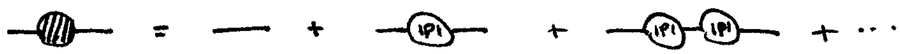
RAMBLING THOUGHTS ON RENORMALIZATION

RECAP OF LECTURES



FULL QUANTUM AMPLITUDE FOR PROPAGATION

"ONE PARTICLE IRREDUCIBLE"

cannot be disconnected by cutting a single line — ie really what you would think of as a blob.

STORY: EVEN THOUGH THESE ARE ASYMPTOTICALLY FREE STATES @ ∞, THEORY IS NOT FREE!

PARTICLES HAVE SELF-INTERACTIONS.

$\downarrow$

CLEARLY THIS IS *NOT* A STATE ASSOCIATED WITH H_{free} THEORY, EVEN ASYMPTOTICALLY.

BUT: WE KNOW THE PUNCHLINE; WE CAN REPACKAGE OUR DEFINITION OF THE THEORY TO MAKE IT LOOK ASYMP. FREE.

ie WE IDENTIFY 

SUM OF DIAGRAMS IN "BARE" THEORY

RENORMALIZED

HOW WE DID THIS:

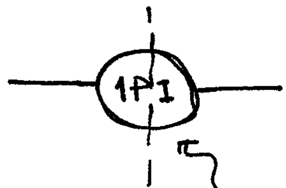
PRACTICALLY this must be COMPUTED ORDER-BY-ORDER IN PERTURBATION THEORY.

$\equiv -i\Pi(p^2)$

$\Rightarrow$  $= \dfrac{i}{p^2 - m^2 - \Pi(p^2) + i\varepsilon}$

$$\frac{i}{p^2 - m^2 - \Pi(p^2) + i\varepsilon}$$

when on-shell, $p^2 = m^2$, $\Pi(m^2)$ PICKS UP IMAGINARY PART FROM OPTICAL THEOREM



WHATEVER INTERMEDIATE STATES

$$\text{Im}\,\Pi(m^2) = -m\Gamma$$

WIDTH

$\Rightarrow$ GIVES BREIT-WIGNER PEAK. (& AVOIDS DIVERGENCE IN SCATTERING)

ok. WHAT ABOUT THE $\mathbb{R}$ PART of $\Pi(p^2)$?

PROPERTIES of THE FREE PROPAGATOR

"classical," "free-level," whatever.
THE PROPAGATOR THAT YOU WERE NAIVELY TRICKED ~~into~~ INTO BELIEVING WAS THE PROPAGATOR.

(the truth will be that it is, but only after a conceptual shift!)

1. POLE @ $p^2 = m^2$ $\longrightarrow$ SINGULARITY @ the mass
2. RESIDUE $= 1$ $\longrightarrow$ CANONICALLY NORMALIZED

SO, WRITE: $\Pi(p^2) = \Pi(m^2) + \Pi'(m^2)\,p^2 + \frac{1}{2}\Pi''(m^2)\,p^4 + \cdots$

$(p^2 - m^2)$

HW: WHAT DOES THIS CORRESPOND TO? (think of effective theory)

$$\frac{i}{p^2 - m^2 - \Pi(m^2) - \Pi'(m^2)\,p^2 + \cdots + i\epsilon}$$

$(p^2 - m^2)$

think about what these mean! (cute exercise)

SHIFTS POLE

SHIFTS RESIDUE

<u>MESSAGE</u>: THIS is *NOT* A FREE THEORY!
<u>BUT</u>, WE CAN MASSAGE IT INTO SOMETHING LIKE THAT.

[<u>SOAP BOX</u>: the key POINT IS THAT FOR THE QUADRATIC PART of THE LAGRANGIAN WE WERE ABLE TO SUM THE GEOMETRIC SERIES.
↑ 're-sum'

THIS MEANS WE CAN TREAT THEM AS A MODIFIED "FREE" ASYMPTOTIC STATES. THIS WILL BE SLIGHTLY MORE CLEAR IN THE PATH INTEGRAL FORMALISM WHERE ONE CAN EXPLICITLY DO THE PATH INTEGRAL FOR THE QUADRATIC PART of THE ACTION.]

NOW THIS IS THE POINT WHEN WORDS BECOME SLIPPERY!
↪ very easy to get confused

[IN SOME SENSE, ISSUE of INTERPRETATION]

CLEARLY "POLE MASS" & "LAGRANGIAN MASS" DIFFER.

- WHICH ONE IS PHYSICAL? (POLE MASS) } m_{phys}
  → THIS IS OBSERVABLE
- WHAT ABOUT "UNPHYSICAL" MASS? } m_0, BARE MASS
  → NOT OBSERVABLE, JUST A TOOL.

THE DIFFERENCE BETWEEN m_{phy} & m_0: QUANTUM-CORRECTED VS. CLASSICAL

- Makes you think: both are sensible masses. NOT NEC. TRUE!
  Q'M CORRECTIONS CAN GIVE ∞'s, BUT m_{phys} = FINITE.
  ⇒ m_0 HAS ∞'s TO CANCEL Q'M CORR! WEIRD? SURE. BUT SINCE m_0 IS MANIFESTLY UNPHYSICAL, SO WHAT?

NOW WE'RE GOING TO START USING "BIG WORDS"
→ words themselves aren't long
but they are loaded w/ meaning & subtlety!

M_0 IS THE BARE MASS.

- THE MASS WE WROTE DOWN AS NAIVE YOUNG QUANTUM FIELD THEORISTS.
- WE THOUGHT IT WAS THE MASS... IT'S NOT. WE CAN SEE THAT THE PHYSICAL MASS IS SOMETHING LIKE $m^2 \sim M_0^2 + \Pi(m^2)$
- THIS MASS IS UNPHYSICAL

↳ DO NOT ATTACH ANY NOSTALGIC SENTIMENT TO IT! IT IS NOT THE OBJECT WE WANT TO BE WORKING WITH.

IT IS $\boxed{\text{NOT}}$ EVEN SOMETHING FAMILIAR!

↳ DON'T THINK THAT THIS IS SOME STEPPING STONE TO THE CLASSICAL OR FREE THEORY! THAT LINE OF THOUGHT IS A RED HERRING.

THIS IS IMPORTANT: M_0^2 CONTAINS INFINITIES
YOU WILL ONLY FIND THIS DISTURBING IF YOU'VE ATTACHED SOME PRIOR INCORRECT IDEAL TO M_0

to repeat: there is nothing holy about the bare mass!

okay? [VERY IMPORTANT]
same holds for bare couplings & bare field.

"THERE IS NO UNIVERSE BUT THIS UNIVERSE → THIS IS ITS LAGRANGIAN"

ON THE OTHER HAND: NEVER USE THE PHRASE "BARE LAGRANGIAN"
THE SIMPLEST WAY TO THINK ABOUT THIS WHOLE PROGRAM:

THERE IS ONLY ONE LAGRANGIAN, $\mathcal{L}$.
WHEN WE WROTE IT DOWN, PERHAPS NAIVELY, IT IS

$$\mathcal{L} = \frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}M_0^2\phi_0^2 + \frac{1}{4!}\lambda_0\phi_0^4 + \cdots \quad \text{etc.}$$

WE WILL REWRITE THIS INTO AN EQUIVALENT EXPR. IN TERMS OF PHYSICAL PARAMETERS, BUT $\mathcal{L}$ IS ALWAYS THE SAME!

$$\mathcal{L}(\phi_0, M_0, \lambda_0) = \mathcal{L}(\phi, m, \lambda) \qquad \text{and later: same under RG flow!}$$
$$= \mathcal{L}(\phi(\mu), m(\mu), \lambda(\mu))$$

LET'S BE VERY CAREFUL. EXPAND $\Pi(p^2)$ ABOUT SOME p_0^2, ARBITRARY

$$\Pi(p^2) = \Pi(p_0^2) + \Pi'(p_0^2)(p^2 - p_0^2) + \cdots$$

NOW PICK $p^0 = m$, THE PHYSICAL MASS, @ WHICH THE PROPAGATOR IS

$$\frac{1}{\underbrace{p^2 - m_0^2 - \Pi(m^2)}_{\equiv -m^2} - \Pi'(m^2)(p^2 - m^2) + \cdots + i\varepsilon}$$

$\equiv -m^2$ BY DEFINITION of THE PHYSICAL MASS!

$$= \frac{1}{(p^2 - m^2)\underbrace{(1 - \Pi'(m^2) + \cdots)} + i\varepsilon} \equiv \frac{iZ}{p^2 - m^2 + i\varepsilon}$$

THIS IS NOW SOME OVERALL PREFACTOR. CALL IT Z^{-1}

SO: WITH RESPECT TO OUR ORIGINAL (NAIVE, BARE) PARAMETERIZATION of THE ONE-TRUE-LAGRANGIAN, OUR PROPAGATOR IS NOT NORMALIZED. IT MUST BE "RE NORMALIZED." ← BUT WE WILL USE THIS WORD IN A CLOSELY RELATED BUT DIFFERENT WAY SOON

RECALL THAT PROP $\sim \langle \phi_0(x)\phi_0(0) \rangle \sim \frac{iZ}{p^2 - m^2}$

then if we wrote $\phi_0(x) \equiv Z^{1/2}\phi(x)$ [DEFINITION]

RENORMALIZED, or ϕ_r (PHYSICAL)

THEN: $\langle \phi(x)\phi(0) \rangle \sim \frac{i}{p^2 - m^2}$ ← CORRECT PROPAGATOR

b/c $\phi_0\phi_0 = Z\,\phi\phi = \frac{iZ}{p^2 - m^2}$

[PRESKILL INSTEAD APPEALS TO 1-PARTICLE UNITARITY; MORE RIGOROUS BUT PERHAPS LESS CLEAR.]

NOW GO BACK TO THE ONE-TRUE-LAGRANGIAN

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi_0)^2 - \tfrac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

$$= \frac{Z}{2}(\partial\phi)^2 - \frac{Z}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}Z^2\phi^4$$

NOT CANONICALLY NORMALIZED!

CAN WE RESCALE FIELD? <u>NO</u>.
WE JUST FIXED FIELD NORMALIZATION TO GET CORRECTLY NORMALIZED PROPAGATOR.

another way to see eg: cannot simultaneously normalize (canonically) the propagator & the quadratic part of the Lagrangian.

CAN WE RESCALE $\mathcal{L}$? <u>NO</u>. THAT'S A <u>DIFFERENT</u> THEORY.

eventually we'll be talking about different Lagrangians in a very specific way.

FOR NOW: @ A GIVEN & FIXED ENERGY SCALE, THERE IS ONLY "ONE TRUE $\mathcal{L}$"!

SO WHAT CAN WE DO? PEEL OFF THE EXCESS & CALL IT AN INTERACTION.

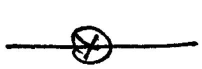
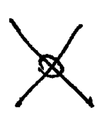
$$= \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

PHYSICAL COUPLING (DEFINED SOMEWHAT ARBITRARILY)

$$\underbrace{+ \underbrace{(Z-1)}_{\delta_Z}\tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\underbrace{(Zm_0^2 - m)}_{\delta_m}\phi^2 - \frac{1}{4!}\underbrace{(\lambda_0 Z^2 - \lambda)}_{\delta_\lambda}\phi^4}$$

<u>COUNTER-TERMS</u>

for now, just a word.
STUFF THAT COMES FROM OUR INABILITY TO CANONICALLY NORMALIZE $\mathcal{L}$ WRT PHYSICAL FIELD + PHYSICAL PARAMETERS.

 $= i(p^2\delta_Z - \delta m)$  $= -i\delta_\lambda$

REMARKS

WE NEVER REQUIRED ANY DIVERGENCES.
ALL WE HAVE DONE IS REFORMULATED THE SAME thy
SO THAT

1. THE FIELDS ARE NORMALIZED

$\Longleftrightarrow$ PROPAGATOR IS NORMALIZED.
(W/RT Q.M. CORRECTIONS)

2. THE PARAMETERS IN THE DEFINITION OF THE THEORY ARE PHYSICALLY MEASURED QUANTITIES.

WILL GO INTO MORE DETAIL. BUT FOR NOW WE CAN SAY M^2 IS "POLE MASS" OR PHYSICAL MASS OF PARTICLE

3. [COST:] EXTRA COUNTER TERM VERTICES.

which are higher order in couplings.

IMPLICIT SO FAR: RENORMALIZATION CONDITIONS.

just a formal statement of #1 & #2

$$-\!\!\bullet\!\!- = \frac{i}{p^2 - M^2 + i\varepsilon} + (\text{analytic})$$

1. POLE @ PHYS MASS
2. RESIDUE IS 1

$$\left.\begin{array}{l} \Pi(M^2) = 0 \\ \Pi'(M^2) = 0 \end{array}\right.$$

you will do this integral in the HW
[THAT'S A HINT]

REN. COND GIVE US A PRESCRIPTION FOR SEPARATING BARE PARAMETERS INTO PHYSICAL PARAMETERS & COUNTER TERMS.

the significance of these guys will become clear soon enough.
BUT TO LOWEST O IN PERT THY, THEY DON'T SHOW UP.

THE RENORMALIZATION CONDITION for COUPLINGS (eg λ) IS MORE ARBITRARY + SUBTLE.

eg.  $= i\lambda$ @ $s = 4m^2,\ t = u = 0$

DEFINE λ @ THRESHOLD

so JUST COLLIDE ϕ @ $E_{cm} = 2m$ AND MEASURE AMPLITUDE, EXTRACT λ!

BUT CALCULATIONALLY MORE CONVENIENT TO DO THIS @ $s = t = u$.

YOU KNOW THIS ISN'T PHYSICAL.
HOW IS THIS A SENSIBLE DEFINITION?
HOW TO MEASURE SUCH A THING?

→ off SHELL EXT MOMENTA
(ANALYTIC CONTINUATION FROM PHYSICAL MOMENTA)

CAN "SORT OF" MEASURE THIS BY LOOKING AT POLES OF HIGHER POINT DIAGRAMS.

eg IN CLASS:  not measured directly
eg $1 \to 2$ for same-mass particles?

BUT:

FULL PROPAGATOR

1PI

$$i\lambda\left(\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right)$$

λ FROM POLES OF PHYS SCATTERING w/ MOMENTA ANALYTICALLY CONTINUED.

- EACH REN. CONDITION ~~IS~~ REQUIRES A PHYSICAL MEASUREMENT.

almost trivial statement: a theory w/ n PARAMETERS (eg Z, m, λ, ...) requires n measurements to fix. $(n+1)$ measurements to test.

POWER COUNTING

ALL OF THIS COUNTER-TERM STUFF BECOMES IMPORTANT @ HIGHER ORDERS IN PERT THY.

↳ δ's ARE CORRECTIONS TO THE TREE LEVEL LAGRANGIAN WRITTEN W/ PHYS. PARAMETERS. THEY ARE PROPORTIONAL TO POWERS OF THE COUPLING.

eg.  $\sim \lambda^2$

↳ GIVES  CORRECTION (CORRECTS — $\sim \lambda^0$)

eg.  $\sim \lambda^3$, GIVES  (CORRECTS $\lambda \sim \lambda^2$)

LOOPS ~~ARE~~ CARRY THEIR OWN CAN OF WORMS

↳ ADDITIONAL UNCONSTRAINED MOMENTUM INTEGRALS

→ WE WILL DEVELOP A TOOLKIT TO DO THESE W/O HAVING TO DO ANNOYING CONTOUR INTEGRALS EACH TIME.

→ DIVERGENCES

↑ OOOH! SCARY!

WE WILL ALSO DEAL W/ THESE. THEY WILL BE EATEN BY THE COUNTER TERMS ~~& WILL DISAPPEAR.~~

↳ MEANING THE BARE PARAMETERS ARE DIVERGENT IN A WAY THAT CANCELS THESE LOOPS.

FOR NOW: <u>QUANTIFY</u> DIVERGENCES:

USEFUL TO DEFINE <u>SUPERFICIAL DEGREE OF DIVERGENCE</u>, D

$$\int d^d k \; \frac{k^n + \cdots}{k^m + \cdots} \to D = d + n - m$$

s.t. $\underline{D<0}$: FINITE

$\underline{D=0}$: PERHAPS LOG-DIVERGENT

$\underline{D>0}$: PERHAPS POWER LAW DIVERGENT.

SOON YOU WILL BE CALCULATING LOOP DIAGRAMS.

TECHNICAL :
1. REGULATE DIVERGENCES
2. ABSORB THEM INTO COUNTER-TERMS
3. MAKE SENSE OF THEM

they don't just disappear!
DIVERGENCES <u>DO</u> LEAVE PHYSICAL TRACES

not just "throwing away" ∞

<u>REMARK</u>: ABILITY TO ABSORB ∞'s INTO COUNTER-TERMS IS IMPORTANT.

YOU CAN ALWAYS DO THIS.

<u>BUT</u>: SOMETIMES YOU HAVE TO DO THIS "A LOT"

QUANTUM GRAVITY $\longrightarrow$ <u>NON RENORMALIZABLE</u>: ALL AMPLITUDES ~~[illegible]~~ REQUIRE c.t. (DIVERGE)
THIS IS "BAD" (not really): NEED ∞ MEASUREMENTS TO SPECIFY THE THEORY!

<u>RENORMALIZABLE</u> : ONLY FINITE # MEAS (# COND)

<u>SUPER REN</u>: ONLY FINITE # of <u>DIAGRAMS</u> DIVERGE
(vs. FINITE # AMP, BUT DIV AMP CAN HAVE DIVERGENCES @ ALL ORDERS IN PERT. THY)

$L = I - V + 1$

IN YOUR HOMEWORK YOU WILL BE ASKED TO COMMENT ON THE SUPERFICIAL DEG of DIVERGENCE of ϕ^4 IN d DIM

↳ think about IMPLICATIONS ON RENORMALIZABILITY.

~~PSET~~ EXERCISE: CONVINCE YOURSELF THAT THE RENORMALIZABILITY of A THEORY CAN BE READ off FROM THE DIMENSION of THE COUPLING CONSTANTS!

btw $L = I - V + 1$

BTW: $[g]$ in $g(\partial\phi)^2\phi^2$?

LOTS (more than expected) of people got this wrong. Now IT'S IMPORTANT.

Ex. 2: PLAY W/ ARBITRARY POLYNOMIAL INTERACTION IN ARB DIM!

Big Picture

· the most beautiful idea in all of physics.

↳ RENORMALIZATION GROUP

Divergence IN A THY, BUT NOT IN NATURE

↳ THEORY BREAKS DOWN AT HIGH SCALES (PRESUMABLY THY is GOOD @ LOW SCALES)

↑ CAN HAVE MANY GOOD THEORIES DESCRIBING SAME PHYSICS @ LOW E, only DIFFER @ Hi E.

SO THE THEORY COMES W/ A CUT OFF, Λ. ← loosely DEFINED
SOME CHARACTERISTIC ENERGY SCALE WHERE THE THEORY BREAKS DOWN.

↳ Λ CAN ALSO SERVE AS A REGULATOR: $\int d^4k \to \int d\Omega \int^\Lambda k^3\,dk$

CALL THIS AN EFFECTIVE THEORY (EFT)

MODERN VIEWPOINT: ALL THEORIES ARE EFFECTIVE.

e.g. $\frac{1}{3!}\phi\varphi^3$ w/ HEAVY ϕ vs. φ^6 THEORY, ← turns out to be non renormalizable → EXERCISE: CONVINCE YOURSELF!

@ ~~ABOUT~~ $E \ll M_\phi$, THEORIES MATCH.
BUT NO LONGER MATCH @ $E \sim M_\phi$. (no divergence - they just had a cutoff)

$\boxed{\text{BTW}}$ → PERT THY: WHEN IS A COUPLING SMALL?

A CUTOFF IS A <u>SCALE</u>.

→ BREAKS <u>SCALE INVARIANCE</u>

↳ RG BOILS DOWN TO THE MANIFESTATION of BROKEN SCALE INVARIANCE. MOST BEAUTIFUL EXPOSITIONS ARE BY COLEMAN (perhaps a little technical for this audience)

EASIER: STEVENSON, <u>DIM ANALYSIS IN FIELD THY</u>

THEORY is NOW DEFINED BY:

- PARTICLE CONTENT
- PARTICLE MASSES (+ ASSUME CANONICAL NORM)
  - ↓ M ↓ Z
- COUPLINGS (λ, or more generally g_i)
  - ↑ physical values ⟺ renormaliz conditions
- CUTOFF SCALE ← Q: is THIS PHYSICAL? Do WE MEASURE IT?

"No" ... in quotation marks.

↑ not phys b/c we never directly measure it. BUT SORT OF PHYSICAL BECAUSE IT HAS TO BE THERE eg WHEN THERE ARE DIVERGENCES.

... DOESN'T APPEAR IN $\mathcal{L}$

... BACK TO "ONE TRUE LAGRANGIAN" IDEA:

HOW IS LOW-E PHYS DEPENDENT ON CUTOFF?

↳ a chef does not need to know gauge thy!

"DECOUPLING"

PHYSICS @ LOW SCALES SHOULD BE INSENSITIVE TO PHYSICS @ Λ

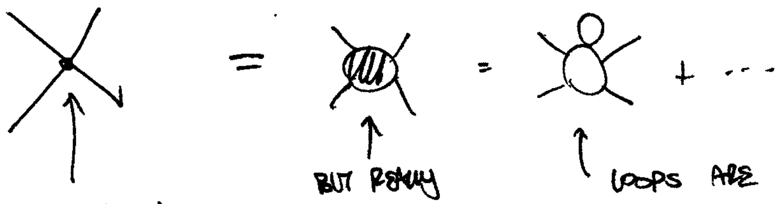
EXPERIMENT

same philosophy in this statement

SO IF I CHANGE $\Lambda \rightarrow$ ~~[illegible]~~ $\left(1 - \frac{\delta\Lambda}{\Lambda}\right)\Lambda$

ALMOST A SCALE TRANSF

then my "experiments" shouldn't get different results.



measure λ

BUT REALLY FULL OF Q'M EFFECTS

LOOPS ARE (POTENTIALLY) DIVERGENT DEPEND ON Λ AS CUTOFF.

BUT λ IS $\lambda_{\underline{PHYS}}$, SHOULDN'T CHANGE.

[this is all <u>HEURISTIC</u>, don't take too seriously]

I CAN <u>UNDO</u> THE CHANGE IN Λ BY DOING A SCALE TRANSFORMATION ON MY THEORY.

in fact, what would this look like?

$$\phi \rightarrow Z^{1/2}\phi \quad !$$

<u>READ</u>: ZEE 2ND ED §III.1, PARAM. OF IGNORANCE

<u>WHAT HAPPENS</u>: IN ORDER TO FIX LOW E PHYS W/RT CHANGING Λ (CHANGING SCALE), THE OTHER PARAMETERS (m, g_i) MUST CHANGE.

↳ COUPLING CONSTANTS "RUN" $\longrightarrow$ VALUE DEPENDS ON ENERGY SCALE @ WHICH YOU PROBE THY!

$$\text{eg } \alpha = 1/137 \longrightarrow \sim 1/128 \text{ @ } M_Z$$

<u>RENORMALIZ COND</u>: BOUNDARY CONDITIONS for DIFF EQ. DESCRIBING THE "FLOW" OF PARAMETERS IN THEORY SPACE!

RG flow

SOME TRIVIAL EXAMPLE OF "CLASSICAL" RG FLOW

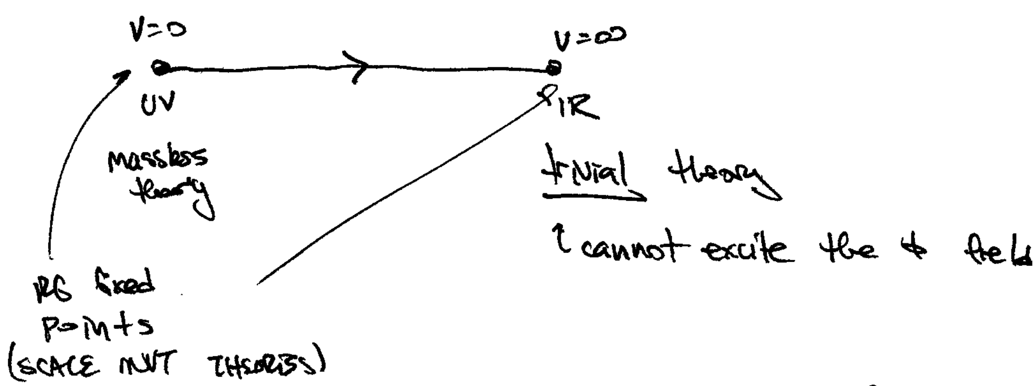
SCALAR THY W/ A MASS m
THINK OF m LIKE A COUPLING

WHEN IS THE COUPLING "SMALL"? MUST BE COMPARED TO A SCALE @ WHICH WE OBSERVE THE THEORY, μ.
THE REAL PARAMETER OF THE THEORY IS

$\nu \equiv m/\mu \leftarrow$ DIMLESS,

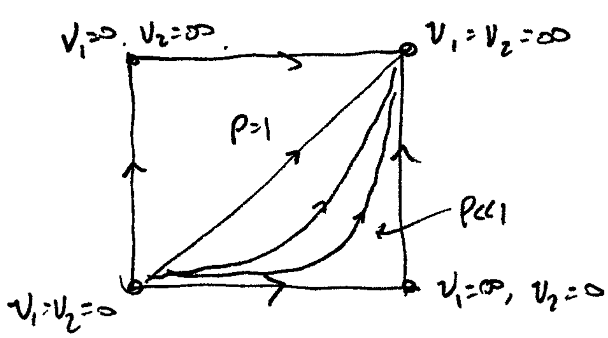
@ HIGH ENERGIES $\mu \gg m$ & $\nu \approx 0$
THEORY IS BASICALLY MASSLESS.

WE CAN "FLOW" FROM HI TO LOW SCALES, $\mu \ll m$.



m_i/μ

THEORY W/ 2 PARTICLES: ν_1, ν_2



$\rho = m_1/m_2$