## **Basic Principles of SRF**

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## Introduction

### An example of real SRF Cryomodule (TRISTAN)



### **Principle of RF particle acceleration**



### **Charged Particle Acceleration**





- **Q1: Why RF loss in SRF Cavities**
- **Q2:** What is the theoretical SRF field limitation
- Q3: What kind of SC material is best for SRF application
- Q4: Thermal Conductivity in Superconducting State and Residual Resistance Ratio (RRR)
- Q5: Real SRF field limitation and Technologies to push gradient

### **Surface Impedance**

For plane wave:  $\mathbf{\dot{E}}_{t} = \mathbf{\ddot{E}}_{o} \exp(i\mathbf{\dot{k}} \cdot \mathbf{\ddot{x}} - i\omega t), \quad \mathbf{\ddot{H}}_{t} = \frac{1}{\mu\omega} \mathbf{\ddot{k}} \times \mathbf{\ddot{E}}_{t}$ here, **k** is orthogonal to **E**<sub>t</sub>.

Surface impedance : 
$$Z = R_s + iX_s = \frac{E_t}{H_t} = \frac{\mu\omega}{k}$$

For good conductor  $\sigma >> 1$ ,  $k = (1 + i)\sqrt{\frac{\mu\sigma\omega}{2}}$ 

$$Z = \mu \omega \sqrt{\frac{2}{\mu \sigma \omega}} \cdot \frac{1}{1+i} = \mu \omega \sqrt{\frac{2}{\mu \sigma \omega}} \cdot \frac{1-i}{2} = (1-i) \sqrt{\frac{\mu \omega}{2\sigma}}$$

$$\mathbf{R}_{\mathbf{s}} = \sqrt{\frac{\mu\omega}{2\sigma}} = \frac{1}{\sigma}\sqrt{\frac{\mu\sigma\omega}{2}} = \frac{1}{\sigma\delta}$$

 $\sigma$  : electric conductivity  $\delta$  : skin depth

## Why RF loss in SRF cavities



**RF** surface resistance in superconductor-Two Fluid model-General equation:  $m \frac{\partial \dot{\mathbf{v}}}{\partial t} = q(\mathbf{E} + \mathbf{v}\mathbf{x}\mathbf{B}) - m\mathbf{v}\mathbf{v}$ Two-fluid model  $f f r r r r r r r J_s = n_s q_s v_s$ ,  $J_n = n_n q_n v_n$  $\mathbf{r}$   $\mathbf{v}\mathbf{x}\mathbf{B} << 1$ Maxwell equation: niglecting the Lorentz term,  $m_{s} \frac{\partial \mathbf{v}_{s}}{\partial t} = q_{s} \mathbf{E}, m_{s} = 2m_{e}, q_{s} = 2 \cdot (-e)$   $m_{e} \frac{\partial \mathbf{v}_{n}}{\partial t} = q_{n} \mathbf{E} - m_{e} \mathbf{v} \mathbf{v}_{n}, q_{n} = -e$  $\overset{\mathsf{f}}{\mathbf{E}} = \overset{\mathsf{f}}{\mathbf{E}}_{o} e^{i\omega t} \implies \overset{\mathsf{f}}{\mathbf{J}}_{s} = \frac{n_{s}q_{s}^{2}}{i\omega m} \overset{\mathsf{f}}{\mathbf{E}} , \qquad \overset{\mathsf{f}}{\mathbf{J}}_{n} = \frac{n_{n}e^{2}}{i(\omega - i\omega)} \overset{\mathsf{f}}{\mathbf{E}}$  $\mathbf{J} = \left(\frac{n_s q_s^2}{i\omega m} + \frac{n_n e^2}{i(\omega - iv)m}\right)\mathbf{E}$  $\mathbf{v} > \mathbf{\omega} \implies \mathbf{J} = \left(\frac{n_n e^2}{\mathbf{v} m_e} - i \frac{n_s q_s^2}{\mathbf{\omega} m_s}\right) \mathbf{E} = (\mathbf{\sigma}_n - i \mathbf{\sigma}_s) \mathbf{E} = \mathbf{\sigma} \mathbf{E}$  $\sigma = \sigma_n - i\sigma_s$ 

$$Z = (1-i)\sqrt{\frac{\mu\omega}{2\sigma}} = (1-i)\sqrt{\frac{\mu\omega}{2(\sigma_n - i\sigma_s)}}$$
$$= (1-i)\sqrt{\frac{\mu\omega}{2}} \cdot \frac{1}{\sqrt{-i\sigma_s(1-\sigma_n / i\sigma_s)}}$$
$$\approx (1-i)\sqrt{\frac{\mu\omega}{2}} \cdot \frac{(1+\frac{\sigma_n}{i2\sigma_s})}{\sqrt{-i\sigma_s}}$$
$$= \sqrt{\mu\omega} \left(\frac{\sigma_n}{2\sigma_s^{3/2}} + i\frac{1}{\sigma_s^{1/2}}\right) \qquad Q\sqrt{-i} = \sqrt{e^{-\frac{\pi}{2}}} = \frac{(1-i)}{\sqrt{2}}$$
$$Z = \frac{\mu\omega\lambda^3}{\delta^2} + i\mu\omega\lambda = \frac{\mu^2\omega^2\lambda^3\sigma_n}{2} + i\omega\mu\lambda$$
Here, 
$$\lambda = \sqrt{\frac{m_s}{n_sq_s^2\mu}} = c\sqrt{\frac{2m}{4n_s\mu e^2}} = c\sqrt{\frac{m_s}{2n_s\mu e^2}} \qquad \text{London penetration depth}$$

### Surface resistance in superconductor

$$\sigma_{n} = \frac{n_{n} \cdot e^{2} \cdot l}{m \cdot v_{F}} = \frac{e^{2} \cdot l}{m \cdot v_{F}} \cdot n_{s}(T = 0) \cdot e^{-\frac{\Delta}{k_{B} \cdot T}}$$

$$R_{S} = \frac{1}{2} \cdot (2\pi)^{2} \cdot \mu^{2} \cdot f^{2} \cdot \lambda_{L}^{3} \cdot l \cdot \frac{n_{s}(0)}{m v_{F}} \cdot e^{-\frac{\Delta}{k_{B} T}}$$
At a finite temperature T  

$$= A \cdot f^{2} \cdot e^{-\frac{\Delta}{k_{B} T}}$$

$$BCS \text{ theory}$$

$$R_{BCS}(T, f) = A(\lambda, \xi, |, Tc) \cdot \frac{f^{2}}{T} \cdot e^{-\frac{\Delta}{k_{B} T}}$$

 $\Delta/k_{\rm B} = 1.76T_{\rm C}$  by BCS theory Higher T<sub>c</sub> material produces lower R<sub>BCS</sub>

Superconducting state

## **R**<sub>BCS</sub> at 2 and 4.25K

Used Harbritter's code



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## Minimum of R<sub>BCS</sub>



### **Measurement of the Surface resistance**



### **Q2: Theoretical SRF field limitation Q3: What kind of SC material is best for SRF application ?**

### Two types of superconductor



 $\frac{\lambda}{\xi} = \kappa$ : Gintzburg - Landau Parameter

### **Vortex state**





**ξ : Coherence length** size of Cooper pair





Figure 19 Triangular lattice of fluxoids through top surface of a superconducting cylinder. The points of exit of the flux lines are decorated with fine ferromagnetic particles. The electron micro-scope image is at a magnification of 8300, by U. Essmann and H. Träuble.

 $\lambda_L$  : London penetration depth

Depth of penetration of the magnetic field

### Hc measurement



### Measurement results with $H_{C1}$ , $H_C$ , $H_{C2}$



### **Abrikosov Theory: Theory for Type-II SC**

$$H_{c} = \frac{\kappa}{\lambda^{2}} \frac{hc}{\sqrt{2}e^{*}} = \frac{\kappa}{\lambda^{2}} \frac{(hc/2e)}{2\pi\sqrt{2}} = \frac{\phi_{0}}{2\pi\sqrt{2}\lambda\xi}$$
$$H_{c2} = \sqrt{2} \frac{\lambda}{\xi} \frac{\phi_{0}}{2\pi\sqrt{2}\lambda\xi} = \frac{\phi_{0}}{2\pi\xi^{2}}$$

$$H_{c1} = \frac{\phi_o}{4\pi\lambda^2} \ln(\frac{\lambda}{\xi} + 0.08)$$

$$\phi_0 = hc / 2e = 2.0678 \times 10^{-7} Gauss \cdot cm^2$$
  
= 2.0678 × 10<sup>-15</sup> T · m<sup>2</sup>

### **T-dependence** of $\lambda$ , $\xi$ , $\kappa$

Abrikosov theory

$$\xi = \sqrt{\frac{\phi_0}{2\pi \cdot H_{c2}}}, \ \lambda = \sqrt{\frac{\phi_0 \cdot H_{c2}}{4\pi \cdot H_c^2}}$$

From both theory and experiment  $\lambda(T)$ ,Hc(T) are:

$$\lambda(\mathbf{T}) = \frac{\lambda(\mathbf{0})}{\sqrt{1 - \left(\frac{\mathbf{T}}{\mathbf{T}_{\mathbf{c}}}\right)^4}} , \mathbf{H}_{\mathbf{c}}(\mathbf{T}) = \mathbf{H}_{\mathbf{c}}(\mathbf{0}) \cdot \left[1 - \left(\frac{\mathbf{T}}{\mathbf{T}_{\mathbf{c}}}\right)^2\right]$$

$$\begin{aligned} \mathbf{H}_{c2}(\mathbf{T}) &= \frac{4\pi\lambda(\mathbf{T})^{2}}{\phi_{o}} \cdot \mathbf{H}_{c}(\mathbf{T})^{2} = \frac{4\pi\lambda(\mathbf{0})^{2} \cdot \mathbf{H}_{c}(\mathbf{0})^{2}}{\phi_{o}} \cdot \frac{\left[1 - (\mathbf{T} / \mathbf{T}_{c})^{2}\right]^{2}}{1 - (\mathbf{T} / \mathbf{T}_{c})^{4}} \\ &= \mathbf{H}_{c2}(\mathbf{0}) \cdot \frac{1 - (\mathbf{T} / \mathbf{T}_{c})^{2}}{1 + (\mathbf{T} / \mathbf{T}_{c})^{2}} \\ \boldsymbol{\xi}(\mathbf{T}) &= \sqrt{\frac{\phi_{o}}{2\pi \cdot \mathbf{H}_{c2}(\mathbf{0})}} \cdot \sqrt{\frac{1 + (\mathbf{T} / \mathbf{T}_{c})^{2}}{1 - (\mathbf{T} / \mathbf{T}_{c})^{2}}} = \boldsymbol{\xi}(\mathbf{0}) \cdot \sqrt{\frac{1 + (\mathbf{T} / \mathbf{T}_{c})^{2}}{1 - (\mathbf{T} / \mathbf{T}_{c})^{2}}} \\ \frac{\lambda(\mathbf{T})}{\boldsymbol{\xi}(\mathbf{T})} &= \kappa(\mathbf{T}) = \frac{1}{\sqrt{2}} \cdot \frac{\mathbf{H}_{c2}(\mathbf{T})}{\mathbf{H}_{c}(\mathbf{T})} = \frac{\mathbf{H}_{c2}(\mathbf{0})}{\sqrt{2 \cdot \mathbf{H}_{c}(\mathbf{0})}} \cdot \frac{1}{1 + (\mathbf{T} / \mathbf{T}_{c})^{2}} = \frac{\kappa(\mathbf{0})}{1 + (\mathbf{T} / \mathbf{T}_{c})^{2}} \end{aligned}$$

### **T-dependence of K** with Lab material



## **Critical field limitation in SRF application**



### High Gradient Limitation of Type-II SRF cavities



$$\mathbf{H}_{\mathbf{cr}}(T) = \frac{\xi(T)}{\lambda(T)} \cdot \sqrt{2} \mathbf{H}_{\mathbf{c}}(T) = \frac{\sqrt{2} \mathbf{H}_{\mathbf{c}}(T)}{\kappa(T)} = \sqrt{2} \frac{\mathbf{H}_{\mathbf{c}}(0)}{\kappa(0)} \cdot \left[1 - \left(\frac{T}{T_{\mathbf{c}}}\right)^4\right] \quad \mathbf{H}_{\mathbf{cr}}^{\mathbf{Nb}}(t) = 1750 \cdot [1 - t^4]$$

Eacc ~ 50 MV/m is still reachable if one changes cavity shape with a smaller Hp/Eacc ( = 35 Oe / [MV/m]) ratio.

### Which material is best ?

Material point of view:

- Smaller heat loading for refrigerator  $\implies$  Higher  $T_c$
- High gradient HRF > HC<sup>RF</sup>, then normal conducting

$$\mathbf{H}_{\mathbf{c}}^{\mathbf{RF}} = \sqrt{2} \cdot \frac{\mathbf{H}_{\mathbf{c}}}{\kappa}, \kappa : \mathbf{G} - \mathbf{L}$$
 parameter

The material with higher Hc and smaller  $\kappa$ -value

If Hc is high enough, Type-I material is better because of the smaller κ-value.

#### Good formability

Materials	Tc [K]	Hc, Hc1	κ	Туре	Fabrication
		[Gauss]			
Pb	7.2	803, -	0.65	Ι	Electroplating
Nb	9.25	1900, 1700	1.5	II	Deep drawing, film
Nb3Sn	18.2	5350, 300	7	II	Film
MgB2	39	4290, 300		II	Film

Niobium has higher Tc, Hc and enough formability. Now, niobium is widely used for RF sc cavity production.

### Q4: Thermal Conductivity in Superconducting State and Residual Resistance Ratio (RRR)

### **Thermal conductivity measurement**



Cryostat

## Thermal conductivity of Nb material at low temperature



# Calculation of thermal conductivity based on Quantum mechanics

$$\kappa_{s}(T) = R(y) \cdot \left[\frac{\rho_{295K}}{L \cdot RRR \cdot T} + a \cdot T^{2}\right]^{-1} + \left[\frac{1}{D \cdot \exp(y) \cdot T^{2}} + \frac{1}{BlT}\right]^{-1}$$
  
e-impurities scatt. e- phonons scatt. lattice - phonnons scatt. lattice - grain boundaries scatt.  
$$L = 2.05E - 8, RRR = 200, \rho_{295K} = 14.5E - 8 \Omega m, a = 7.52E - 7$$
  
$$-y = \alpha \cdot \frac{T_{c}}{T}, \alpha = 1.53, T_{c} = 9.25K, T \le 0.6 \cdot T_{c}$$

$$D = 4.27E - 3$$
,  $B = 4.34E3$ ,  $l = 50\mu m$ 

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### Calculated $\kappa_{sc}(T)$



Thermal conductivity at 2K with Nb is ~1/15 of that of stainless at room. Temp.(15W/m\*K)) or 1/6800 of that of copper at 4.2K (6800W/m\*K).

### **RRR** measurement



 $C_{H} = 1ppm, C_{C} = 5ppm, C_{N} = 5ppm, C_{O} = 7ppm, C_{Ta} = 400ppm (99.9582\%)$ RRR = 188.8

### Linear relationship between $\kappa_{SC}$ (2K, 4.25K) and RRR



**RRR** is a good parameter to evaluate thermal conductivity of superconductor.

### Q5: Real SRF field limitation and Technologies to push gradient

### **Real SRF Cavity Performance**



Eacc

### **Various Surface Defects**



0.5 mm

**Foreign materials** 



Cracks

### **Surface Defect**



Picture of the defect area



R15

R 21

R22

1st grinding

R31

R16

⊿R=35Ω

R28)

'R26

. ≪-35mm →

## **Mechanism of Thermal Instability**



# **RRR Dependence of Quench Field** Quench Field : $Hq = \sqrt{\frac{4\kappa(Tc - Tb)}{r_D \cdot R_s(Tb)}} \propto RRR^{\frac{3}{4}} \cdot \sqrt{\frac{(Tc - Tb)}{r_D \cdot Rs(300K)}}$



Needs control the defects with 1mm size, Use high pure niobium material with RRR>200

## Multipacting



Diagnostics: Temperature mapping & X-ray mapping

### **Onset Field of One-point MP** Scale law on RF frequency with the maltipacting levels **Cyclotron frequency :** $\omega = \frac{e \cdot H}{dt}$ $c \cdot m$ $2\pi \cdot f(1P - nth) = \frac{e \cdot H(1P - nth)}{c \cdot m},$ $T(1P - nth) = \frac{1}{f(1P - nth)} = \frac{2\pi \cdot c \cdot m}{e \cdot H(1P - nth)} = n \cdot T_{RF} = \frac{n}{f_{RF}}$ **n=3** n=1 **n=2** $\frac{H(1P - nth)}{f_{RF}} = \frac{constant}{n}, \quad n=1, 2, 3 \quad \cdot \quad \cdot \quad [Oe/Hz]$ Spherical shape suppresses the one **Experiment : Onset field** $\frac{H(1P - nth, [Oe])}{f_{RF}[MHz]} = \frac{0.3}{n}$ [Oe/MHz] point multipcting. **Example**; 1300MHz, Hp/Eacc = 43.8 [Oe/(MV/m)]**1P-1**<sup>st</sup> order • • $H_{RF}(1P-1^{st}) = 0.3 \times 1300 = 390 \text{ Oe}$ $Eacc(1P-1^{st}) = 390/43.8=8.9 \text{ MV/m}$ **1P-2**<sup>nd</sup> **order** • • • **Eacc**(**1P-2**<sup>nd</sup>) = **4.5** MV/m

### **Two-point MP**



### **T-mapping of Tow-point MP**



### **Cures against MP**



### **Field Emission**

#### 10.372 $S_0 = -7.275$ cm 7.890 5.408 $\rho$ (cm) 2.926 0.444 Emission site -2.039 1200 mK 4.521 -4.504 -2.022 0.461 2.943 10.389 12.871 15.354 5.425 7.907 z (cm)

#### Non-resonant electron loading due to field emitted electrons by tunneling effect

### Heating on meridian

### **Q-slope: Partially Disappeared Heating Spots by Baking on CP Cavity**



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## **Q-slope: Disappeared Heating Spots by Baking on EP Cavity**



### **Oxygen Diffusion**



Oxygen on top diffuse into bulk by baking (120°C for 48 hr).

### **Loss Mechanism**

### Interface Tunnel Exchange(ITE Model) By J.Halbritter



Fig. 1: Nb surface with crack corrosion by oxidation by Nb<sub>2</sub>O<sub>5</sub> volume expansion (factor 3). Nb<sub>2</sub>O<sub>5-y</sub>-NbO<sub>x</sub> weak links/segregates (y, x < 1) extend up to depths between 0.01 – 1/1-10 µm for good – bad Nb quality and weak - strong oxidation [8]. Embedded in the adsorbate layer of H<sub>2</sub>O/C<sub>x</sub>H<sub>y</sub>OH ( $\geq$  2 nm) being chemisorbed by hydrogen bonds to NbO<sub>x</sub>(OH)<sub>y</sub>, adsorbate covered dust is found. This dust yields enhanced field emission (EFE [7]) summarized in Sect. 3.1.



**Fig. 3:** Band structure at Nb-NbO<sub>x</sub>-Nb<sub>2</sub>O<sub>5-y</sub> interfaces with E<sub>c</sub>-E<sub>F</sub> =  $\phi \approx 0.1 - 1$  eV as barrier heights for tunneling along crystallographic shear planes (~ 0.1 eV) or of Nb<sub>2</sub>O<sub>5-y</sub> crystallites (~ 1 eV). Added is the superconducting energy gap  $\Delta^*(z) < \Delta_o$ being reduced in NbO<sub>x</sub> clusters or interfaces and being normal conducting  $\Delta^*$  ( $z_L \ge 0.5$  nm) in localized states of Nb<sub>2</sub>O<sub>5-y</sub>. By their volume expansion those clusters locally enhance T\* and  $\Delta^*$ >  $\Delta_o$  in adjacent Nb by the uniaxal strain yielding a smeared BCS DOS.

Disease	Phenomena	Cures
Thermal instability	Quench at bad spot	Mechanical grinding, Use high pure niobium material Sever material control
Multipacting	Q-drop at discrete field levels (Electron resonant loading), Heating around equator section X-ray	Make clean surface Use spherical shape
Field emission	Exponential Q-drop with gradient (Electron non resonant loading) Heating on meridian X-ray	Make clean and smooth surface Use ultrapure water Use clean room assembly High pressure water rinsing
Hydrogen Q-disease	Low Q from low field, Depends on cooling speed	Annealing
Q-slope	Exponential Q-degradation without / with x-ray	Baking