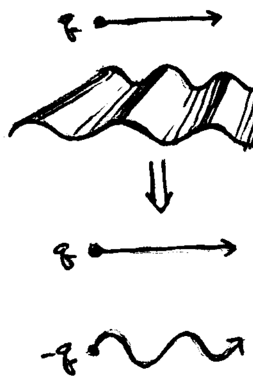
5. <u>THE PHYSICS</u>: THE CHARGE ITSELF IS NOT ACCELERATING SO IT DOES NOT RADIATE. HOWEVER, IT INDUCES A CHARGE ON THE CORRUGATED CONDUCTING SHEET. THIS INDUCED CHARGE, WHICH FOLLOWS THE ORIGINAL CHARGE, <u>DOES</u> OSCILLATE WITH THE WIGGLES OF THE CONDUCTING SHEET.



YOU KNOW FROM P120 THAT YOU CAN USE THE METHOD OF IMAGES TO DEAL WITH CONDUCTING SURFACES. (SO IT IS THE IMAGE CHARGE THAT IS OSCILLATING!)

THIS IS A TRICKY PROBLEM, SO THESE SOLUTIONS WILL PROVIDE TWO APPROACHES AND NOTE A PARTICULARLY TRICKY PART OF ONE OF THEM.

<u>APPROACH #1</u>: naive application of METHOD OF IMAGES

"DISTANT OBSERVER" ⇒ IGNORE EFFECTS THAT "SMEAR OUT" THE IMAGE CHARGE. i.e. TREAT THE IMAGE CHARGE AS IN THE DIAGRAM ABOVE (a charge that wiggles with the corrugation)

THE SYSTEM OF THE CHARGE & IMAGE CHARGE FORM AN OSCILLATING DIPOLE WITH FREQUENCY

$$f_0 = v/L$$

NOTE, HOWEVER, THAT THE DIPOLE IS <u>MOVING</u> WITH A CONSTANT VELOCITY v.

(9.10) $f = c/\lambda$; c = SPEED OF WAVE, λ = WAVELENGTH

SO, NAIVELY $\boxed{\lambda = \dfrac{cL}{v}}$ $\boxed{7/10}$

5, CONTD.

APPROACH 1*: enlightened application of METHOD OF IMAGES

UNFORTUNATELY THE ANSWER ABOVE IS INCOMPLETE. BECAUSE THE DIPOLE IS MOVING, WE MUST ACCOUNT FOR THE DOPPLER EFFECT (for light)

it is a shame that Griffiths does not discuss the Doppler effect for light, but one can re-derive the result:

DERIVATION OF THE DOPPLER EFFECT FOR LIGHT

FIRST CONSIDER THE TIME DILATION ASSOCIATED WITH THE SOURCE.

THE PROPER TIME BETWEEN WAVE CRESTS IS $t_o = 1/f_o$

IN THE LAB FRAME THERE IS TIME DILATION BTWN. OBSERVED CRESTS

$$t = \gamma t_o \quad ; \quad \gamma = (1-\beta^2)^{-1/2} \quad , \quad \beta = v/c$$

LET T BE THE TIME (in the LAB FRAME) BETWEEN SUCCESSIVE CRESTS.

THEN:
$$T = t - \frac{vt}{c}$$

pf./ write this as $cT = ct - vt$

DISTANCE LIGHT TRAVELS IN THE LAB FRAME w/ RELATIVE MOTION OF OBSERVER & SOURCE = DISTANCE LIGHT WOULD HAVE TRAVELLED IN THE LAB FRAME WITHOUT RELATIVE MOTION MINUS THE DISTANCE COVERED BY THE RELATIVE MOTION

IN OUR CASE:

$$T = t - \frac{v\cos\theta\, t}{c}$$
$$= \frac{t_o}{\sqrt{1-\beta^2}} - \frac{\beta\cos\theta\, t_o}{\sqrt{1-\beta^2}}$$
$$= \frac{t_o}{\sqrt{1-\beta^2}}\left(1 - \beta\cos\theta\right)$$

SO WE CAN MORE PROPERLY WRITE THE FREQUENCY AS:

$$f = \frac{1}{T} = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} f_o$$

$$\Rightarrow \boxed{\lambda = \frac{cL}{v} \cdot \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}}$$

NOTE: $\lambda \approx \frac{cL}{v}\left(1 - \frac{v}{c}\cos\theta\right)$ SO THAT OUR NAIVE ANSWER IS CORRECT FOR $v \ll c$ (zeroth ORDER IN β)

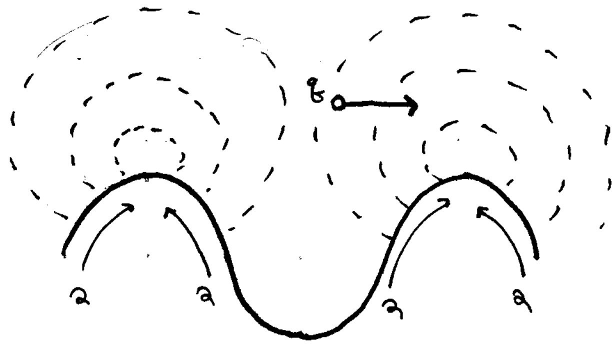
+1 for recognizing DOPPLER SHIFT

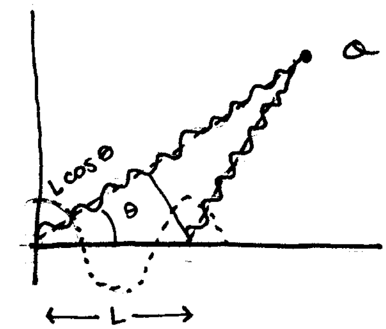
+2 for correct answer.

5, CONTD.

APPROACH 2: SUPER-CLEVER DIFFRACTION APPROACH

AN ALTERNATE, AND SUPER-CLEVER, WAY TO GET THE SAME RESULT IS TO TREAT EACH PEAK OF THE CORRUGATED CONDUCTING SHEET AS A BREMSSTRAHLUNG SOURCE AS THE CHARGE MOVES OVER THEM.





WE CAN THUS TREAT THIS AS A 2-SLIT INTERFERENCE PATTERN (remember PHYSICS 45/65 ?). THE RADIATION DETECTED BY AN OBSERVER AT A POINT O IS AT A WAVELENGTH SUCH THAT THE TWO SOURCES INTERFERE CONSTRUCTIVELY.

THERE IS ONE TRICKY PART: WE HAVE TO ACCOUNT FOR THE TIME DIFFERENCE $\Delta t = L/v$ BETWEEN WAVES BEING EMITTED AT ADJACENT PEAKS:

$$\frac{L}{v} - \frac{L\cos\theta}{c} = m\frac{\lambda}{c} \qquad \text{for } m \in \mathbb{Z}$$

$$\Rightarrow \lambda_m = \frac{L}{m}\left(\frac{c}{v} - \cos\theta\right)$$

for $m = 1$,

$$\boxed{\lambda_1 = \frac{cL}{v}(1 - \beta\cos\theta)}$$ AS ABOVE.

17/10

6. THIS IS EXACTLY THE CASE AS SECTION 11.1.2 !

a) GRIFFITHS CITES THREE APPROXIMATIONS

3/10

1. $a \ll |\vec{x}|$
2. $a \ll \lambda$ OR $a \ll c/\omega$
3. $|\vec{x}| \gg \lambda$ OR $|\vec{x}| \gg c/\omega$

b) $$\langle \vec{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2\theta}{r^2} \hat{r} \qquad \text{(eq. 11.21)}$$

5/10

this is the average energy per unit time, per unit area

WE WANT THE DIFFERENTIAL POWER RADIATED PER STERADIAN

$$\frac{d\langle P \rangle}{d\Omega} = r^2 \langle \vec{S} \rangle$$

OR YOU CAN LOOK AT: $\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int \frac{\sin^2\theta}{r^2} r^2 \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}$ (11.22)

$$\Rightarrow \boxed{\frac{d\langle P \rangle}{d\Omega} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \sin^2\theta}$$

where $p_0 = 2q_0 d$ (11.4)

c) $a \sim 10^{-9}$ m

?/10

VISIBLE LIGHT : $\lambda \sim 10^{-7}$ APPROX VALID
X-RAYS : $\lambda \sim 10^{-10}$ NOT VALID

7.

a) STRATEGY: FIGURE OUT THE VELOCITY IN FRAME S
THEN TRANSFORM TO THE FRAME OF SHIP 1.

IN S: VELOCITY OF PACKAGE IS $\vec{u} = u_x \hat{x} + u_y \hat{y}$

IN ORDER FOR THE PACKAGE TO HIT SHIP (2), $u_y = c/2$ +1
WE ARE GIVEN THAT $|\vec{u}| = 3/4\, c$

$$\Rightarrow u_x = \sqrt{|\vec{u}|^2 - u_y^2} = \sqrt{\frac{5}{16}}\, c \qquad +2$$

THE VELOCITY TRANSFORMATIONS (eq. 12.45)
ADAPTED TO A BOOST $v = -c/2$ IN THE $\hat{y}$ DIRECTION
GIVE:

$$u'_y = \frac{u_y - v}{1 - v u_y / c^2} = \frac{c/2 + c/2}{1 + 1/4} = \frac{4}{5} c \qquad +4$$

$$u'_x = \frac{u_x}{\gamma(1 - v u_y / c^2)} = \frac{\sqrt{5}/4\, c \sqrt{1 - 1/4}}{1 + 1/4} = \sqrt{\frac{3}{20}}\, c$$

THEN THE ANGLE θ' THAT AN OBSERVER IN SHIP 1
SHOULD FIRE THE PACKAGE IS GIVEN BY

$$\tan \theta' = \frac{u'_y}{u'_x} = \frac{8}{\sqrt{15}} \qquad +2$$

$$\Rightarrow \boxed{\theta' = \mathrm{Tan}^{-1} \frac{8}{\sqrt{15}}}$$

b) $$|\vec{u}'| = \sqrt{u_x'^2 + u_y'^2} = \boxed{\sqrt{\frac{79}{100}}\, c} \qquad +1$$

8. $\eta^2 \equiv \eta^\mu \eta_\mu$ is LORENTZ INVARIANT.

$\vec{V}$ is a 3-VECTOR, $|\vec{V}|$ is INVARIANT UNDER EUCLIDEAN ROTATIONS (but not LORENTZ), AND η^μ is A 4-VECTOR (COVARIANT, but not INVARIANT)

USING (eq. 12.108):

$$\bar{\vec{E}} \cdot \bar{\vec{B}} = \bar{E}_x \bar{B}_x + \bar{E}_y \bar{B}_y + \bar{E}_z \bar{B}_z$$
$$= E_x B_x + \gamma^2 (E_y - vB_z)(B_y + v/c^2 E_z) + \gamma (E_z + vB_y)(B_z - v/c^2 E_y)$$
$$= E_x B_x + \gamma^2 \{ E_y B_y + v/c^2 E_y E_z - vB_y B_z - v^2/c^2 E_z B_z - v/c^2 E_y E_z + vB_y B_z - v^2/c^2 E_y B_y \}$$
$$= E_x B_x + \gamma^2 [E_y B_y (1 - v^2/c^2) + E_z B_z (1 - v^2/c^2)]$$
$$= E_x B_x + E_y B_y + E_z B_z$$
$$= \vec{E} \cdot \vec{B}$$

$$\bar{\vec{E}}^2 - c^2 \bar{\vec{B}}^2 = \bar{E}^2 - \bar{B}^2 \quad \leftarrow \text{set } c^2 = 1$$
$$= [E_x^2 - \gamma^2 (E_y - vB_z)^2 + \gamma^2 (E_z - vB_y)^2] - [B_x^2 + \gamma^2 (B_y + vE_z)^2 + \gamma^2 (B_z - vE_y)^2]$$
$$= E_x^2 + \gamma^2 (E_y^2 - 2E_y v B_z + v^2 B_z^2 + E_z^2 + 2E_z v B_y + v^2 B_y^2 - B_y^2 - 2vB_y E_z - v^2 E_z^2 - B_z^2 + 2vB_z E_y - v^2 E_y^2) - B_x^2$$
$$= E_x^2 - B_x^2 + \gamma^2 [E_y^2 (1 - v^2) + E_z^2 (1 - v^2) - B_y^2 (1 - v^2) - B_z^2 (1 - v^2)]$$
$$= (E^2 - B^2)$$

9 a) $t = \gamma\tau$ $\leftarrow \tau = 10^{-6}$ s
$= d/v$ $\leftarrow d = 10^4$ m

$$d/v = \gamma\tau$$
$$\Downarrow$$
$$d^2 - d^2 v^2/c^2 = v^2\tau^2$$
$$cd^2 - d^2v^2 = v^2c^2\tau^2$$
$$\Downarrow$$
$$v^2 = \frac{c^2d^2}{c^2\tau^2 + d^2} \Rightarrow v = 2.998\times10^8 \text{ m/s} = 0.99955c$$

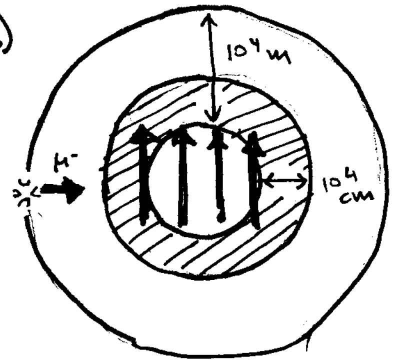
$$\Rightarrow \gamma = \left(1 - \frac{d^2}{c^2\tau^2+d^2}\right)^{-1/2} = 33.3483$$

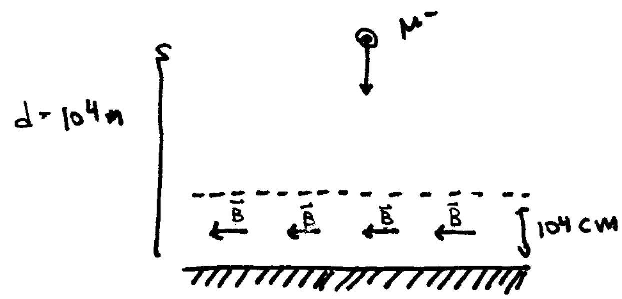
$$E = \gamma mc^2$$

+6

$$\boxed{E \approx 3.334 \text{ GeV}}$$

b)





MUON TRAVELLING AT $v = \sqrt{\frac{c^2d^2}{c^2\tau^2+d^2}}$

NO DEFLECTION OVER $(d - 10^4 \text{ cm})$ BEFORE HITTING REGION OF EFFECTIVE MAGNETIC FIELD.

SO ONLY IN B-FIELD FOR $\Delta t = \frac{10^4 \text{ cm}}{v}$

IN THE B-FIELD REGION THE LORENTZ FORCE PROVIDES THE CENTRIPETAL ACCELERATION FOR CYCLOTRON MOTION.

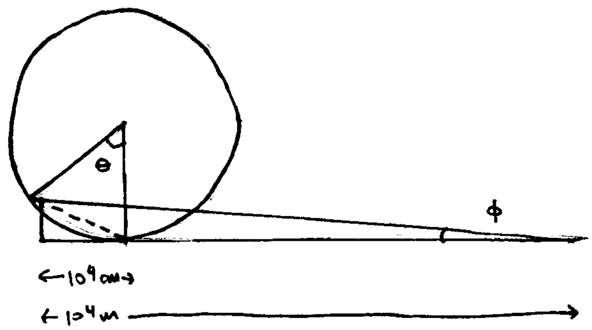
+6

9b) CONTD

USING THE CYCLOTRON FORMULA (5.3), $QVB = m V^2/R$

$$\Rightarrow R = \frac{mV}{QB} \quad \text{where } Q = e$$

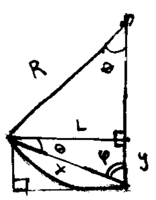
THE MUON TRAVELS 10^4 CM ALONG THE CIRCUMFERENCE OF A CIRCLE OF RADIUS R



$V = 2.9998 \times 10^8$ m/s
$m = 1.78 \times 10^{-28}$ kg
$Q = 1.6 \times 10^{-19}$ C
$B = 1$ gauss $= 10^{-4}$ kg/c·sec

$\Rightarrow R = 3336.45$ m
$\Rightarrow \theta = 10^4\text{ cm}/R = 0.03$ rad

DEFLECTION IS IN THE $\hat{\phi}$ DIRECTION (take S→N AXIS OF EARTH TO BE $\hat{z}$)



$\varphi = \frac{1}{2}(\pi - \theta)$
$= 1.5558$ rad

$\leftarrow L = R\sin\theta \quad \Rightarrow L = 100.078$ m
$L = x \sin\varphi \quad \Rightarrow \boxed{x = 100.089 \text{ m}}$
$y = x\sin\theta \quad \Rightarrow y = 3.00222$ m
$= L\frac{\sin\theta}{\sin\varphi}$

$$\phi = \tan^{-1}\left(\frac{y}{10^4\text{ m}}\right) = \boxed{3.00222 \times 10^{-4} \text{ rad}} \quad \text{in } \hat{\phi} \text{ dir.}$$

9c) THRESHOLD : P & γ HAVE JUST ENOUGH ENERGY TO PRODUCE A PION (LEAVING PROTON)

$$\Rightarrow \quad E_p + E_\gamma = m_p + m_\pi$$

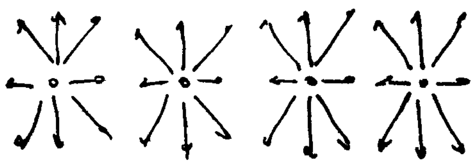
$$\boxed{\begin{array}{c} E_p = m_p + m_\pi - E_\gamma \\ \text{OR} \\ (KE)_p = m_\pi - E_\gamma \end{array}}$$

+3

10 a) IN THE LAB FRAME, THE e^- BUNCH WE SEE IS LORENTZ CONTRACTED. i.e. IN THE REST FRAME OF THE BUNCH, THE e^-'s ARE MUCH FURTHER APART FROM ONE ANOTHER (BY A FACTOR OF $\gamma = 10^5$).

NO CREDIT FOR SAYING THERE IS A MAGNETIC FORCE SUCH AS THAT FOR 2 CURRENT-CARRYING WIRES. IT IS THE LONGITUDINAL REPULSION THAT IS PROBLEMATIC.

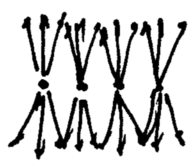
b) REST FRAME



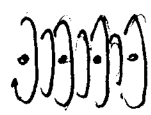
$\vec{E}$

No $\vec{B}$

LAB FRAME



$\vec{E}$



$\vec{B}$

$\rightarrow \vec{v}$

OTHER SIMILAR DIAGRAMS ACCEPTABLE.

c) USING THE RESULT FROM example 2.1 IN GRIFFITHS

$$E \approx \frac{1}{4\pi\varepsilon_0} \frac{2.5\times10^{10} e}{z\sqrt{z^2 + L^2/4}} \quad \Big\} \quad z = 100\ \mu m, \quad L = c\tau = 300\ \mu m$$

$$(5.36) \quad B \approx \frac{\mu_0 I}{2\pi z} = \frac{\mu_0 (2.5\times10^{10} e) c}{2\pi z L}$$

ACTUALLY, USE GAUSS/AMPERE'S LAW

$$\boxed{\begin{array}{ll} E \approx & 2\times10^9\ \text{V/m} \\ B \approx & 8\ \text{T} \end{array}}$$

$B \sim E/c$ ✓

IF CROSS SECTION EXPANDS, BUT Q FIXED, NO CHANGE

IF BUNCH COMPRESSED WHILE Q FIXED, THEN L CHANGES ⇒ E & B CHANGE.