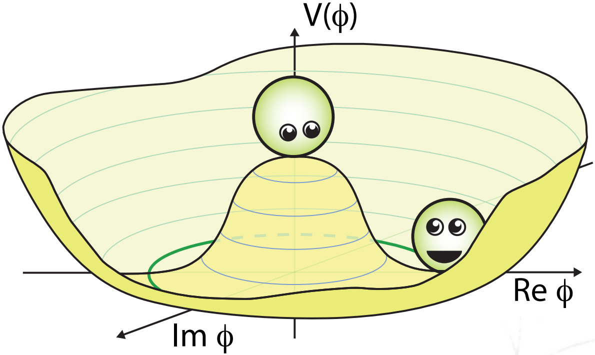
Global symmetry at $f \sim 500$ GeV with coupling $\frac{1}{f^2}h^2(\partial a)^2$



Bellazzini, Csáki, Falkowski, Hubisz, Shao, Weiler: 0906.3026, 1012.1316; Luty, Phalen, Pierce: 1012.1347

Can we bury the Higgs through a decays, but dig up dark matter in χ?

Goldstone Boson Review



Global ~~U(1)~~ ⇒ massless pseudoscalar

Shift symmetry ⇒ derivative coupling

Nonlinear Σ Model (NLΣM)

e.g. chiral perturbation theory

QCD is a theory of $\begin{cases} \text{quarks, gluons} & (E \gg \Lambda_{\text{QCD}}) \\ \text{pseudoscalar mesons } (\pi\text{s}) & (E \ll \Lambda_{\text{QCD}}) \end{cases}$

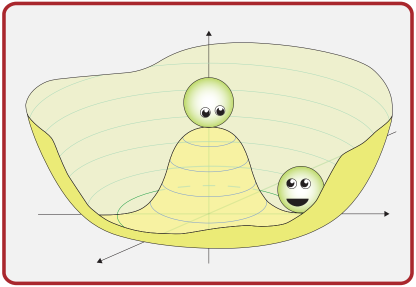
$$\langle \bar{q} q \rangle : SU(3)_L \times SU(3)_R \to SU(3)_V$$

Nonlinear realization

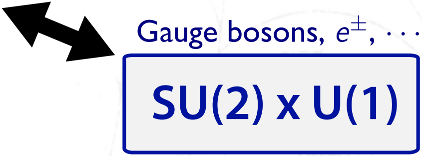
$$U(x) = \exp\left(2i\pi^a(x) T^a / f\right) \qquad \mathcal{L} = \frac{f^2}{4} \text{Tr} \left|\partial U\right|^2$$

m_{q_i}s explicitly break flavor symmetry, $m_\pi \neq 0$

Couple NLΣM to the [low energy] SM



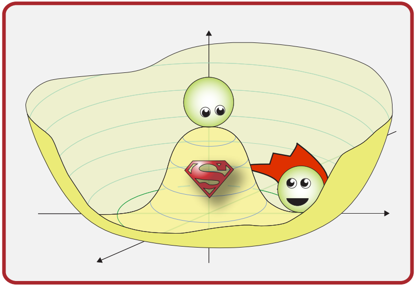
NLΣM ($\pi(x)$ theory)



Gauge bosons, $e^{\pm}, \cdots$

SU(2) x U(1)

Our construction



Goldstone Supermultiplet

q, g, γ, h

MSSM

The Goldstone Supermultiplet

$$A = \frac{1}{\sqrt{2}}\left(\overset{\text{sGoldstone}}{s} + i\,\overset{\text{Goldstone boson}}{a}\right) + \sqrt{2}\theta\,\overset{\text{Goldstone fermion}}{\chi} + \theta^2 F$$

Carries the low-energy degrees of freedom of the UV fields,

$$\Phi_i = f_i e^{q_i A/f} \qquad\qquad f^2 = \sum_i q_i^2 f_i^2$$

~~SUSY~~ $\Rightarrow$ explicit s mass, $m_\chi \approx q_i \langle F_i \rangle / f$, a massless

a mass through small supersymmetric explicit ~~U(1)~~ terms

A simple example of a $\cancel{U(1)}$ sector

UV theory

$$K = \overline{N}^\dagger\overline{N} + N^\dagger N + S^\dagger S \qquad W = S\left(\overline{N}N - \mu^2\right)$$

$$N \sim fe^{+A/f}$$

$$\overline{N} \sim fe^{-A/f}$$

Effective theory

$$K = \cosh(A + A^\dagger) \qquad W = 0$$

SUSY Breaking and the χ mass

Tamvakis-Wyler Thm. Phys. Lett B 112 (1982) 451; Phys. Rev. D 33 (1986) 1762

Global symmetry: $W[\Phi_i] = W[e^{i\alpha q_i}\Phi_i]$ so that

$$0 = \frac{\partial W[e^{i\alpha q_i}\Phi_i]}{\partial\alpha} = \sum_j W_j q_j \Phi_j ,$$

Taking a derivative $\partial/\partial\Phi_i$ gives:

$$0 = \frac{\partial}{\partial\Phi_i}\left(\sum_j W_j q_j \Phi_j\right)\Bigg|_{\langle\Phi\rangle} = \sum_j W_{ij} q_j f_j + W_i q_i$$

$\chi = \sum_i q_i f_i \psi_i / f$ mass depends on the vevs of U(1)-charged F-terms in the presence of soft ~~SUSY~~ terms

Assuming no D-term mixing with gauginos

SUSY Breaking and the χ mass

If R symmetry unbroken: $R[\chi] = -1$ & no Majorana mass

- Soft scalar masses preserve R
- *A*-terms are holomorphic and generally break R symmetry

Assuming A_i, $m_i < f_i$, generic size is $|F_i| \approx A_i f_i$

$$m_\chi \sim A_i q_i$$

Often the *A*-terms are suppressed relative to other soft terms, so it's reasonable to expect χ to be the LSP.

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

But one might worry (1104.0692) about Planck-scale operators giving an irreducible contribution to m_χ,

$$\int d^4\theta \frac{(A+A^\dagger)^2(X+X^\dagger)}{M_{\mathrm{Pl}}} \sim m_{3/2}\chi\chi$$

However...

The A-term contribution to m_χ is equivalent to F-term mixing between U(1) charged fields and the ~~SUSY~~ spurion, X.

SUSY Breaking and the χ mass

Contribution from Planck 'sloperators'

For concreteness, consider gravity mediation with $m_{\text{soft}} \sim F/M_{\text{Pl}}$.

$$K = \sum_i Z(X, X^\dagger)\Phi_i^\dagger \Phi_i$$

Analytically continue into superspace hep-ph/9706540

$$\Phi \to \Phi' \equiv Z^{1/2}\left(1 + \frac{\partial \ln Z}{\partial X} F\theta^2\right)\Phi$$

Canonical normalization generates A-terms:

$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

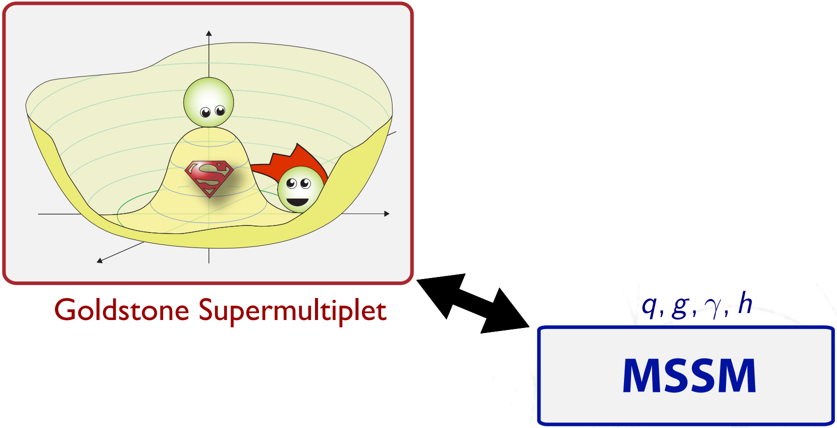
SUSY Breaking and χ mass

$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

Completely incorporates F-term mixing of the form $FF_i^\dagger \Phi_i$.
Assuming $A_i, m_i < f_i$, generic size is $|F_i| \approx A_i f_i$ so that $m_\chi \sim A_i q_i$.

Not a problem when $\cancel{U(1)}$ sector sequestered from ~~SUSY~~ So indeed reasonable to consider $m_a \lesssim m_\chi \ll m_s$.

Parameterize couplings to the MSSM



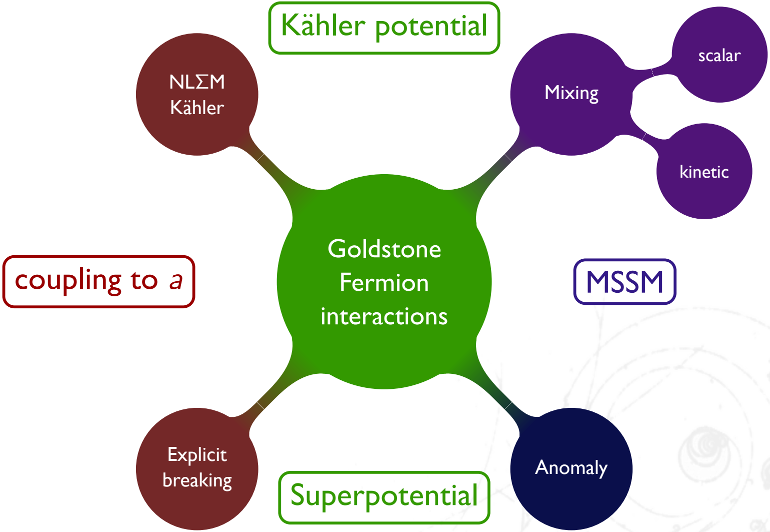
Goldstone Supermultiplet

q, g, γ, h

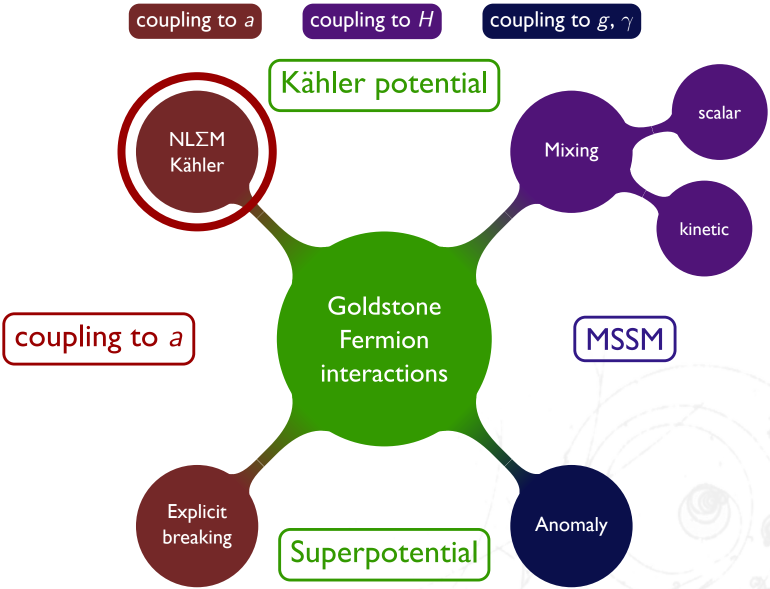
MSSM

Interactions: Overview

coupling to a coupling to H coupling to g, γ



Interactions: Overview



coupling to a

coupling to H

coupling to g, γ

Kähler potential

NLΣM Kähler

Mixing

scalar

kinetic

coupling to a

Goldstone Fermion interactions

MSSM

Explicit breaking

Superpotential

Anomaly

Interactions: NLΣM Kähler potential

Non-linearly realized global U(1) leads to interactions of the Goldstone fields in through the Kähler terms:

$$\frac{\partial^2 K}{\partial A \partial A^\dagger} = 1 + b_1 \frac{q}{f}(A + A^\dagger) + \cdots \qquad b_1 = \frac{1}{qf^2}\sum_i q_i^3 f_i^2$$

Note the manifest shift-invariance. This leads to:

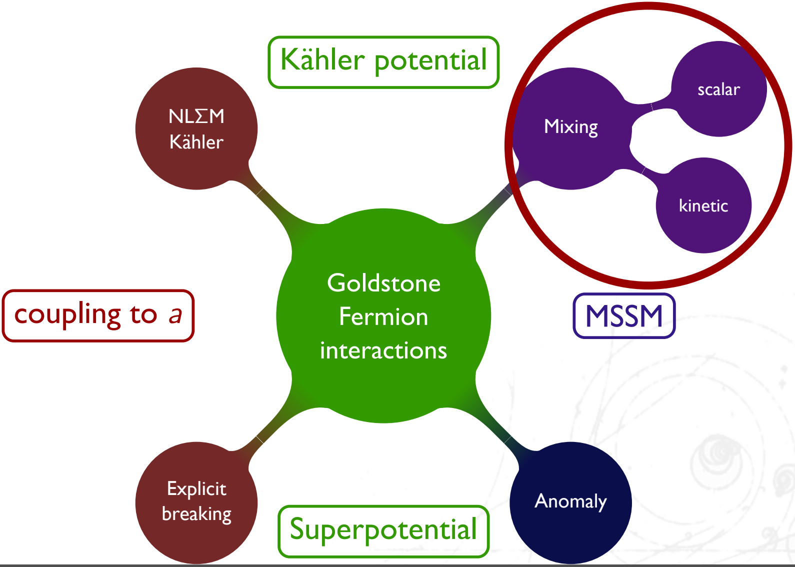
$$\mathcal{L} = \text{(usual kinetic terms)} \left(1 + b_1 \frac{\sqrt{2}}{f} s + \cdots \right) + \frac{1}{2\sqrt{2}}\left(b_1 \frac{1}{f} + b_2 \frac{\sqrt{2}}{f^2} s + \cdots \right) \boxed{(\overline{\chi}\gamma^\mu\gamma^5\chi)\,\partial_\mu a} + \cdots$$

b_1 controls the annihilation cross section.

Zumino, Phys. Lett. B 87 (1979) 203

Interactions: Overview

coupling to a coupling to H coupling to g, γ



Kähler potential

NLΣM Kähler

Mixing

scalar

kinetic

coupling to a

Goldstone Fermion interactions

MSSM

Explicit breaking

Superpotential

Anomaly

Interactions: scalar mixing

MSSM fields are uncharged under the global U(1), but may mix with the Goldstone multiplet through higher-order terms in K:

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(c_1 H_u H_d + \cdots\right) + \frac{1}{2f^2}\left(A + A^\dagger\right)^2\left(c_2 H_u H_d + \cdots\right)$$

The new scalar interactions take the form

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\boxed{\bar{\chi}\not{\partial}\chi}\right]\left(1 + \boxed{c_h \frac{v}{f} h} + \cdots\right)$$

c_h depends on c_i and the Higgs mixing angles.

c_h controls direct detection

$c_h \to (m_h/m_s)^2$ in the large m_s limit.

We neglect mixing with the heavy higgses.

Interactions: other mixing

The higher order terms in K also induce kinetic $\widetilde{H}$–χ mixing.

$$\mathcal{L} \supset i\epsilon_u \boxed{\overline{\chi}\gamma^\mu\partial_\mu \widetilde{H}_u^0} + i\epsilon_d \boxed{\overline{\chi}\gamma^\mu\partial_\mu \widetilde{H}_d^0} + \text{h.c.}$$

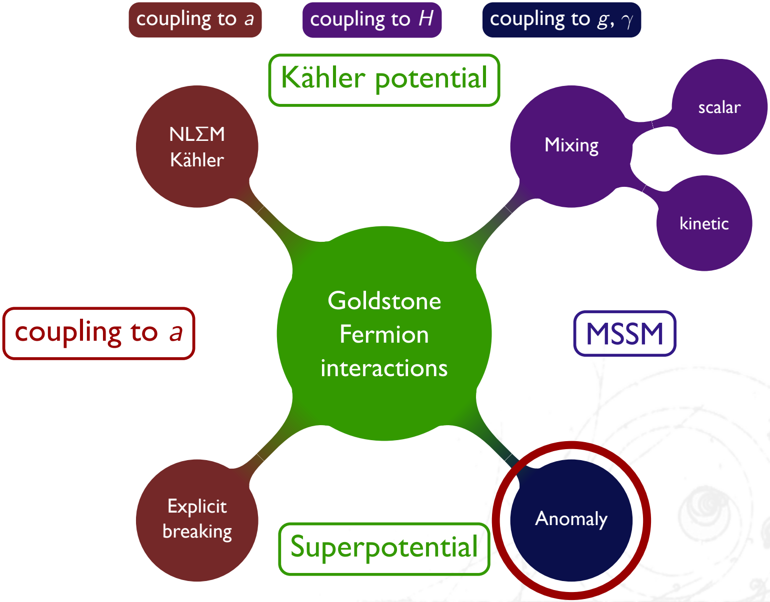
where $\epsilon \sim v/f$. For large μ: χ has a small $\widetilde{H}$ component of $\mathcal{O}(vm_\chi/f\mu)$.

Mixing with other MSSM fields is suppressed. Assuming MFV,

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(\frac{Y_u}{M_u}\overline{Q}H_u U + \cdots\right)$$

where the scales $M_{u,d,\ell}$ are unrelated to f or v and can be large and dependent on the UV completion

Interactions: Overview



coupling to a

coupling to H

coupling to g, γ

Kähler potential

NLΣM
Kähler

Mixing

scalar

kinetic

coupling to a

Goldstone
Fermion
interactions

MSSM

Explicit
breaking

Superpotential

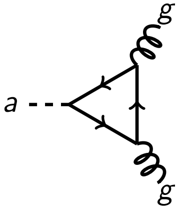
Anomaly

Interactions: anomaly

Fermions Ψ charged under global U(1) and Standard Model

$$\mathcal{L}_{\text{an}} \supset \frac{c_{\text{an}}}{f\sqrt{2}} \left(\boxed{a G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}} + 2\overline{\chi} G^a_{\mu\nu} \sigma^{\mu\nu} \gamma^5 \lambda^a \right)$$

$$\boxed{c_{\text{an}} = \frac{\alpha}{8\pi} q_\Psi N_\Psi}$$



$\mathrm{U(1)\,SU(3)}_c^2$

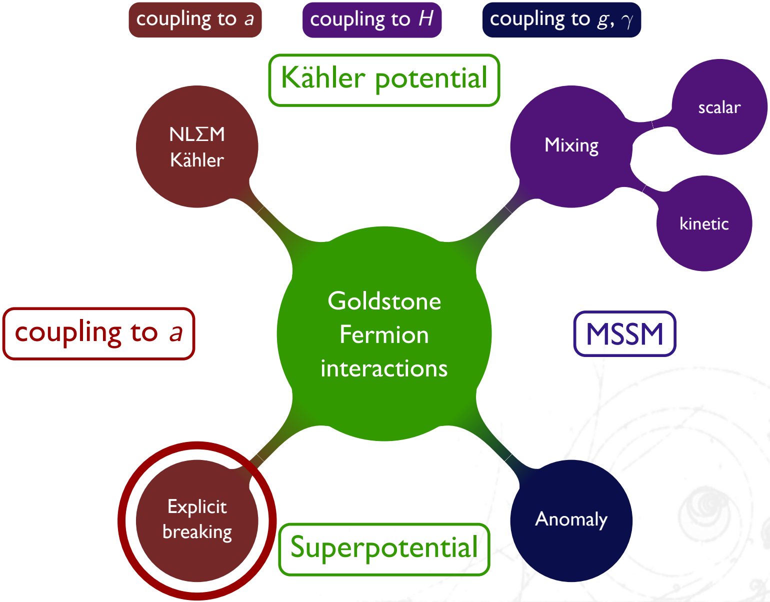
$\mathrm{U(1)\,U(1)}_{\text{QED}}^2$

Integrating out λ^a generates χ couplings to gluons, photons

$$\mathcal{L} \supset -\left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right) \boxed{\overline{\chi}\chi\, GG} - i \left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right) \boxed{\overline{\chi}\gamma^5\chi\, G\tilde{G}}$$

These contribute to collider and astro operators.

Interactions: Overview



Interactions: explicit breaking

Include explicit $\cancel{U(1)}$ spurion $R_\alpha = \lambda_\alpha f$ with $\lambda_\alpha \ll 1$

$$W_{\cancel{U(1)}} = f^2 \sum_\alpha R_{-\alpha} e^{aA/f}$$

Perserve SUSY $\Rightarrow$ at least two spurions with opposite charge.

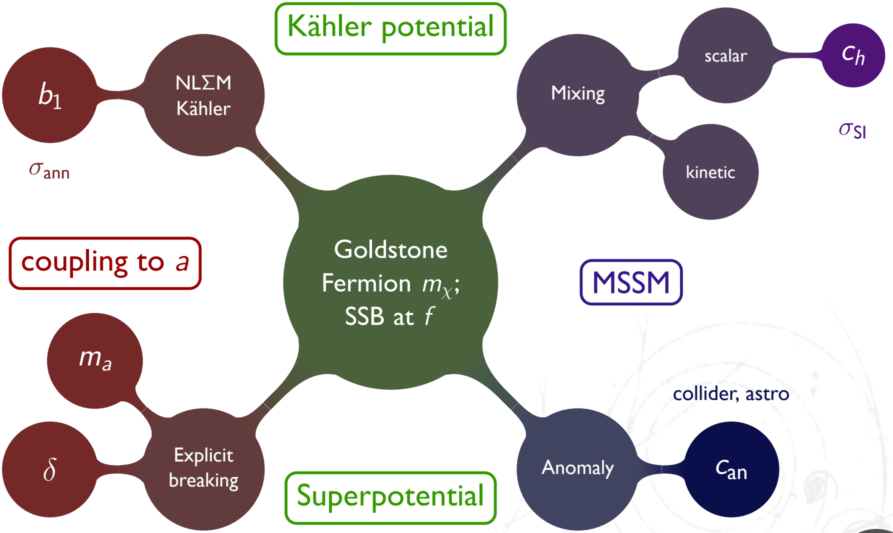
This generates $m_a = m_\chi = m_s$ and couplings

$$\mathcal{L} \supset \underbrace{-\frac{m_a}{2\sqrt{2}f}(\alpha+\beta)}_{\delta}\, i\, \boxed{a\overline{\chi}\gamma^5\chi} + \underbrace{\frac{m_a}{8f^2}(\alpha^2+\alpha\beta+\beta^2)}_{\rho}\, a^2\overline{\chi}\chi$$

By integration by parts this is equivalent to a shift in the b_1 coefficient from the Kähler potential

Main parameters

coupling to a | coupling to H | coupling to g, γ



Kähler potential

b_1

σ_{ann}

NLΣM Kähler

Mixing

scalar

kinetic

c_h

σ_{SI}

coupling to a

Goldstone Fermion m_χ; SSB at f

MSSM

m_a

δ

Explicit breaking

Superpotential

Anomaly

collider, astro

c_{an}

Parameter space scan

$$\textbf{Abundance}: \langle \sigma v \rangle \approx \frac{{b_1}^4}{8\pi} \frac{T_f}{m_\chi} \frac{m_\chi^2}{f^4} \approx 1\ \text{pb}$$

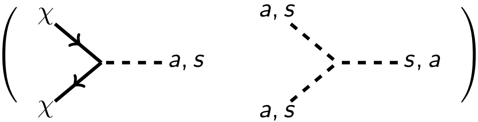
p-wave: $b_1 \gtrsim 1$, all other parameters take natural values

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	$[0, 2]$
c_{an}	Anomaly coefficient	0.06
c_h	Higgs coupling	$[-1, 1]$
δ	Explicit breaking $ia\overline{\chi}\gamma^5\chi$ coupling	3/2

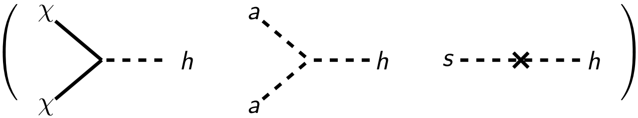
$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\overline{\chi}\not{\partial}\chi\right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f}\left(\overline{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a + \frac{c_{\text{an}}}{f\sqrt{2}} aG\widetilde{G} + i\delta a\overline{\chi}\gamma^5\chi$$

Main Interactions summary

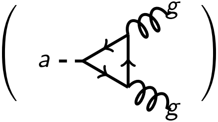
$$\frac{b_1}{f} \times$$



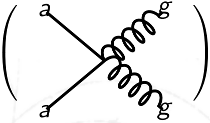
$$\frac{v}{f^2} \times$$



$$\frac{\alpha_S N}{8\pi f} \times$$

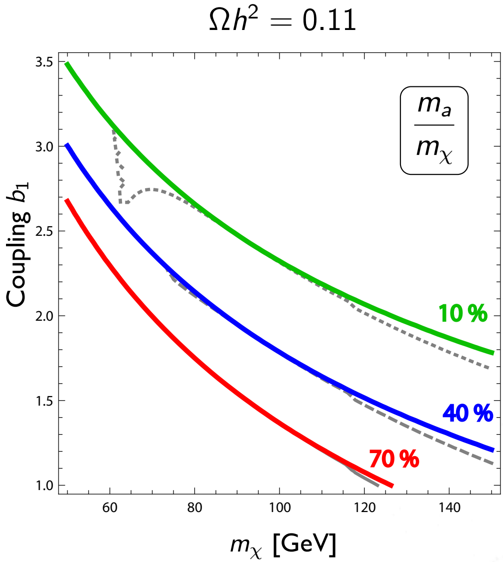


$$\left(\frac{\alpha_S N}{8\pi f}\right)^2 \frac{1}{M_\lambda} \times$$



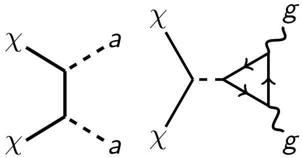
$$m_a < m_\chi \ll m_s$$

Annihilation: Contours of fixed Ω



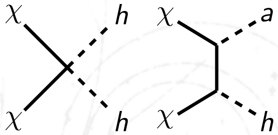
Dominant contribution

Kähler, anomaly, ~~U(1)~~



Subleading

Mixing with Higgs



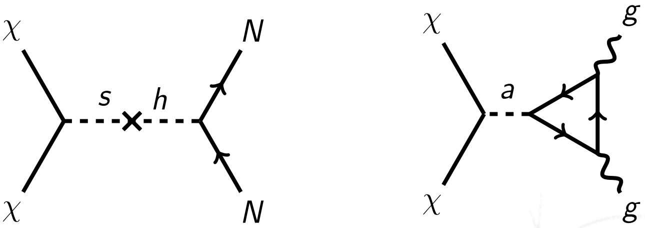
Negligible

$\chi\chi \to s \to aa$, $\chi\chi \overset{t,u}{\to} hh$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h v}{2f}\overline{\chi}\not{\partial}\chi h - \frac{c_{an}{}^2}{2M_\lambda f^2}\overline{\chi}\chi GG - \frac{i c_{an}{}^2}{2M_\lambda f^2}\overline{\chi}\gamma^5\chi G\widetilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}}\overline{N}N\overline{\chi}\chi$,

$$G_{\text{nuc}} = c_h\frac{\lambda_N}{2\sqrt{2}}\left(\frac{m_\chi m_N}{m_h^2 f^2}\right) + \frac{4\pi c_{an}{}^2}{9\alpha_s}\frac{m_N}{M_\lambda f}\left(1 - \sum_{i=u,d,s} f_i^{(N)}\right)$$

Direct Detection

Higgs exchange typically dominates by a factor of $\mathcal{O}(10^3)$.

$$\sigma_{\text{SI}}^{\text{H}} \approx \boxed{2\cdot 10^{-45}\ \text{cm}^2}\, c_h^2 \left(\frac{125\ \text{GeV}}{m_h}\cdot\frac{700\ \text{GeV}}{f}\right)^4 \left(\frac{m_\chi}{100\ \text{GeV}}\cdot\frac{\mu_\chi}{\text{GeV}}\cdot\frac{\lambda_N}{0.5}\right)^2$$

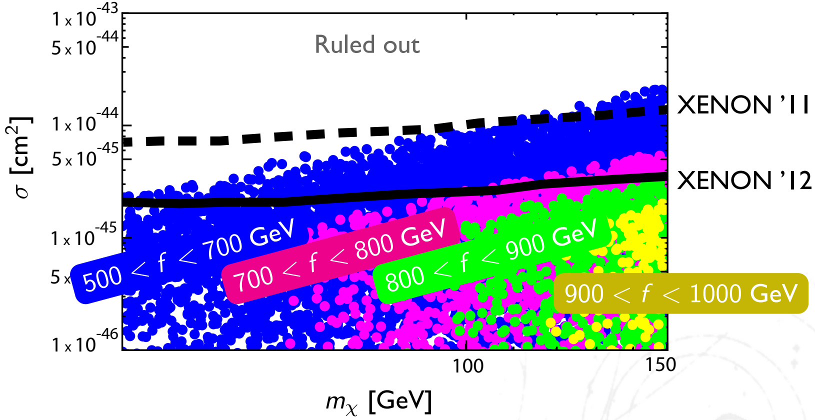
Compare this to the MSSM Higgs with $\mathcal{L} = \frac{1}{2}cg\overline{\chi}\chi h$:

$$\sigma_{\text{SI}}^{\text{MSSM}} \sim \frac{c^2g^2}{2\pi}\frac{\lambda_N^2\mu^2 m_N^2}{m_h^2 v^2} \approx c^2 \times 10^{-42}\ \text{cm}^2$$

Natural suppression: $(m_\chi v/f^2)^2$

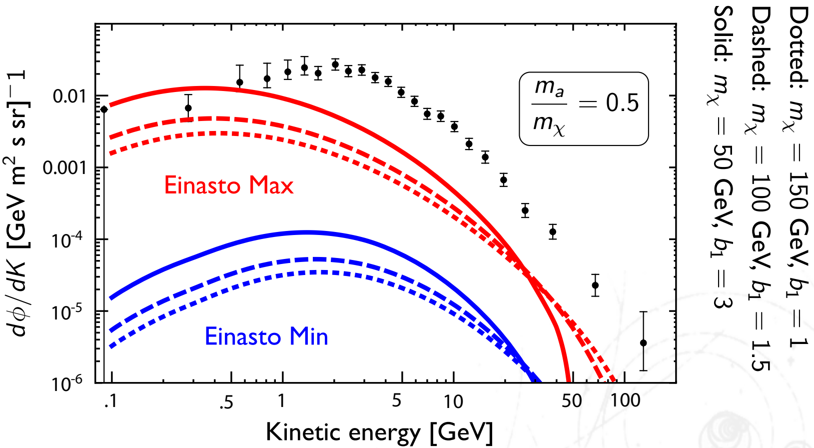
Is it enough to avoid current direct detection bounds?

Parameter space scan & direct detection



Indirect detection: $\overline{p}$ flux vs. PAMELA

$f = 700$ GeV, $Q_\Psi = 2$, $\delta = \frac{3}{2}$, $N_\Psi = 5$



Dotted: $m_\chi = 150$ GeV, $b_1 = 1$

Dashed: $m_\chi = 100$ GeV, $b_1 = 1.5$

Solid: $m_\chi = 50$ GeV, $b_1 = 3$

Using Einasto DM Halo profile in 1012.4515, 1009.0224

Indirect detection: Fermi-LAT

Annihilation is *p*-wave, but this is suppressed at late times.
Indirect detection from anomaly diagrams:

γ-ray line search: 30 – 200 GeV

- $\chi\chi \to a \to \gamma\gamma$ via anomaly
- $\mathcal{O}(10)$ smaller than bound even for extreme parameters

Diffuse γ-ray spectrum: 20 – 100 GeV

- $\chi\chi \to a \to gg \to \pi^0$s
- $\mathcal{O}(10)$ smaller than bound

`http://fermi.gsfc.nasa.gov/science/symposium/2011/program`

Goldstone fermions at the LHC

Collider production through gluons.

ISR monojets: sensitive to $\sigma_{\mathrm{SI}}^{N} \sim 10^{-46}$ cm^2 with 100 fb^{-1}.

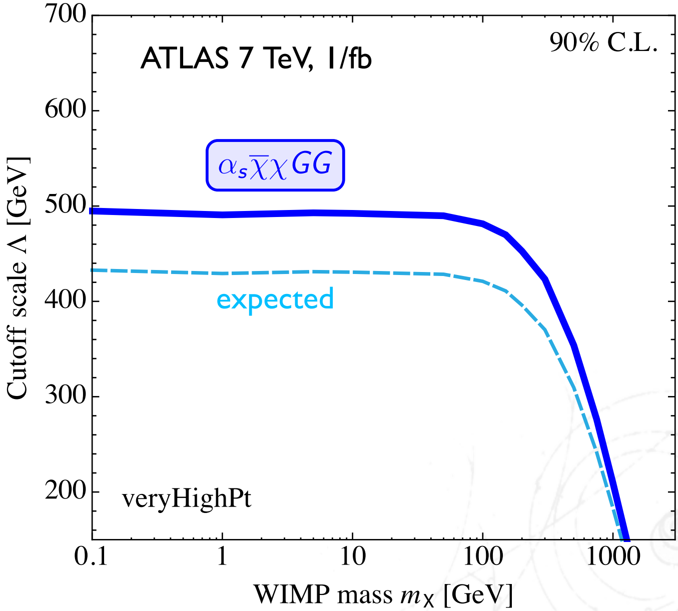
The dim-7 anomaly operators are too small:

$$\mathcal{L} \supset -\frac{c_{\mathrm{an}}{}^{2}}{2M_\lambda f^2}\overline{\chi}\chi G G - \frac{i\, c_{\mathrm{an}}{}^{2}}{2M_\lambda f^2}\overline{\chi}\gamma^5\chi G\widetilde{G}$$

$gg \to a^* \to \chi\chi$ may be within 5σ reach with 100 fb^{-1}

1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196, 1109.4398

Goldstone fermions at the LHC



Fox, Harnik, Kopp, Tsai: 1109.4398

The $\gamma\gamma$ games

Experimental progress, model-building directions:

1. $m_h = 125$ GeV, possible enhancement in $h \to \gamma\gamma$?
2. FERMI line at 130 GeV, possibly $\chi\chi \to \gamma\gamma$?

Can we realize this in Goldstone Fermion DM? Maybe.

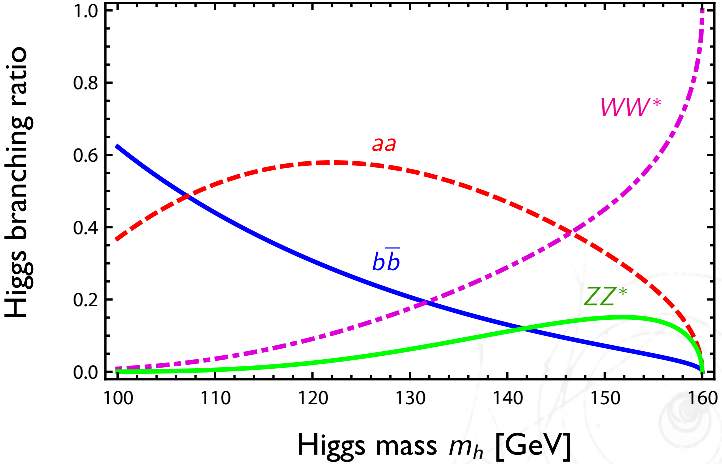
Work in progress with B. Bellazzini, M. Cliche

1. Goldstone boson decays modify Higgs branching ratios
2. Anomaly coupling, SUSY framework to control spectrum for narrow box-shaped γ-ray spectrum.

Non-standard Higgs decays

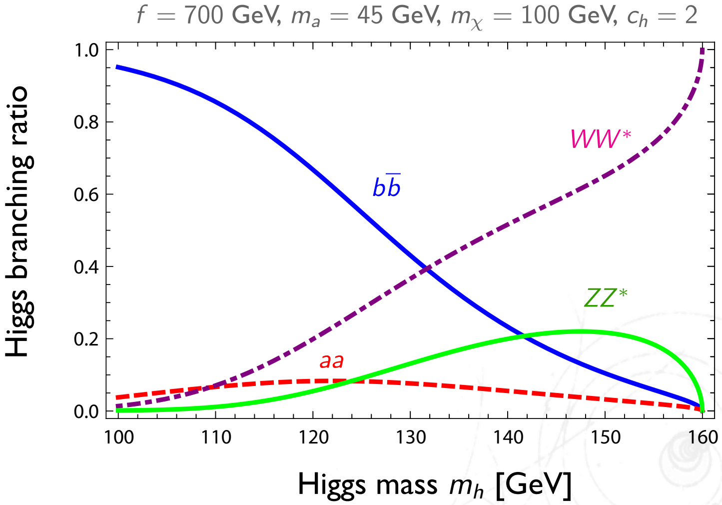
Hard to completely bury the Higgs. LEP: Br(SM)$\gtrsim$ 20% $\Rightarrow m_h \gtrsim 110$ GeV

$f = 500$ GeV, $m_a = 45$ GeV, $m_\chi = 100$ GeV, $c_h = 2$



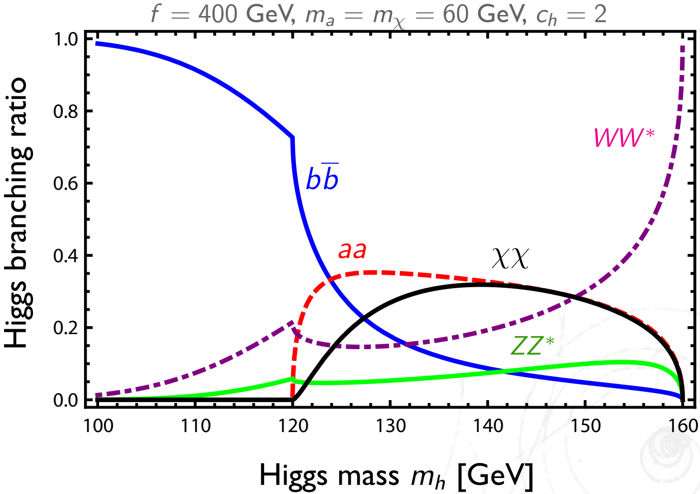
Non-standard Higgs decays

For larger f, can suppress $h \to aa$

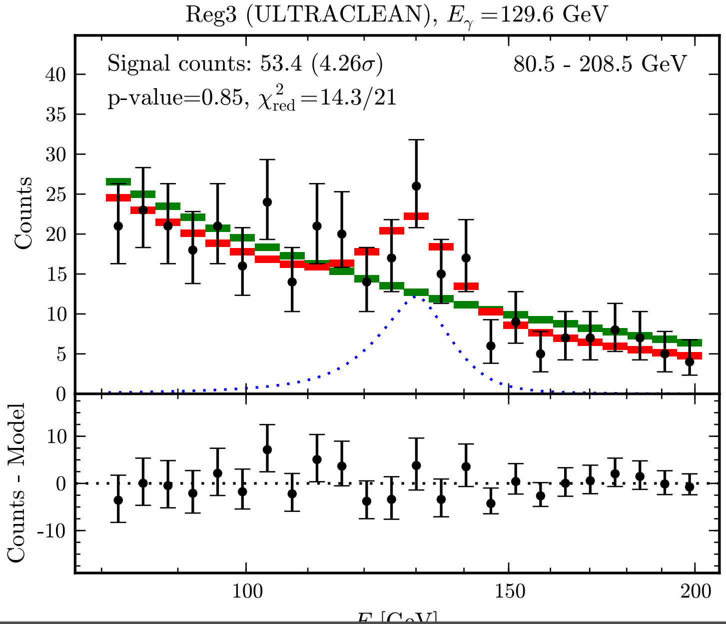


Non-standard Higgs decays

Partially buried & invisible: Suppressed SM channels, MET, $\Gamma_{\text{tot}} < 1$



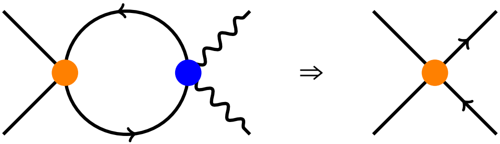
The 130 GeV Weniger Line



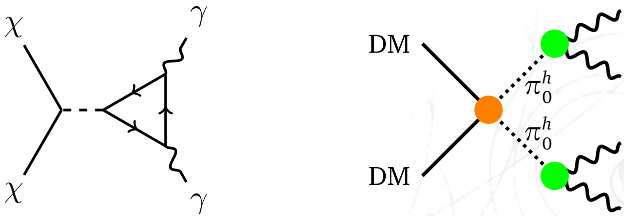
1204.2797

Model building for the Weniger Line

Problem: photon continuum Cohen, Lisanti, Slatyer, Wacker (1207.0800)

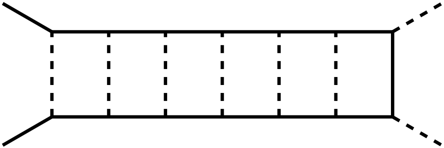


Tuned pseudoscalar processes Fan & Reece (1209.1097)



Sommerfeld enhancement for singular potentials

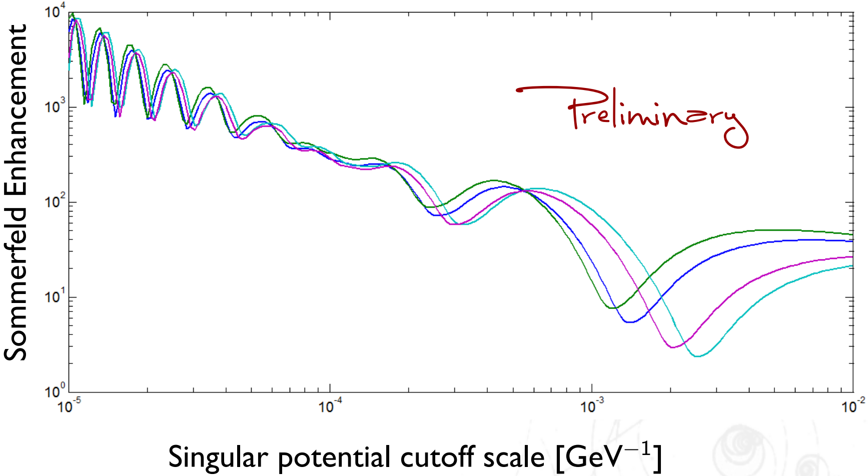
$\mathcal{O}$(few) sufficient to open up parameter space.
Larger enhancement may be used for $\gamma\gamma$ signal



Pseudoscalar exchange $\Rightarrow 1/r^3$ potential, need to regulate and renormalize effective non-relativistic theory.

Need to clarify UV sensitivity, viability of matching.
0810.0713, 0902.0688, 0907.0235

Pseudoscalar Yukawa enhancement

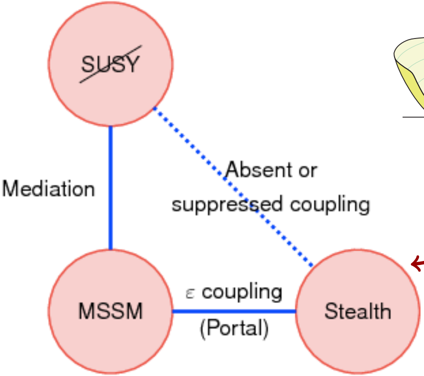


Preliminary

Other applications: Stealth SUSY limit

10^5 GeV $\lesssim f < 10^8$ GeV

$m_\chi \to 0$



~~SUSY~~

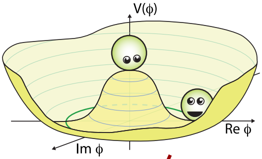
Mediation

Absent or suppressed coupling

MSSM

ε coupling (Portal)

Stealth



$M_{\text{SUSY}} \sim M_{\text{EWK}}$

$M_{\cancel{\text{SUSY}}} \sim \varepsilon M_{\text{EWK}}$

Fan, Reece, Ruderman: 1201.4875

Conclusions

Executive summary: Goldstone Fermion dark matter

- SSB: global U(1) ⇒ Goldstone boson *a* and fermion χ
- χ is LSP and DM, *a* can modify Higgs branching ratios

Simple extension of MSSM with natural WIMP dark matter

- Kähler $\chi\chi a$ interaction controls abundance
- Higgs mixing, anomaly controls direct detection

Further directions:

- *p*-wave Sommerfeld enhancement
- Non-abelian generalization
- $h, \chi\chi \to \gamma\gamma$ hints

Extra Slides

Examples of Linear Models

Simplest example:

$$W = yS\left(\overline{N}N - \mu^2\right) + \underbrace{N\overline{\phi}\phi}_{\text{anomaly}} + \underbrace{SH_uH_d}_{\text{mixing}} + \underbrace{W_{\text{explicit}}}_{\text{explicit}\,\cancel{U(1)}}$$

Example with $|b_1| \geq 1$:

$$W = \lambda XYZ - \mu^2 Z + \frac{\widetilde{\lambda}}{2} Y^2 N - \widetilde{\mu}\overline{N}N$$

$q_Z = 0$, $q_N = -q_{\overline{N}} = -2q_Y = 2q_X$. Goldstone multiplet:

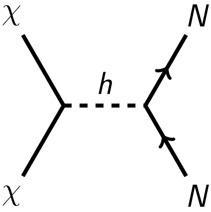
$$A = \sum_i \frac{q_i f_i \psi_i}{f} = \frac{q_Y}{f}\left(Yf_Y - Xf_X + 2\overline{N}F_{\overline{N}}\right)$$

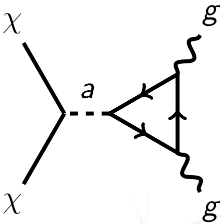
$$b_1 = \frac{-f_X^2 + f_Y^2 + 8f_{\overline{N}}^2}{f_X^2 + f_Y^2 + 4f_{\overline{N}}^2}$$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h v}{2f}\bar{\chi}\not{\partial}\chi h - \frac{{c_{\text{an}}}^2}{2M_\lambda f^2}\bar{\chi}\chi GG - \frac{i{c_{\text{an}}}^2}{2M_\lambda f^2}\bar{\chi}\gamma^5\chi G\widetilde{G}$$





Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}}\overline{N}N\bar{\chi}\chi$,

$$G_{\text{nuc}} = c_h\frac{\lambda_N}{2\sqrt{2}}\left(\frac{m_\chi m_N}{m_h^2 f^2}\right) + \frac{4\pi {c_{\text{an}}}^2}{9\alpha_s}\frac{m_N}{M_\lambda f}\left(1 - \sum_{i=u,d,s} f_i^{(N)}\right)$$

Direct Detection

Some details:

$$G_{\chi N} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi {c_{an}}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

For reduced mass $\mu_\chi = (m_\chi^{-1} + m_N^{-1})^{-1}$,

$$\sigma_{\text{SI}}^{\text{Higgs}} = \frac{4\mu_\chi^2}{A^2\pi} \left[G_{\chi p} Z + G_{\chi n}(A - Z) \right]$$

$$\sigma_{\text{SI}}^{\text{H}} \approx 3{\cdot}10^{-45}\ \text{cm}^2 c_h^2 \left(\frac{115\ \text{GeV}}{m_h} \right)^4 \left(\frac{700\ \text{GeV}}{f} \right)^4 \left(\frac{m_\chi}{100\ \text{GeV}} \right)^2 \left(\frac{\mu_\chi}{1\ \text{GeV}} \right)^2 \left(\frac{\lambda_N}{0.5} \right)^2$$

$$\sigma_{\text{SI}}^{\text{glue}} \approx 2 \cdot 10^{-48}\ \text{cm}^2 \left(\frac{700\ \text{GeV}}{M_\lambda} \right)^2 \left(\frac{700\ \text{GeV}}{f} \right)^4 \left(\frac{N_\Psi}{5} \right)^4 \left(\frac{q_\Psi}{2} \right)^4 \left(\frac{\mu}{1\ \text{GeV}} \right)^2$$

using $c_{an} = \alpha_s q_\Psi N_\Psi / 8\pi$

Why are the $\chi\chi \to aa$ annihilations p-wave?

If the initial state is a particle-antiparticle pair with zero total angular momentum and the final state is CP even, then the process must vanish when $v = 0$.

Under CP a particle/antiparticle pair picks up a phase $(-)^{L+1}$. When $v = 0$ momenta are invariant and thus the initial state gets an overall minus sign. Since final state is CP even, the amplitude must vanish in this limit. For Dirac particles P is sufficient, but for Majorana particles CP is the well-defined operation.

This is why $\chi\chi \to G\widetilde{G}$ is s-wave while $\chi\chi \to aa$ is p-wave.

Nuclear matrix element and matching

The nucleon matrix element at vanishing momentum transfer:

$$M_N = \langle \Theta^\mu_\mu \rangle = \langle N | \sum_{i=u,d,s} m_i \overline{q}_i q_i + \frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} | N \rangle$$

from: Shifman, Vainshtein, Zakharov. Phys. Lett 78B (1978)

$\beta = -9\alpha^2/2\pi + \cdots$ contains only the light quark contribution, M_N is the nucleon mass. The GG matches onto the nucleon operator $\overline{N}N$.

$$M_N f^{(N)}_{i=u,d,s} = \langle N | m_i \overline{q}_i q_i | N \rangle \qquad f^{(N)}_g = 1 - \sum_{i=u,d,s} f^{(N)}_i$$

Nuclear matrix element and matching

$$\frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} \longrightarrow M_N \left(1 - \sum_{i=u,d,s} f_i^{(N)}\right) \overline{N} N$$

Where $f_{u,d}^{(N)} \ll f_s^{(N)} \approx 0.25$. For a detailed discussion, see 0801.3656 and 0803.2360.

Image Credits and Colophon

- 'Zombie arm' illustration from `http://plantsvszombies.wikia.com`
- Beamer theme Flip, available online `http://www.lepp.cornell.edu/~pt267/docs.html`
- All other images were made by Flip using TikZ and Illustrator