P3327: LAST SECTION (12, I think) 16 NOV '12

ANNOUNCEMENTS

- Last section
- OH ON MON 19 AS USUAL
- **FINAL** FRI DEC 7
- **REVIEW SESSION?** PICK ONE
  - M DEC 3 5:15
  - T DEC 4 any time
  - U DEC 2 > 3pm

~~IF YOU REALLY WANT WE CAN DO TWO (BUT Y'ALL BRINGING FOOD)~~

WHAT YOU WANT		WHAT YOU GET
HOW TO DO HW	→	OH ON MONDAY ✓ → WILL POST SAMPLE HW
METICULOUSLY DONE SAMPLE PROBLEMS W/ EVERY FACTOR OF TWO	→	TRIVIAL SAMPLE PROBLEMS (for your own good & mine)
LATEST LHC RESULTS & INTRO TO SUSY	→	HAND WAVY (P)REVIEW of SPECIAL RELATIVITY [see PI talk]
HOW TO DO FINAL	→	REVIEW SESSIONS
TO GO SEE BOND?	→	SEE CLOUD ATLAS INSTEAD (OR READ IT)
GRADED HW	→	✓

Antenna - shit

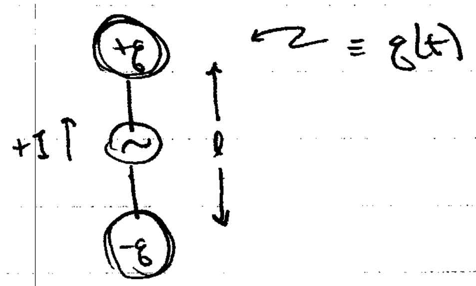
not $r \gg \lambda$

RADIATION APPROX: $r \gg \lambda$

HW 9-3: show: t-dep ELEC DIPOLE HAS A NEAR FIELD TERM THAT IS THE BIOT-SAVART LAW.

still $r \gg l$

IDEA:



$$\left.\begin{aligned} \dot{q} &= I(t) \\ p &= q\,l \end{aligned}\right\} I = \frac{\dot{p}}{l}$$

THEN FROM LAST TIME: RETARDED FIELDS

$R = r - s$

$$\underline{E} = \int d^3s \left(\frac{[\rho]\hat{R}}{R^2} + \frac{[\dot{\rho}]\hat{R}}{cR} - \frac{[\dot{J}]}{c^2 R} \right)$$

$$\underline{B} = \int d^3s \left(\frac{[J]\times\hat{R}}{cR^2} + \frac{[\dot{J}]\times\hat{R}}{c^2 R} \right)$$

Biot savart has something to do with this, i think :)

IDENTIFY: $\underline{J}\, d^3s = I\, d\underline{l} = \dot{p}\,\hat{l} = \dot{\underline{p}}$

→ technically: doing the integrals & assuming the wire has negligible cross section (or use $r \gg l$)

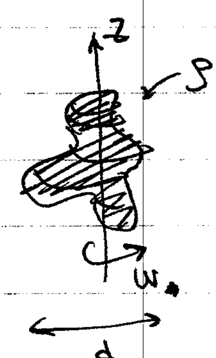
$R \rightarrow r$ (if you want, taylor expand)

THEN IT'S TRIVIAL.

$$\underline{B} = \underbrace{\int \frac{[\mathbf{I}\, d\underline{l}] \times \hat{r}}{c r^2}}_{\text{BIOT SAVART}} + \cdots$$

$$= \frac{[\dot{p}] \times \hat{r}}{c r^2} \quad \Big\} \text{ less obvious form.}$$

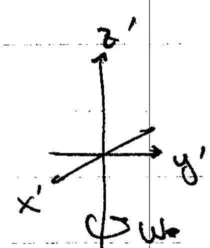
PROBLEMS & SOLUTIONS IN E&M (Lim) **#4067**



non-relativistic: $\omega_0 d \ll c$

WHAT FREQUENCIES of EM RAD ARE OBSERVED @ $r \gg d$?

(go to NLO)



define rotating (PRIMED) COORDINATES

RELATE to NON-ROTATING COORDS

$$\begin{cases} x = x' C_{\omega_0 t} - y' S_{\omega_0 t} \\ y = x' S_{\omega t} + y' C_{\omega t} \\ z = z' \end{cases}$$

<u>MONOPOLE</u>: no rad.

<u>DIPOLE MOMENT</u>: $\underline{p}(t) = \int \rho\, \underline{s}\, d^3s$

integrate over $s = (x', y', z')$

$$\underline{p} = \int d^3s \; \rho \cdot (x' C_{\omega t} - y' S_{\omega t})\,\hat{x} + \rho \cdot (x' S_{\omega t} + y' C_{\omega t})\,\hat{y} + \rho \, z' \,\hat{z}$$

$$= (p'_{x'} C_{\omega t} - p'_{y'} S_{\omega t})\,\hat{x} + (p'_{x'} S_{\omega t} + p'_{y'} C_{\omega t})\,\hat{y} + p'_{z'}\,\hat{z}$$

$\underline{p}'$ IS DIPOLE MOMENT IN ROTATING FRAME (CONSTANT BY CONSTRUCTION)

$\Rightarrow \underline{p}(t)$ oscillates w/ freq ω.
(OBVIOUS)

MAGNETIC DIPOLE RADIATION? none $\rightarrow$ steady current.

ELECTR
QUADRUPOLE

$$Q_{ij} = \int d^3s \, (3 s'_i s'_j - s'^2 \delta_{ij}) \, \rho$$

CAN YOU SEE WHAT HAPPENS?
NOW PRODUCT of COORDS
eg.

$$Q_{12} \sim \int d^3s \; 3 (x' C_{\omega t} - y' S_{\omega t})(x' S_{\omega t} + y' C_{\omega t})$$

PRODUCT of TRIGS $\sim e^{\pm i\omega t} e^{i\omega t} \longrightarrow$ CAN GIVE $e^{i(2\omega)t}$

thus: xy terms $\longrightarrow$ freq $\sim 2\omega$

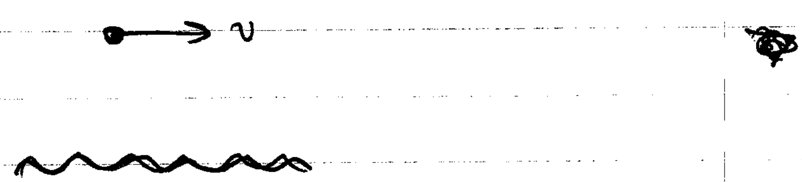
xz terms $\longrightarrow$ freq $\sim \omega$

REMARK in § 9.6 of HM, QUADRUPOLE RAD:

$$\text{POWER} \rightarrow \frac{\langle P \rangle_{\text{QUAD}}}{\langle P \rangle_{\text{Di}}} \sim \left(\frac{d}{\lambda}\right)^2 \sim (d\omega)^2$$

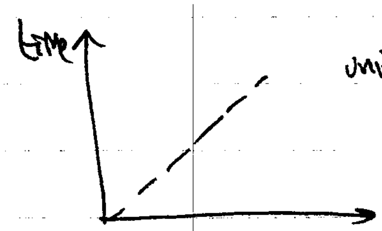
but this is boring.

EX: CHARGED PARTICLE w/ VELOCITY v IN X-DIR. MOVES ABOVE CORRUGATED CONDUCTING SHEET:



DISTANT OBSERVER DETECTS EM RAD @ ANGLE θ w/rt $\underline{v}$. ? λ of RADIATION

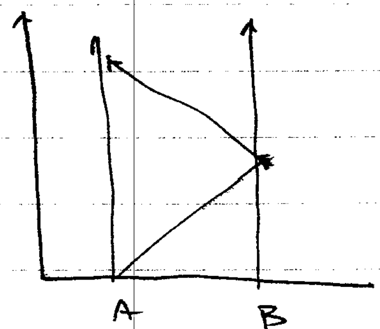
Special Relativity – GRAPHICALLY



units s.t. $c = 1$

space, say x

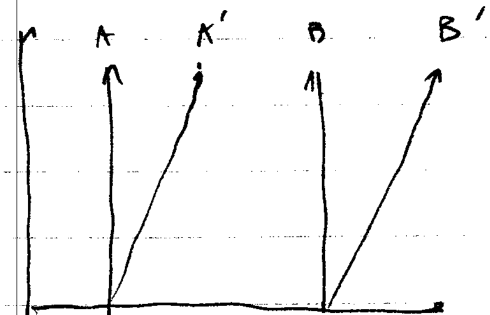
LIGHT TRAVELS @ 45° LINES

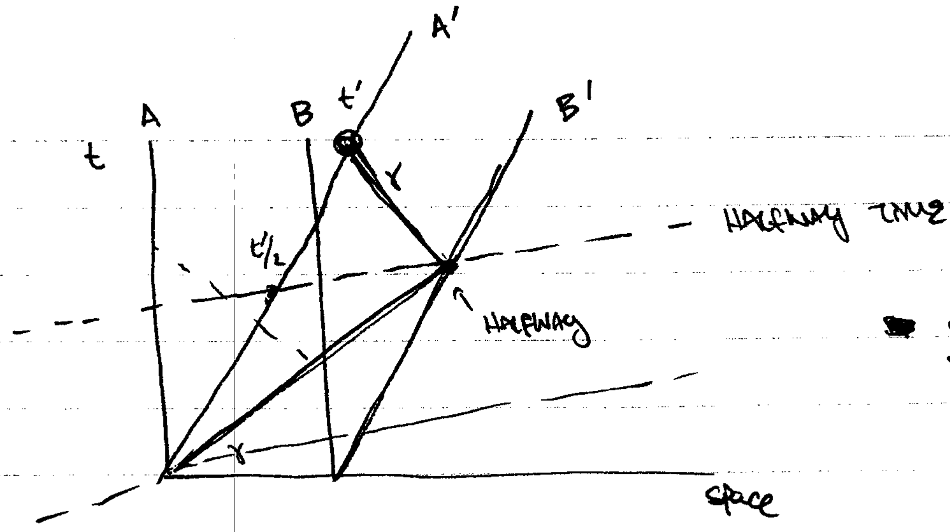


BOUNCING LIGHT SIGNALS

<u>MOVING FRAMES</u> : LORENTZ TRANSFORMATION.

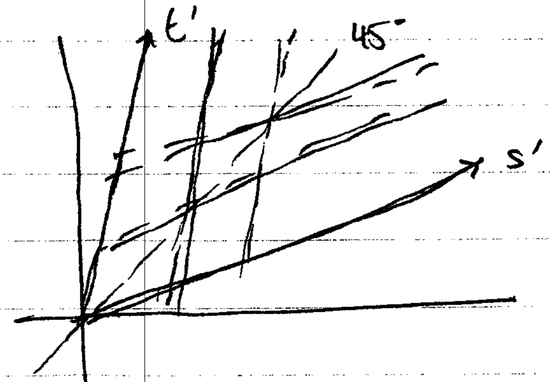
↳ inertial





c = const = c'
still 45° lines

SO EVIDENTLY THE AXES IN THE BOOSTED FRAME HAVE CHANGED (w/r/t UNPRIMED FRAME)



$\cancel{s' = \gamma(s - vt)}$

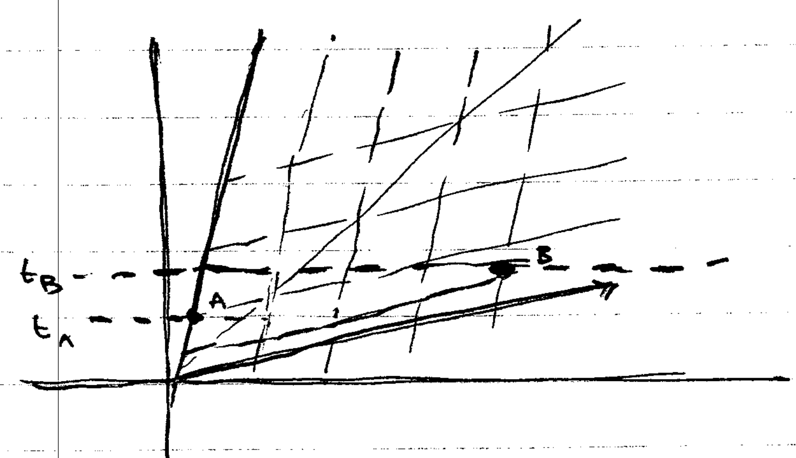
$\cancel{ct' = \gamma(ct - s)}$

$$\begin{cases} s' = \gamma\left(s - \beta\,(ct)\right) \\ ct' = \gamma\left(ct - \beta s\right) \end{cases}$$

AS YOU KNOW: $\gamma = (1 - \beta^2)^{-1/2}$

$\beta = v/c$

CAUSALITY

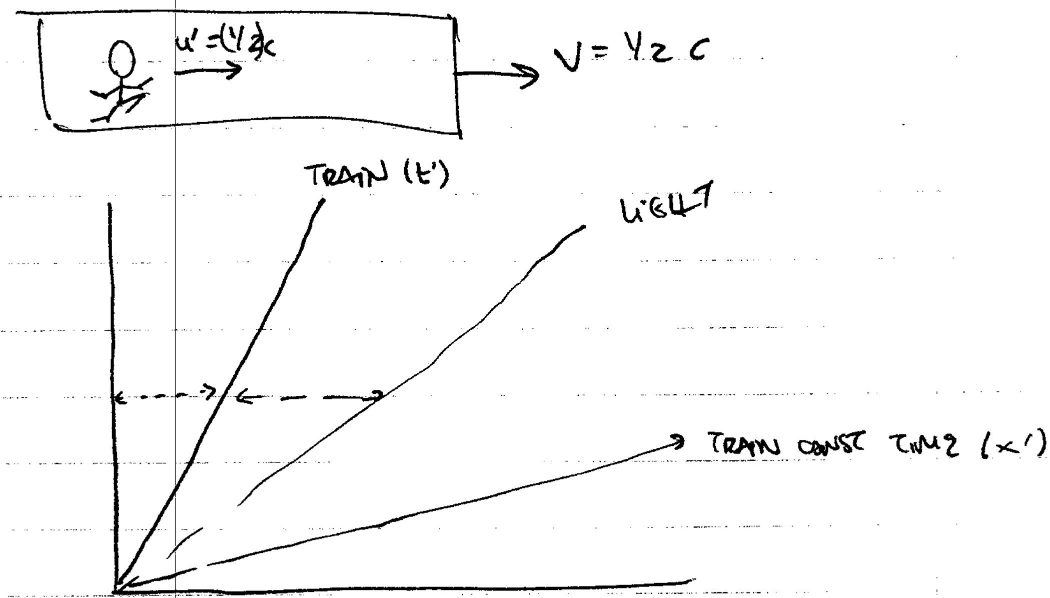


cf LIGHTCONE

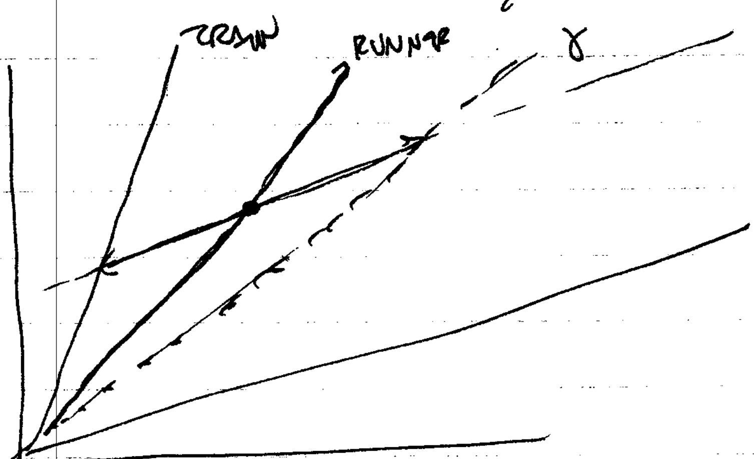
INSIDE LIGHTCONE IS CAUSAL

$t_B > t_A$ BUT $t'_A > t'_B$

ADDING VELOCITIES GEOMETRICALLY



HOW TO DRAW BIT TRIP RUNNER / JAMES BOND ?

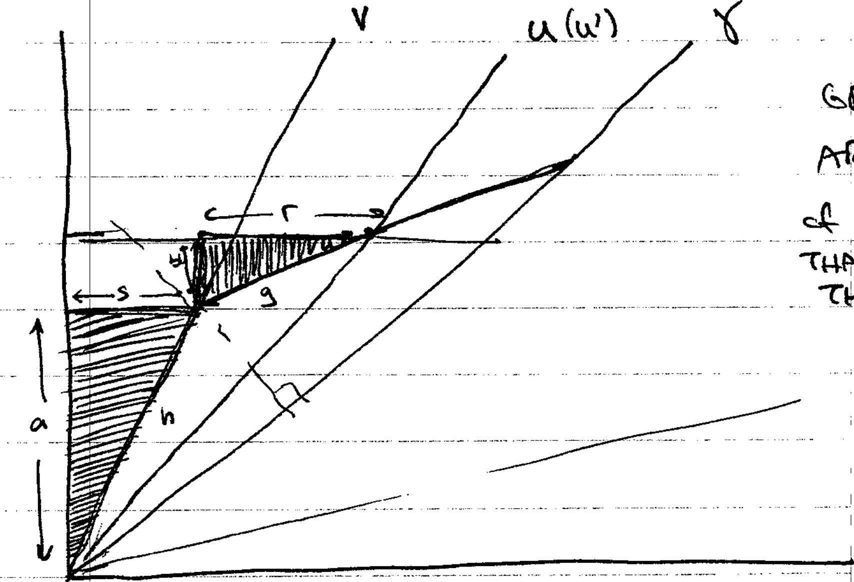


THE VELOCITY ADDITION FORMULA

TRAIN w/ VELOCITY V

RUNNER w/ VELOCITY u' w/rt TRAIN

? u OF RUNNER w/rt OBSERVER ?



GREEN TRIANGLES ARE SIMILAR

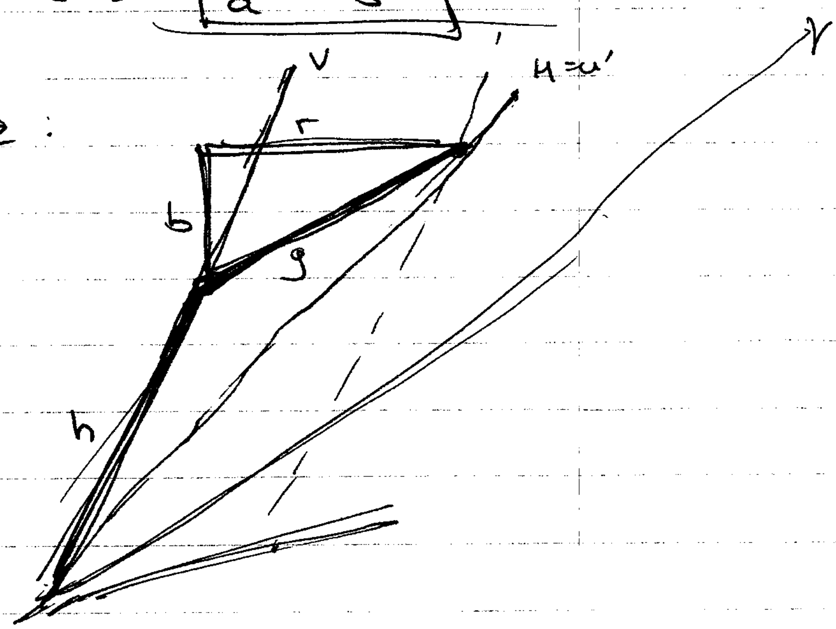
of LINE ⊥ TO γ THAT SEPARATES THE TRIANGLES.

- $\frac{s}{a} = \frac{s = vt}{a = ct}$ ← DIST TRAVELLED BY TRAIN ← DIST TRAVELLED BY LIGHT PULSE

  $= \beta$

- $\frac{r}{b} = \frac{a}{s}$ ⟷ $\boxed{\frac{r}{a} = \frac{b}{s}}$

IN TRAIN FRAME :



$\boxed{\frac{g}{h} = \frac{u'}{c}}$

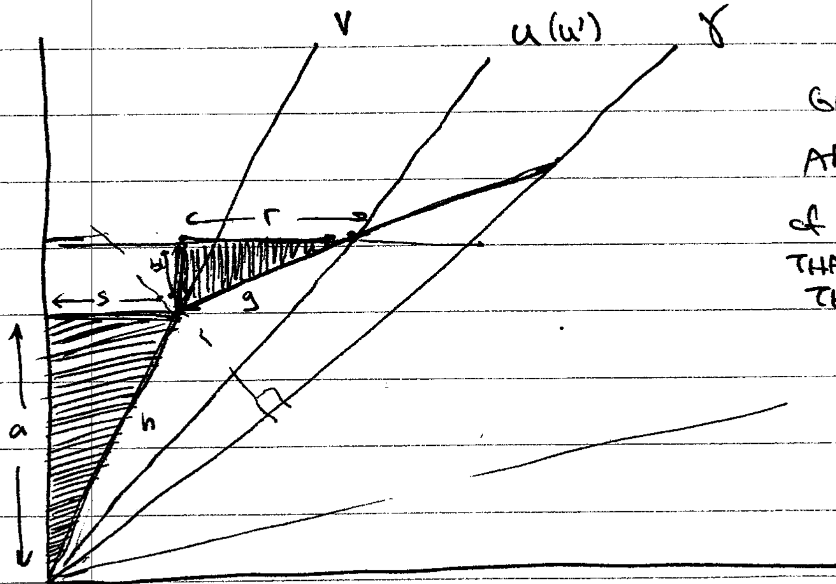
same arg:

THE VELOCITY ADDITION FORMULA

TRAIN w/ VELOCITY V

RUNNER w/ VELOCITY u' w/rt TRAIN

? u OF RUNNER w/rt OBSERVER ?



GREEN TRIANGLES ARE SIMILAR

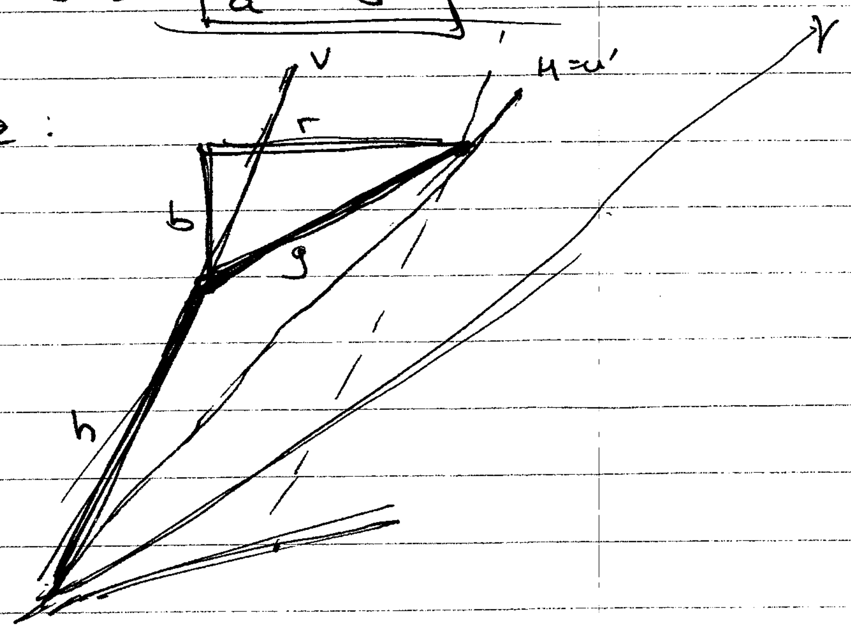
of LINE ⊥ TO γ THAT SEPARATES THE TRIANGLES.

- $\dfrac{s}{a} = \dfrac{s = vt}{a = ct}$ ← DIST TRAVELLED BY TRAIN / ← DIST TRAVELED BY LIGHT PULSE

  $= \beta$

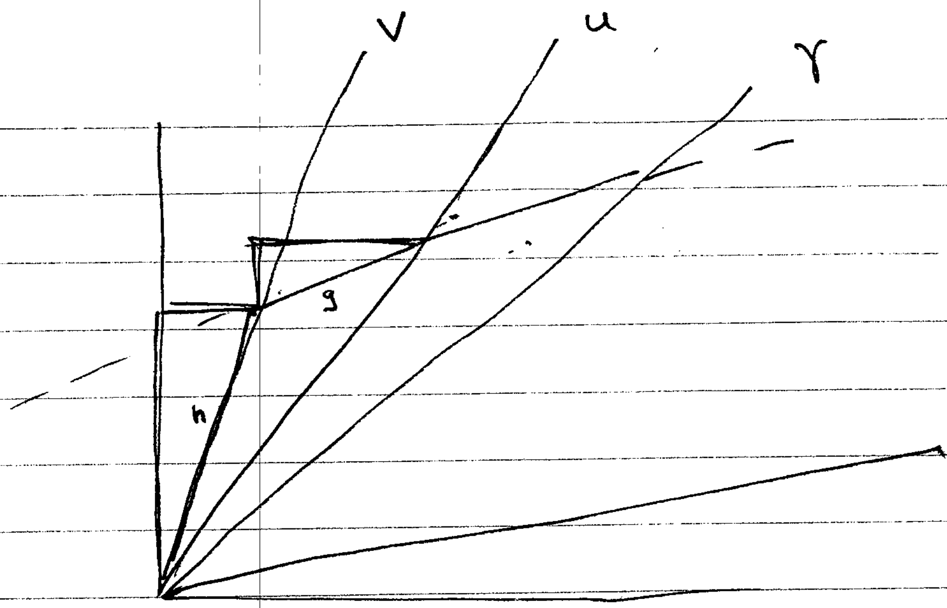
- $\dfrac{r}{b} = \dfrac{a}{s} \quad \longleftrightarrow \quad \boxed{\dfrac{r}{a} = \dfrac{b}{s}}$

IN TRAIN FRAME:



- $\boxed{\dfrac{g}{h} = \dfrac{u'}{c}}$

same arg.



h = DIST LIGHT COVERS IN RED FRAME

$= 2g$

g = DIST TRAVELLED BY ~~GIRL~~ RUNNER IN TRAIN.

~~$\Rightarrow$ ... $\frac{w'}{c}$ ... DIST LIGHT TRAVELS / DIST ... TRAVELS~~

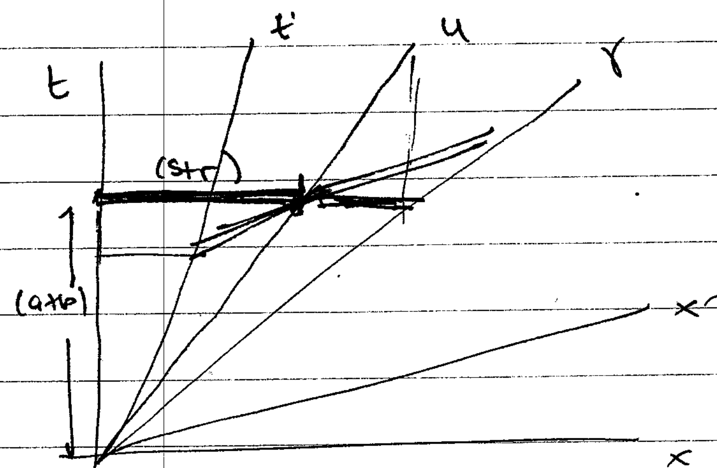
FROM ABOVE

since $\frac{g}{h} = \frac{w}{c}$ $\Rightarrow$ $\boxed{\frac{w'}{c} = \frac{b}{s}}$ $= \boxed{\frac{r}{a}}$

from similarity of triangles

Algebra: $\begin{cases} \boxed{b = \dfrac{u's}{c} \\ r = \dfrac{u'a}{c}} \end{cases}$

- IN THE OBSERVERS FRAME



(s+r) = u t DIST TRAVELLED BY RUNNER
(a+b) = c t DIST TRAVELLED BY LIGHT

$$\boxed{\frac{u}{c} = \frac{s+r}{a+b}}$$

$$\frac{b}{a} = \frac{v}{c}$$

$$u = c\,\frac{s+r}{a+b}$$

$$= \cancel{c}\,\frac{\cancel{a}\frac{v}{\cancel{c}} + \cancel{a}\frac{u'}{\cancel{c}}}{\cancel{a} + \frac{u'}{c}\cancel{a}\frac{v}{c}} = \boxed{\frac{u'+v}{1+\frac{u'v}{c^2}}}$$

OBSERVE:

1. $V = \frac{1}{2}c$, $u' = \frac{1}{2}c \Rightarrow \boxed{u = \frac{4}{5}c}$

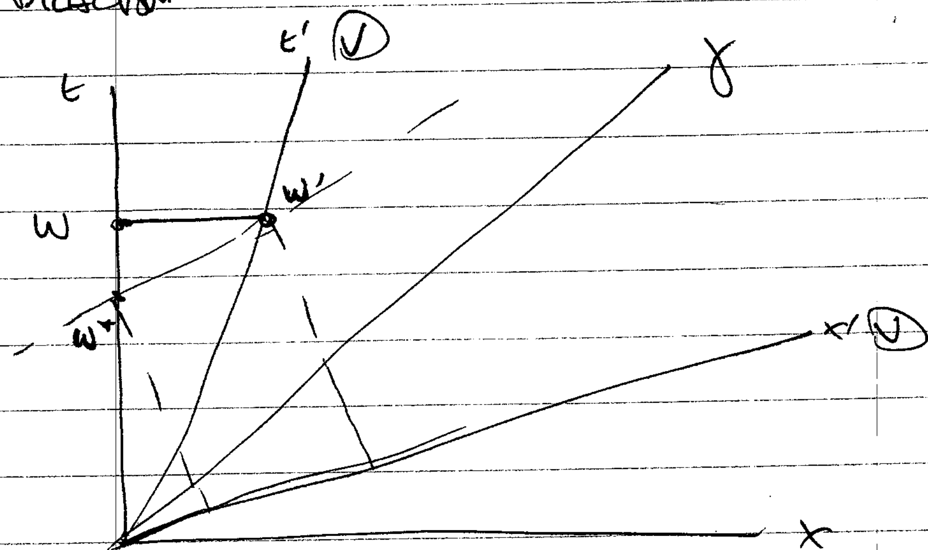
2. for $\frac{u'}{c}, \frac{v}{c} \ll 1$

REDUCES TO NEWTONIAN.

3. $u' = c \Rightarrow u = c$ ✓

~~LORENTZ TRANSFORMATIONS~~

TIME DILATION

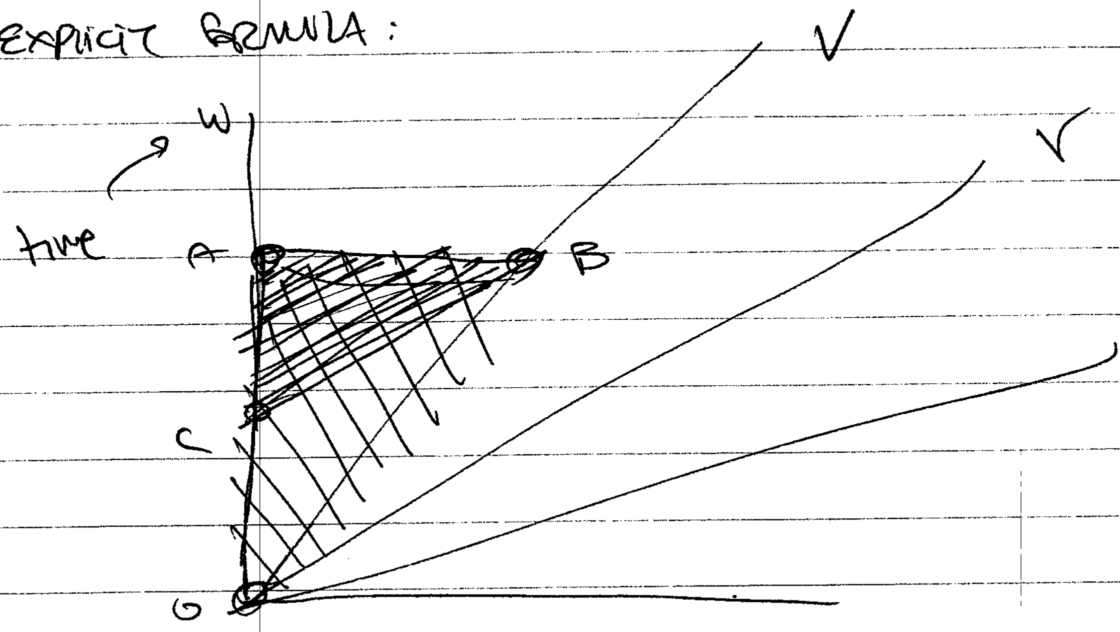


define: γ VIA $w' = \gamma w^*$

CLAIM: SYMMETRY: $w = \gamma w'$ $\Big\}$ $w = \gamma^2 w^*$

note: $\gamma^2 > 1$; $\underbrace{w' < w}_{\text{moving clock runs more slowly}}$ $(w' = w/\gamma)$

EXPLICIT FORMULA:



SHADED ABO & 2x SHADED ABC

TRIANGLES ARE SIMILAR

$$\Rightarrow \frac{AB}{AO} = \frac{AC}{AB}$$

$$= \frac{vt}{ct} = \beta$$

$$= \frac{AB}{W}$$

$$\boxed{AB = \frac{WV}{c} = \beta W}$$

$$\boxed{AC = W - W^*}$$

$$AB^2 = AO\, AC$$

$$\beta^2 W^2 = W\,(W - W^*)$$

$$W\beta^2 = W - W^* \Rightarrow W(\beta^2 - 1) = -W^*$$

$$\Rightarrow \boxed{\gamma^2 = \frac{1}{1-\beta^2}} \qquad \boxed{W = W^*/(1-\beta^2)}$$

SIMILARLY : YOU CAN PLAY & GET LORENTZ TRANSFORMATIONS.

$$x' = \gamma(x - \beta(ct)) \qquad ct' = x^{0\prime} = \gamma(x^0 - \beta x^1)$$
$$ct' = \gamma(ct - \beta x) \qquad x = x^1 = \gamma(x^1 - \beta x^0)$$

$$\begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x^0 \\ 1 \end{pmatrix}$$

$$x^\mu = \underbrace{\Lambda^\mu{}_\nu}_{} x^\nu$$

4x4 MATRIX
"ROTATION" IN SPACETIME.

RECALL OUR VECTOR ANALYSIS FROM SECTION 1?

DOT PRODUCT: $\underline{\underline{x \cdot y}} = x^\mu y_\mu = x^\mu y^\nu \eta_{\mu\nu}$

$$\begin{pmatrix} + & & \\ & - & \\ & & - \end{pmatrix}$$

this length
is preserved
→ MINKOWSKI SPACE