Warped Penguins from an Extra Dimension

Flip Tanedo

Based on arXiv:0910.soon
In collaboration with C. Csáki, Y. Grossman, Y. Tsai.

Cornell University

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Penguin etymology

"For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time. Then, one evening, after working at CERN, I stopped on my way back to my apartment to visit some friends living in Meyrin where I smoked some illegal substance. Later, when I got back to my apartment and continued working on our paper, I had a sudden flash that the famous diagrams look like penguins. So we put the name into our paper, and the rest, as they say, is history."

-John Ellis (hep-ph/9510397)

The March of the Penguins

Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

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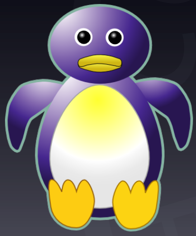
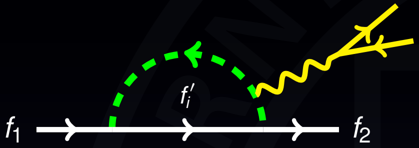


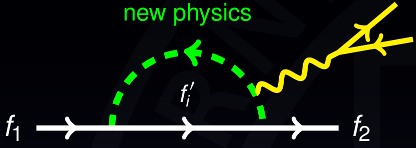
The March of the Penguins



Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

The March of the Penguins



Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

The $\mu \to e\gamma$ Penguin

Experimental Bound

$\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (MEGA)

$\mathrm{Br}(\mu \to e\gamma) < 10^{-13}$ (MEG)

$$\mathcal{M} = a\frac{e}{16\pi^2} H \cdot L\sigma^{\mu\nu}\left(yy^\dagger y\right)\overline{E}F_{\mu\nu}$$

Gauge invariance $\to$ Chiral structure $\to$ Mass insertion

Gauge choice: $\xi = \infty$, decouple Goldstones.

Won't have to worry about effects of bulk gauge fields.

"Is $\mu \to e\gamma$ really a penguin?"

Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

A penguin with no feet...
... is still a penguin!

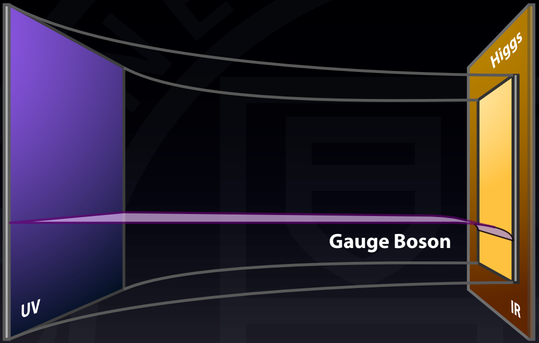
- Flavor-changing
- Occurs on-shell
- Mass insertion

Randall-Sundrum in one slide

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99);

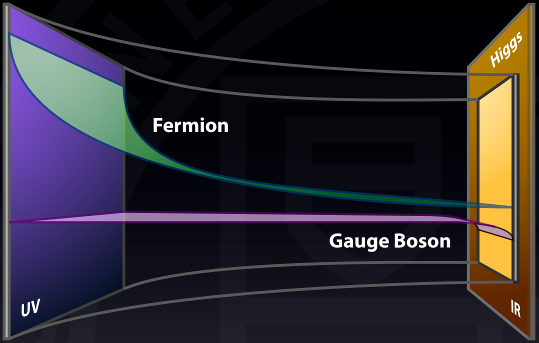
Randall-Sundrum in one slide



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99);

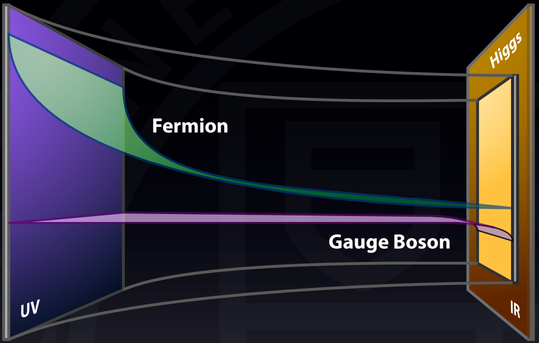
Randall-Sundrum in one slide



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99); Grossman, Neubert (00); Gherghetta, Pomarol (00); **Bulk Higgs:** Agashe, Contino, Pomarol (04); Davoudiasl, Lille, Rizzo (05)

Randall-Sundrum in one slide



$$Y_{ij}^{(4D)} = f_i^{(0)} Y_{ij}^{(5D)} f_j^{(0)}$$

Flavor: Huber, Shafi (03); Burdman (03); Kalil, Mohapatra (04); Agashe, Perez, Soni (04); Chen (05); Agashe, Blechman, Petriello (06); Davidson, Isidori, Uhlig (07); Csáki, Falkowski, Weiler (08) Chen, H.B. Yu (08); Chen, Mahanthappa, F. Yu (09) , ...

Comments on Flavor in RS

Bulk mass $c = MR$ determines zero-mode localization.

$$Y_{ij}^{(4D)} = f_{c_i}^{(0)} Y_{ij}^{(5D)} f_{c_j}^{(0)} \qquad f_c = \sqrt{\frac{1-2c}{1-(R/R')^{1-2c}}}$$

Anarchy: $Y_{ij}^{(5D)} \sim Y_* \sim 2$ (Agashe, Delgado, May, Sundrum 03)

Note that the c_i introduce additional flavor structure, but no tree-level FCNCs between zero-modes.

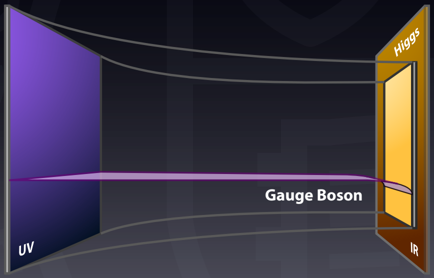
The T parameter becomes too large (mixing with KK modes)

- Gauge $SU(2)_R$ in bulk (Agashe, Delgado, May, Sundrum 03)
- Add brane kinetic terms (Tait et al. 03; Davoudiasl et al. 03)

$\mu \to e\gamma$ in RS

First thorough analysis by Agashe, Blechman, Petriello (06):

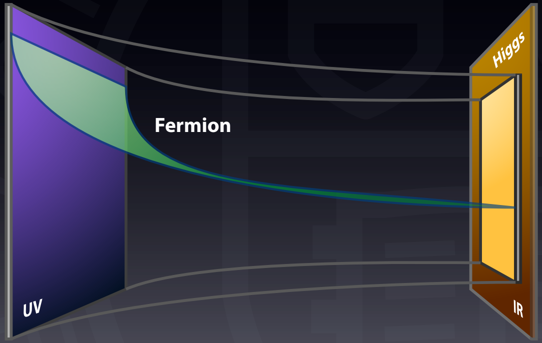
- Brane-localized Higgs: $\mu \to e\gamma$ appears $\log \Lambda$ divergent
- Bulk Higgs scenario: UV finite
- Tension between $\mu \to e\gamma$ and $\mu \to 3e$, $\mu \to e$.
  Y_*: upper bound from $\mu \to e\gamma$, lower bound from $\mu \to 3e$



$\mu \to e\gamma$ in RS

The problem

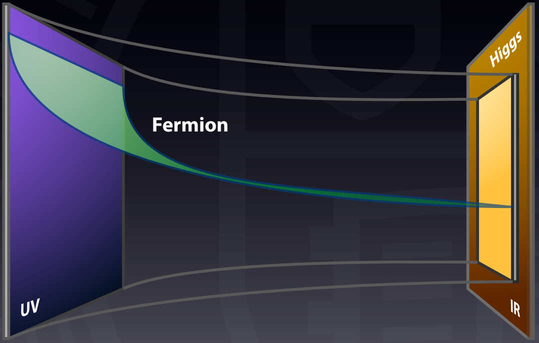
Previous analyses suggest $\mu \to e\gamma$ is logarithmically **UV sensitive** in the RS model. Tricky: sum over infinite KK tower.



$\mu \to e\gamma$ in RS

Our claim

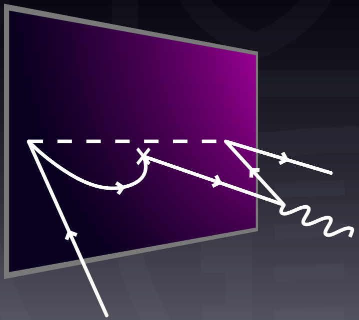
$\mu \to e\gamma$ is manifestly **UV finite** in models of an extra dimension on an interval with chiral boundary conditions, e.g. RS.



$\mu \to e\gamma$ in RS

Very heuristic motivation

- UV effects represent very localized phenomena
- The brane-Higgs forces localization onto the IR brane
- IR brane: ordinary, renormalizable 4D theory
- No tree-level dipole operator, no possible counter term
- Leading order (loop-level) contribution must be finite



$\mu \to e\gamma$ in the Standard Model

4D diagrams with Dirac fermions.

Naïve expectation: $\log \Lambda$ divergent, find $\mathcal{M} \sim M^{-2}$

Cancellation mechanisms (degree of divergence)

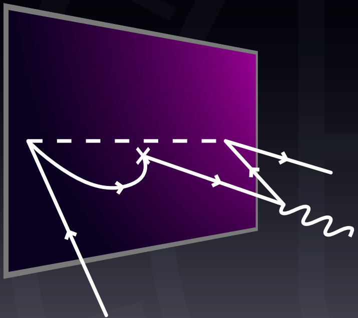
1. **Gauge invariance**: Ward identity forces $q_\mu \mathcal{M}^\mu = 0$, thus the amplitude must depend non-trivially on q_μ. (-1)
2. **Lorentz invariance**: no divergences odd in the loop momentum. (-1)

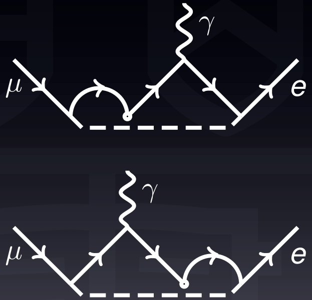
$$\int d^4k \; \not{k}/k^{2n} = 0.$$

3. **Chiral structure**: Magnetic transition requires mass insertion.

5D Diagrams

Recall: brane-localized Higgs responsible for Yukawa interaction and chirality-flipping mass insertion.





$\mu \to e\gamma$ in an extra dimension

Problem: in 5D we have one additional momentum integral (sum over KK modes) *and* we lose (5D) Lorentz invariance.

Status of cancellation mechanisms (degree of divergence)

0. Additional loop momentum (+1).
1. Gauge invariance still holds (same).
2. 5D Lorentz invariance is **broken**! (+1)
3. **Chiral structure** saves the day. (-1)

It appears that we're in trouble and we're really UV sensitive. But a new cancellation mechanism becomes operative from our boundary conditions.

$\mu \to e\gamma$ in an extra dimension

Conventional wisdom: Don't do loops in RS*.
* - usually don't have to: tree level flavor effects from bulk masses

But if you really **must**...

How to do KK Loops

- Do a Kaluza-Klein reduction to 4D fields
- Diagonalize infinite-dimensional mass matrices
  Practically: diagonalize first few modes
- Perform 4D loops as usual
- Sum over KK modes
  Practically: sum over the first few
- Pray that your approximations make sense
  Practically: Show that subsequent terms are small

$\mu \to e\gamma$ in an extra dimension

UV dependence enters at the cutoff scale, $\Lambda \gg k = 1/R$.
Thus to study finiteness, it is sufficient to consider a flat XD.

Don't have to worry about warping effects, analysis is simpler.

The divergences of warped XD are exactly those of flat XD.

This still sucks

- Brane localized terms (e.g. Higgs) can be tricky
- Sum over infinite number of KK modes
- Diagonalize infinite dimensional mass matrices

This is fine when you're calculating something that you already know manifestly finite, but it can be very tricky when determining finiteness

$\mu \to e\gamma$ in an extra dimension

Alternative: Mixed position-momentum space propagators
Work in position space in XD, momentum space in Minkwoski.
Flat: Puchwein, Kunszt (03); RS: Carena, Delgado, Ponton, Tait, Wagner (04)

'Natural' thing to do on an interval

- Brane localized terms are easy
- No infinite sum over KK modes, finite *dz* integral
- Mass matrices are finite-dimensional

Still have to be careful about picking a flavor basis; the zero-modes aren't in the 'canonical' 5D basis where c_is diagonal.

$\mu \to e\gamma$ in an extra dimension

Quick, heuristic derivation of the propagator:

$$(-\not{p} + i\gamma^5\partial_5 + m)\Delta(p, z, z') = i\delta(z - z')$$

'Hard' to solve. Trick: solve 'squared Dirac operator' equation,
This is now a *scalar* differential equation.

$$\left(p^2 - \partial_5^2 + m^2\right)F(p, z, z') = i\delta(z - z')$$

and we automatically get a solution to Dirac Green's function:

$$\Delta(p, z, z') = (\not{p} - i\gamma^5\partial_5 + m)F(p, z, z')$$

General solution takes the form: $({\chi_p}^2 = p_{4\text{D}}^2 - m^2)$

$$F(p, z, z') = A(p, z')\cos(\chi_p z) + B(p, z')\sin(\chi_p z)$$

Coefficients fixed by boundary conditions.

$\mu \to e\gamma$ in an extra dimension

Status of cancellation mechanisms (degree of divergence)

0. Additional loop momentum (+1).
1. Gauge invariance still holds (same).
2. 5D Lorentz invariance is **broken**! (+1)
3. **Chiral structure** saves the day. (-1)

The point: we can now understand how # 2 fails.

$$\Delta(k, z, z') = \left(\not{k} - i\gamma^5\partial_5 + m\right) F(k, z, z')$$

The $i\gamma^5\partial_5$ term brings out a factor of $\chi_k \sim \sqrt{k^2}$, which is *even* in k. Thus we can have non-zero loop integrands which are even functions of k while being odd powers of k.

$$\int \frac{d^4k}{(2\pi)^4} \frac{\chi_k}{k^{2n}} \neq 0.$$

$\mu \to e\gamma$ in an extra dimension

Status of cancellation mechanisms (degree of divergence)

0. Additional loop momentum (+1).
1. Gauge invariance still holds (same).
2. 5D Lorentz invariance is **broken**! (+1)
3. **Chiral structure** saves the day. (-1)

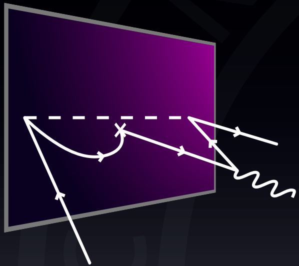
But now **chiral boundary conditions** can save us.

The $i\gamma^5\partial_5$ term in Δ swaps good-chirality Weyl spinors with wrong-chirality Weyl spinors:

$$\chi_L \leftrightarrow \psi_L$$

These wrong-chirality spinors vanish on the brane by chiral boundary conditions. Thus the $d^4k\, \chi_k/k^{2n}$ integrals indeed vanish. We end up with a result that goes as M^{-1}.

Something you see in 5D, but not 4D



This is an example of an effect that you can see in the 5D formalism that is obfuscated in the 4D KK reduction.

Chiral boundary conditions are hidden in the diagonalization of an infinite dimensional mass matrix.

The difference: terms that are 'small' at finite KK mode can appear small but summed over an infinite number of modes. In the 5D we see that those 'small' things are actually exactly zero.

Diagrams and Operator

For this talk, ignore the charged Higgs loop. ($\mathcal{O}(3\%)$ correction.)

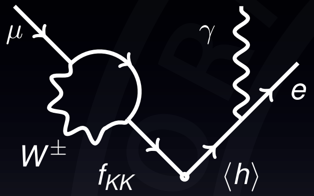
The the only diagrams at leading order in mass insertion are

These contribute to the gauge invariant operator

$$a_{kl}\frac{e_5}{16\pi^2}H\cdot L_i\sigma^{\mu\nu}\left(y_{ij}y^\dagger{}_{k\ell}y_{\ell j}\right)\overline{E}_jF_{\mu\nu}$$

Diagrams and Operator (Counter-examples?)

We can try drawing 'crazy' diagrams or 'crazy' operators



$$\mathcal{O}_1 = \frac{b_1}{16\pi^2}\overline{L}_i \not{D}(yy^\dagger)L_j$$

$$\mathcal{O}_2 = \frac{b_2}{16\pi^2}\overline{E}_i \not{D}(yy^\dagger)E_j$$

Naïvely these satisfy requirements (chirality, gauge invarince). However, vector operators do not contribute to this process, they cannot be massaged into gauge invariant effective operators. We require an *internal* mass insertion.

See, e.g. Lavoura (03) or Cheng & Li

Calculation

Gauge invariance (Ward identity) constrains the form of the amplitude calculated from diagrams

$$\mathcal{M}^\mu = a\,\overline{u}(p_2)\left[(p_1^\mu + p_2^\mu) - \gamma^\mu(m_1 + m_2)\right]u(p_1) + \cdots$$

Dropping terms that vanish upon contraction with γ polarization.

Use equations of motion on external spinors to massage this into the tensorial operator.

$$a_{kl}\frac{e_5}{16\pi^2}H\cdot L_i\sigma^{\mu\nu}\left(y_{ij}{y^\dagger}_{k\ell}y_{\ell j}\right)\overline{E}_jF_{\mu\nu}$$

It is sufficient to look at the p^μ coefficient of the amplitude. This is nice because you avoid looking at naïvely divergent vector terms which end up cancelling.

Results (preliminary)

Neutral Higgs contribution (M_H in GeV), upper bound on Y_*

$(c_L, -c_R)$	$(0.55, 0.55)$	$(0.55, 0.65)$	$(0.55, 0.75)$	$(0.90, 0.85)$
$M_H = 200$	0.2586	0.2583	0.2581	0.2581
$M_H = 400$	0.2667	0.2661	0.2655	0.2650

Result: $Y_* < 0.3$

Slight tuning relative to 'natural' value of $Y_* \sim 2$
c.f. Agashe, Delgado, May, Sundrum (03)

Compare to Agashe, Blechman, Petriello (06),
Lower bound on Y_* from $\mu \to 3e$, $Y_* > 1$

What's next?

1. **Quark penguin**: $b \to s\gamma$, a staple in B physics
Previous analysis: Gedalia, Isidori, Perez (09)

2. **Bulk Higgs**: generalize calculation
In progress: Agashe, Okui, Zhu (09)
Our analysis: should also be UV finite (a bit tricky)

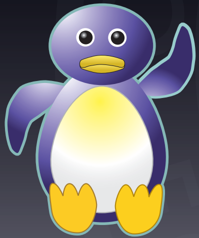
Reassess model-building prospects for RS flavor.

Neutrino masses and mixings in RS; Chen, H.B. Yu (08); Csáki, Delaunay, Grojean, Grosman (08); Chen, Mahanthappa, F. Yu (09)

Summary

- $\mu \to e\gamma$ is **UV finite** in RS
- Reason: chiral boundary conditions
- Subtlety: only manifest using 5D methods
- 'Mild tension' with $\mu \to 3e$ in minimal model

Implications for flavor model-building in RS-type models?



Thanks for having me!

Miscellaneous notes

Previous result: Agashe, Blechman, Petriello (06)

$$\text{Br}(\mu \to e\gamma)_{\text{hep-ph/0606021}} \propto \left(\frac{Y_*}{M_{\text{KK}}}\right)^4 \ln^2 \Lambda$$

So $Y_* \sim \dfrac{1}{\sqrt{\ln \Lambda}}$ from $\mu \to e\gamma$ in previous result.