

Charmonium: Above the Open Charm Threshold

Estia Eichten
Fermilab

- ◆ QCD Dynamics Near Threshold
- ◆ The ψ (3770)
- ◆ Other Charmonium States
- ◆ X(3872)
- ◆ Y(4260) and Beyond
- ◆ To Do List

QCD Dynamics Near Threshold

- QCD dynamics is much richer than present phenomenological models -
Lattice QCD
- Gluon/String dynamics
- Light quark loops and strong decays

Below Threshold

Narrow states allow precise experimental probes of the subtle nature of QCD

NRQCD: $\langle v^2/c^2 \rangle \approx 0.3$

Potential models:

- masses
- spin splittings
- EM transitions
- hadronic transitions
- direct decays

Lattice QCD:

- masses
 - spin splittings
 - EM transitions
- variety of approaches

Supports and will supplant potential models

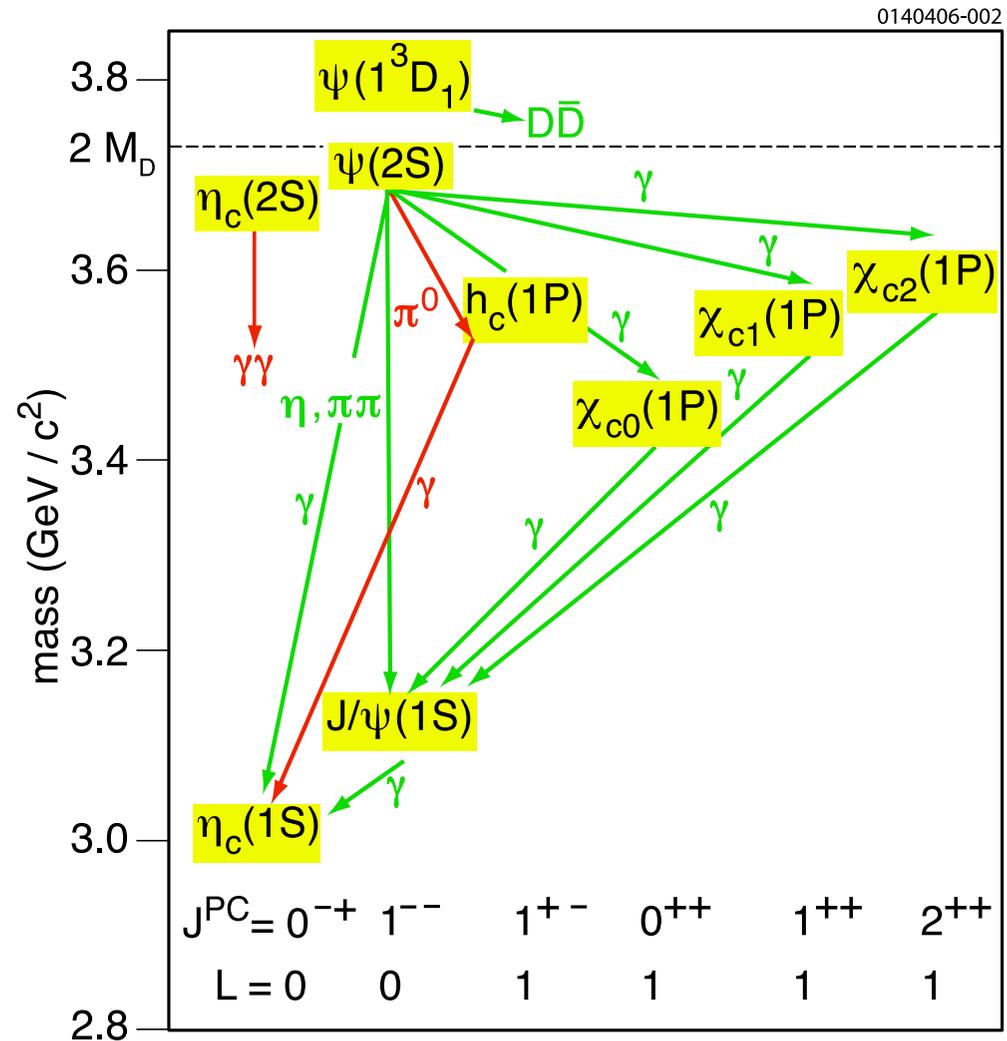


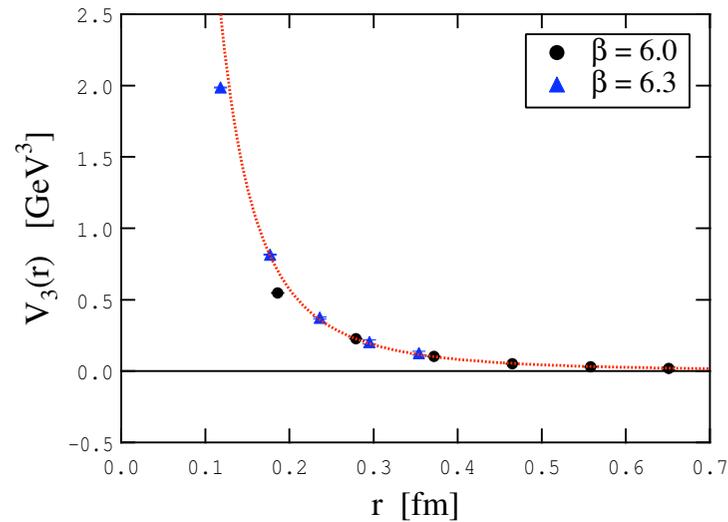
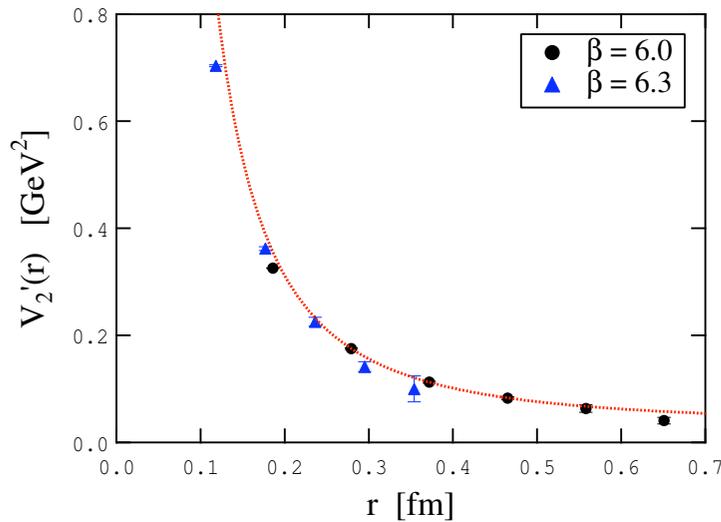
FIGURE 8. Transitions among low-lying charmonium states. From Ref. [65].

Multi-level algorithm allows **lattice determination** of potentials with unprecedented precision

Y. Koma, M. Koma and H. Wittig

Quenched

[PRL 97 (2006) 122003]



Heavy quark potential

To $O(1/m^2)$

$$\begin{aligned}
 V(r) = & V^{(0)}(r) + \left(\frac{1}{m_1} + \frac{1}{m_2} \right) V^{(1)}(r) + O\left(\frac{1}{m^2} \right) \\
 & + \left(\frac{\vec{s}_1 \vec{l}_1}{2m_1^2} - \frac{\vec{s}_2 \vec{l}_2}{2m_2^2} \right) \left(\frac{V^{(0)}(r)'}{r} + 2 \frac{V^{(1)}(r)'}{r} \right) + \left(\frac{\vec{s}_2 \vec{l}_1}{2m_1 m_2} - \frac{\vec{s}_1 \vec{l}_2}{2m_1 m_2} \right) \frac{V^{(2)}(r)'}{r} \\
 & + \frac{1}{m_1 m_2} \left(\frac{(\vec{s}_1 \vec{r})(\vec{s}_2 \vec{r})}{r^2} - \frac{\vec{s}_1 \vec{s}_2}{3} \right) V^{(3)}(r) + \frac{\vec{s}_1 \vec{s}_2}{3m_1 m_2} V^{(4)}(r)
 \end{aligned}$$

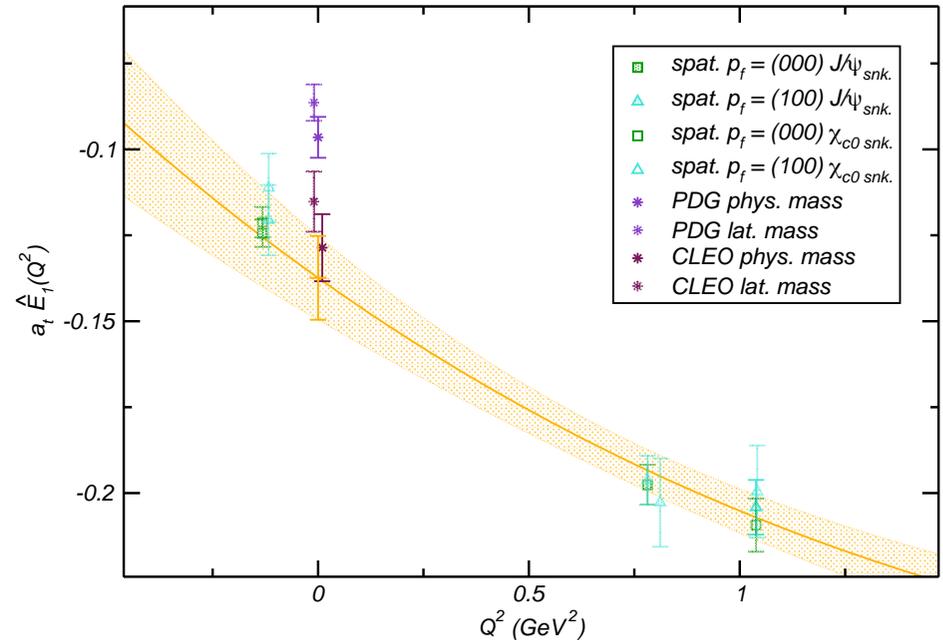
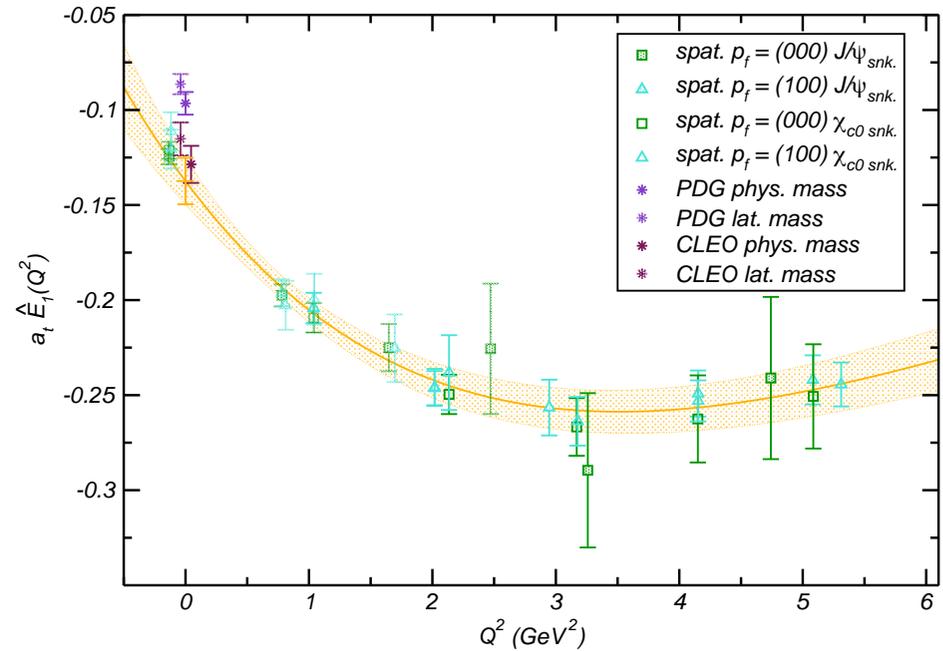
Fine and hyper-fine splitting

Recent LQCD results

Dudek, Edwards, Richards

[PR D73:07450 (2006)]

$\chi_{c0} \rightarrow J/\psi\gamma$



| E1 | $\chi_{c0} \rightarrow J/\psi\gamma$ | $\chi_{c1} \rightarrow J/\psi\gamma$ | $h_c \rightarrow \eta_c\gamma$ |
|--|--------------------------------------|--------------------------------------|--------------------------------|
| β/MeV | 542(35) | 555(113) | 689(133) |
| ρ/MeV | 1080(130) | 1650(590) | ∞ |
| $\Gamma_{\text{phys.mass}}^{\text{lat.mass}}/\text{keV}$ | 288(60) | 600(178) | 663(132) |
| $\Gamma_{\text{CLEO}}^{\text{PDG}}/\text{keV}$ | 232(41) | 487(122) | 601(55) |
| | 115(14) | 303(44) | - |
| | 204(31) | 364(31) | - |

| M1 | $J/\psi \rightarrow \eta_c\gamma$ | M2 | $\chi_{c1} \rightarrow J/\psi\gamma$ |
|--|-----------------------------------|--------------------|--------------------------------------|
| β/MeV | 540(10) | β/MeV | 617(142) |
| $\Gamma_{\text{phys.mass}}^{\text{lat.mass}}/\text{keV}$ | 1.61(7) | $\frac{M2}{E1}$ | -0.199(121) |
| $\Gamma_{\phi\phi}^{\text{PDG}}/\text{keV}$ | 2.57(11) | expt. | -0.002($^{+8}_{-17}$) |
| | 1.14(33) | | |
| | 2.9(1.5) | | |

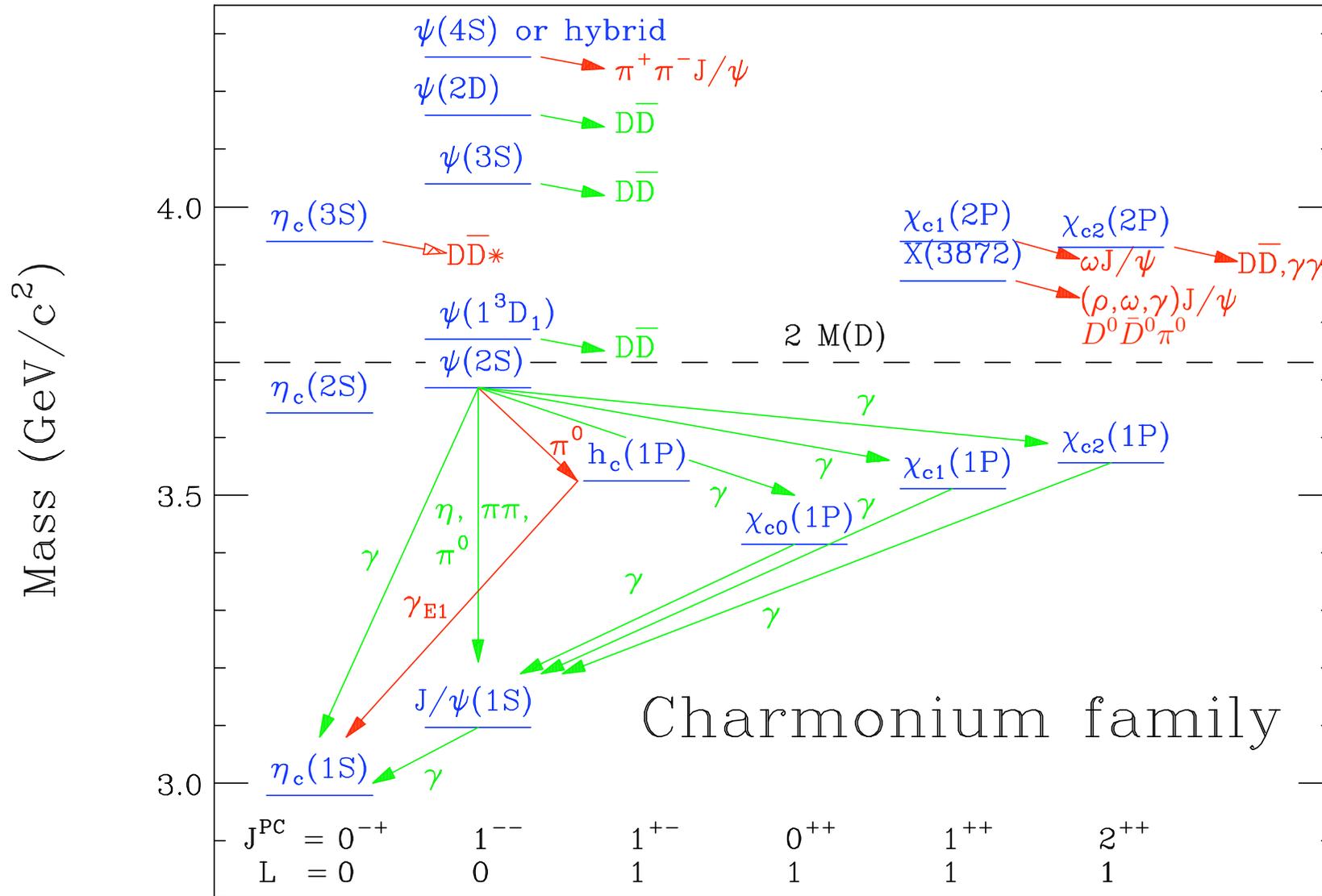
Promising but still work to do:

quenched
ground states
extrapolations

$Q^2 \rightarrow 0$

$a \rightarrow 0$

Above Threshold



Hard to extract states in the threshold region in LQCD

Excited charmonium states

Strong decay channels -- resonances:

Nearby Thresholds

TABLE I: Thresholds for decay into open charm and nearby hidden-charm thresholds.

| Channel | Threshold Energy (MeV) |
|--|------------------------|
| $D^0 \bar{D}^0$ | 3729.4 |
| $D^+ D^-$ | 3738.8 |
| $D^0 \bar{D}^{*0}$ or $D^{*0} \bar{D}^0$ | 3871.5 |
| $\rho^0 J/\psi$ | 3872.7 |
| $D^\pm D^{*\mp}$ | 3879.5 |
| $\omega^0 J/\psi$ | 3879.6 |
| $D_s^+ D_s^-$ | 3936.2 |
| $D^{*0} \bar{D}^{*0}$ | 4013.6 |
| $D^{*+} D^{*-}$ | 4020.2 |
| $\eta' J/\psi$ | 4054.7 |
| $f^0 J/\psi$ | ≈ 4077 |
| $D_s^+ \bar{D}_s^{*-}$ or $D_s^{*+} \bar{D}_s^-$ | 4080.0 |
| $a^0 J/\psi$ | 4081.6 |
| $\varphi^0 J/\psi$ | 4116.4 |
| $D_s^{*+} D_s^{*-}$ | 4223.8 |

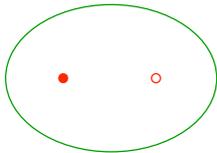
Gluon/String Dynamics

Heavy Quark Limit - Static Energy

Short distance: Perturbative QCD

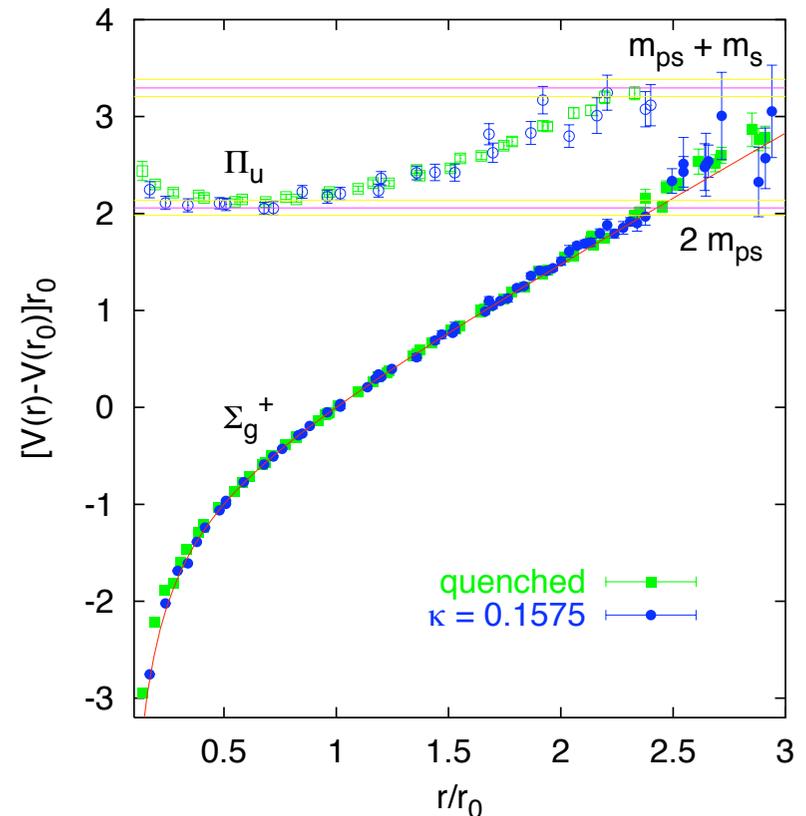
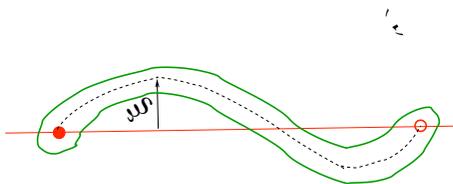
singlet: $-4/3 \alpha_s / r$

octet: $2/3 \alpha_s / r$ gluelumps



Large distance: String

σr NG string behaviour



Operators for excited gluon states

TABLE I: Operators to create excited gluon states for small $q\bar{q}$ separation R are listed. \mathbf{E} and \mathbf{B} denote the electric and magnetic operators, respectively. The covariant derivative \mathbf{D} is defined in the adjoint representation [10].

| gluon state | J | operator |
|-----------------|-----|---|
| $\Sigma_g^{+'}$ | 1 | $\mathbf{R} \cdot \mathbf{E}, \quad \mathbf{R} \cdot (\mathbf{D} \times \mathbf{B})$ |
| Π_g | 1 | $\mathbf{R} \times \mathbf{E}, \quad \mathbf{R} \times (\mathbf{D} \times \mathbf{B})$ |
| Σ_u^- | 1 | $\mathbf{R} \cdot \mathbf{B}, \quad \mathbf{R} \cdot (\mathbf{D} \times \mathbf{E})$ |
| Π_u | 1 | $\mathbf{R} \times \mathbf{B}, \quad \mathbf{R} \times (\mathbf{D} \times \mathbf{E})$ |
| Σ_g^- | 2 | $(\mathbf{R} \cdot \mathbf{D})(\mathbf{R} \cdot \mathbf{B})$ |
| Π_g' | 2 | $\mathbf{R} \times ((\mathbf{R} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\mathbf{R} \cdot \mathbf{B}))$ |
| Δ_g | 2 | $(\mathbf{R} \times \mathbf{D})^i (\mathbf{R} \times \mathbf{B})^j + (\mathbf{R} \times \mathbf{D})^j (\mathbf{R} \times \mathbf{B})^i$ |
| Σ_u^+ | 2 | $(\mathbf{R} \cdot \mathbf{D})(\mathbf{R} \cdot \mathbf{E})$ |
| Π_u' | 2 | $\mathbf{R} \times ((\mathbf{R} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\mathbf{R} \cdot \mathbf{E}))$ |
| Δ_u | 2 | $(\mathbf{R} \times \mathbf{D})^i (\mathbf{R} \times \mathbf{E})^j + (\mathbf{R} \times \mathbf{D})^j (\mathbf{R} \times \mathbf{E})^i$ |

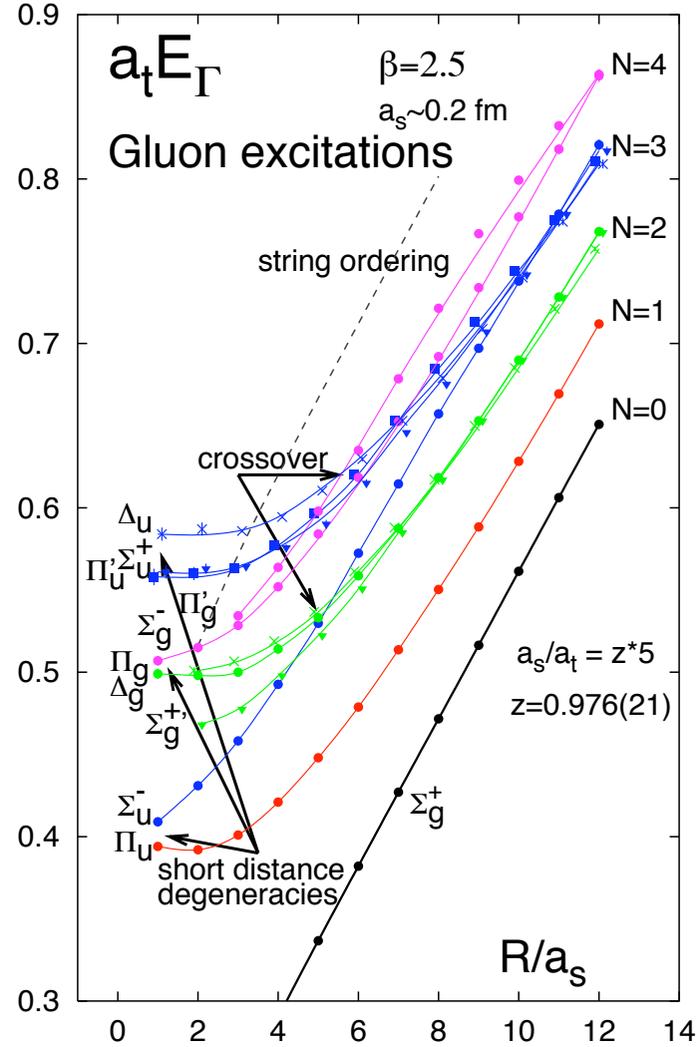


FIG. 2: Short-distance degeneracies and crossover in the spectrum. The solid curves are only shown for visualization. The dashed line marks a lower bound for the onset of mixing effects with glueball states which requires careful interpretation.

Hybrid Potentials

Solve the Schoedinger Equation for each potential

$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \left\{ \frac{\langle L_{Q\bar{Q}}^2 \rangle}{2\mu r^2} + V_{Q\bar{Q}}(r) \right\} u(r) = E u(r),$$

where

$$\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad \mathbf{S} = \mathbf{s}_Q + \mathbf{s}_{\bar{Q}}, \quad \mathbf{L} = \mathbf{L}_{Q\bar{Q}} + \mathbf{J}_g$$

$$\langle L_{Q\bar{Q}}^2 \rangle = L(L+1) - 2\Lambda^2 + \langle \mathbf{J}_g^2 \rangle$$

eigenstates

$$|LSJM; \lambda \eta\rangle + \varepsilon |LSJM; -\lambda \eta\rangle$$

where $\varepsilon = \pm 1$, $\Lambda = |\lambda|$

$$P = \varepsilon (-1)^{L+\Lambda+1}, \quad C = \eta \varepsilon (-1)^{L+S+\Lambda}$$

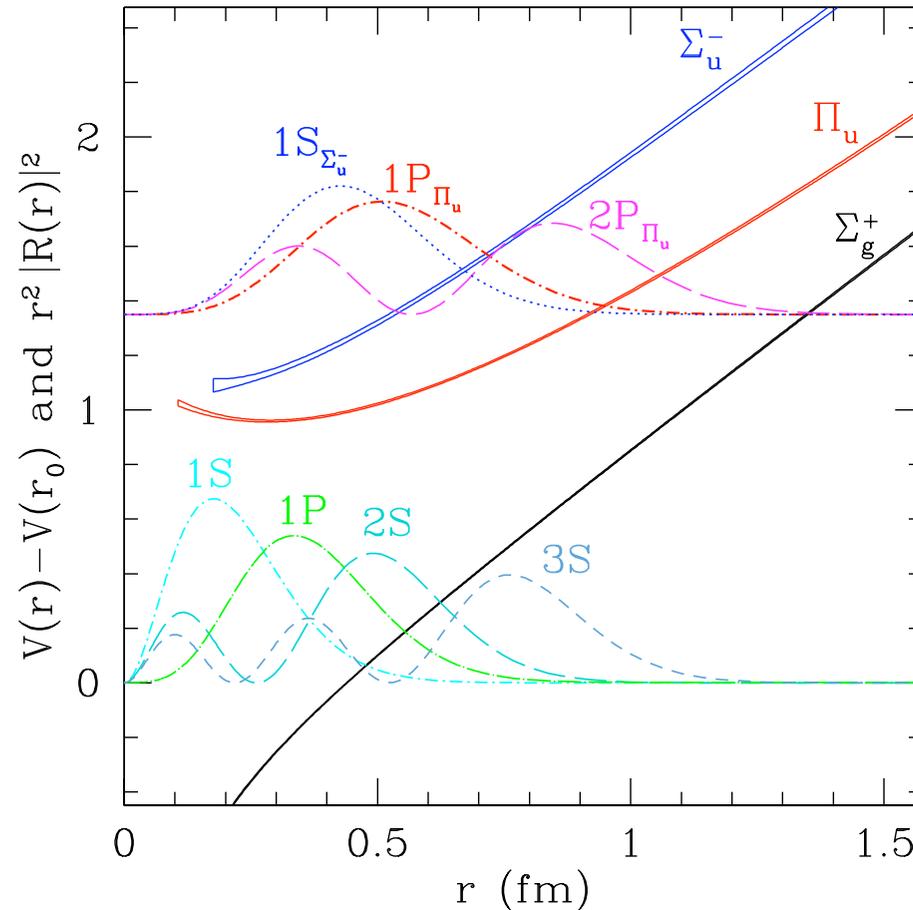
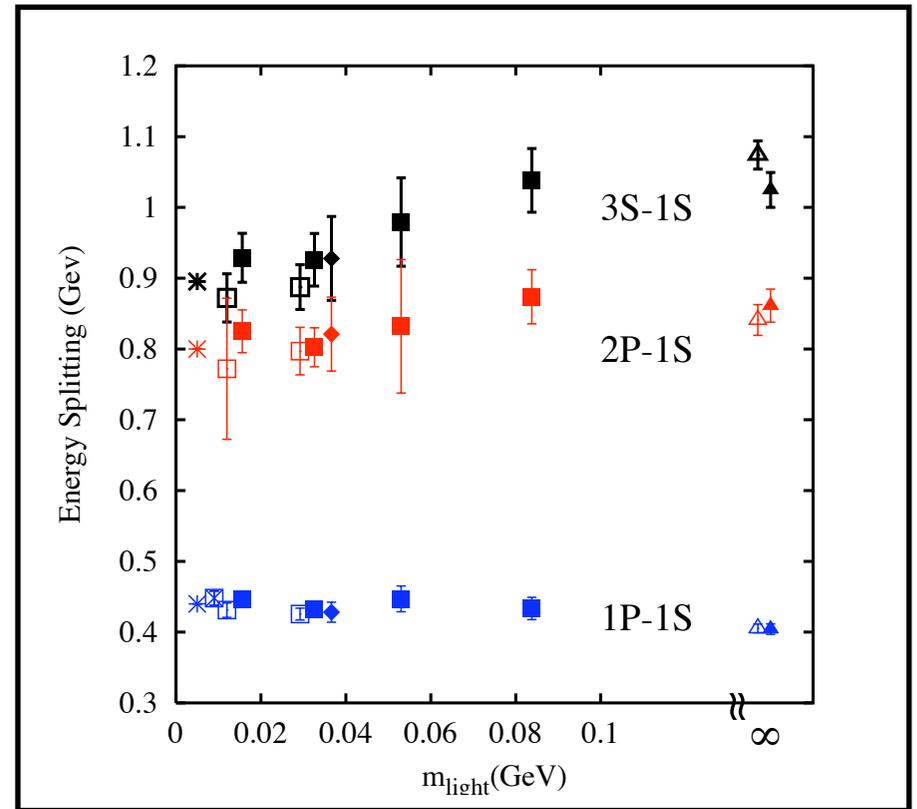
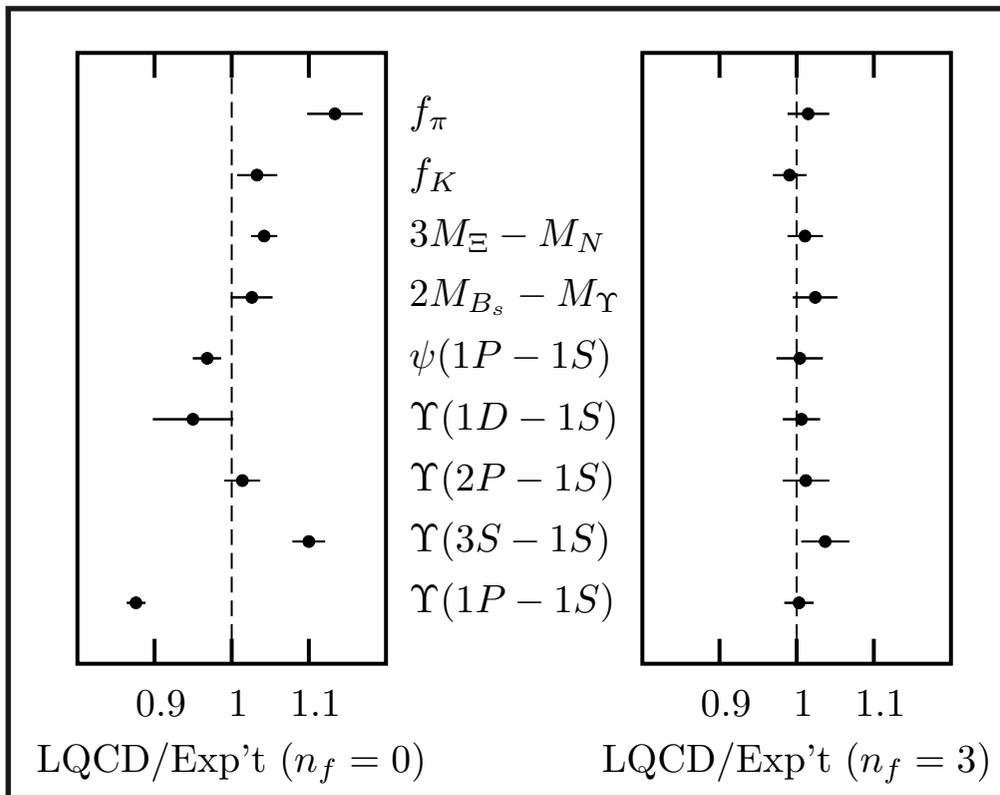


Figure 2. Wavefunctions and potentials for the various hybrid/meson states.

Light quark loops

Effects on spectrum clearly seen in LQCD



C.T. H. Davies et al. [HPQCD, Fermilab Lattice, MILC, and UKQCD Collaborations], PRL 92, 022001 (2004)

Including Light Quark Effects

$$[\mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_I]\psi = \omega\psi$$

$$\mathcal{H}_0 \quad Q\bar{Q}$$

NRQCD (without couplings light quarks)

$$\mathcal{H}_I \quad Q\bar{Q} \longrightarrow Q\bar{q} + q\bar{Q}$$

light quark pair creation

Cornell model (CCCM)

$$\mathcal{H}_I = \frac{3}{8} \sum_a \int : \rho_a(\mathbf{r}) V(\mathbf{r} - \mathbf{r}') \rho_a(\mathbf{r}') : d^3r d^3r'$$

Vacuum Pair Creation model
(QPC)

$$\mathcal{H}_I = \gamma \int \bar{\psi}\psi(\mathbf{r}) d^3r$$

$$\mathcal{H}_2 \quad Q\bar{q} + q\bar{Q}$$

meson pair interactions

Lattice effort to extract couplings

$$C(t) = \begin{pmatrix} C_{QQ}(t) & C_{QB}(t) \\ C_{BQ}(t) & C_{BB}(t) \end{pmatrix} = e^{-2m_Q t} \begin{pmatrix} \text{[diagram: empty box]} & \sqrt{n_f} \text{[diagram: box with wavy line]} \\ \sqrt{n_f} \text{[diagram: box with wavy line]} & -n_f \text{[diagram: box with wavy line]} + \text{[diagram: two wavy lines]} \end{pmatrix}, \quad (1)$$

transition amplitude

$$g = \left. \frac{dC_{QB}(t)}{dt} \right|_{t=0} \frac{1}{\sqrt{C_{BB}(0)C_{QQ}(0)}}.$$

difficult to extract accurately

G.S. Bali, H. Neff, T. Düssel, T. Lippert and K. Schilling [SESAM Collaboration], *Phys. Rev. D* **71**, 114513 (2005) [arXiv:hep-lat/0505012].

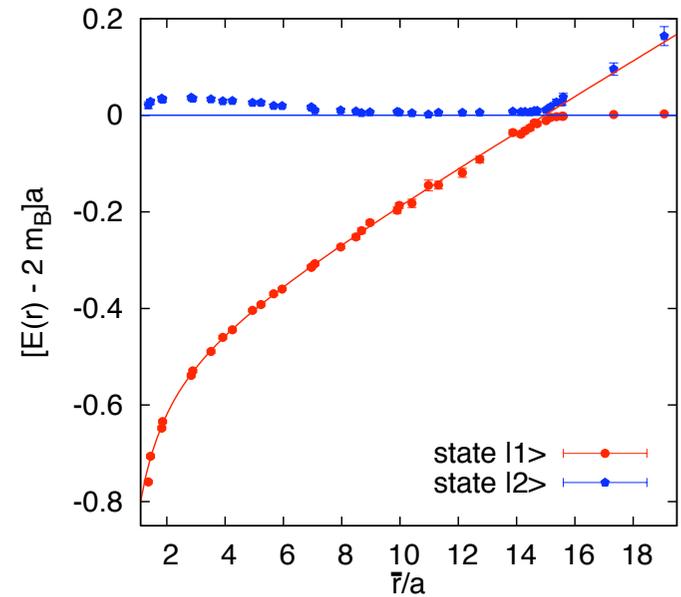


FIG. 13: The two energy levels, as a function of \bar{r} , normalized with respect to $2m_B$ (horizontal line). The curve corresponds to the three parameter fit to $E_1(\bar{r})$, Eqs. (80)–(82), for $0.2 \text{ fm} \leq \bar{r} \leq 0.9 \text{ fm} < r_c$.

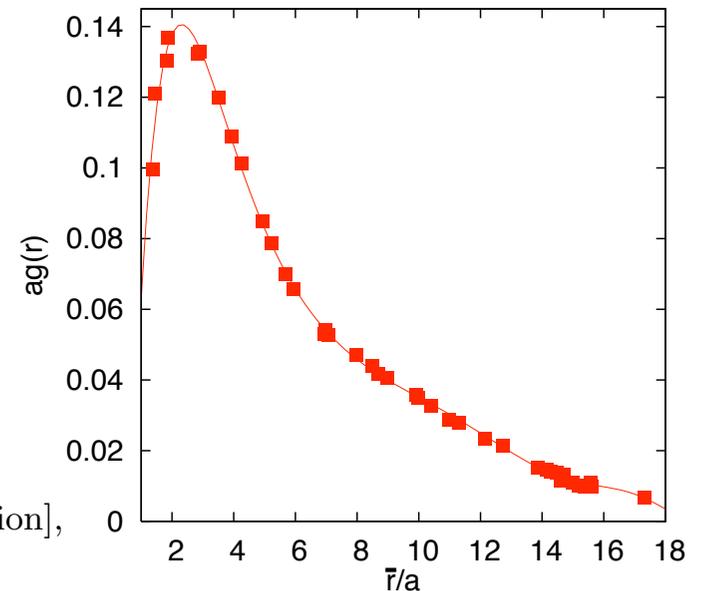


FIG. 18: The transition rate g between $|B\rangle$ and $|Q\rangle$ states, as a function of \bar{r} .

Coupling to open-charm channels

Phenomenological approach:

$$\mathcal{H}_I = \frac{3}{8} \sum_a \int : \rho_a(\mathbf{r}) V(\mathbf{r} - \mathbf{r}') \rho_a(\mathbf{r}') : d^3r d^3r'$$

$$\rho^a = \bar{c} \gamma^0 t^a c + \bar{q} \gamma^0 t^a q$$

CCCM

Calculate pair-creation amplitudes,

Evaluate $\langle {}^3 D_2 | \mathcal{H}_I | D \bar{D}^* \rangle$, etc.

ELQ 2004

Solve coupled-state system

$$\psi = \psi_0 + \psi_2$$

$$\bar{c}c \quad \bar{D}D$$

solve $\left[\mathcal{H}_0 + \mathcal{H}_I^\dagger \frac{1}{\omega - \mathcal{H}_2 + i\epsilon} \mathcal{H}_I \right] \psi_0 = \omega \psi_0$

for ω and ψ_0

Statistical Factors in Strong Decays

TABLE II: Statistical recoupling coefficients C , defined by Eq. D19 of Ref. [10], that enter the calculation of charmonium decays to pairs of charmed mesons. Paired entries correspond to $\ell = L - 1$ and $\ell = L + 1$.

| State | $D\bar{D}$ | $D\bar{D}^*$ | $D^*\bar{D}^*$ |
|---------|--------------------|--------------------------------|----------------------------------|
| 1S_0 | – : 0 | – : 2 | – : 2 |
| 3S_1 | – : $\frac{1}{3}$ | – : $\frac{4}{3}$ | – : $\frac{7}{3}$ |
| 3P_0 | 1 : 0 | 0 : 0 | $\frac{1}{3}$: $\frac{8}{3}$ |
| 3P_1 | 0 : 0 | $\frac{4}{3}$: $\frac{2}{3}$ | 0 : 2 |
| 1P_1 | 0 : 0 | $\frac{2}{3}$: $\frac{4}{3}$ | $\frac{2}{3}$: $\frac{4}{3}$ |
| 3P_2 | 0 : $\frac{2}{5}$ | 0 : $\frac{6}{5}$ | $\frac{4}{3}$: $\frac{16}{15}$ |
| 3D_1 | $\frac{2}{3}$: 0 | $\frac{2}{3}$: 0 | $\frac{4}{15}$: $\frac{12}{5}$ |
| 3D_2 | 0 : 0 | $\frac{6}{5}$: $\frac{4}{5}$ | $\frac{2}{5}$: $\frac{8}{5}$ |
| 1D_2 | 0 : 0 | $\frac{4}{5}$: $\frac{6}{5}$ | $\frac{4}{5}$: $\frac{6}{5}$ |
| 3D_3 | 0 : $\frac{3}{7}$ | 0 : $\frac{8}{7}$ | $\frac{8}{5}$: $\frac{29}{35}$ |
| 3F_2 | $\frac{3}{5}$: 0 | $\frac{4}{5}$: 0 | $\frac{11}{35}$: $\frac{16}{7}$ |
| 3F_3 | 0 : 0 | $\frac{8}{7}$: $\frac{6}{7}$ | $\frac{4}{7}$: $\frac{10}{7}$ |
| 1F_3 | 0 : 0 | $\frac{6}{7}$: $\frac{8}{7}$ | $\frac{6}{7}$: $\frac{8}{7}$ |
| 3F_4 | 0 : $\frac{4}{9}$ | 0 : $\frac{10}{9}$ | $\frac{12}{7}$: $\frac{46}{63}$ |
| 3G_3 | $\frac{4}{7}$: 0 | $\frac{6}{7}$: 0 | $\frac{22}{63}$: $\frac{20}{9}$ |
| 3G_4 | 0 : 0 | $\frac{10}{9}$: $\frac{8}{9}$ | $\frac{2}{3}$: $\frac{4}{3}$ |
| 1G_4 | 0 : 0 | $\frac{8}{9}$: $\frac{10}{9}$ | $\frac{8}{9}$: $\frac{10}{9}$ |
| 3G_5 | 0 : $\frac{5}{11}$ | 0 : $\frac{12}{11}$ | $\frac{16}{9}$: $\frac{67}{99}$ |



Effects on the spectrum

Coupling to virtual channels induces spin-dependent forces in charmonium near threshold, because $M(D^*) > M(D)$



| State | Mass | Centroid | Splitting (Potential) | Splitting (Induced) |
|----------|---------------------|---------------------|--------------------------|------------------------|
| 1^1S_0 | 2979.9 ^a | 3067.6 ^b | -90.5 ^e | +2.8 |
| 1^3S_1 | 3096.9 ^a | | +30.2 ^e | -0.9 |
| 1^3P_0 | 3415.3 ^a | 3525.3 ^c | -114.9 ^e | +5.9 |
| 1^3P_1 | 3510.5 ^a | | -11.6 ^e | -2.0 |
| 1^1P_1 | 3524.4 ^f | | +0.6 ^e | +0.5 |
| 1^3P_2 | 3556.2 ^a | | +31.9 ^e | -0.3 |
| 2^1S_0 | 3638 ^a | 3674 ^b | -50.1 ^e | +15.7 |
| 2^3S_1 | 3686.0 ^a | | +16.7 ^e | -5.2 |
| 1^3D_1 | 3769.9 ^a | (3815) ^d | -40 | -39.9 |
| 1^3D_2 | 3830.6 | | 0 | -2.7 |
| 1^1D_2 | 3838.0 | | 0 | +4.2 |
| 1^3D_3 | 3868.3 | | +20 | +19.0 |
| 2^3P_0 | 3881.4 | (3922) ^d | -90 | +27.9 |
| 2^3P_1 | 3920.5 | | -8 | +6.7 |
| 2^1P_1 | 3919.0 | | 0 | -5.4 |
| 2^3P_2 | 3931 ^g | | +25 | -9.6 |
| 3^1S_0 | 3943 ^h | (4015) ⁱ | -66 ^e | -3.1 |
| 3^3S_1 | 4040 ^a | | +22 ^e | +1.0 |

The $\psi(3770)$

pdg 2007

Mass m = 3772.4 ± 1.1 MeV , (S = 1.8)
Full width Γ = 25.2 ± 1.8 MeV

Decay width in good agreement
with theory

Parameterizing the $\psi(3770)$ as a simple
mixture of $|1D\rangle$ and $|2S\rangle$ state is inadequate

Production in e^+e^- due to relativistic terms:

(a) Expansion of EM current

$$j_c^i = s_1 \psi^\dagger \sigma^i \chi + \frac{s_2}{m_c^2} \psi^\dagger \sigma^i \mathcal{D}^2 \chi \quad \text{S-wave}$$

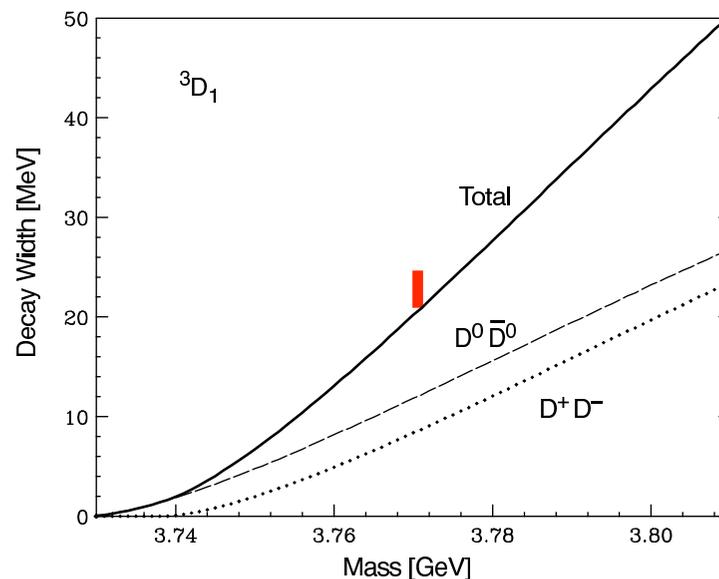
$$+ \frac{d_2}{m_c^2} \psi^\dagger \sigma^j \left[\frac{1}{2} (\mathcal{D}^i \mathcal{D}^j + \mathcal{D}^j \mathcal{D}^i) - \frac{1}{3} \delta^{ij} \mathcal{D}^2 \right] \chi + \dots \quad \text{D-wave}$$

(b) S-D mixing terms - short range

(c) Induced mixing from D^*-D mass difference - long range

$$\psi(3772) = 0.10 |2S\rangle + 0.01 e^{+0.22i\pi} |3S\rangle + \dots$$

$$+ 0.69 e^{-0.59i\pi} |1D\rangle + 0.10 e^{+0.27i\pi} |2D\rangle + \dots$$



CCC Model

Decays into open charm

The ratio, $R^{0/+}$, of $D^0\bar{D}^0$ to D^+D^- production deviates from one due to isospin violating terms:

- (a) up-down mass difference
- (b) EM interactions

-> $m(D^+) - m(D^0) = 4.78 \pm 0.10$ MeV

-> different final state interactions

$$R^{0/+}$$

| PDG07 | p^3 | CCCM |
|-----------------|-------|------|
| 1.28 ± 0.14 | 1.47 | 1.36 |

The shape of the resonance differs from the usual Breit-Wigner:

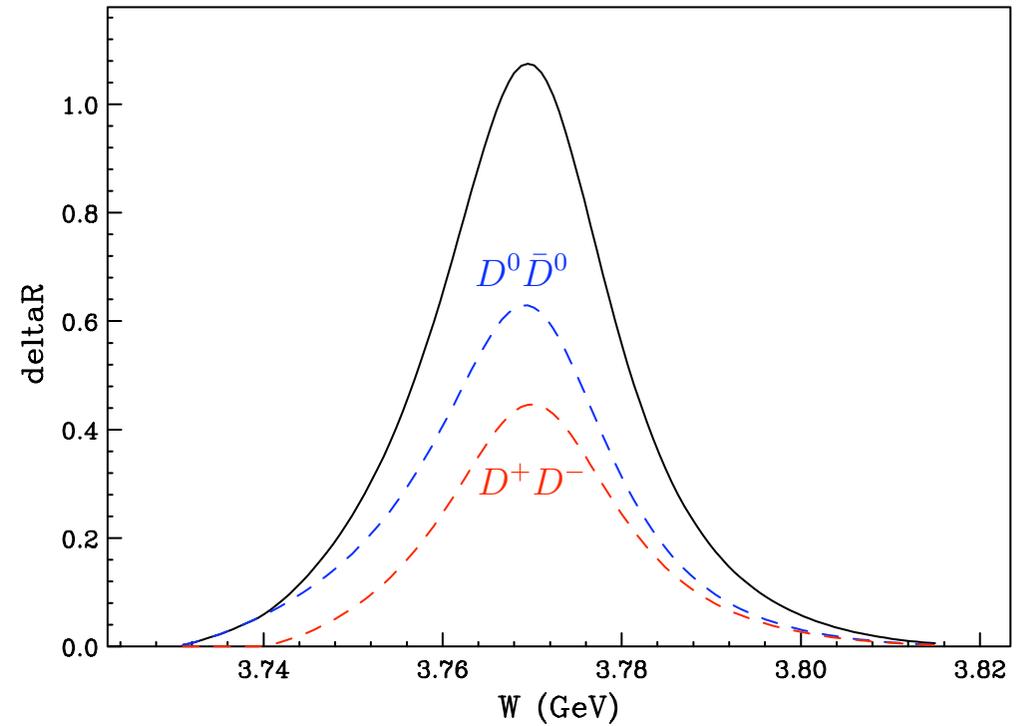
- (1) width $\Gamma(p)$ not pure p wave
- (2) interference with $2S$ state.



$$\Gamma(p) \sim A \frac{p^3}{\Lambda^2} \exp\left(-\frac{p^2}{\Lambda^2}\right)$$

$$A = .18 \quad \Lambda = .57 \text{ GeV}$$

$$p_0 = 283 \text{ MeV} \quad p_+ = 250 \text{ MeV}$$



Two very important measurements:

(1) Resonance shape

(2) Ratio of charge to neutral DD final states

$$R^{c/n} = \frac{\sigma(e^+e^- \rightarrow P^+P^-)}{\sigma(e^+e^- \rightarrow P^0\bar{P}^0)}$$

over the whole resonance region

G.P. Lepage, Phys.Rev. D **42**, 3251 (1990).

N. Byers and E. Eichten, Phys.Rev. D **42**, 3885 (1990).

R. Kaiser, A.V. Manohar, and T. Mehen, Report hep-ph/0208194, Aug. 2002 (unpublished)

M.B. Voloshin, Mod.Phys.Lett. A **18**, 1783 (2003).

M.B. Voloshin, Phys.Atom.Nucl. **68**, 771 (2005) [Yad.Fiz. **68**, 804 (2005)].

S. Dubynskiy, A. Le Yaouanc, L. Oliver, J.-C. Raynal, and M. B. Voloshin [arXiv:0704.0293]

QED
quark mass
differences

phase shifts

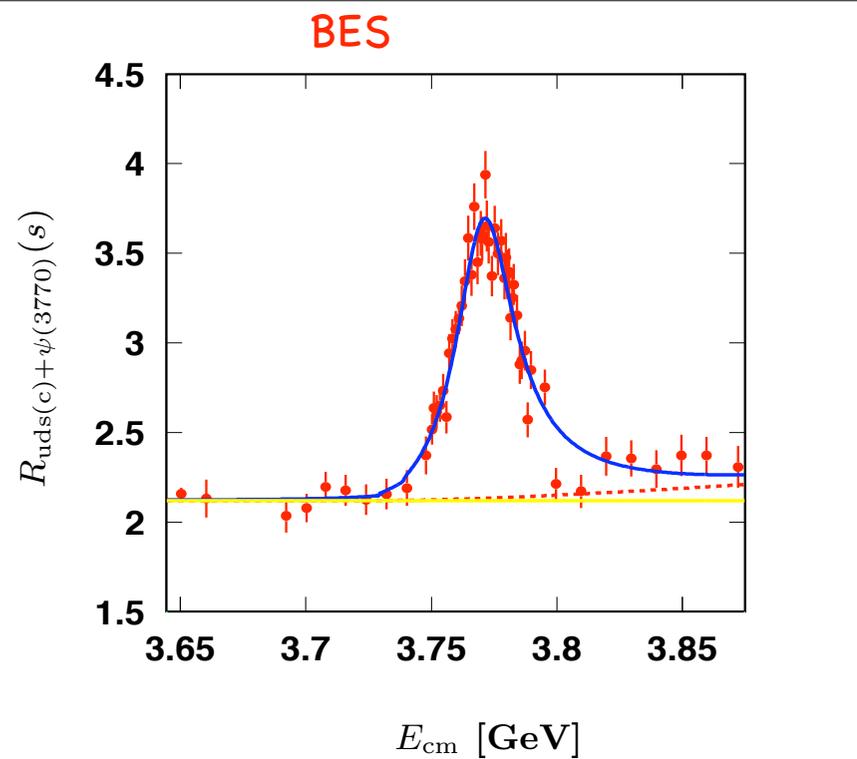


FIG. 1: The $R_{uds(c)+\psi(3770)}(s)$ versus the c.m. energy (text).

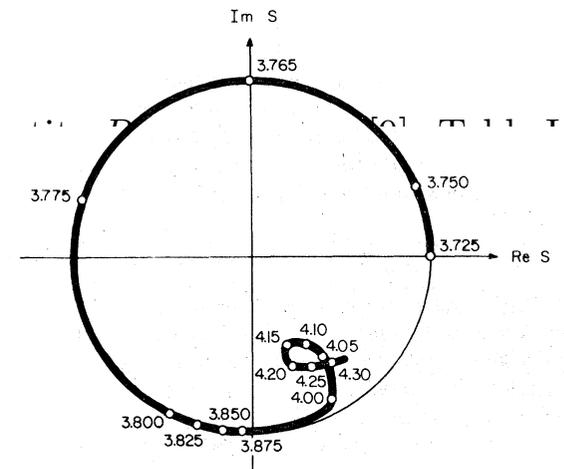


FIG. 9. Argand plot of the $D\bar{D}S$ matrix in the 1^{--} state. The rather narrow elastic 3D_1 resonance $\psi(3772)$ is clearly in evidence, as is an inelastic resonance at ~ 4.15 GeV due to the $3^3S_1 c\bar{c}$ state. The parameters are the same as in Figs. 7 and 8.

E. Eichten, K. Gottfried, T. Kinoshita, K. Lane and T.M. Yan PR D17, 3090 (1978)

Non DD decays of the $\psi(3770)$

- $X J/\psi$

Theory expectation for $\pi^+\pi^-J/\psi$: 0.1-0.7%

| | | |
|---------------------------------------|-----------------------------|------|
| $\psi'' \rightarrow \pi^+\pi^-J/\psi$ | $0.34 \pm 0.14 \pm 0.09$ | BES |
| | $0.189 \pm 0.020 \pm 0.020$ | CLEO |
| $\psi'' \rightarrow \pi^0\pi^0J/\psi$ | $0.080 \pm 0.025 \pm 0.016$ | CLEO |
| $\psi'' \rightarrow \eta^0J/\psi$ | $0.087 \pm 0.033 \pm 0.022$ | CLEO |

- ΥX_{cJ}

Good agreement with theory expectations including relativistic effects

| Mode | E_γ (MeV) [55] | Predicted (keV) | | | | | CLEO (keV) [136] |
|-------------------|--------------------------|-----------------|-----|-----|-----|--------------|---------------------|
| | | (a) | (b) | (c) | (d) | (e) | |
| $\gamma\chi_{c2}$ | 208.8 | 3.2 | 3.9 | 4.9 | 3.3 | 24 ± 4 | < 21 |
| $\gamma\chi_{c1}$ | 251.4 | 183 | 59 | 125 | 77 | 73 ± 9 | 70 ± 17 |
| $\gamma\chi_{c0}$ | 339.5 | 254 | 225 | 403 | 213 | 523 ± 12 | 172 ± 30 |

- light hadrons

No evidence for direct decays to light hadrons seen yet.

Puzzle of missing decays

$$\sigma_{\psi(3770)} = 6.38 \pm 0.08^{+0.41}_{-0.30} \text{ nb}$$

$$\sigma_{\psi(3770)} - \sigma_{\psi(3770) \rightarrow D\bar{D}} = -0.01 \pm 0.08^{+0.41}_{-0.30} \text{ nb}$$

CLEO

$$\sigma_{\psi(3770)} = 7.25 \pm 0.27 \pm 0.34 \text{ nb}$$

BES

No evidence of unexpected rates for non DD decays

| Decay Mode | $\sigma_{\psi(3770) \rightarrow f}$ [pb] | $\sigma_{\psi(3770) \rightarrow f}^{\text{up}}$ [pb] | $\mathcal{B}_{\psi(3770) \rightarrow f}^{\text{up}}$ [$\times 10^{-3}$] |
|---------------------------|---|---|--|
| $\phi\pi^0$ | $< 3.5^{tn}$ | < 3.5 | < 0.5 |
| $\phi\eta$ | $< 12.6^{tn}$ | < 12.6 | < 1.9 |
| $2(\pi^+\pi^-)$ | $7.4 \pm 15.0 \pm 2.8 \pm 0.8$ | < 32.5 | < 4.8 |
| $K^+K^-\pi^+\pi^-$ | $-19.6 \pm 19.6 \pm 3.3 \pm 2.1^z$ | < 32.7 | < 4.8 |
| $\phi\pi^+\pi^-$ | $< 11.1^{tn}$ | < 11.1 | < 1.6 |
| $2(K^+K^-)$ | $-2.7 \pm 7.1 \pm 0.5 \pm 0.3^z$ | < 11.6 | < 1.7 |
| ϕK^+K^- | $-0.5 \pm 10.0 \pm 0.9 \pm 0.1^z$ | < 16.5 | < 2.4 |
| $p\bar{p}\pi^+\pi^-$ | $-6.2 \pm 6.6 \pm 0.6 \pm 0.7^z$ | < 11.0 | < 1.6 |
| $p\bar{p}K^+K^-$ | $1.4 \pm 3.5 \pm 0.1 \pm 0.2$ | < 7.2 | < 1.1 |
| $\phi p\bar{p}$ | $< 5.8^{tn}$ | < 5.8 | < 0.9 |
| $3(\pi^+\pi^-)$ | $16.9 \pm 26.7 \pm 5.5 \pm 2.4$ | < 61.7 | < 9.1 |
| $2(\pi^+\pi^-)\eta$ | $72.7 \pm 55.0 \pm 7.3 \pm 8.2$ | < 164.7 | < 24.3 |
| $2(\pi^+\pi^-)\pi^0$ | $-35.4 \pm 24.6 \pm 6.6 \pm 4.0^z$ | < 42.3 | < 6.2 |
| $K^+K^-\pi^+\pi^-\pi^0$ | $-36.9 \pm 43.8 \pm 12.8 \pm 4.2^z$ | < 75.2 | < 11.1 |
| $2(K^+K^-)\pi^0$ | $18.1 \pm 7.7 \pm 0.7 \pm 2.0^n$ | < 31.2 | < 4.6 |
| $p\bar{p}\pi^0$ | $1.5 \pm 3.9 \pm 0.5 \pm 0.1$ | < 7.9 | < 1.2 |
| $p\bar{p}\pi^+\pi^-\pi^0$ | $26.0 \pm 13.9 \pm 2.6 \pm 3.2$ | < 49.7 | < 7.3 |
| $3(\pi^+\pi^-)\pi^0$ | $-12.7 \pm 55.9 \pm 8.7 \pm 1.8^z$ | < 92.8 | < 13.7 |

BES [hep-ex/0705.2276]

The remaining D states

$${}^3D_2 \quad {}^1D_2$$

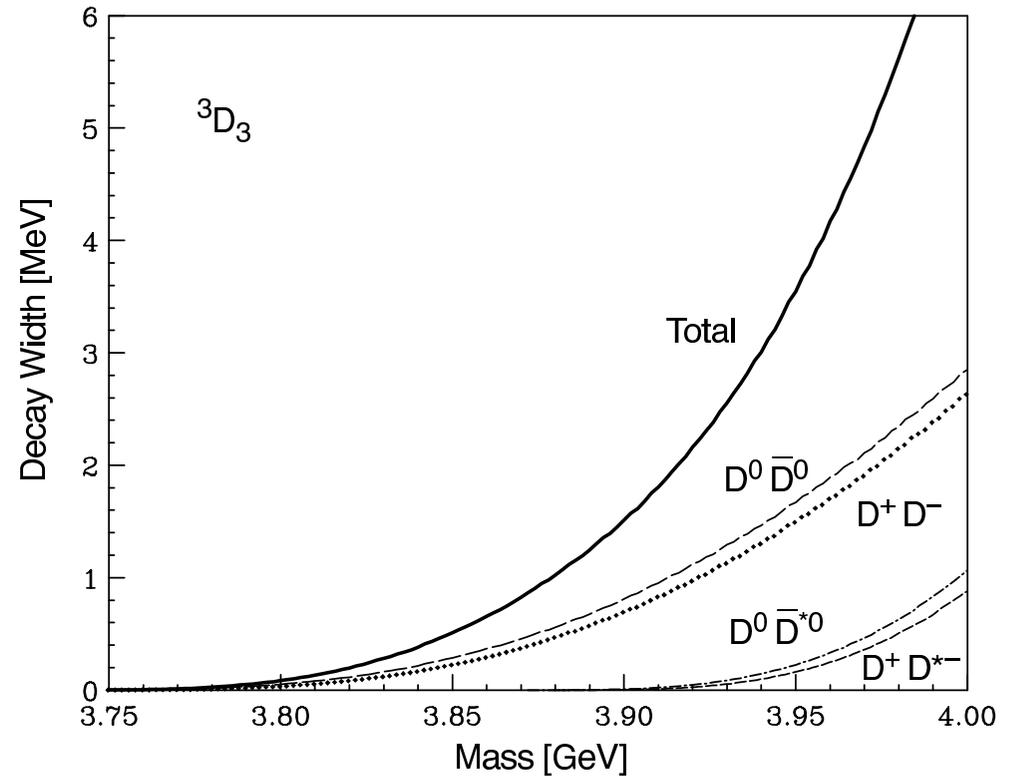
No strong decays below

$$D\bar{D}^* + \bar{D}D^* \quad \text{threshold}$$

3D_3 decay width small
search in $D\bar{D}$ channel

All remaining 1D states are
narrow

How to produce these states?



Other Charmonium States ?
X(3943), Y(3940), Z(3930), ...

Basic Questions in Charm Threshold Region:

Is it a new state? What are its properties?

Charmonium or not?

If not what? New spectroscopy?

Comments on ΔR

This rich structure arises simply from the 3S and 2D states

Interference between the 3S and 2D plays an important role.

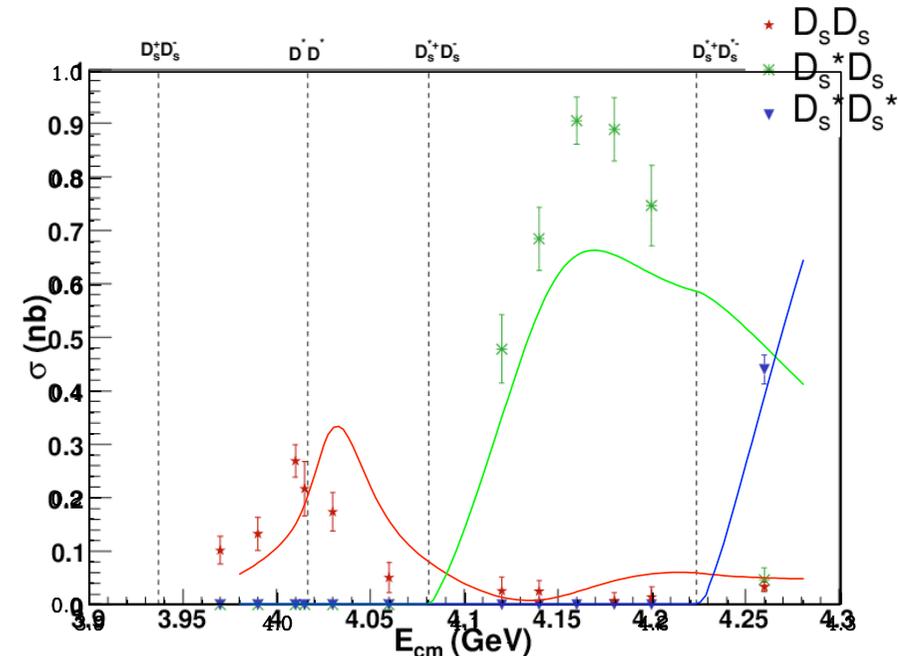
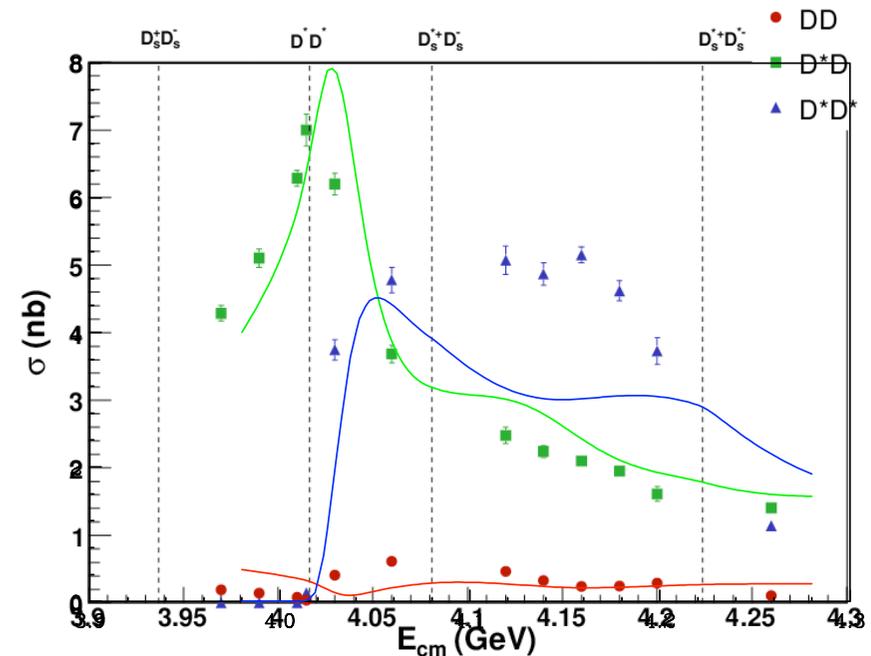
Decay amplitudes for radially excited states have oscillatory structure

The peaks for individual final states do not coincide

Determining the number and properties of the resonances is impossible without a detail decay model.

A Caution for All

CLEO-c [hep-ex/0606016](https://arxiv.org/abs/hep-ex/0606016)



Updated Cornell Coupled Channel Model

Likely charmonium states:

★ Z(3930) - Observed by Belle in $\Upsilon\Upsilon$ production

Decay mode DD

Mass = $3929 \pm 5(\text{stat}) \pm 2(\text{sys})$ MeV

Width = $29 \pm 10(\text{stat}) \pm 2(\text{sys})$ MeV

$J^{PC} = 0^{++}$ or 2^{++}

DD angular distribution favors $J=2$

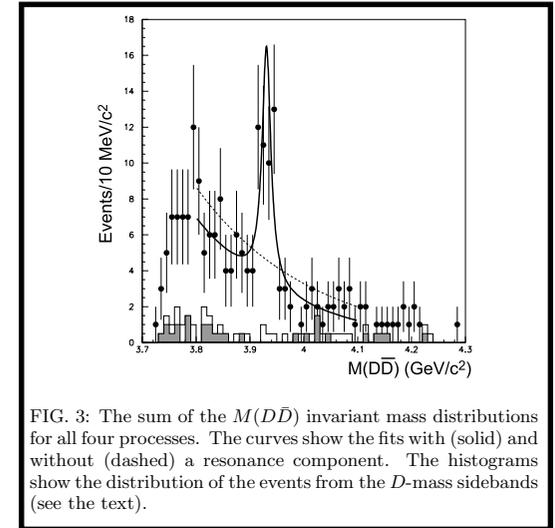


FIG. 3: The sum of the $M(D\bar{D})$ invariant mass distributions for all four processes. The curves show the fits with (solid) and without (dashed) a resonance component. The histograms show the distribution of the events from the D -mass sidebands (see the text).

[PRL 96, 082003 (2006)]



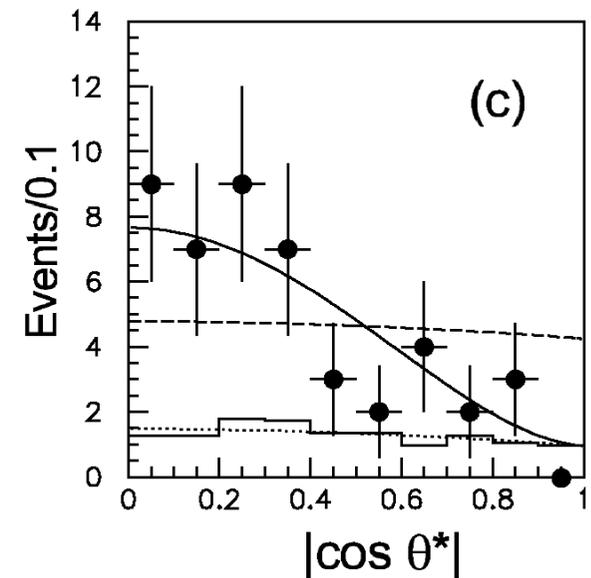
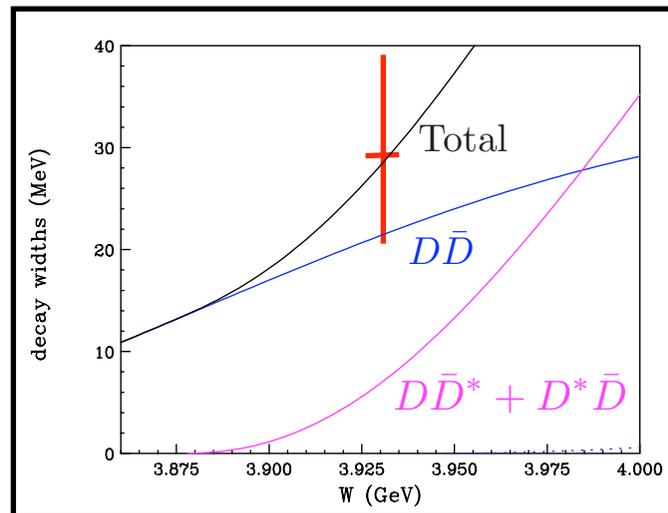
2^3P_2 χ'_{c2}

$\Gamma = 29$ MeV Model

$$\frac{D\bar{D}^* + D^*\bar{D}}{D\bar{D}} = 0.32$$

$$\frac{D^+\bar{D}^-}{D^0\bar{D}^0} = 0.95$$

$0.74 \pm 0.43 \pm 0.16$ exp



Other 2P States

| | Mass | | Spin Splittings | | Widths |
|------------|-------------------|------------|-----------------|-------|-------------------|
| 2^3P_0 | 3881.4 | | -90 | +27.9 | $D\bar{D}$ 61.5 |
| 2^3P_1 | 3920.5 | $(3922)^d$ | -8 | +6.7 | $D\bar{D}^*$ 81.0 |
| 2^1P_1 | 3919.0 | | 0 | -5.4 | $D\bar{D}^*$ 59.5 |
| ✓ 2^3P_2 | 3931 ^g | | +25 | -9.6 | $D\bar{D}$ 21.5 |
| | | | | | $D\bar{D}^*$ 7.1 |
| | | | | | total 28.8 |

★ $Y(3940)$ – Observed by Belle in B decays

Seen in decay mode $\omega J/\psi$

Significant branching fraction:

$$\mathcal{B}(B^+ \rightarrow K^+ Y(3940)) \times \mathcal{B}(Y(3940) \rightarrow \omega J/\psi) = 7.1 \pm 1.3 \pm 3.1 \times 10^{-5}$$

$$\mathcal{B}(B^+ \rightarrow K^+(c\bar{c})) \sim 6 - 10 \times 10^{-4} \text{ per mode}$$

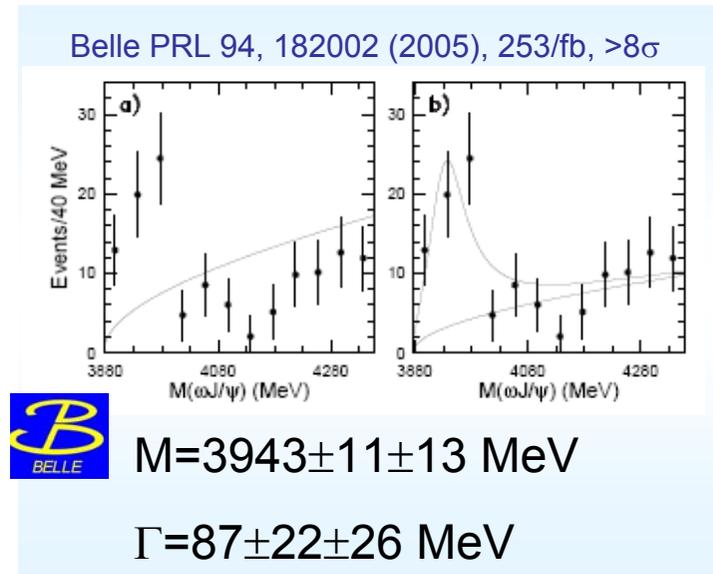
so

$$\mathcal{B}(Y(3940) \rightarrow \omega J/\psi) \sim 0.1$$

2^3P_1 interpretation:

Problems with mass and decay mode.

Main decay mode should be $D\bar{D}^*$
Present bound?



$Y(3940)$ confirmed by Babar

Babar [Cibinetto EPS 2007]

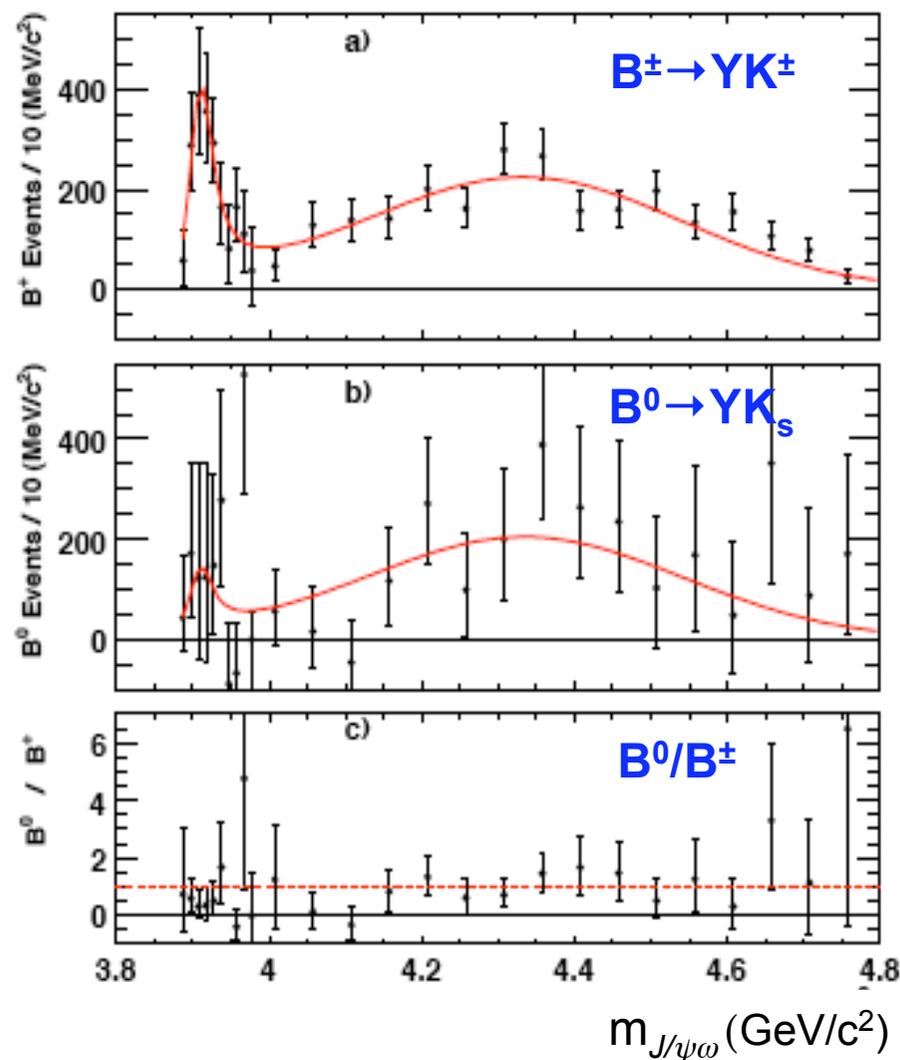
$$M(Y) = (3914.3_{-3.4}^{+3.8}(\text{stat})_{-1.6}^{+1.6}(\text{syst})) \text{ MeV}/c^2$$

$$\Gamma(Y) = (33_{-8}^{+12}(\text{stat})_{-0.6}^{+0.6}(\text{syst})) \text{ MeV}.$$

- ~30MeV lower mass than Belle's
- Narrower width
- Preliminary BF estimate similar to the Belle's ($\sim 10^{-5}$)

✓ 2^3P_1 interpretation:

Mass near expected value



★ X(3943) - Observed by Belle in recoil against J/ψ

Mass = $3942^{+7}_{-6} \pm 6$ MeV

Width = $37^{+26}_{-15} \pm 12$ MeV

(update EPS 2007)

Not a 3P_0 state

$BR(D\bar{D}) < 41\%$ @ 90%cl

$BR(D\bar{D}^* + D^*\bar{D}) > 45\%$ @ 90%cl

✓ Likely the η_c'' state

Width ≈ 50 MeV (CCC Model)

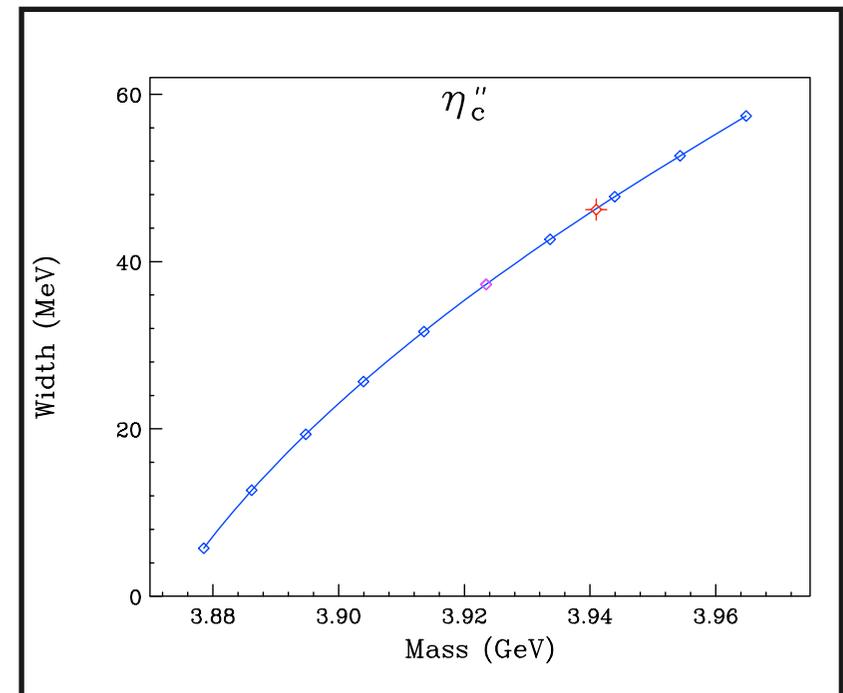
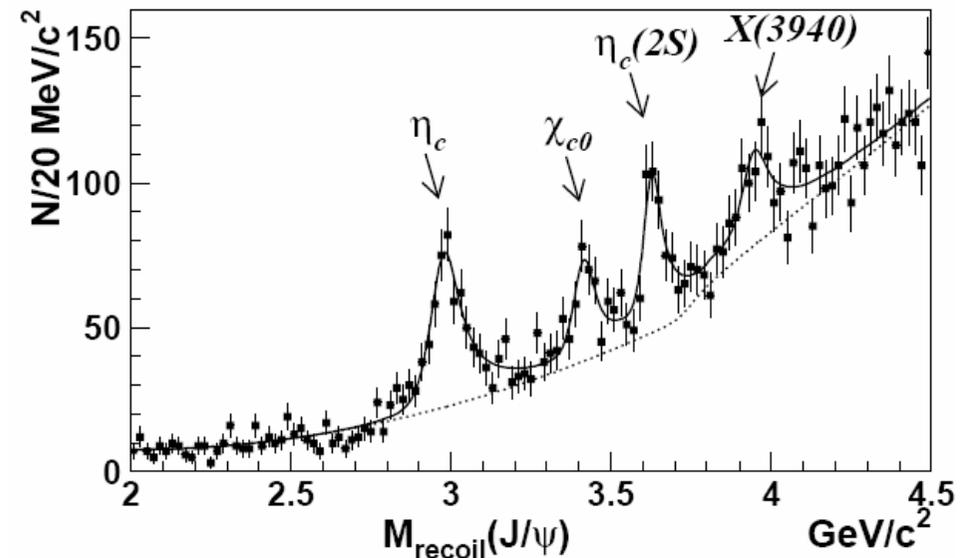
but

$M(\psi(4040) - X(3943)) \approx 100$ MeV (Large)

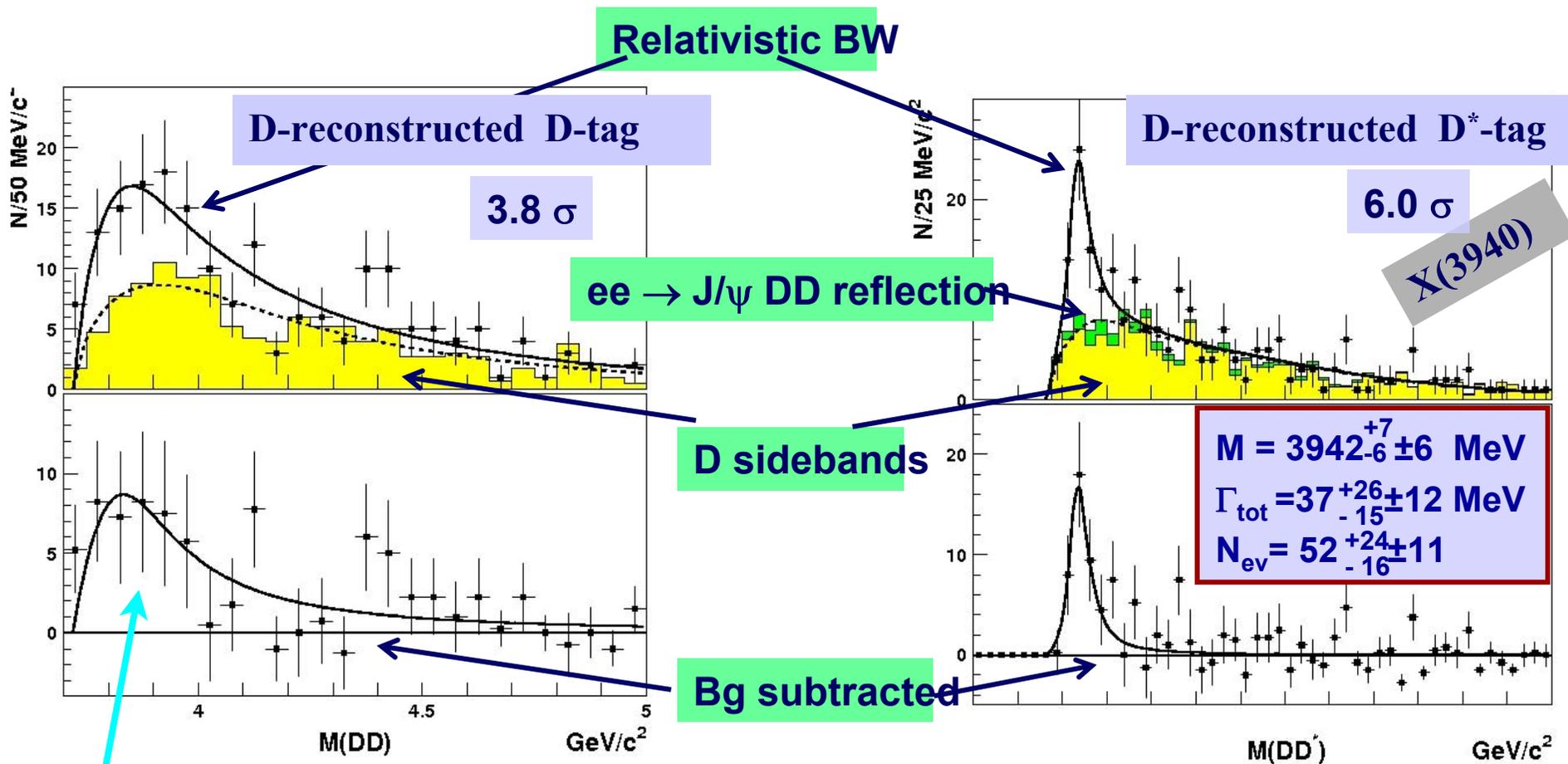
Requires bare splitting: 88 MeV

Including DD_p channels: Expected to add significant spin splitting

Phys. Rev. Lett. 98, 082001 (2007)



Belle - recoil against J/ψ



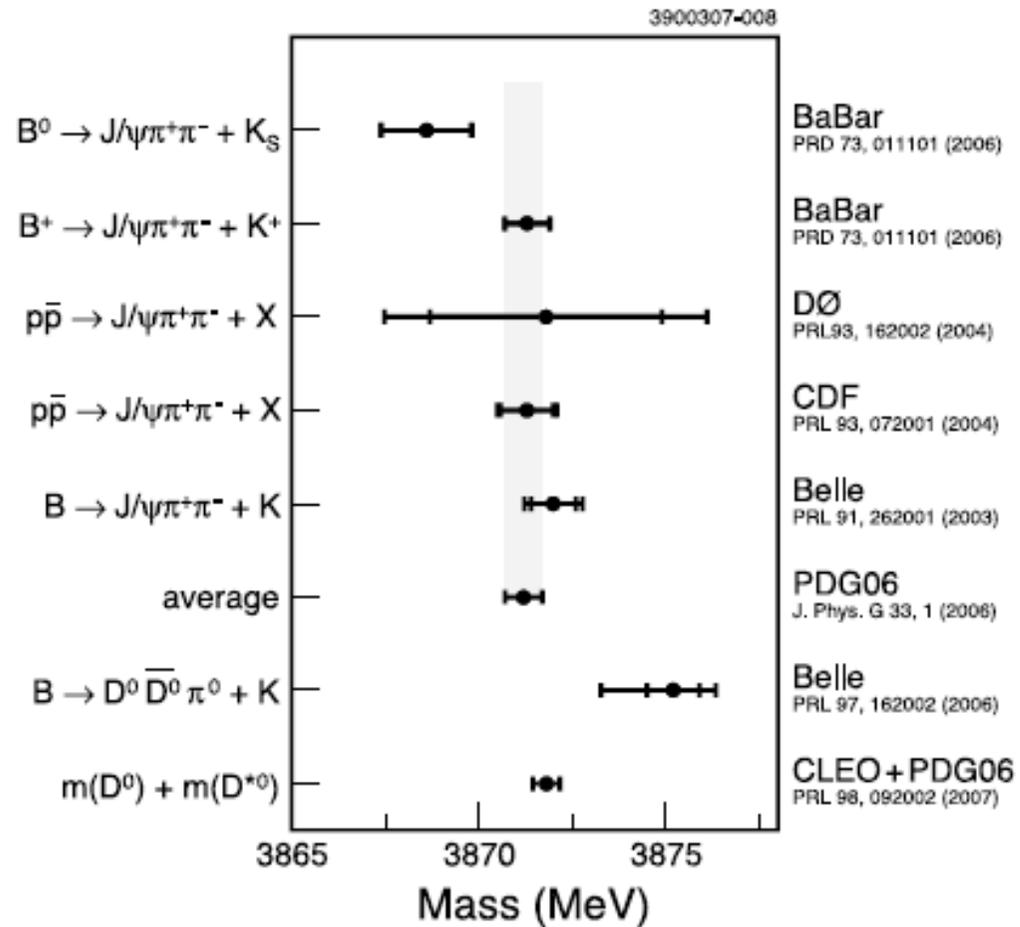
EPS-HEP 2007, Manchester, July 2007

P.Pakhlov

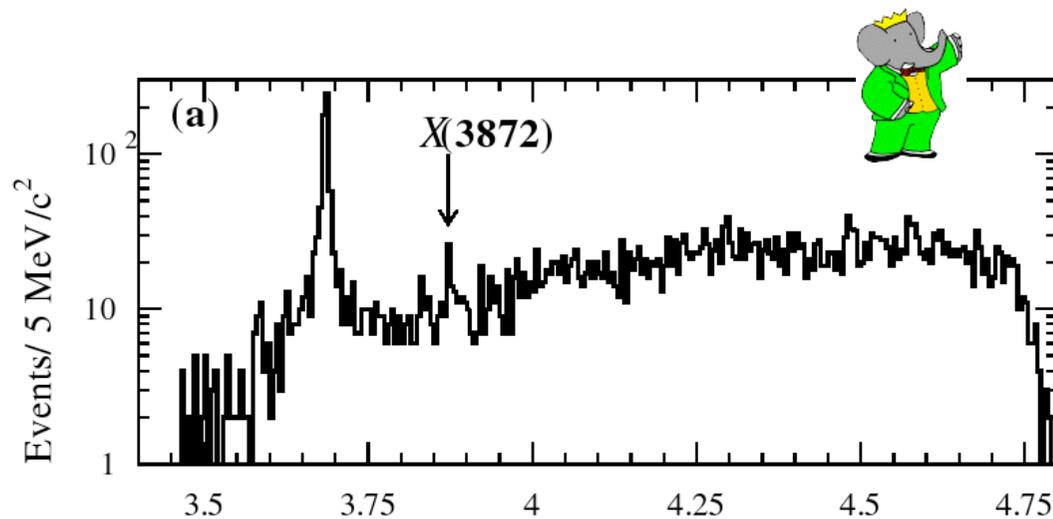
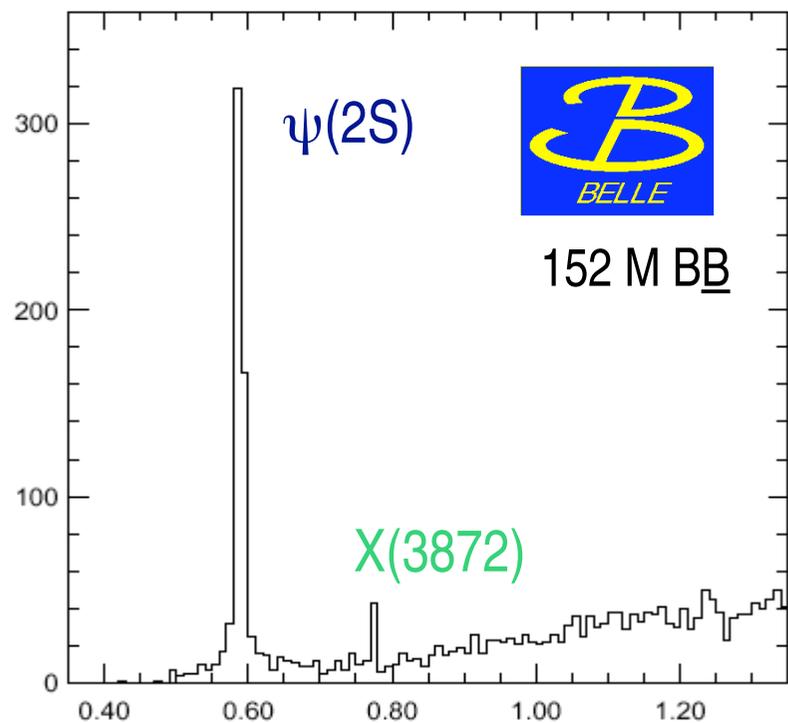
2^3P_0 ? expected mass 3881,
width = 62:

X(3872)

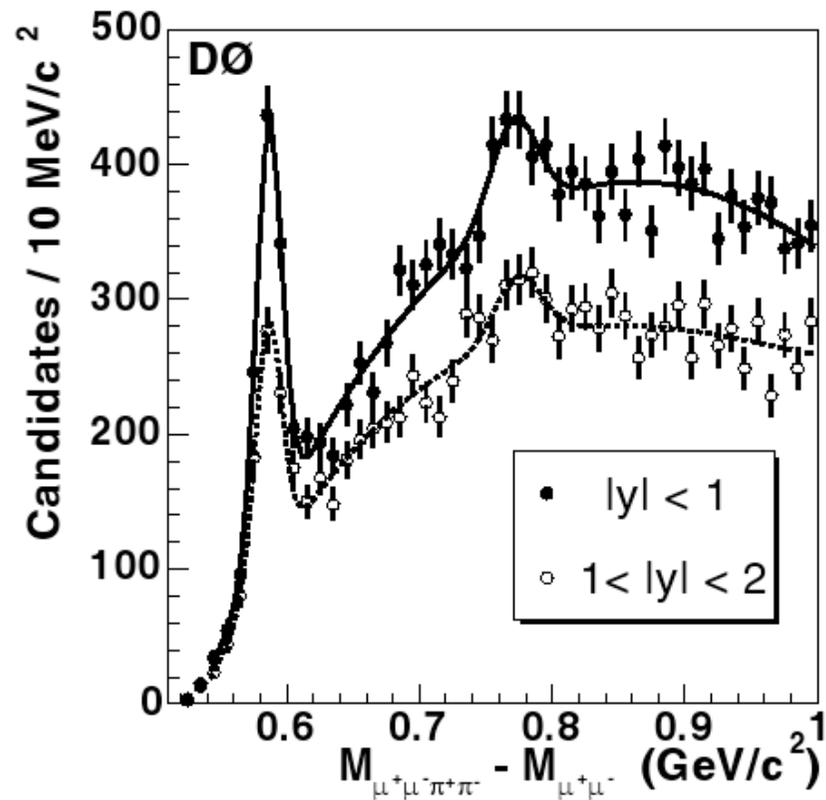
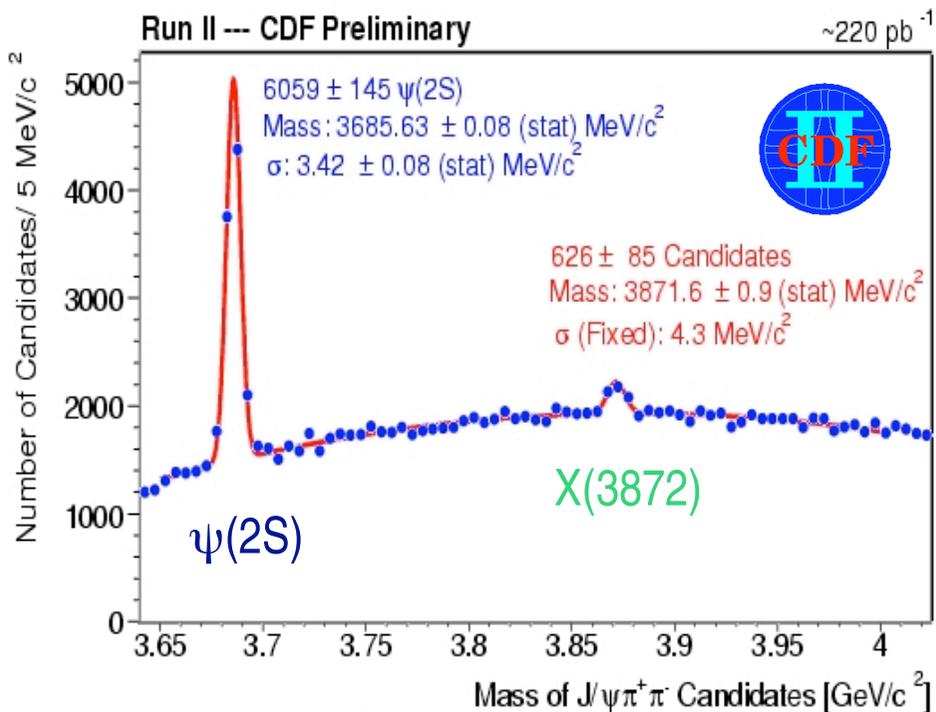
Mass = 3871.2 ± 0.6
 Width < 2.3 90% cl



Discovery



• Use $\sim 220 \text{ pb}^{-1}$ Run II Data

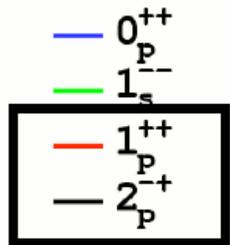


$\pi^+\pi^-$ mass distribution fits $\rho J/\psi$ ($L=0$)

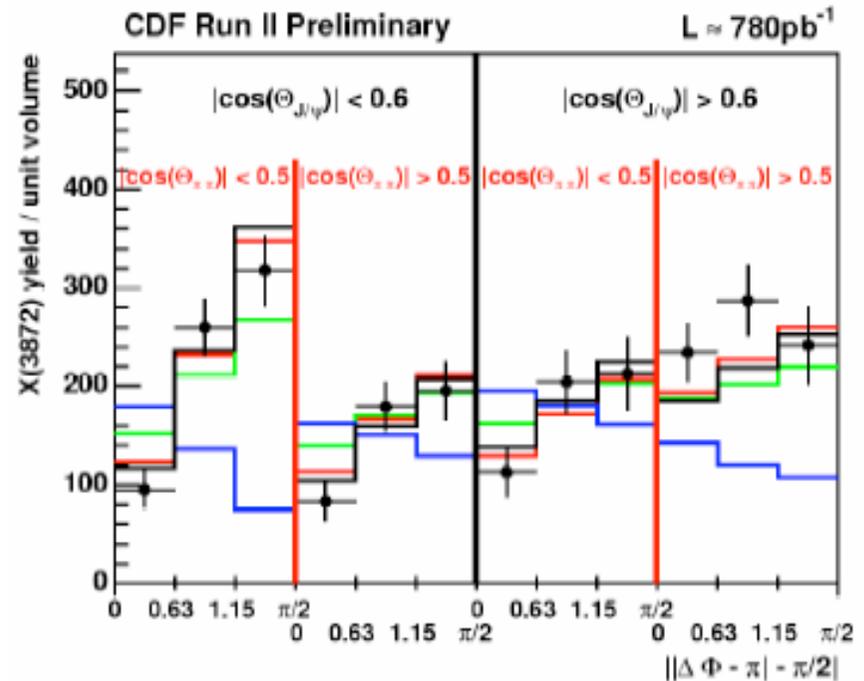
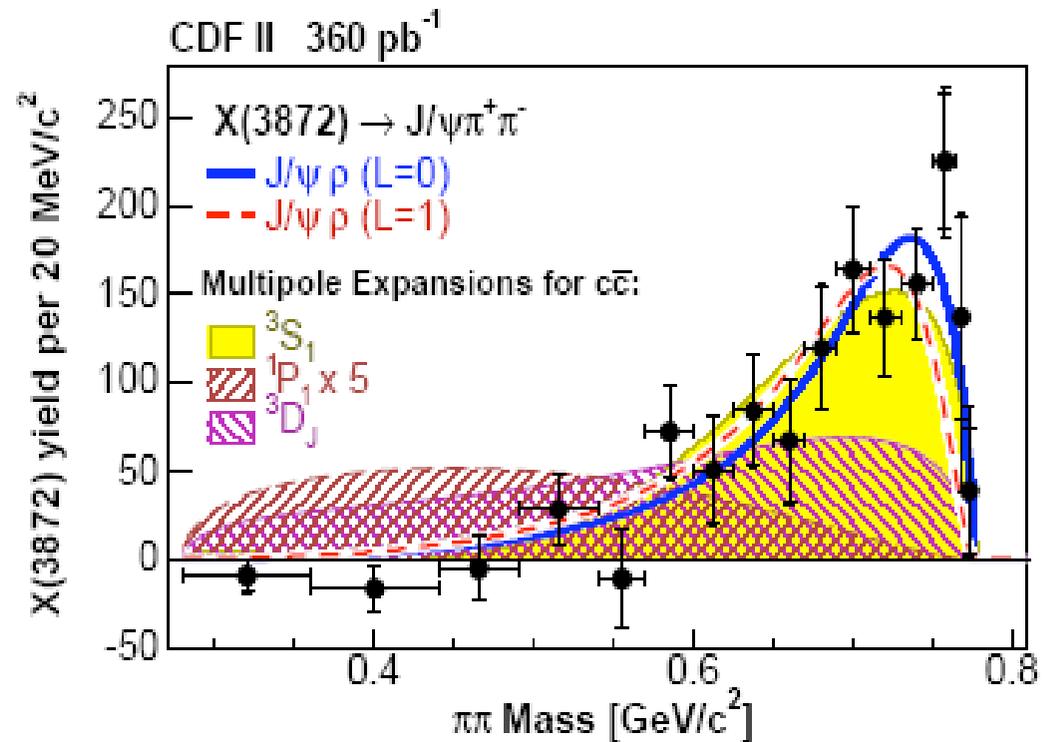
$$J^{PC} = 1^{++}$$

Strongly favored

- X(3872)
- data points
- acc. corrected prediction for



| J^{PC} | χ^2 prob. |
|----------|----------------------|
| 1^{++} | 27.8% |
| 2^{-+} | 25.8% |
| 1^{--} | 0.02% |
| 2^{+-} | $5.5 \cdot 10^{-5}$ |
| 1^{+-} | $3.8 \cdot 10^{-5}$ |
| 2^{--} | $3.8 \cdot 10^{-5}$ |
| 3^{+-} | $3.8 \cdot 10^{-5}$ |
| 3^{--} | $2.4 \cdot 10^{-5}$ |
| 2^{++} | $1.1 \cdot 10^{-5}$ |
| 1^{-+} | $4.1 \cdot 10^{-6}$ |
| 0^{-+} | $3.5 \cdot 10^{-17}$ |
| 0^{+-} | $< 1 \cdot 10^{-20}$ |
| 0^{++} | $< 1 \cdot 10^{-20}$ |



hep-ex/0701014

Other decay modes:

$$\frac{X(3872) \rightarrow \omega J/\psi}{X(3872) \rightarrow \rho J/\psi} = 1.0 \pm 0.4 \pm 0.3$$

Belle

Measure of
isospin breaking

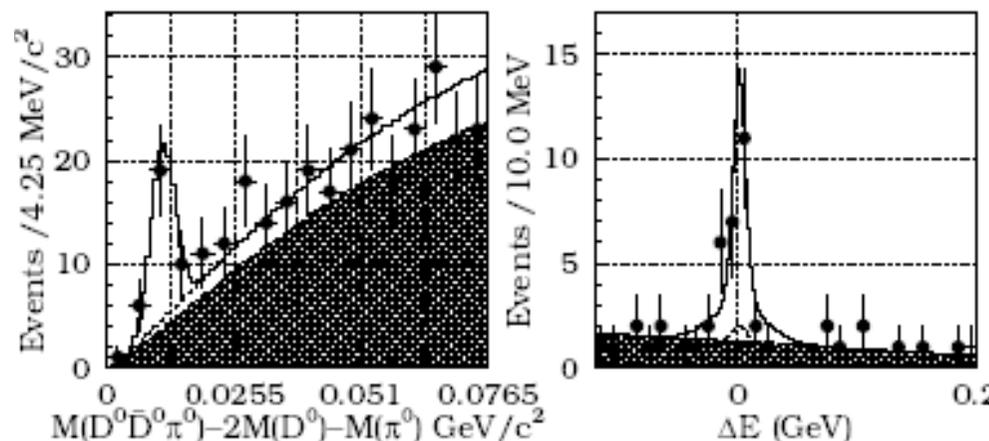
$$\frac{X(3872) \rightarrow \gamma J/\psi}{X(3872) \rightarrow \pi^+ \pi^- J/\psi} = 0.19 \pm 0.07$$

Belle + BaBar

C = +1

$$\frac{X(3872) \rightarrow \pi^0 D^0 \bar{D}^0}{X(3872) \rightarrow \pi^+ \pi^- J/\psi} \approx 10$$

Belle



$$M = 3875.4 \pm 0.7 \begin{matrix} +0.7 \\ -1.7 \end{matrix} \pm 0.8 \text{MeV}$$

DD* "Binding Energy?":

$$M - (m_{D^0} + m_{D^{*0}}) = +4.3 \pm 0.7 \begin{matrix} +0.7 \\ -1.7 \end{matrix} \text{ MeV}$$

Recent developments

CLEO precise D^0 mass measurement [PRL 98, 092002 (2007)]

$$1864.847 \pm 0.150 \pm 0.095 \text{ MeV}$$

$$M(X) - M(D^0) - M(D^{0*}) = -0.6 \pm 0.6 \text{ MeV}$$

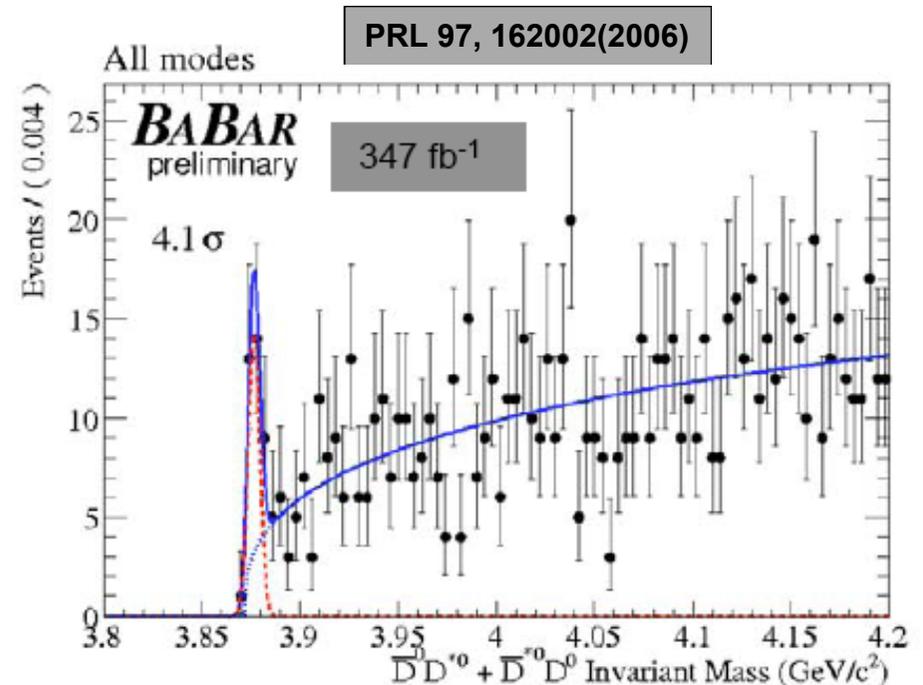
BaBar confirms Belle decay $D^0 D^{*0} \pi^0$
with X mass:

$$M = 3875.4_{-2.0}^{+1.2} \pm 0.7 \text{ MeV}/c^2$$

Two key features of $X(3872)$:

X is extremely close to threshold

$D^0 D^{*0} \pi^0$ mode above threshold



Options for $X(3872)$

(225 papers)

$D^0\bar{D}^{0*}$ molecule:

Tornqvist (8-03, 2-04); Close and Page (9-03); Pakvasa and Suzuki (9-03); Voloshin (9-03, 8-04, 9-05, 5-06); Wong (11-03); Braaten and Kusunoki (11-03; 2-04; 12-04, 6-05, 7-05, 9-06); Swanson (11-03, 6-04, 10-04); Braaten, Kusunoki, and Nussinov (4-04); Kalashnikova (6-05); AlFiky, Gabbiani, and Petrov (6-05); El-Hady (3-06), **Chiu and Hsieh (3-06)**; Zhang, Chiang, Shen and Zou (4-06); Melikhov and Stech (6-06)

tetraquark: $(\bar{c}\bar{q})_3(qc)_\bar{3}$

Vijande, Fernandez, and Valcarce (7-04); Maiani, Piccinini, Polosa, and Riquer (12-04); Ishida, Ishida and Maeda (9-05); Ebert, Faustov and Galkin (12-05); Karliner and Lipkin (1-06); Chiu and Hsieh (3-06)

hybrid: $(\bar{c}gc)$

Close and Page (9-03); Li (10-04)

threshold cusp:

Bugg (10-04)

tetraquark: $(\bar{c}c)_8(\bar{q}q)_8$

Hogassen, Richard and Sorba (11-05);
Buccella, Hogassen, Richard and Sorba (8-06)

Viabile options

Tetraquarks **No**

No partner states found
Why so close to threshold?

Measurements:

- $\Delta m = (2.7 \pm 1.3 \pm 0.2) \text{ MeV}/c^2$ in $B \rightarrow J/\psi \pi^+ \pi^-$
- $\Delta m = (0.7 \pm 1.9 \pm 0.3) \text{ MeV}/c^2$ in $B \rightarrow \bar{D}^0 D^{*0} K$

BaBar: Phys. Rev. D73 (2006) 011101
BaBar: Preliminary

Hybrids **No**

Decays to DD^* unexpected
Why so close to threshold?

Charmonium 2^3P_1 **No (but may play a role)**

Why so close to threshold?
Mass about 50 MeV to high
Isospin issues

$Y(3940)$ may be this 2P state

Threshold cusp **May play a role**

Molecule **Some problems**

Expect $\frac{\mathcal{B}(B^0 \rightarrow X + K^0)}{\mathcal{B}(B^+ \rightarrow X + K^+)} \sim 0.1$

Measurements:

- $R(B^0/B^+) = 0.50 \pm 0.30 \pm 0.05$ in $B \rightarrow J/\psi \pi^+ \pi^-$
- $R(B^0/B^+) = 2.23 \pm 0.93 \pm 0.55$ in $B \rightarrow \bar{D}^0 D^{*0} K$

BaBar: Phys. Rev. D73 (2006) 011101
BaBar: Preliminary

What is the binding force?

Pion exchange much too feeble

M.Suzuki

In a two body system with short range interactions and an S-wave bound state sufficiently close to threshold

Universal properties depending only on the large scattering length (a)

Braaten and Hammer
[cond-mat/0410417]

This applies to the X(3872)

Braaten and Kusunoki

If $a > 0$ one bound state

$$\frac{1}{a} = \gamma_r + i\gamma_i \quad E_X = \gamma_r^2 / (2\mu) \quad \mu = \frac{M(D^0)M(D^{0*})}{M(D^0) + M(D^{0*})}$$
$$\Gamma_X = 2\gamma_r\gamma_i / \mu$$
$$\psi(r) = \frac{\exp(-\gamma_r r)}{r} \quad \sigma(E) = \frac{\pi}{\gamma_r^2 + (\gamma_i + \sqrt{2\mu E})^2}$$

Very large average separation between the charm quark and antiquark

Since this behavior is universal it gives no insight into how the bound state forms

For molecular interpretation cross section for $D^0 D^{*0} \pi^0$

$$\sigma(E) = \frac{\pi}{\gamma_r^2 + (\gamma_i + \sqrt{2\mu E})^2}$$

One possibility is that the nearby 2^3P_1 state with its strong coupling to DD^* provides the needed binding.

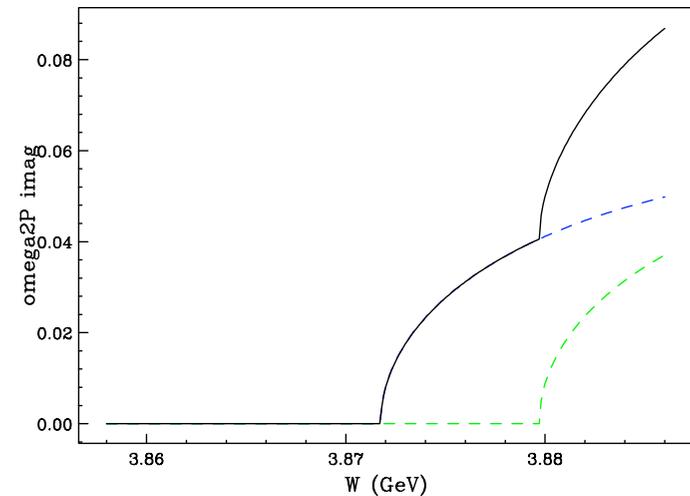
$$G(W) \sim \frac{1}{[W - M + \Omega(W) + i\epsilon]}$$

Assume:

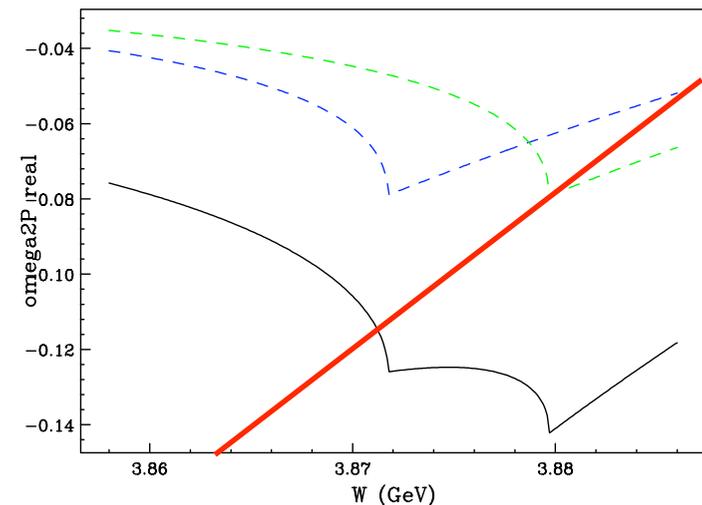
Pole at 0.6 MeV below threshold
Small non DD^* width - 400 KeV

Fit to BaBar data ?

$Im(\Omega)$



$Re(\Omega)$



Obtain for $D^0 D^0 \pi^0$ final state

Even though the X state is slightly below threshold.

More complicated than Braaten and Kusunoki
Real part of Ω varies rapidly

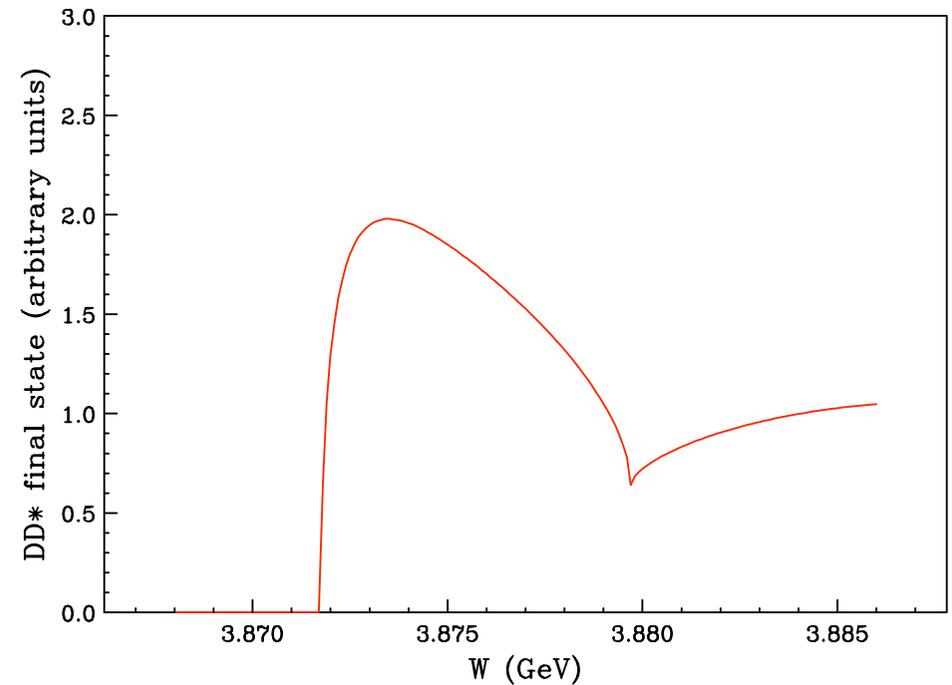
Required conditions for this behaviour:

S wave threshold

Decay into two very narrow hadrons

Nearby state $|M_S - M(\text{threshold})| \leq \Gamma_S$

with sufficiently strong coupling to decay channel.



Y(4260) and Beyond

Y(4260)

Production:

Seen by BaBar in
ISR production

$$J^{PC} = 1^{--}$$

$$\text{Mass: } 4259 \pm 8 \begin{matrix} +2 \\ -6 \end{matrix} \text{ MeV}$$

$$\text{Width: } 88 \pm 23 \begin{matrix} +6 \\ -4 \end{matrix} \text{ MeV}$$

Confirmed by CLEO and Belle

Decays: $\pi^+ \pi^- J/\psi$

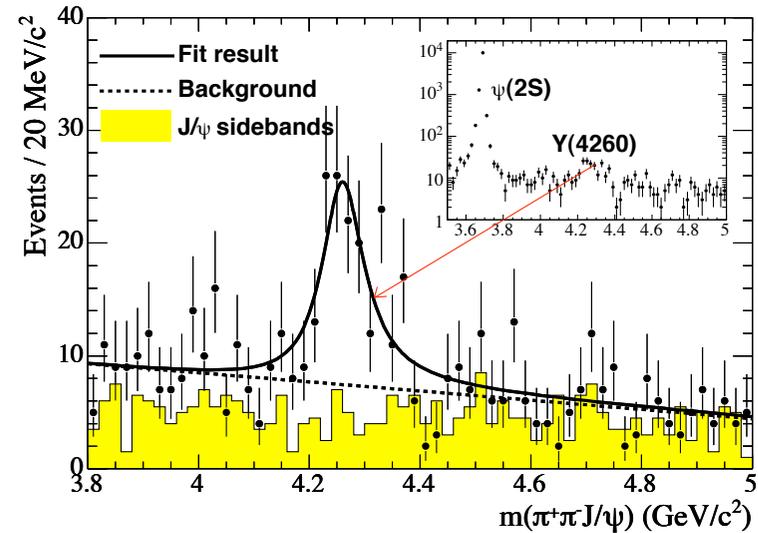
$\pi^0 \pi^0 J/\psi$

$K^+ K^- J/\psi$

discovery mode

CLEO

BaBar



small ΔR

consistent with
isospin zero

NOT a charmonium state

4S state: $\Delta R \sim 2.5$ for 4S at
Ruled out the Y(4260) mass

2D (4160):

Ruled out

Lattice calculations:

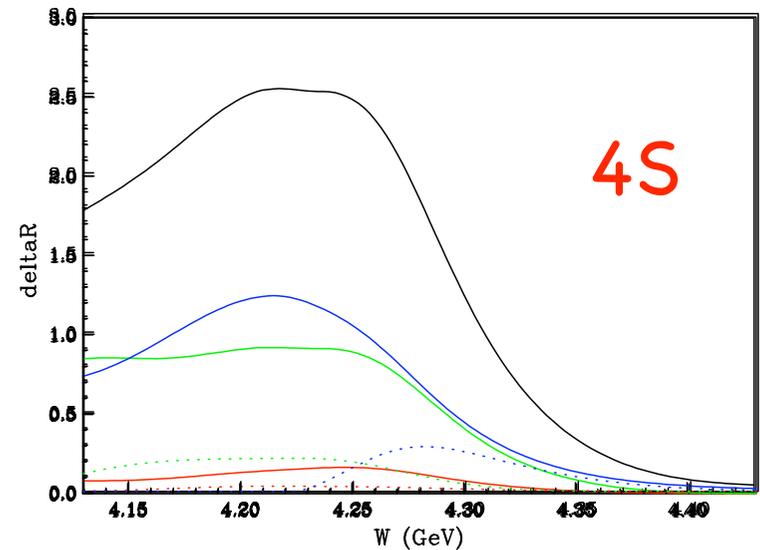
$$M(1^{-+}) = M(1^{--}) \text{ (leading order in } 1/m_c)$$

McNeile review
ICHEP 2006

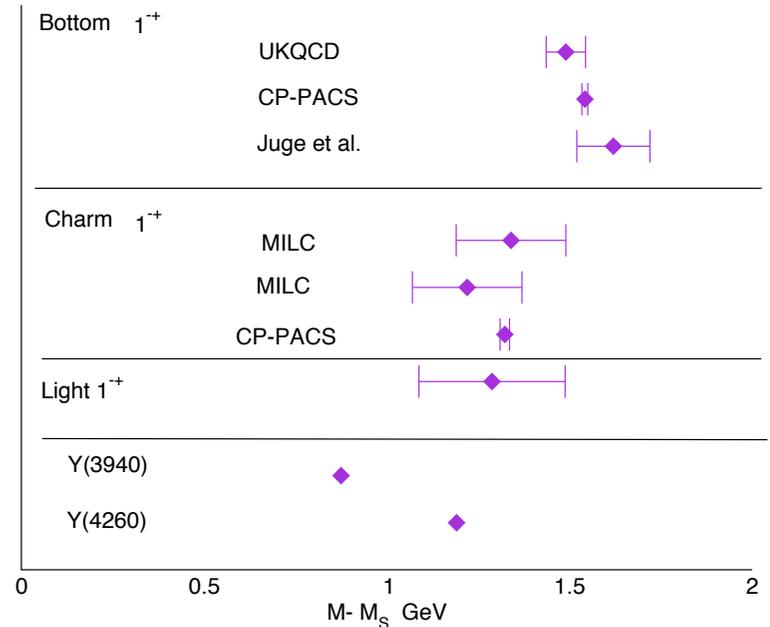
Early attempts of various groups give
 conflicting results for direct mass calculations

Chiu and Hsieh [hep-lat/0512029]

Luo and Liu [hep-lat/0512044]



M - M_S mass splitting
 (M_S is spin averaged mass)



Y(4350)

Seen by BaBar
in the decay mode

$$\pi^+ \pi^- \psi(2S)$$

Mass:

$$4354 \pm 16 \text{ MeV}$$

Width:

$$106 \pm 9 \text{ MeV}$$

Recently Confirmed by Belle

$$M(Y(4360))$$

$$4361 \pm 9 \pm 9$$

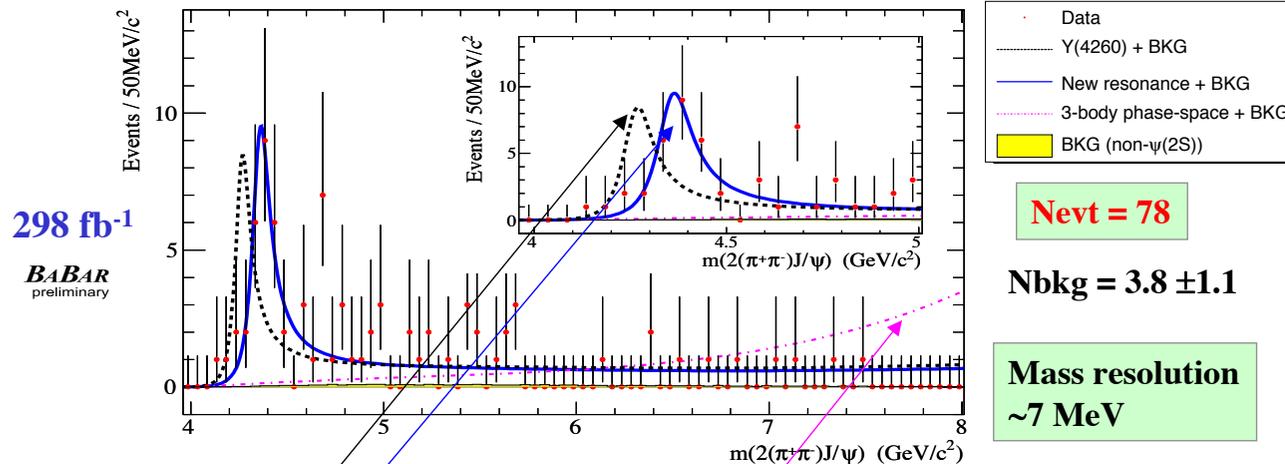
$$\Gamma_{\text{tot}}(Y(4360))$$

$$74 \pm 15 \pm 10$$

...but it's not the Y(4260)...

Fit to $m(2(\pi^+\pi^-)J/\psi)$ to avoid combinatorics.

Try S-wave 3-body phase space, old and new resonance, **cannot find a good fit**



Incompatible with Y(4260), $\psi(4415)$, or S-wave 3-body phase-space production

Assuming a **single resonance** \Rightarrow $\text{mass}=(4354 \pm 16) \text{ MeV}/c^2$, $\Gamma=(106 \pm 19) \text{ MeV}$ (statistical errors only)
still **insufficient** to fully describe the spectrum ($\chi^2\text{-prob} = 1.4 \times 10^{-4}$)
compared with $\chi^2\text{-prob} = 1.6 \times 10^{-8}$ for Y(4260), 4.2×10^{-9} for $\psi(4415)$

Options for $Y(4260)$

(62 papers)

hybrid: $(\bar{c}gc)$

Close and Page (7-05); Kou and Pene (7-05); Zhu (7-05); Juge, O'Cais, Oktay, Peardon and Ryan (10-05); Luo and Liu (12-05); Chiu and Hsieh (12-05); Swanson (9-05, 1-06); Barnes (10-05); Eichten, Lane and Quigg (11-05); S. Godfrey (5-06); Buisseret and Mathieu (7-06);

threshold effect:

Beveren and Rupp (5-06); Rosner (8-06)

tetraquark: $(\bar{c}q)_1(\bar{q}c)_1$, $(\bar{c}\bar{q})_3(qc)_{\bar{3}}$, or $(\bar{c}c)_8(\bar{q}q)_8$

Liu, Zeng and Li, (7-05); Bigi, Maiani, Piccinini, Polosa and Riquer (10-5); Yuan, Wang and Mo (11-05); Ebert, Faustov and Galkin (12-05); Maiani, Riquer, Piccinini and Polosa (3-06); Stancu (7-06); Cui, Chen, Deng and Zhu (7-06); Buccella, Hogassen, Richard and Sorba (8-06)

Y(4260)

Molecular state - **Unlikely**

Threshold effects -

$D^*D \pi$ and $D^*D^*\pi$
measurements

BES and Belle:

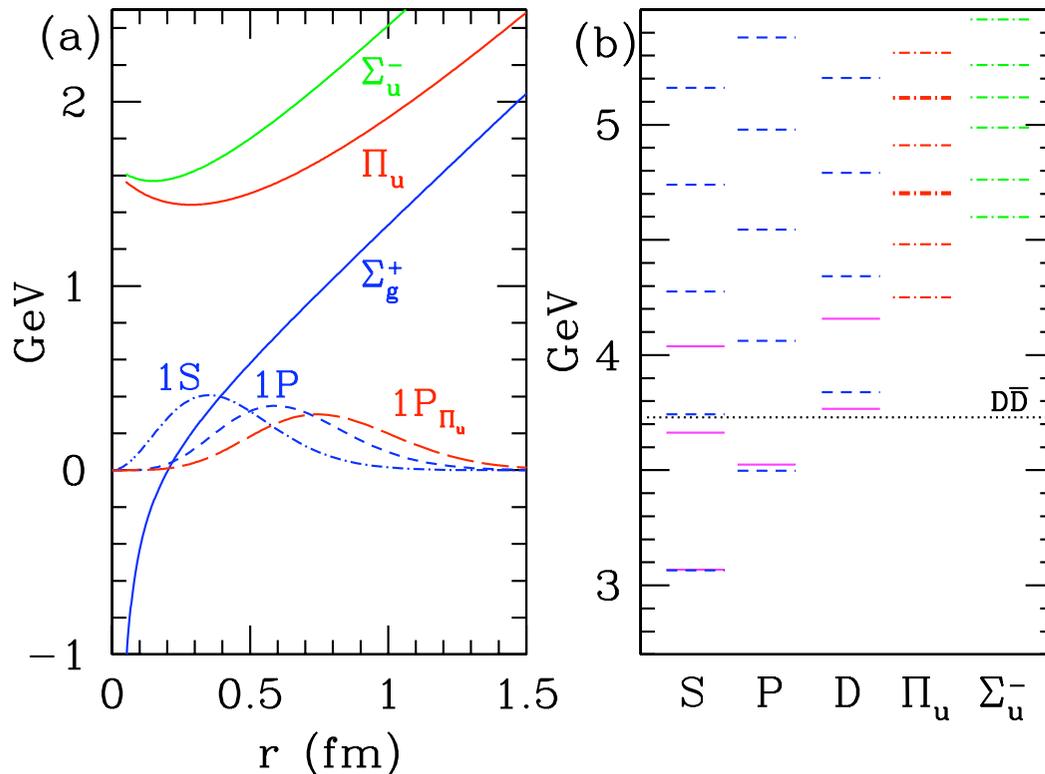
Do not support these ideas

| Channel | Threshold Energy | Width | |
|------------------------|------------------|-----------|--------|
| $D_s^{*+} D_s^{*-}$ | 4223.8 | - | P wave |
| $D\bar{D}_1(3/2^+)$ | 4286.5 | 20.3(1.7) | D wave |
| $D\bar{D}_1(1/2^+)$ | 4306(32) | 329(76) | S wave |
| $D\bar{D}_2(3/2^+)$ | 4327.5 | 43.8(2.0) | D wave |
| $D^* \bar{D}_0(1/2^+)$ | 4315(36) | 276(66) | D wave |

Hybrid - **Attractive**

Close and Page [PL B628 (2005)]

Zhu [PL B625 (2005)]



Charmonium

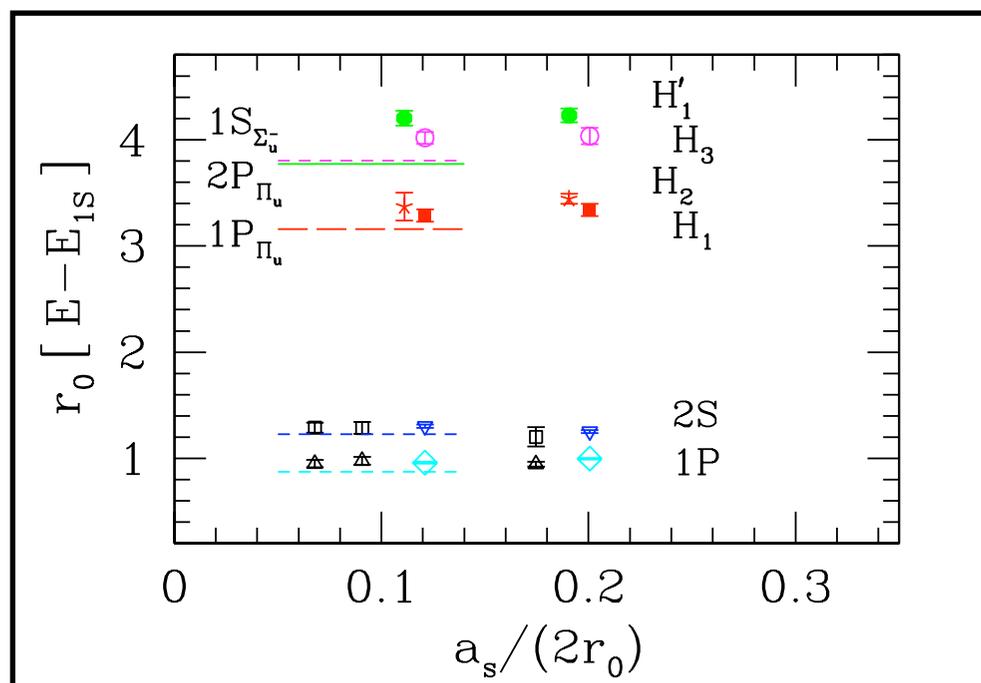
Juge, Kuti, Morningstar
[nucl-th/0307116]

Expect triplet partners

| J^{PC} | | Degeneracies | Operator |
|----------|--------------|--------------------------|--|
| 0^{-+} | S wave | 1^{--} | $\chi^\dagger (\mathbf{D}^2)^p \psi$ |
| 1^{+-} | P wave | $0^{++}, 1^{++}, 2^{++}$ | $\chi^\dagger \mathbf{D} \psi$ |
| 1^{--} | H_1 hybrid | $0^{-+}, 1^{-+}, 2^{-+}$ | $\chi^\dagger \mathbf{B}(\mathbf{D}^2)^p \psi$ |
| 1^{++} | H_2 hybrid | $0^{+-}, 1^{+-}, 2^{+-}$ | $\chi^\dagger \mathbf{B} \times \mathbf{D} \psi$ |
| 0^{++} | H_3 hybrid | 1^{+-} | $\chi^\dagger \mathbf{B} \cdot \mathbf{D} \psi$ |

Quenched Spectrum

How many narrow?



Belle

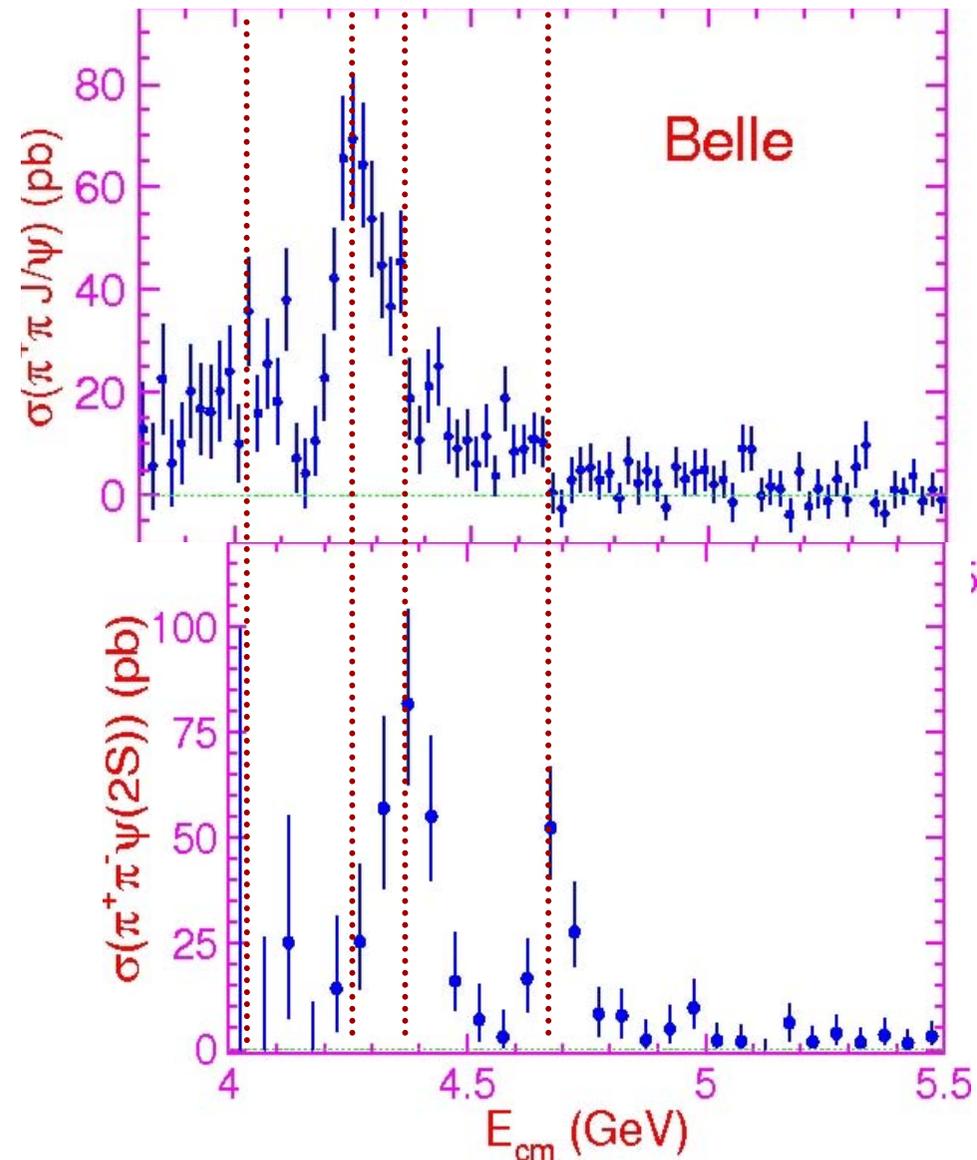
New state $Y(4660)$ observed
by Belle in $\pi^+ \pi^- \psi'$

$$\begin{array}{ll} M(Y(4660)) & 4664 \pm 11 \pm 5 \\ \Gamma_{\text{tot}}(Y(4660)) & 48 \pm 15 \pm 3 \end{array}$$

Very exciting

$Y(4260)$ and $Y(4350)$ might be
one wide state with energy
dependent branching ratios.
(compare 3S region)

$Y(4660)$ is a radial excitation
of the charm quarks state
(analog of ψ' to J/ψ)



To Do List

Summary and To Do List

We are closer to a theoretical understanding the charm threshold region than it may appear.

The X(3872) is likely $D^0 D^{0*}$ bound state with binding provided by nearby 2^3P_1 state.

The [Y(4260), Y(4350)] and Y(4660) highly suggestive of the hybrid nature of these states.

Lattice calculations will provide insight into theoretical issues.

NRQCD and HQET allows scaling from c to b systems. This will eventually provide critical tests of our understanding of new charmonium states.

Answers in many cases will require the next generation of heavy flavor experiments - BES III, LHCb and Super-B factories.

A list of **experimental** and **theoretical** questions:

1 For experiment:

- Measure $R^{+/-}(E)$ in the $\psi(3770)$ resonance region.
- Observe $\psi(3770) \rightarrow \gamma\chi_{c2}$
- Angular distribution of $X(3940) \rightarrow D + D^*$ to distinguish 0^{-+} and 0^{++} .
- The measurement of the $D^0\bar{D}^0\pi^0$ decay mode of the X(3872) by Belle and Babar is very important to understanding the nature of X(3872). Can more information about the shape of the enhancement be obtained?
- The Y(4260) and/or Y(4350) are above threshold for decays to D^*D_P states. These various decays play an important role in understanding the nature of these states. What limit can you put on the ratio of such decays to the $\pi\pi J/\psi^{(\prime)}$ discovery modes?
- Confirm the Y(4660) in $\pi\pi J/\psi'$. Look for other modes 'light hadrons' + $J/\psi^{(\prime)}$ and $\omega + \chi_{cJ}$.
- Look for Y(4260), Y(4350), and Y(4660) in $\pi^+\pi^-\psi^{(\prime)}$ at hadron colliders.

2 For theory:

- Compute ΔR_c in the region near the $\psi(3770)$ resonance. This will provide a detailed model for fitting the total cross section.
- Include $D^{(*)} + \bar{D}_P$ final states in coupled channel calculations.
- Investigate the excitation spectrum for hybrid states using the JKM static potential.

3 For lattice:

- The combination of the static energy for hybrids and the SE for obtaining the masses is very practical. If the $Y(4260)$ is a hybrid state, then there is a triplet of nearby states expected (0^-+ , 1^-+ , 2^-+). The splitting comes from including the heavy quark spin fine structure. How could this be calculated? Even the sign would be useful.
- Much better calculations of the masses of low-lying four quark ($Q\bar{q}q\bar{Q}$) states are needed. What is the prospect obtaining them in the near future. Could a more indirect approach be used to decide if any of the diquark combinations are sufficiently attractive to bind?
- The combination of the static energy for hybrids and the SE for obtaining the masses is very practical. If the $Y(4260)$ is a hybrid state, then there is a triplet of nearby states expected (0^-+ , 1^-+ , 2^-+). The splitting comes from including the heavy quark spin fine structure. How could this be calculated? Even the sign would be useful.