

NOTE: FOR ALL GRADING ISSUES, PLEASE CONTACT FLIP TANEKO  
AT [flip@stanford.edu](mailto:flip@stanford.edu)

## 1. GRIFFITHS 1.2

$$m = \frac{h}{r_0 c}$$

$$\approx 3.5 \times 10^{-28} \text{ kg}$$

$$\approx \boxed{386 m_e}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\approx \boxed{197 \text{ MeV}}$$

$$m_{\pi} \approx \boxed{140 \text{ MeV}}$$

from 2004 PARTICLE PHYSICS BOOKLET

"Not too bad," AS ONE OF YOUR COLLEAGUES PUT IT.

2. GRIFFITHS 1.10 (TABLE IIb OF REFERENCE: M. ROOS RMP 85, 314 (1963).)

See attached copy of table IIb.

ALSO ATTACHED: J. EXPTL. THEORET. PHYS. (USSR) 43, 1543 (1962)

↳ This is reference 52 of M. Roos' paper, it was cited in many of the mesons in his table IIb that did not "stand the test of time."

In case you were curious why so many of the table IIb mesons turned out to be false, consider fig 1 & 2 in the AMUTDINOV paper. (EXPERIMENTALISTS MAY BE SLIGHTLY MORE INCENSED THAN THEORISTS...)



TABLE IIb. Mesonic Resonant States, March 1963. (Continued)

Symbol	Charge	Isospin	Spin	Parity G-parity	S	Mass		Full width $\Gamma$ (MeV)	Life- time $\Gamma^{-1}$ (1/ $m_{\pi^{\pm}}$ )	Production		Decay			References	
						(MeV)	( $m_{\pi^{\pm}}$ )			Process	$k_{lab}$ (MeV)	Modes	Branching ratio (%)	Q (MeV)		
$\eta$	0	0	0	-	+	0	548.5 ± 0.6	3.93	≤ 7	> 20	$\pi p$	685	$\pi^+\pi^-\pi^0$ $\pi^+\pi^-\gamma$ $3\pi^0$ $\pi^0\gamma$ $2\gamma$ others	25 ± 10 7 ± 2 68 ± 10	135 270 144 414 549	46
$\varphi_2$	0	0	0			0	520 ± 20	3.7	70 ± 30	2.0	$\pi^-p$	639	$\pi^+\pi^-$		240	47
$\psi_2$	-	2	0			0	440	3.1			$\pi^-p$	735	$\pi^-\pi^-$	100	160	52
$\psi_2$	0		0			0	420-440	3.1				515	$\pi^-\pi^+$		160	52
$\psi_2$	++		0			0	440	3.1				975	$\pi^+\pi^+$	100	160	52
$\varphi_1$	0	0	0			0	395 ± 10	2.8	50 ± 20	2.8	$\pi^-p$	446	$\pi^+\pi^-$		115	47
$\psi_1$	-	2	0			0	330	2.4			$\pi^-p$	537	$\pi^-\pi^-$	100	50	52
$\psi_1$	0		0			0	330	2.4				340	$\pi^-\pi^+$		50	52
$\psi_1$	++		0			0	330	2.4				790	$\pi^+\pi^+$	100	50	52
$\omega_{ANC}$	0	0	0			0	317 ± 6	2.3	≤ 16	> 0	$pd$		$\pi^+\pi^-$		38	49

and DuMond, *Nuovo Cimento* 5, 541 (1957) and *Fundamental Constants of Physics* (Interscience Publishers, Inc., New York, 1957).

8. The neutron mean life is based on the value  $11.7 \pm 0.3$  min for the half life, by A. N. Sosnovskij, P. E. Spivak, Y. A. Prokoviev, I. E. Kutikov, and Y. P. Dobrynin, *Nucl. Phys.* 10, 395 (1959).

9.  $m(K^+)$  is from the compilation of E. O. Okonov, *Fortschr. Physik* 8, 42 (1960), who also gives the  $K^-$  mass as  $493.9 \pm 0.4$  MeV.

$\tau(K^+)$  is a weighted average of the following results in  $10^{-8}$ sec:  $1.224 \pm 0.013$ , compilation of Barkas & Rosenfeld (cf. reference 3),  $1.231 \pm 0.011$ , A. M. Boyarski, E. C. Loh, L. Q. Niemela, D. M. Ritson, R. Weinstein and S. Ozaki, *Phys. Rev.* 128, 2398 (1962).

The  $K^+$  branching ratios are from B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, *Phys. Rev. Letters* 7, 346 (1961); and G. Giacomelli, D. Monti, G. Quarenzi, A. Quarenzi-Vignudelli, W. Püschel, and J. Tietge, *Phys. Letters* 3, 346 (1963). Note that these branching ratios (except for  $\tau$ ) disagree with the weighted averages obtained from emulsion experiments, as quoted by Crawford (cf. reference 6):

$$\begin{aligned} \mu_2 &= 57.4 \pm 2.0, \\ \pi_2 &= 25.6 \pm 1.5, \\ \mu_3 + e_3 + \tau &= 11.0 \pm 1.0, \\ \tau &= 5.7 \pm 0.2. \end{aligned}$$

10. J. Button and B. C. Maglic, *Phys. Rev.* 127, 1297 (1962).

11.  $m(K^0)$  is from A. H. Rosenfeld, F. T. Solmitz and R. D. Tripp, *Phys. Rev. Letters* 2, 110 (1959).  $\mu(K^0)$  is from E. O. Okonov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* 42, 1554 (1962).

$\tau(K^0)$  is a weighted average, based on the following recent results only (in  $10^{-10}$ sec), all quoted by Crawford (cf. reference 6):

$0.94 \pm 0.05$ , Crawford *et al.* (cf. below),  
 $0.90 \pm 0.05$ , A. F. Garfinkel, Report Nevis 104 (1962). [Thesis, Columbia University, Physics Department (unpublished).]

$0.885 \pm 0.025$ , R. L. Golden, G. Alexander, J. A. Anderson, F. S. and B. B. Crawford, L. J. Lloyd, G. W. Meisner, and L. Price (to be published).

The  $K^0 \rightarrow \pi^0\pi^0$  branching ratio is a weighted average of the following results in percent:

$28.5 \pm 3.6$  by F. S. Crawford, M. Cresti, R. Douglass, M. Good, G. Kalbfleisch, M. L. Stevenson, and H. Ticho, *Phys. Rev. Letters* 2, 266 (1959),

$29.4 \pm 2.1$ , M. Chretien, V. Fisher, H. R. Crouch, R. E. Lanou, J. T. Massimo, A. M. Shapiro, J. P. Averell, A. E. Brenner, D. R. Firth, L. G. Hyman, M. E. Law, R. H. Milburn, E. E. Ronat, K. Strauch, J. C. Street, J. J. Szymanski, L. Guerriero, I. A. Pless, L. Roesenson, and G. A. Saladin (to be published), quoted by Crawford (cf. reference 6),

$33.5 \pm 1.4$ , J. L. Brown, J. A. Kadyk, G. H. Trilling, B. P. Roe, D. Sinclair, and J. C. Vander Welde, *Phys. Rev.* (to be published),

$26.0 \pm 2.4$ , Anderson *et al.* (cf. reference 6),  
 $29 \pm 3$ , weighted average of earlier results, reported at the 1960 Rochester Conference.

12.  $\tau(K_S^0)$  is a weighted average of the following results in  $10^{-8}$ sec

$6.8 (+2.6/-1.5)$ , Crawford (cf. reference 6),  
 $8.1 (+3.3/-2.4)$ , M. Bardon, K. Lande, L. M. Lederman, and W. Chinowsky, *Ann. Phys.* 5, 156 (1958).  
 $5.1 (+2.4/-1.3)$ , S. E. Darmon, A. Rousset and W. Six, *Phys. Letters* 3, 57 (1962).

The branching ratios have been obtained from the following results:

$R_1 = \frac{K_S^0 \rightarrow 3\pi^0}{K_S^0 \rightarrow \text{all charged}} = 0.38 \pm 0.07$ , M. H. Anikina, M. S. Zhuravleva, D. M. Kotliarevsky, Z. S. Mandyavidze, A. M. Mestvirishvili, D. Neagu, E. O. Okonov, N. S. Petrov, A. M. Rosanova, V. A. Rusakov, G. G. Tachtamishvili, and L. V. Chekhaidze, *Proceedings of the 1962 Annual International Conference on High-Energy Physics at Geneva* (CERN, Geneva, Switzerland, 1962), p. 452.

$R_2 = \frac{K_S^0 \rightarrow \pi^+\pi^-\pi^0}{K_S^0 \rightarrow \text{all charged}} = 0.127 \pm 0.020$ , D. Luers, I. S. Mitra, W. J. Willis, and S. S. Yamamoto, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles in 1961* (CEN Saclay, Seine et Oise, France, 1962), Vol. I, 241 (1961).

$R_2 = 0.134 \pm 0.018$ , Anikina, *et al.*

$R_3 = \frac{K_S^0 \rightarrow \pi e \nu_e}{K_S^0 \rightarrow \text{all charged}} = 0.458 \pm 0.048$ , Luers, *et al.*  
 $R_3 = 0.415 \pm 0.120$ , A. Astier, L. Blaskovic, M. M. de Courreges, B. Equer, A. Lloret, P. Rivet, and J. Slaud, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles in 1961* (CEN Saclay, Seine et Oise, France, 1962), Vol. I, 227 (1961). A value on  $R_3 = (0.185^{+0.031})$  by Astier *et al.* has not been used.

13.  $\tau(\pi^0)$  is from G. von Dardel, D. Dekkers, R. Mermod, J. D. van Putten, M. Vivargent, G. Weber, and K. Winter, *Phys. Letters* 4, 51 (1963).

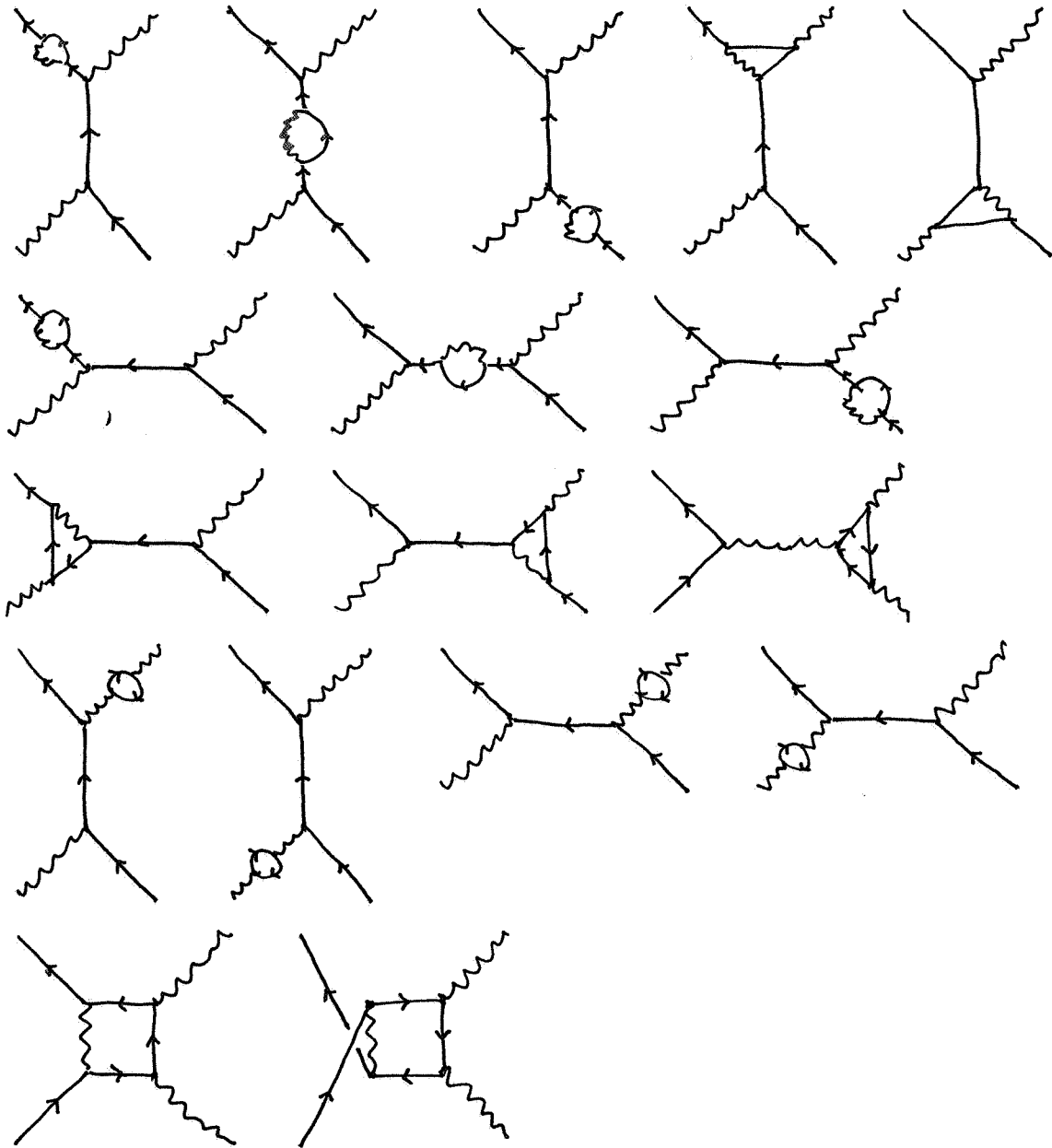
The relative frequency for  $\gamma e^+e^-$  decay is from J. Tietge and W. Püschel, *Phys. Rev.* 127, 1324 (1962).

14.  $\mu(\mu^+)$  is from G. Charpak, F. Farley, R. L. Garwin, T. Muller, J. C. Sens, and A. Zichichi, *Phys. Letters* 1, 16 (1962).

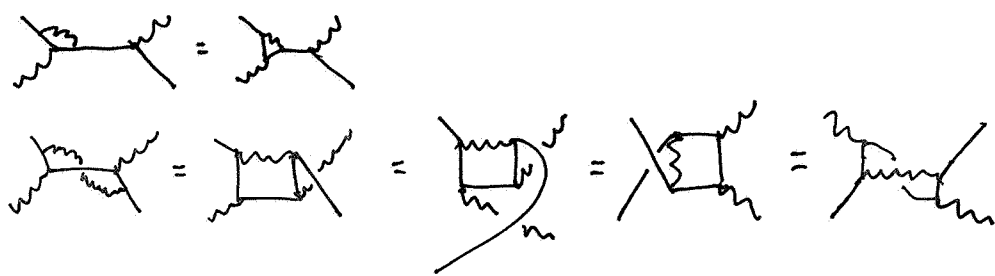
$m(\mu^+)$  is a combined value of the latest measurements, as given by G. McD. Bingham, *Nuovo Cimento* 27, 1352 (1963).

$\tau(\mu^+)$  is a weighted average from R. A. Lundy, *Phys. Rev.* 125, 1686 (1962).

3. GRIFFITHS 2.3, I WILL FOLLOW GRIFFITHS AND HAVE TIME FLOW UPWARDS



NOTES



4. GRAFFITHS 2.5

$$(a) \begin{array}{l} \Xi^- \rightarrow \Lambda + \pi^- \\ (dss) \rightarrow (uds) + (d\bar{u}) \end{array} \quad \Delta S = -1$$

$$\begin{array}{l} \Xi^- \rightarrow n + \pi^- \\ (dss) \rightarrow (udd) + (d\bar{u}) \end{array} \quad \Delta S = -2$$

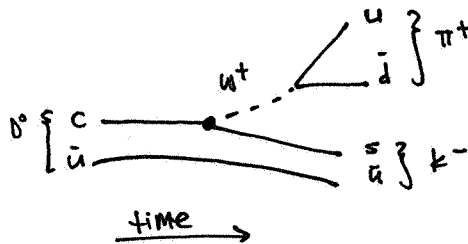
WE EXPECT PREFERENTIAL DECAY TO  $\Lambda + \pi^-$   
TO MINIMIZE STRANGENESS VIOLATION.

INDEED, THE 2004 PARTICLE PHYSICS BOOKLET LISTS

$$\left( \frac{\Gamma_{\Xi^- \rightarrow \Lambda + \pi^-}}{\Gamma_{\text{tot}}} \right) = 99.9\%$$

$$(b) \begin{array}{l} D^0 \rightarrow K^- + \pi^+ \\ (c\bar{u}) \rightarrow (s\bar{u}) + (u\bar{d}) \end{array}$$

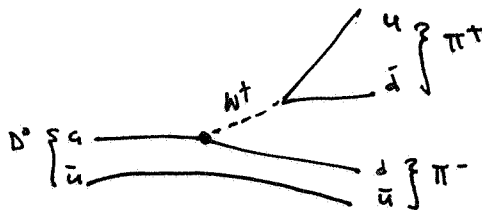
AMPLITUDE  $\propto V_{cs}$   
PDG 2004:  $\frac{\Gamma}{\Gamma_{\text{tot}}} = 3.8\%$



$$D^0 \rightarrow \pi^- + \pi^+$$

$$(c\bar{u}) \rightarrow (d\bar{u}) + (u\bar{d})$$

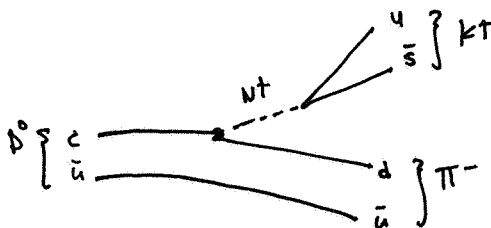
AMPLITUDE  $\propto V_{cd}$   
PDG 2004:  $1.38 \times 10^{-3}\%$



$$D^0 \rightarrow K^+ + \pi^-$$

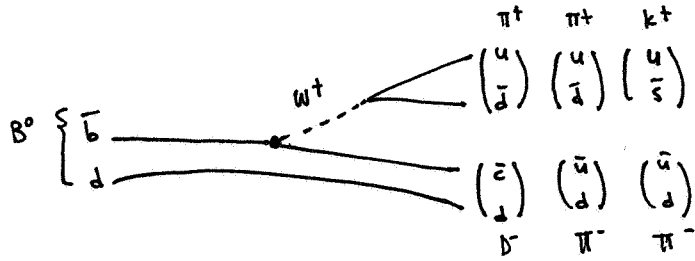
$$(c\bar{u}) \rightarrow (u\bar{s}) + (d\bar{u})$$

AMPLITUDE  $\propto V_{cd} V_{us}$   
PDG 2004:  $1.4 \times 10^{-4}\%$



SINCE  $V_{cs} > V_{cd}$ , WE EXPECT THE  $K^+ \pi^-$  MODE  
TO OCCUR MORE THAN THE OTHER TWO

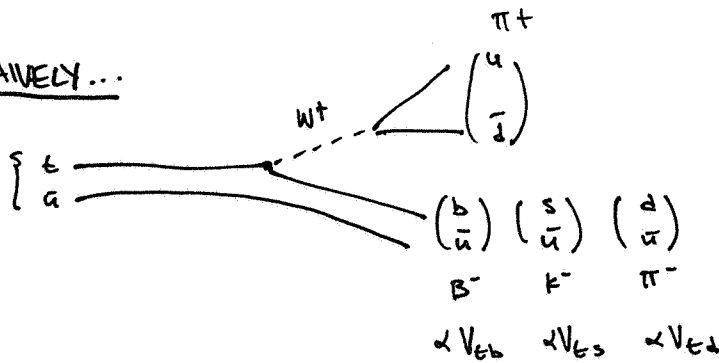
(c)



$$\begin{aligned}
 B^0 &\rightarrow \pi^+ + D^- && \propto V_{cb} \\
 B^0 &\rightarrow \pi^+ + \pi^- && \propto V_{ub} \\
 B^0 &\rightarrow K^+ + \pi^- && \propto V_{ub}
 \end{aligned}$$

$V_{cb} > V_{ub}$ , so we expect  $B^0 \rightarrow \pi^+ + D^-$  to dominate

NAIVELY...



$$V_{tb} > V_{ts} > V_{td}$$

so  $(t\bar{u}) \rightarrow \pi^+ + B^-$  dominates among these

HOWEVER: GRIFFITHS WAS WRITTEN BEFORE THE MASS OF THE TOP QUARK WAS DETERMINED; IT TURNS OUT THAT THE TOP IS SO MASSIVE THAT IT DOES NOT FORM MESONS!

5. GRIFFITHS 2.7

(a)  $p + \bar{p} \rightarrow \pi^+ + \pi^0$

(c)  $\Sigma^0 \rightarrow \Lambda + \pi^0$

(e)  $e^+ + e^- \rightarrow \mu^+ + \mu^-$

(g)  $\Delta^+ \rightarrow p + \pi^0$

(i)  $e + p \rightarrow \nu_e + \pi^0$

(k)  $p \rightarrow e^+ + \gamma$

(m)  $n + \bar{n} \rightarrow \pi^+ + \pi^- + \pi^0$

(o)  $K^- \rightarrow \pi^- + \pi^0$

(q)  $\Sigma^0 \rightarrow \Lambda + \gamma$

(s)  $\Xi^0 \rightarrow p + \pi^-$

(u)  $\pi^0 \rightarrow \gamma + \gamma$

(b)  $\eta \rightarrow \gamma + \gamma$

(d)  $\Sigma^- \rightarrow n + \pi^-$

(f)  $\mu^- \rightarrow e^- + \bar{\nu}_e$

(h)  $\bar{\nu}_e + p \rightarrow n + e^+$

(j)  $p + p \rightarrow \Sigma^+ + n + K^0 + \pi^+ + \pi^0$

(l)  $p + p \rightarrow p + p + p + \bar{p}$

(n)  $\pi^+ + n \rightarrow \pi^- + p$

(p)  $\Sigma^+ + n \rightarrow \Sigma^- + p$

(r)  $\Xi^- \rightarrow \Lambda + \pi^-$

(t)  $\pi^- + p \rightarrow \Lambda + K^0$

(v)  $\Sigma^- \rightarrow n + e + \bar{\nu}_e$

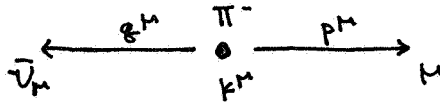
- a) NO - CHARGE CONSERVATION  
 b) YES - EM  
 c) No - ENERGY CONSERVATION  
 d) YES - WEAK  
 e) YES - EM  
 f) No - LEPTON #  
 g) YES - STRONG  
 h) YES - WEAK  
 i) No - BARYON #  
 j) YES - STRONG  
 k) No - BARYON #  
 l) YES - STRONG  
 m) YES - STRONG  
 n) No - CHARGE CONSERVATION  
 o) YES - WEAK  
 p) No - CHARGE CONSERVATION  
 q) YES - EM  
 r) YES - WEAK  
 s) No -  $\Delta S = 2$   
 t) YES - STRONG  
 u) YES - EM  
 v) YES - WEAK

6. GRIFFITHS 3.18

(a)  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

WORK IN THE CM FRAME

- LET  $K^\mu$  = INITIAL 4-MOMENTUM OF  $\pi^-$   
 $q^\mu$  = FINAL 4-MOMENTUM OF  $\bar{\nu}_\mu$   
 $P^\mu$  = FINAL 4-MOMENTUM OF  $\mu^-$



- ASSUME  $\bar{\nu}_\mu$  MASSLESS  $\Rightarrow q^0 = |\vec{q}|$   $\hat{q} = \vec{q}/|\vec{q}|$   
 $\Rightarrow q^\mu = (E_{\nu}, E_{\nu} \hat{q})$   
 CM FRAME IS  $\pi^-$  REST FRAME  $\Rightarrow K^\mu = (m_{\pi}, \vec{0})$   
 CONSERVATION OF 4-MOMENTUM:  $K^\mu = q^\mu + P^\mu$  (\*)

(\*)  $\Rightarrow P^\mu = (\underbrace{m_{\pi} - E_{\nu}}_{E_{\mu}}, \underbrace{-E_{\nu} \hat{q}}_{\vec{p}})$

(\*)  $\Rightarrow q^\mu = K^\mu - P^\mu$   
 $q^2 = K^2 - 2K^\mu P_\mu + P^2$

$0 = m_{\pi}^2 - 2m_{\pi} E_{\mu} + m_{\mu}^2 = m_{\pi}^2 - 2m_{\pi}(m_{\pi} - E_{\nu}) + m_{\mu}^2$   
 $\Rightarrow E_{\mu} = \frac{m_{\pi}^2 + m_{\mu}^2}{2m_{\pi}} \qquad \Rightarrow E_{\nu} = \frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}}$

USING  $|\vec{p}| = -E_{\nu}$

AND  $v_{\mu} = \frac{|\vec{p}|}{E_{\mu}} \Rightarrow -\frac{E_{\nu}}{E_{\mu}} = \frac{m_{\pi}^2 - m_{\mu}^2}{m_{\pi}^2 + m_{\mu}^2}$

$\gamma = \frac{1}{\sqrt{1 - v^2}} = \frac{m_{\pi}^2 + m_{\mu}^2}{\sqrt{(m_{\pi}^2 + m_{\mu}^2)^2 - (m_{\pi}^2 - m_{\mu}^2)^2}}$   
 $= \frac{m_{\pi}^2 + m_{\mu}^2}{\sqrt{4m_{\pi}^2 m_{\mu}^2}}$   
 $= \frac{m_{\pi}^2 + m_{\mu}^2}{2m_{\pi} m_{\mu}}$

(CONTINUED)  $\Rightarrow$



$\mu$  LIFETIME IN LAB (CM) FRAME =  $\gamma \tau$

$\Rightarrow$  DISTANCE TRAVELLED  $d = \gamma v \tau$

$$\begin{aligned} \text{PLUG IN } m_{(\mu)} &= 140 \text{ MeV} \\ m_{(e)} &= 106 \text{ MeV} \\ \tau_{(\mu)} &= 2.20 \times 10^{-6} \text{ s} \end{aligned}$$

$$d = 186 \text{ m.}$$

(b) TRICK QUESTION! (Yeah, I know, that's awful, isn't it?)

THE HINT IS IN THE CAPTION OF FIG. 1.7 - THE IMAGE COMES FROM A BOOK TITLED "THE STUDY OF ELEMENTARY PARTICLES BY THE PHOTOGRAPHIC METHOD."

THE VALUE FOR  $d$  WE CALCULATED IS FOR VACUUM. IN ORDER FOR THIS PHOTOGRAPH TO BE TAKEN, THE INTERACTION OCCURRED IN A MEDIUM (ILFORD EMULSION), WHICH ACCOUNTS FOR THE DISCREPANCY.

NO POINTS WERE DEDUCTED FOR MISSING PART (b).

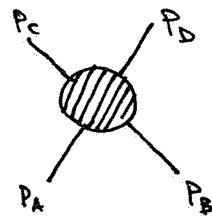
7. GRIFFITHS 3.22, a & d ONLY

SET  $c=1$  LIKE ANY REASONABLE PARTICLE PHYSICIST.

$$S = (P_A + P_B)^2 = (P_C + P_D)^2$$

$$t = (P_A - P_C)^2$$

$$u = (P_A - P_D)^2$$



$$(a) \quad S + t + u = P_A^2 + 2P_A \cdot P_B + P_B^2 \\ + P_A^2 - 2P_A \cdot P_C + P_C^2 \\ + P_A^2 - 2P_A \cdot P_D + P_D^2$$

$$P_i^2 = m_i^2 ; i = A, B, C, D$$

$$= 3m_A^2 + m_B^2 + m_C^2 + m_D^2 \\ - 2P_A \cdot (P_B - (P_C + P_D))$$

$$\leftarrow P_C + P_D = P_A + P_B \\ (\text{CONS. OF 4 MOMENTUM})$$

$$= 3m_A^2 + m_B^2 + m_C^2 + m_D^2 \\ - 2P_A \cdot (P_B - (P_A + P_B)) \\ \underbrace{\hspace{10em}} \\ = -2m_A^2$$

$$= \boxed{m_A^2 + m_B^2 + m_C^2 + m_D^2}$$

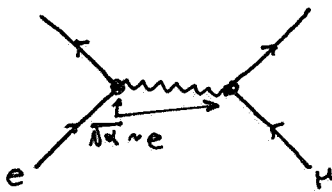
(d) IN THE CM FRAME,  $\vec{P}_A = -\vec{P}_B$

$$\Rightarrow P_A + P_B = (E_A + E_B, 0, 0, 0)$$

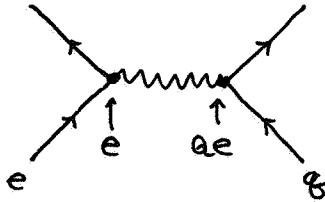
$$S = (P_A + P_B)^2 = (E_A + E_B)^2 = E_{CM}^2$$

$$\boxed{E_{CM} = \sqrt{S}}$$

8.



$$\sigma \sim |M|^2 \sim \frac{\alpha^2}{s} \sim \frac{e^4}{s}$$



SUM RUNS OVER KINEMATICALLY ALLOWED QUARK SPECIES, i.e.  $q$  SUCH THAT  $2m_q \leq \sqrt{s}$

$$\sigma \sim |M|^2 \sim \sum_{\substack{q \text{ s.t.} \\ 2m_q \leq \sqrt{s}}} \frac{3Q^2 e^4}{s}$$

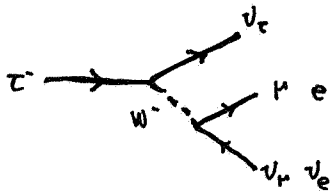
$Q$  = CHARGE OF QUARK  $q$  IN UNITS OF  $e$

$$= \begin{cases} 2/3 & \text{FOR } q = u, c, t \\ -1/3 & \text{FOR } q = d, s, b \end{cases}$$

FACTOR OF 3 COMES FROM SUMMING THE (IDENTICAL) CONTRIBUTION FROM 3 POSSIBLE QUARK COLORS.

$$\Rightarrow \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_{\substack{q \text{ s.t.} \\ 2m_q \leq \sqrt{s}}} 3Q^2$$

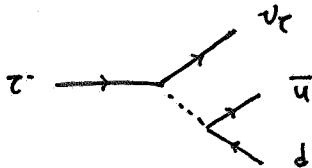
9. ALL DECAYS OF  $Z^-$  INVOLVE THE WEAK FORCE (in particular, the  $W^-$  boson). CONSIDER ONLY THE LOWEST ORDER TREE LEVEL DIAGRAMS.



LEPTONIC DECAYS

$$Z^- \rightarrow \nu_c \mu^- \bar{\nu}_\mu$$

$$\rightarrow \nu_c e^- \bar{\nu}_e$$



SEMILEPTONIC DECAYS

$$Z^- \rightarrow \nu_c d \bar{u} \quad (\times 3 \text{ COLORS})$$

DO NOT CONSIDER...

- top/bottom modes (VIOLATES ENERGY CON.)
  - strange modes
  - charm modes
- (VIOLATES STRANGENESS OR OTHERWISE REQUIRES OFF-DIAGONAL ELEMENTS OF  $V_{CKM}$  → HIGHER ORDER)

ASSUME THESE DECAYS ARE ROUGHLY ON THE SAME SCALE THEN, TO THIS ORDER,  $B(Z^- \rightarrow \nu_c \mu^- \bar{\nu}_\mu) \sim 1/5 \approx 20\%$ .

2004 PARTICLE PHYSICS BOOKLET SAYS  $B(Z^- \rightarrow \nu_c \mu^- \bar{\nu}_\mu) \sim 17\%$  (NOT BAD AT ALL!)

10. (a)  $P^+ \longrightarrow \bullet \quad P^+ \quad q^M = (m, \vec{0})$

$$K^M = (E, \sqrt{E^2 - m^2} \hat{k})$$

$$\hat{k} = \vec{k} / |\vec{k}|$$

$$S = (K^M + q^M)^2$$

$$= (E + m, \sqrt{E^2 - m^2} \hat{k})^2$$

$$= (E + m)^2 - (E^2 - m^2)$$

$$= 2Em + 2m^2$$

$$\boxed{\sqrt{S} = \sqrt{2m(E+m)}}$$

(b)  $P^+ \longrightarrow \bullet \quad \longleftarrow \bullet \quad P^+ \quad q^M = (E, -\sqrt{E^2 - m^2} \hat{k})$

$$K^M = (E, \sqrt{E^2 - m^2} \hat{k})$$

$$S = (2E, 0)^2$$

$$= 4E^2$$

$$\boxed{\sqrt{S} = 2E}$$

$\pi\pi$  INTERACTIONS IN THE MULTIPLE PRODUCTION OF PIONS IN  $\pi p$  COLLISIONSM. S. AINUTDINOV, S. M. ZOMBKOVSKIĬ, S. Ya. NIKITIN, Ya. M. SELEKTOR, and  
V. N. SHULYACHENKO

Institute of Theoretical and Experimental Physics, Academy of Sciences, U.S.S.R.

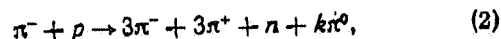
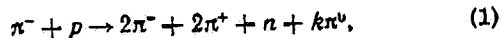
Submitted to JETP editor June 20, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 43, 1543-1546 (October, 1962)

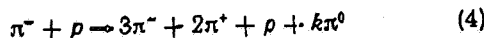
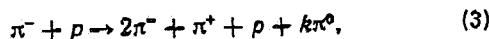
Resonance states were found for two  $\pi$ -mesons of the same charge and the mass values of 0.33; 0.44; 0.58; 0.76; and 0.99 BeV in an investigation of  $\pi\pi$  interactions in  $\pi^-p$  collisions at an energy of 7.2 BeV. Similar resonances were found for the  $\pi^+\pi^-$  and  $\pi^-\pi^0$  systems at the same mass values. An assumption is put forward of the possibility of isotopic and mechanical spin degeneracy in strong interactions.

WE investigated the  $\pi\pi$  interaction in multiple pion production in  $\pi^-p$  collisions. The energy of the primary negative pions was 7.2 BeV. The energy spread of the negative pions in the primary beam is satisfactorily described by a Gaussian distribution with half width 0.8 BeV. Measurements were made with the aid of a liquid-hydrogen bubble chamber 25 cm in diameter, placed in a 13.5-kG magnetic field.

Altogether 13,000 photographs were obtained. The reactions investigated were



where  $k$  is the unknown number of  $\pi^0$  mesons. The reactions



were eliminated by identifying the protons by measuring the momenta and estimating the ionization. In the reactions (1) and (2) segregated in this manner there may be a small admixture of reactions (3) and (4), when the proton momentum exceeds 1.5 BeV/c. For reactions (1) and (2), distributions over the effective masses were plotted (the total energy of two pions in their own c.m.s.) for all possible combinations of two pions. All three possible charge states were investigated (the  $\pi^-\pi^-$ ,  $\pi^+\pi^+$ , and  $\pi^+\pi^-$  meson combinations).

Ideograms corresponding to the distributions over the effective masses for reaction (1) are shown in Fig. 1. The determination of the effective masses was carried out over the measurements of the momenta and angles of emission of

the pions. The mean square value of the error in the determination of the effective mass was  $\pm 25$  MeV.

As follows from Fig. 1, sharp maxima are observed in the constructed ideograms at mass values 0.33, 0.44, 0.58, 0.76, and 0.99 BeV. Maxima were observed in all three charge states within the limits of errors at the same values of the masses. It must be noted that the first two maxima in the ideogram for the  $\pi^+\pi^-$  combinations were not resolved.

The results obtained point to the possibility of realizing resonances in the two-pion system with the mass values indicated above.

To check whether the same pion participates simultaneously in several maxima, the following processing was carried out: two-charge combinations contained in each maximum on the curves of Fig. 1a and b were picked out. For these pions, a distribution was plotted over the effective masses of the  $\pi^+\pi^-$  combinations. The distributions obtained turn out to be smooth, within the limits of statistical errors. From the smoothness of the distributions it follows that the same pion does not participate in two maxima.

The distribution over the effective masses for reaction (2) is shown in Fig. 2. What is striking is the presence of almost all maxima observed in the analysis of four-prong events. The relative intensity of the maxima in the case of six-prong stars, however, is appreciably different from the intensity of the maxima in four-prong stars and the maxima at high values of the masses are partially suppressed.

The table lists the values of the masses of the resonances (in BeV) for three charge states of four- and six-prong stars. From the analysis of

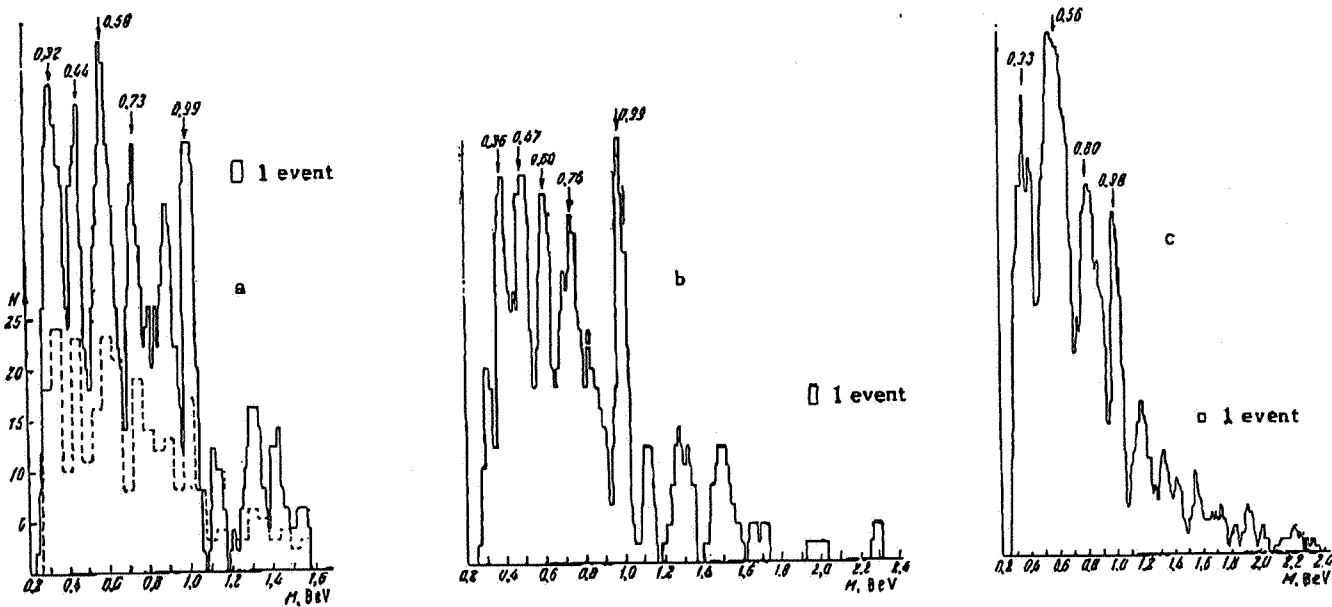


FIG. 1. Distribution over the effective masses for combinations in four-prong stars: a -  $\pi^-\pi^-\pi^-$  combinations, 280 combinations, 280 stars (the dashed line is the histogram); b -  $\pi^+\pi^+\pi^-$  combinations, 247 combinations, 247 stars; c -  $\pi^+\pi^-\pi^-$  combinations, 916 combinations, 229 stars.

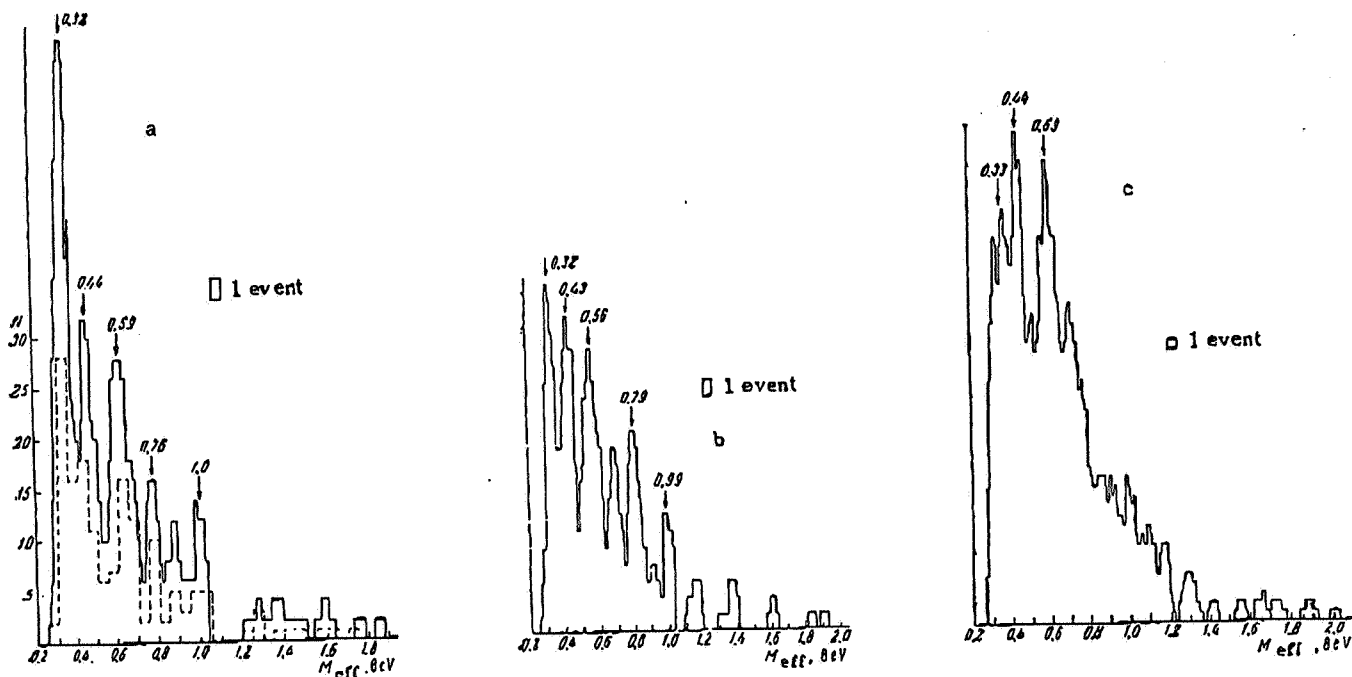


FIG. 2. Distribution over the effective masses for combinations in six-prong stars: a -  $\pi^-\pi^-\pi^-\pi^-$  combinations (the dashed line is the histogram), 156 combinations, 52 stars; b -  $\pi^-\pi^+\pi^-\pi^-$  combinations, 156 combinations, 52 stars; c -  $\pi^+\pi^-\pi^-\pi^-$  combinations, 468 combinations, 52 stars.

the systematic errors it follows that the true value of the masses of the resonances can differ from those listed in the table by not more than 30 MeV.

Resonance maxima are observed at mass values 0.33, 0.44, 0.58, 0.76, and 0.99 BeV. Of these, the three resonances at values of  $M_{eff}$  equal to 0.58,

0.76, and 0.99 BeV were observed earlier in the charge states 0 and 1<sup>[1-3]</sup>. The maxima at  $M_{eff} = 0.33$  and 0.44 BeV call for an additional analysis aimed at determining whether they are due to resonances in the system.

Four-prong stars			Six-prong stars			$M_{\text{eff. av.}}$
Charge states			Charge states			
(--)	(++)	(+-)	(--)	(++)	(+-)	
0.32	0.38	0.33	0.32	0.32	0.33	0.33
0.44	0.47	—	0.44	0.43	0.44	0.44
0.58	0.59	0.56	0.59	0.56	0.59	0.58
0.73	0.74	0.79	0.77	0.79	—	0.76
0.99	0.99	0.98	1.0	0.99	—	0.99

The existence of resonant states in doubly-charged systems of two pions at close values of the masses was also recently observed in  $\pi\pi$  collisions at 2.8 BeV by Shalamov and Grashin<sup>[4]</sup>.

The available experimental data evidently indicate that there exist two systems of resonances in systems with identical mass values, but with different values of isotopic and mechanical spins, that is, in strong interactions one observes degeneracy in the isotopic and mechanical spins.

The authors are deeply grateful to A. I. Alikhanov for numerous discussions of this work and for a large number of valuable suggestions, which were made during the course of these discussions.

Much work on the reduction of these data was made by the scanning group headed by D. I. Tumanova and by a group from the mathematics division headed by R. S. Guter. The authors take this opportunity to express their deep gratitude to them.

The authors are grateful to V. I. Veksler for affording the possibility of obtaining the photographs on the accelerator of the Joint Institute for Nuclear Research.

<sup>1</sup>Pickup, Ayer, and Salant, Phys. Rev. Lett. 5, 161 (1960).

<sup>2</sup>Anderson, Bang, Burke, Carmony, and Schmitz, Revs. Modern Phys. 33, 431 (1960).

<sup>3</sup>Erwin, March, Walker, and West, Phys. Rev. Lett. 6, 628 (1961).

<sup>4</sup>Ya. Ya. Shalamov and A. F. Grashin, JETP, in press.

Translated by J. G. Adashko  
281