Physics 3318: Analytical Mechanics

Lecture 1: themes of the course

- consequences of symmetry
- variational principles
- new physics in phase space

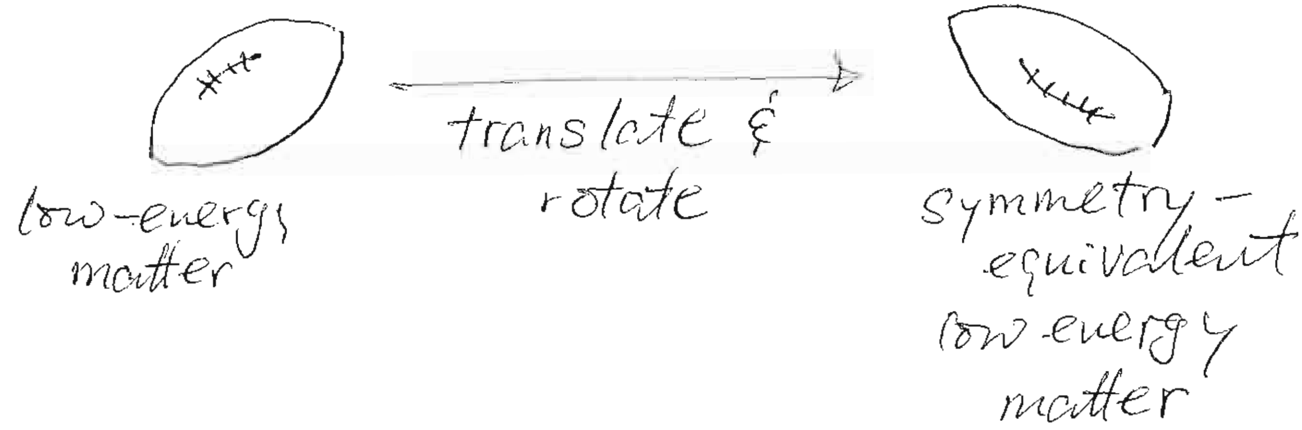
Symmetry

Q: Why all the fuss about "rigid bodies"?

$$\left.\begin{matrix}\text{translations}\\ \text{rotations}\end{matrix}\right\} = \text{fundamental symmetries of space}$$

football = low energy, equilibrium state of matter

= "rigid-body"



⟹ Would be no concept of rigid-body if trans. & rot. not symmetries of space

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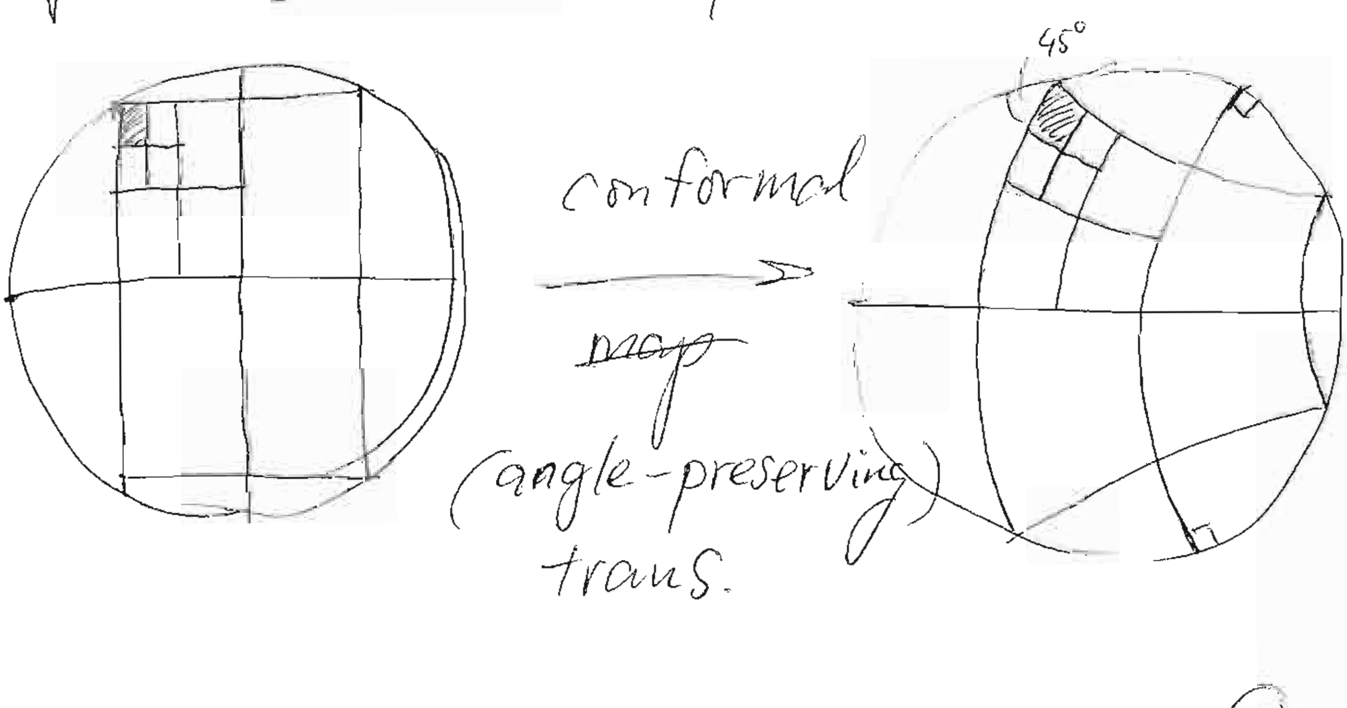
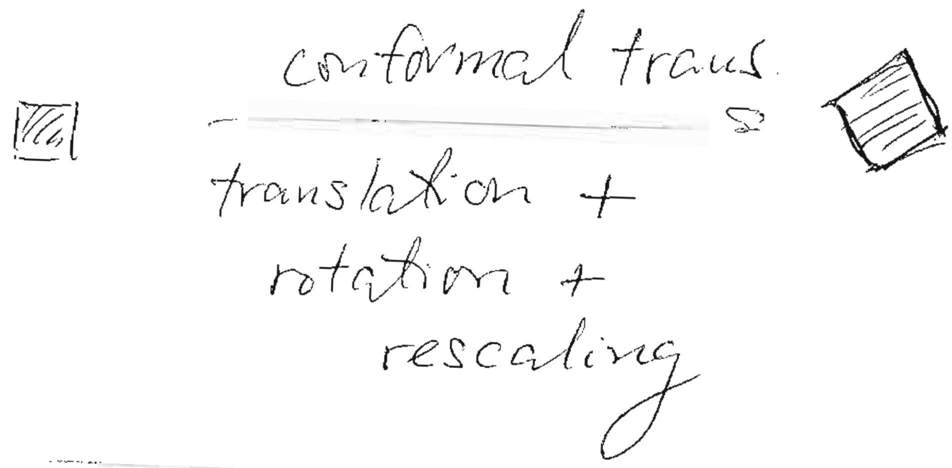
What if space had more symmetries?

Example: <u>conformal</u> ~~maps~~ trans. of 2D disk



limit of infinite subdivisions:



Fact: for each symmetry of space there is a corresponding <u>conservation</u> <u>law</u>

translations $\longrightarrow$ momentum cons.

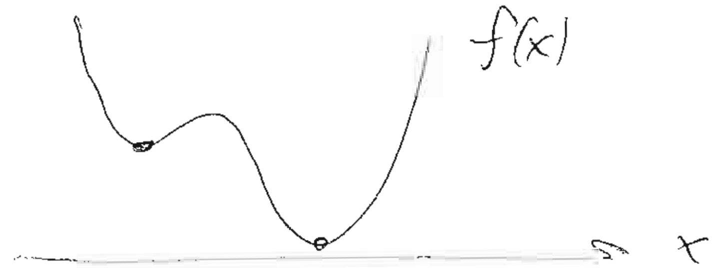
rotations $\longrightarrow$ ang. momentum cons.

What new conservation laws would there be if space had conformal symmetry?

Variational principles :

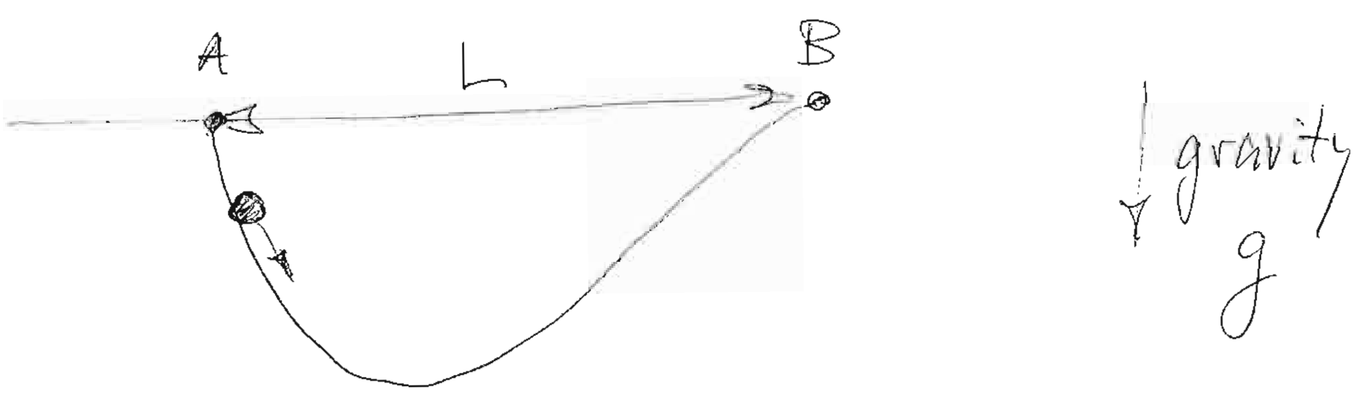
"Xtreme calculus"

Ordinary calculus: find min. of a function of a few variables



In mechanics we often minimize with respect to <u>all</u> <u>possible functions</u>!

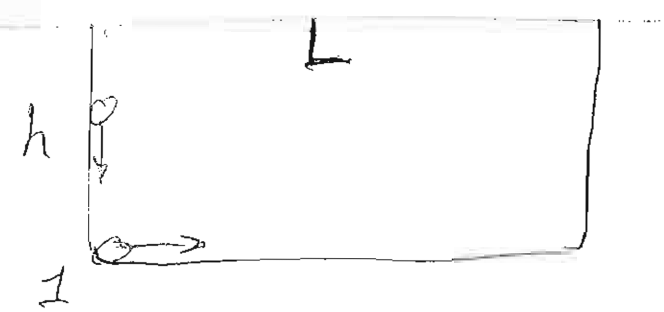
Example: design "ramp" which takes particle from A to B in <u>shortest time</u>



Q : How can you form a time from the given information? (L & g)

A : $\sqrt{L/g}$

rectangular ramps :



T_1 = time to fall to 1

$= \sqrt{2h/g}$

v_1 = speed at 1

$= \sqrt{2gh}$

total time $T = 2\sqrt{\frac{2h}{g}} + \frac{L}{\sqrt{2gh}}$

$$T = \sqrt{L/g}\,\Big(\underbrace{2\sqrt{\tfrac{h}{L}}}_{x} + \underbrace{\tfrac{1}{2}\sqrt{\tfrac{L}{h}}}_{1/x}\Big)$$

$$f(x) = x + 1/x \qquad f' = 1 - \frac{1}{x^2} = 0$$

$$x = 1$$

$$2\sqrt{\tfrac{h}{L}} = 1 \Rightarrow h = \tfrac{1}{4}L$$

$$T_{rect} = \underbrace{2\sqrt{2}}_{\text{can reduce this number with a curved ramp}}\sqrt{L/g}$$

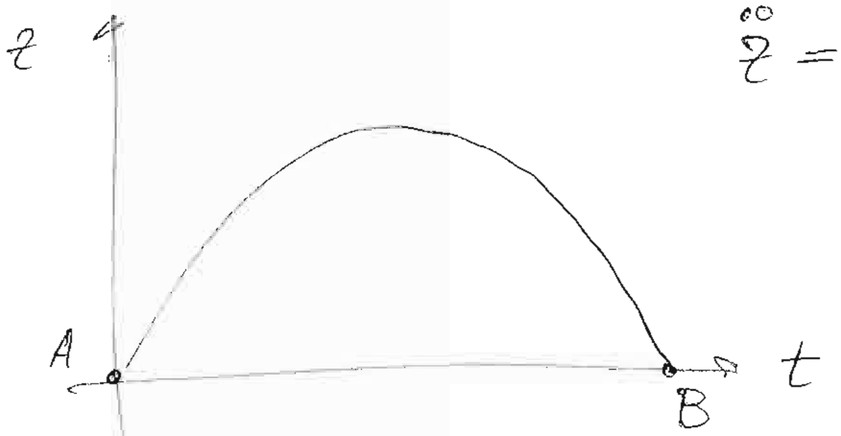
can reduce
this number
with a <u>curved</u> <u>ramp</u>

We will find optimal path (later)
using the <u>calculus of variations</u>.

The calculus of variations is also the basis of an alternative formulation of the laws of mechanics.

<u>Newton</u>: space-time trajectory (of a particle) is determined by its curvature, which is m^{-1} times the instantaneous force

chalk thrown upward:



$$\ddot{z} = \frac{-mg}{m}$$

<u>Variational Principle</u>: space-time trajectory is the curve that <u>extremizes</u> the <u>action</u>

between A & B

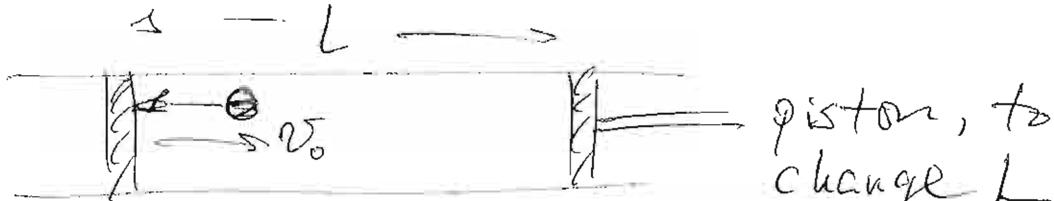
You have already encountered a quantity with the units of action in quantum mechanics : $\hbar$

Q : what are the units of Planck's constant ?

A : $\Delta x \cdot \Delta p$

Phase Space

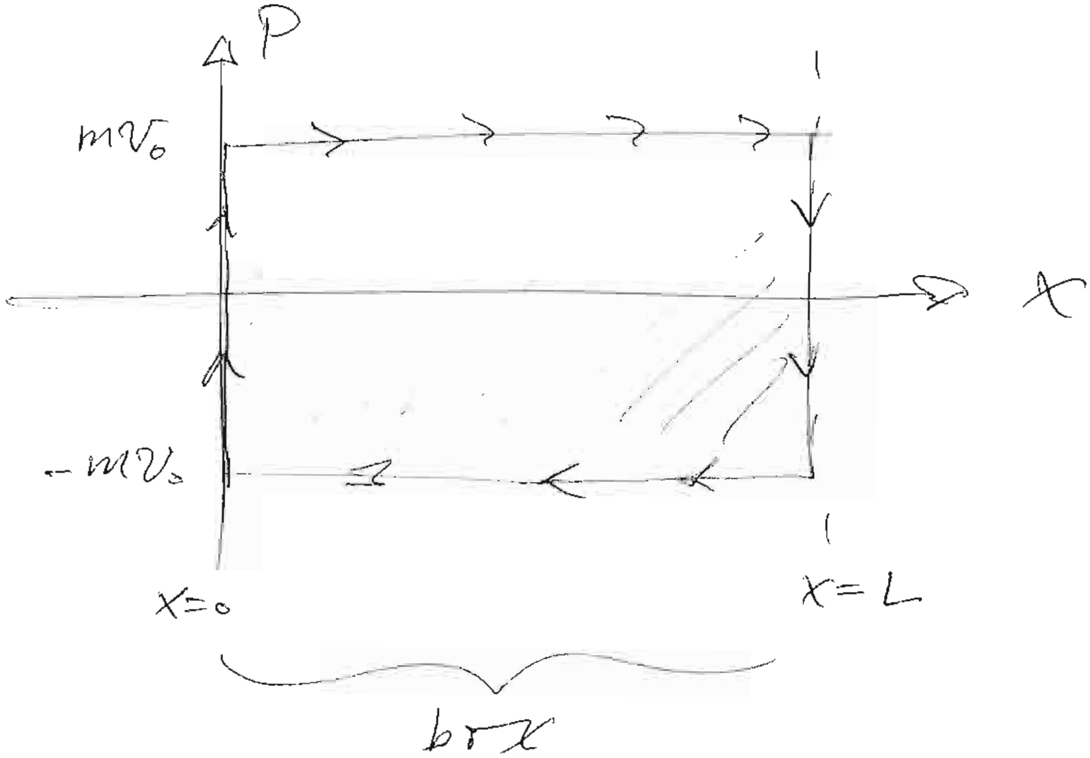
Consider a particle bouncing between the two walls of a 1D "box":



all collisions elastic

phase-space diagram of trajectory

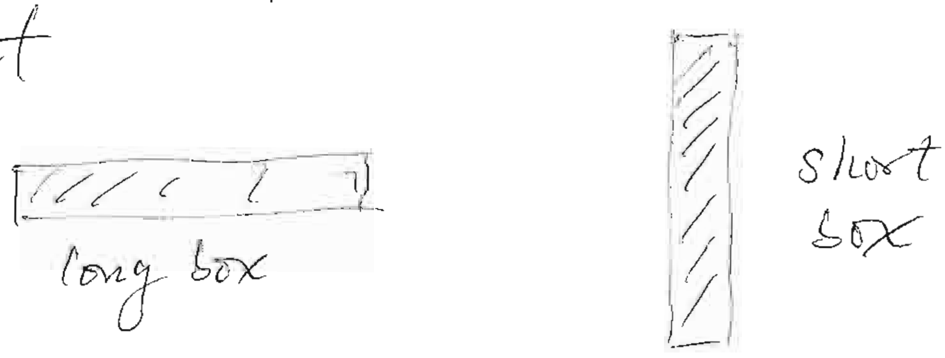
$x(t)$, $p(t)$



$$S = \text{action} = \text{area enclosed}$$

$$= L \times 2mv_0$$

<u>Interesting fact 1</u>: when piston is moved slowly, S remains constant



Interesting fact 2 : the classical trajectory is like a very highly excited quantum system; the level of excitation is approx

$$N \simeq S/\hbar = \frac{m v_0 L}{\hbar}$$

$m = \text{gm} \quad v_0 = \frac{\text{cm}}{\text{sec}} \quad L = \text{cm}$

$$N = \frac{1 \text{ erg sec}}{10^{-27} \text{ erg sec}} = 10^{27}$$

Corollary : a mechanical system will show quantum effects when masses, velocities, lengths are so small that $S \sim \hbar$