PONDEROMOTIVE INSTABILITIES AND MICROPHONICS

A TUTORIAL

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Some Definitions

- Ponderomotive effects: changes in frequency caused by the electromagnetic field (radiation pressure)
  - Static Lorentz detuning (cw operation)
  - Dynamic Lorentz detuning (pulsed operation)

- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

Note: The two are not completely independent.

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

Some History

- M. M. Karliner, V. E. Shapiro, I. A. Shekhtman
  Instability in the Walls of a Cavity due to Ponderomotive Forces of the Electromagnetic Field
  Soviet Physics – Technical Physics, Vol. 11, No. 11, May 1967

- V. E. Shapiro
  Ponderomotive Effects of Electromagnetic Radiation
  Soviet Physics JETP, Vol. 28, No. 2, February 1969

- M. M. Karliner, V. M. Petrov, I. A. Shekhtman
  Vibration of the Walls of a Cavity Resonator under Ponderomotive Forces in the Presence of Feedback
  Soviet Physics – Technical Physics, Vol. 14, No. 8, February 1970

Stability conditions derived by comparing the rate of transfer of energy from the electromagnetic mode to the mechanical mode with the dissipation rate of the mechanical mode.

Analysis valid when decay time of the electromagnetic mode much less than period of mechanical mode ($\tau\, \Omega_{\mu} << 1$), or when the rate of transfer of energy is very high

Some More History

- D. Schulze

  Mechanical Instabilities of a Superconducting Helical Structure due to Radiation Pressure

  Proc. 1970 Proton Linac Conference, Batavia

  Ponderomotorishe Stabilität von Hochfrequenzresonatoren und Resonatorregelungssystemen

  Dissertation, KFK-1493, December 1971

  Ponderomotive Stability of R.F. Resonators and Resonator Control Systems (ANL-TRANS-944, November 1972)

  Analysis of stability of generator-driven superconducting resonator (arbitrary $\tau\,\Omega_{\mu}$) using control systems methods (Laplace transforms, transfer functions) with and without phase and amplitude feedback

  Analysis of damping of mechanical vibrations by ponderomotive forces

Some More History

- J. R. Delayen
  Phase and Amplitude Stabilization of Superconducting Resonators
  Dissertation, Caltech, 1978

  Analysis of ponderomotive instabilities of resonators operated in self-excited loops, with and without feedback, using control systems methods.

  Included I-Q detection

  Used methods of stochastic analysis to quantify performance of feedback system in the presence of microphonics and ponderomotive effects

  Later extended to include effects due to presence of beam loading and electronic damping.

Ponderomotive Effects

- Adiabatic theorem applied to harmonic oscillators (Boltzmann-Ehrenfest)

If $\varepsilon = \frac{1}{\omega^2}\frac{d\omega}{dt} \ll 1$, then $\frac{U}{\omega}$ is an adiabatic invariant to all orders

$$\Delta\left(\frac{U}{\omega}\right) \Big/ \left(\frac{U}{\omega}\right) \sim o(e^{-d/\varepsilon}) \quad \Rightarrow \quad \boxed{\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U}} \qquad \text{(Slater)}$$

Quantum mechanical picture: the number of photons is constant: $U = N\hbar\omega$

$$U = \int_V dV \left[\frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r})\right] \text{ (energy content)}$$

$$\Delta U = -\int_S dS\, \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[\frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r})\right] \text{ (work done by radiation pressure)}$$

Ponderomotive Effects

$$\frac{\Delta\omega}{\omega} = -\frac{\int_S dS\, \vec{n}(\vec{r})\cdot\vec{\xi}(\vec{r}) \left[\frac{\mu_0}{4}H^2(\vec{r}) - \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}{\int_V dV \left[\frac{\mu_0}{4}H^2(\vec{r}) + \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}$$

Expand wall displacements and forces in normal modes of vibration $\phi_\mu(\vec{r})$ of the resonator

$$\int_S dS\, \phi_\mu(\vec{r})\, \phi_\nu(\vec{r}) = \delta_{\mu\nu}$$

$$\xi(\vec{r}) = \sum_\mu q_\mu \phi_\mu(\vec{r}) \qquad q_\mu = \int_S \xi(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

$$F(\vec{r}) = \sum_\mu F_\mu \phi_\mu(\vec{r}) \qquad F_\mu = \int_S F(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

Ponderomotive Effects

Equation of motion of mechanical mode μ

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial \Phi}{\partial \dot{q}_\mu} = F_\mu \qquad L = T - U \qquad \text{(Euler-Lagrange)}$$

$$U = \frac{1}{2}\sum_\mu c_\mu q_\mu^2 \quad \text{(elastic potential energy)}$$

c_μ: elastic constant

$$T = \frac{1}{2}\sum_\mu c_\mu \frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(kinetic energy)}$$

Ω_μ: frequency

$$\Phi = \sum_\mu \frac{c_\mu}{\tau_\mu}\frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(power loss)}$$

τ_μ: decay time

$$\boxed{\ddot{q}_\mu + \frac{2}{\tau_\mu}\dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu}F_\mu}$$

Ponderomotive Effects

The frequency shift $\Delta\omega_\mu$ caused by the mechanical mode μ is proportional to q_μ

$$\boxed{\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^2\Delta\omega_\mu = -\frac{\omega_0}{c_\mu}\left(\frac{F_\mu}{U}\right)^2\Omega_\mu^2\, U = -k_\mu\Omega_\mu^2 V^2}$$

Total frequency shift: $\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$

Static frequency shift: $\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2\sum_\mu k_\mu$

Static Lorentz coefficient: $k = \sum_\mu k_\mu$

Ponderomotive Effects – Mechanical Modes

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu{}^2\Delta\omega_\mu = -\Omega_\mu^2 k_\mu V_o{}^2 + \cancel{n(t)}$$

Fluctuations around steady state:

$$\Delta\omega_\mu = \Delta\omega_{\mu o} + \delta\omega_\mu$$

$$V = V_0(1+\delta v)$$

Linearized equation of motion for mechanical mode:

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o{}^2\delta v$$

The mechanical mode is driven by fluctuations in the electromagnetic mode amplitude.

Variations in the mechanical mode amplitude causes a variation of the electromagnetic mode frequency, which can cause a variation of its amplitude.

$\Rightarrow$ Closed feedback system between electromagnetic and mechanical modes, that can lead to instabilities.

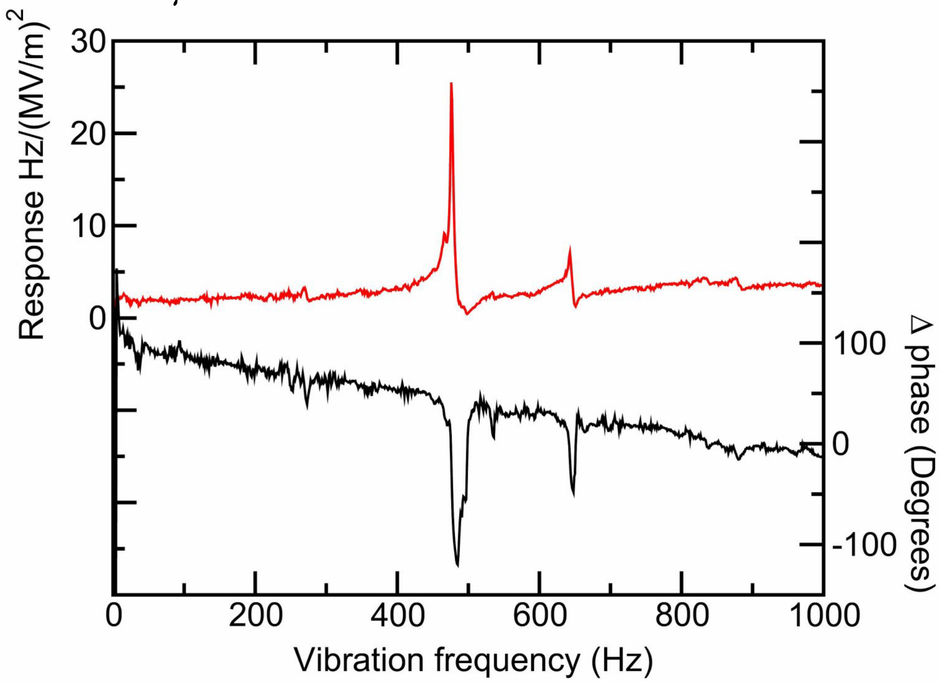
Lorentz Transfer Function

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o{}^2 \delta v$$

$$\delta\omega_\mu(\omega) = \frac{-2\Omega_\mu^2 k_\mu V_o{}^2}{\left(\Omega_\mu^2 - \omega^2\right) + \frac{2}{\tau_\mu} i\omega}\,\delta v(\omega)$$

TEM-class cavities
ANL, single-spoke,
354 MHz, β=0.4

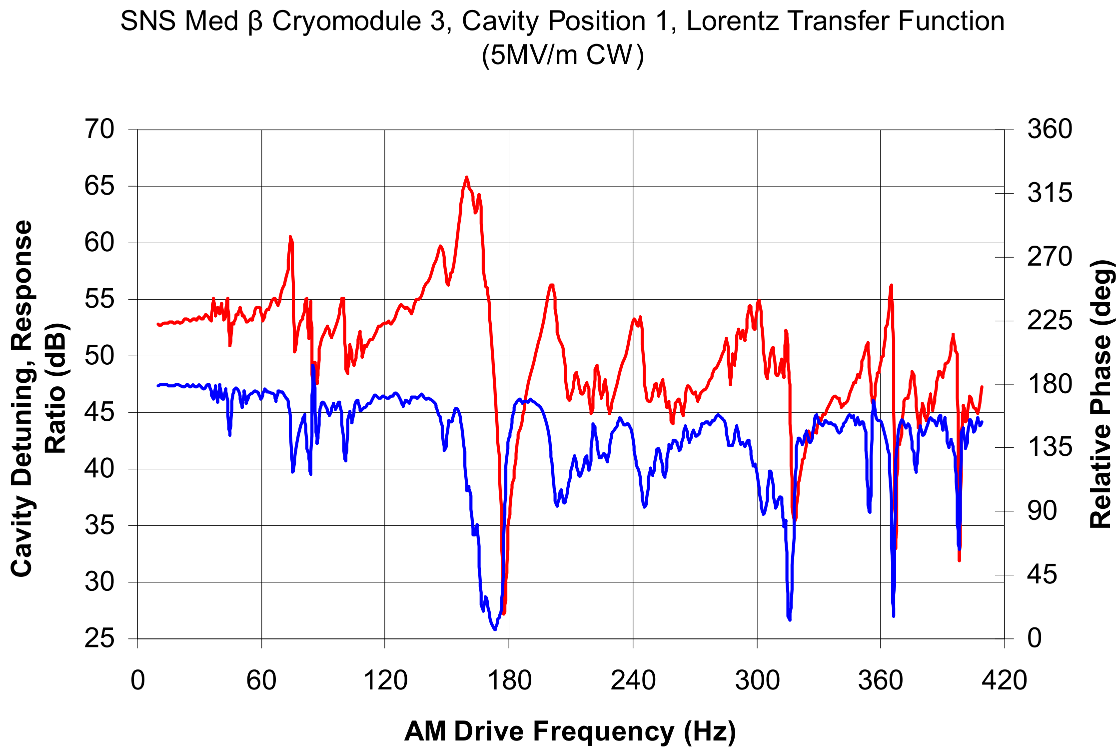
simple spectrum with few modes



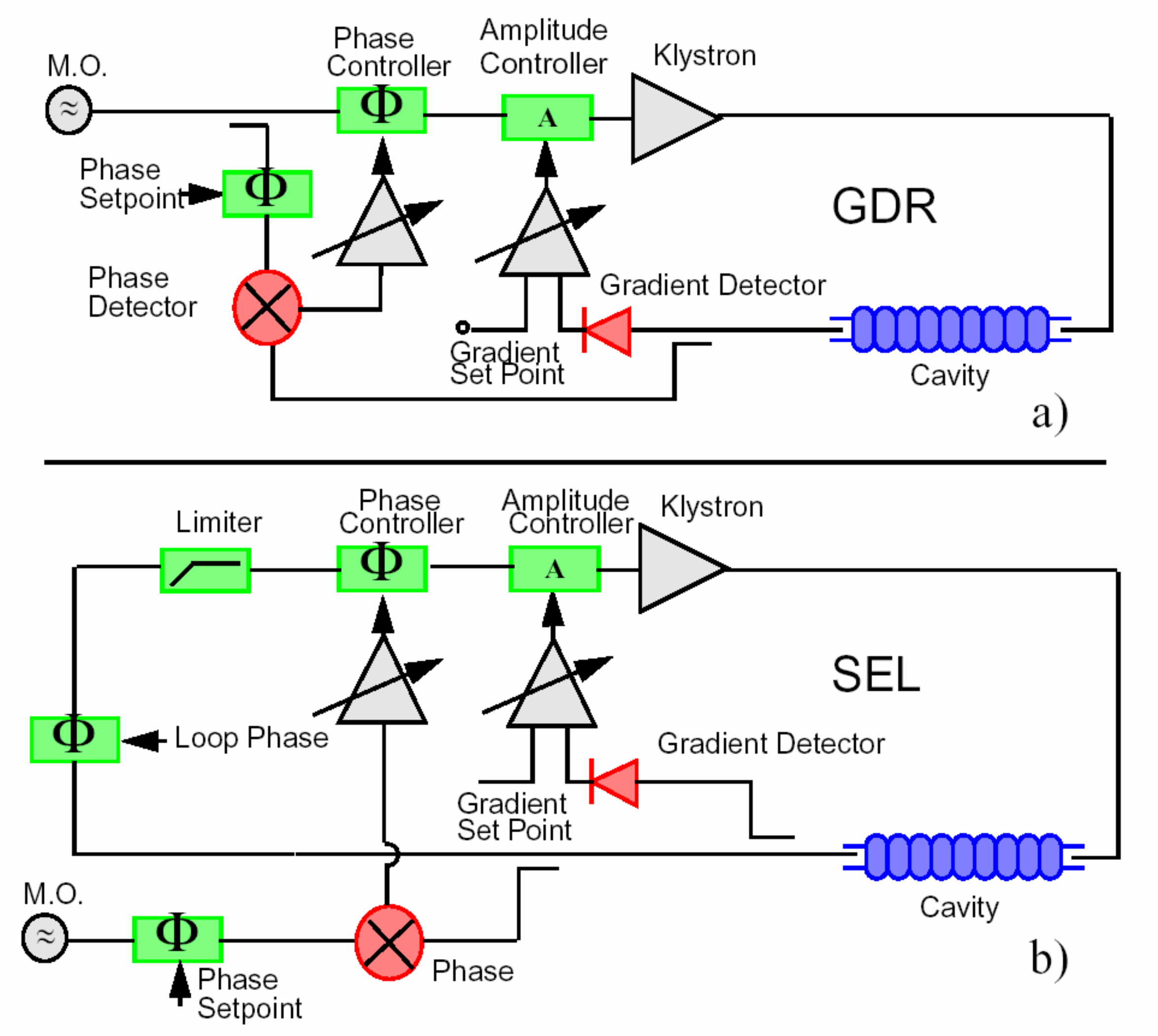
Lorentz Transfer Function

TM-class cavities (Jlab, 6-cell elliptical, 805 MHz, β=0.61)

- Rich frequency spectrum from low to high frequencies
- Large variations between cavities



GDR and SEL



M.O.
Phase Controller
Amplitude Controller
Klystron
Phase Setpoint
GDR
Phase Detector
Gradient Detector
Gradient Set Point
Cavity
a)
Limiter
Phase Controller
Amplitude Controller
Klystron
SEL
Loop Phase
Gradient Detector
Gradient Set Point
Cavity
M.O.
Phase Setpoint
Phase
b)

Generator-Driven Resonator

- In a generator-driven resonator the coupling between the electromagnetic and mechanical modes can lead to two ponderomotive instabilities

- **Monotonic instability** : Jump phenomenon where the amplitudes of the electromagnetic and mechanical modes increase or decrease exponentially until limited by non-linear effects

- **Oscillatory instability** : The amplitudes of both modes oscillate and increase at an exponential rate until limited by non-linear effects

Generator-Driven Resonator

Approximate stability criteria **in the absence** of feedback:

- Monotonic: $$-y\,k_\mu V_o^2 < \frac{1}{2\tau}$$
- Oscillatory: $$y\,k_\mu V_o^2 < \frac{1}{2\tau_\mu}\frac{\left(1+\tau^2\,\Omega_\mu^2\right)^2}{\tau^2\Omega_\mu{}^2}$$

where $y = \tau\left(\omega_g - \omega_{co}\right)$: normalized detuning

The monotonic instability can occur on the low frequency side when the Lorentz detuning is of the order of an electromagnetic bandwidth.
The oscillatory instability can occur on the high frequency side when the Lorentz detuning is of the order of a mechanical bandwidth.

Amplitude feedback can stabilize system with respect to ponderomotive instabilities

Self-Excited Loop

- Resonators operated in self-excited loops in the absence of feedback are free of ponderomotive instabilities. An SEL is equivalent to the ideal VCO.
  - Amplitude is stable
  - Frequency of the loop tracks the frequency of the cavity

- Phase stabilization can reintroduce instabilities, but they are easily controlled with small amount of amplitude feedback

Self-Excited Loop

Approximate stability criteria **with** phase feedback

- Monotonic: $$-y\,k_\mu V_0^2 < \frac{k_a+1}{2\tau}$$
- Oscillatory: $$y\,k_\mu V_0^2 < \frac{1}{2\tau_\mu}\frac{(k_a+1)^2 k_\varphi}{\tau^2\,\Omega_\mu^2(k_\varphi+k_a+1)}$$

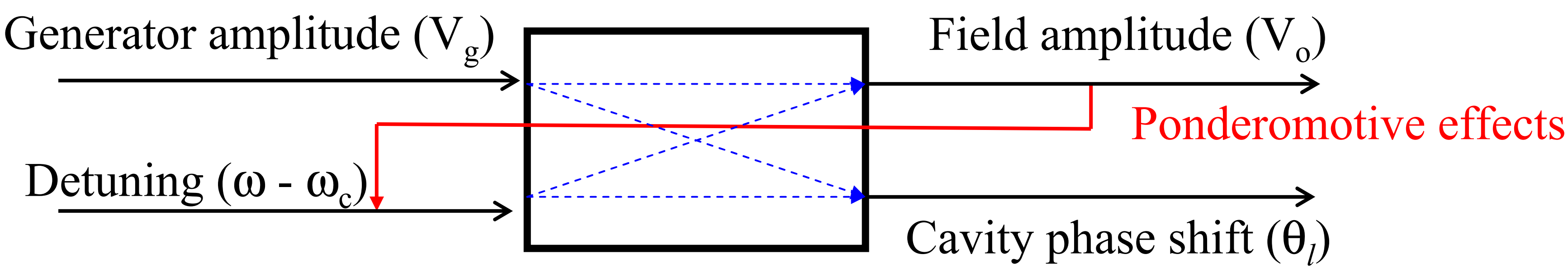
where: $y=\tan\theta_l$ no beam loading $k_a, k_\varphi \gg 1,\ \tau\Omega_\mu$ $\dfrac{\tau}{\tau_\mu} \ll 1$

The stability boundary can be pushed arbitrarily far with amplitude feedback

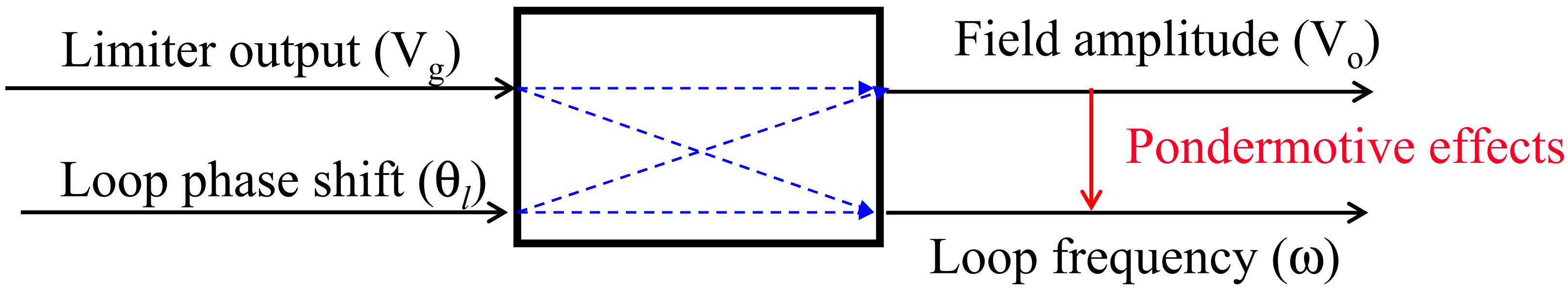
General stability criteria without above restrictions (including beam loading) exist

Input-Output Variables

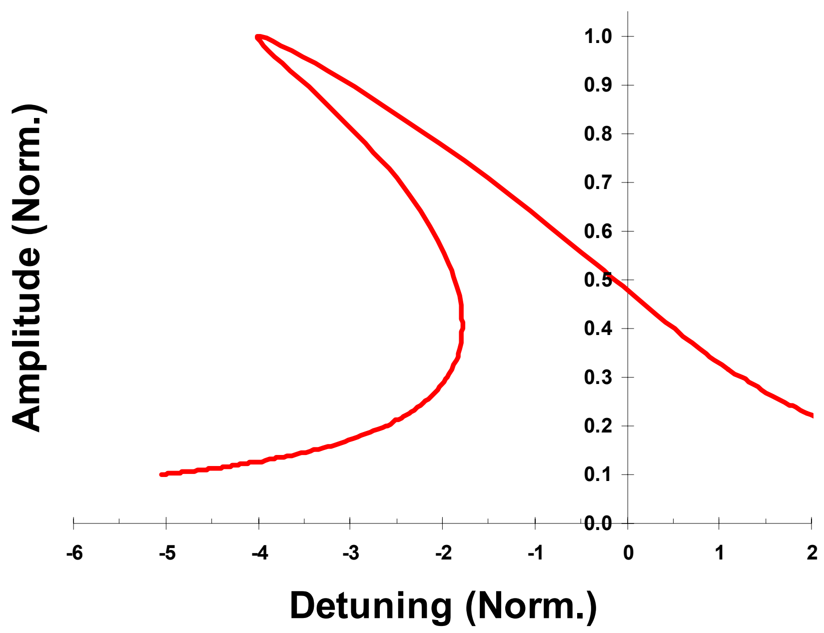
- Generator - driven cavity

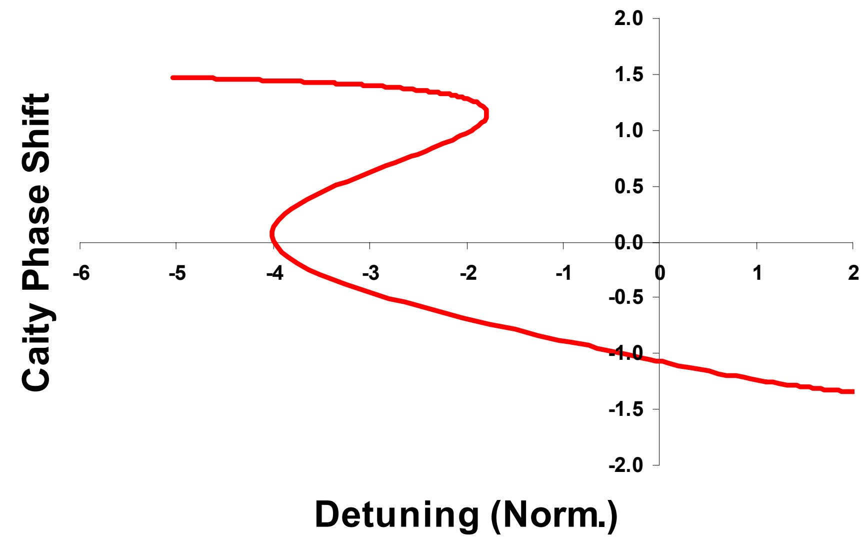


- Cavity in a self-excited loop

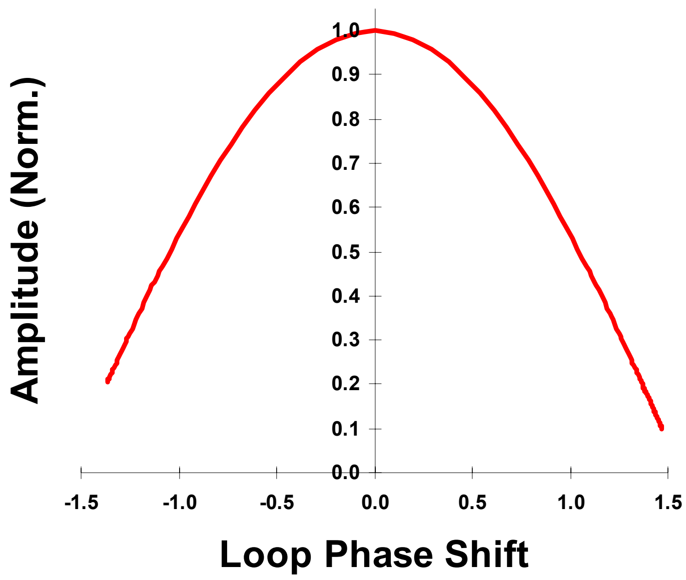


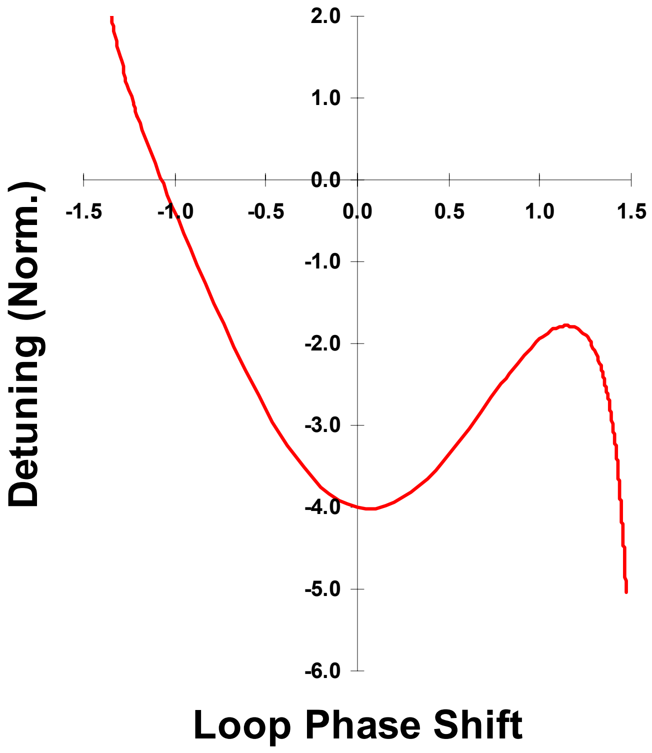
Input-Output Variables
Generator-Driven Resonator





Input-Output Variables
Self-Excited Loop





Ponderomotive Instabilities in GDR

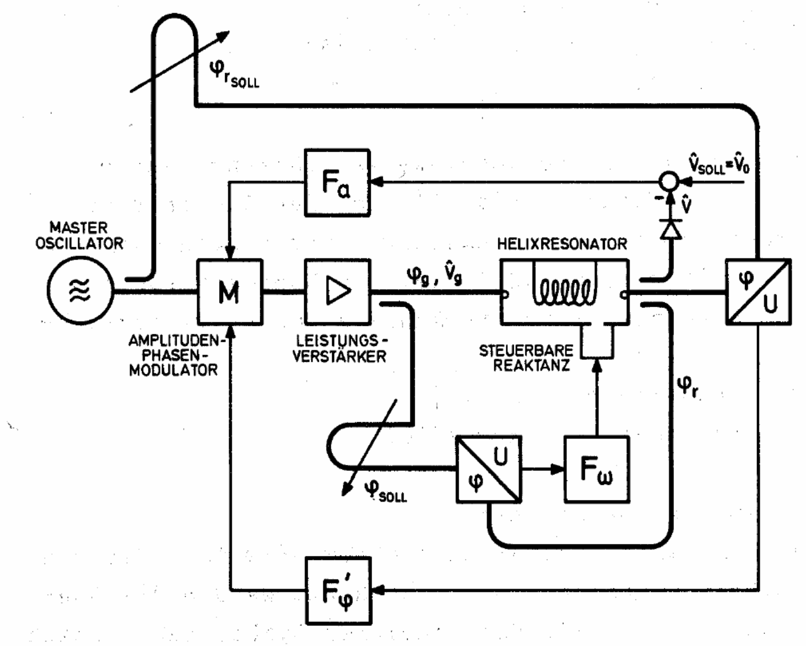


Bild 6.17: Blockschaltbild des Regelungssystems einer unabhängigen Hf-Sektion

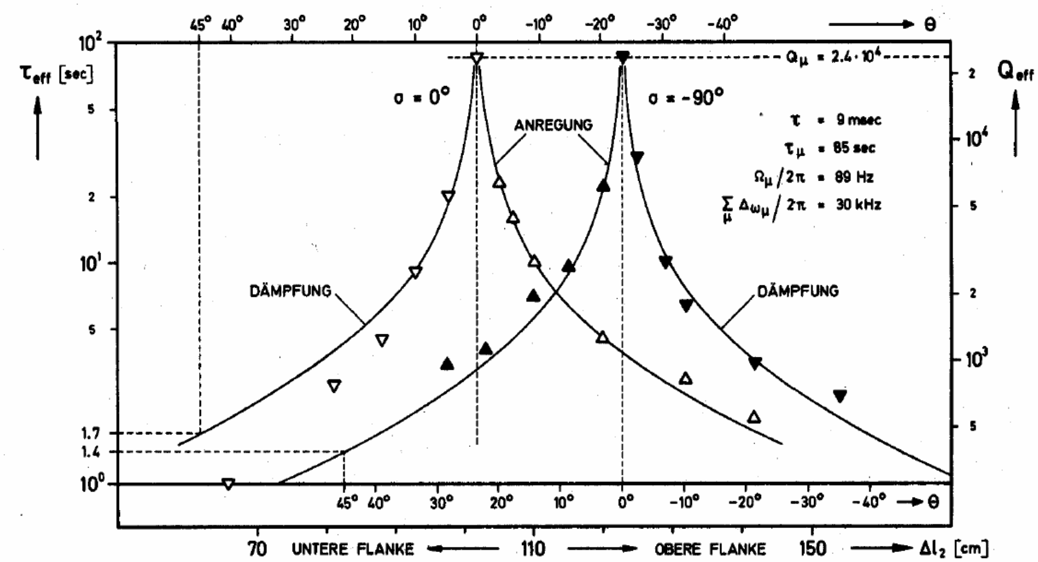
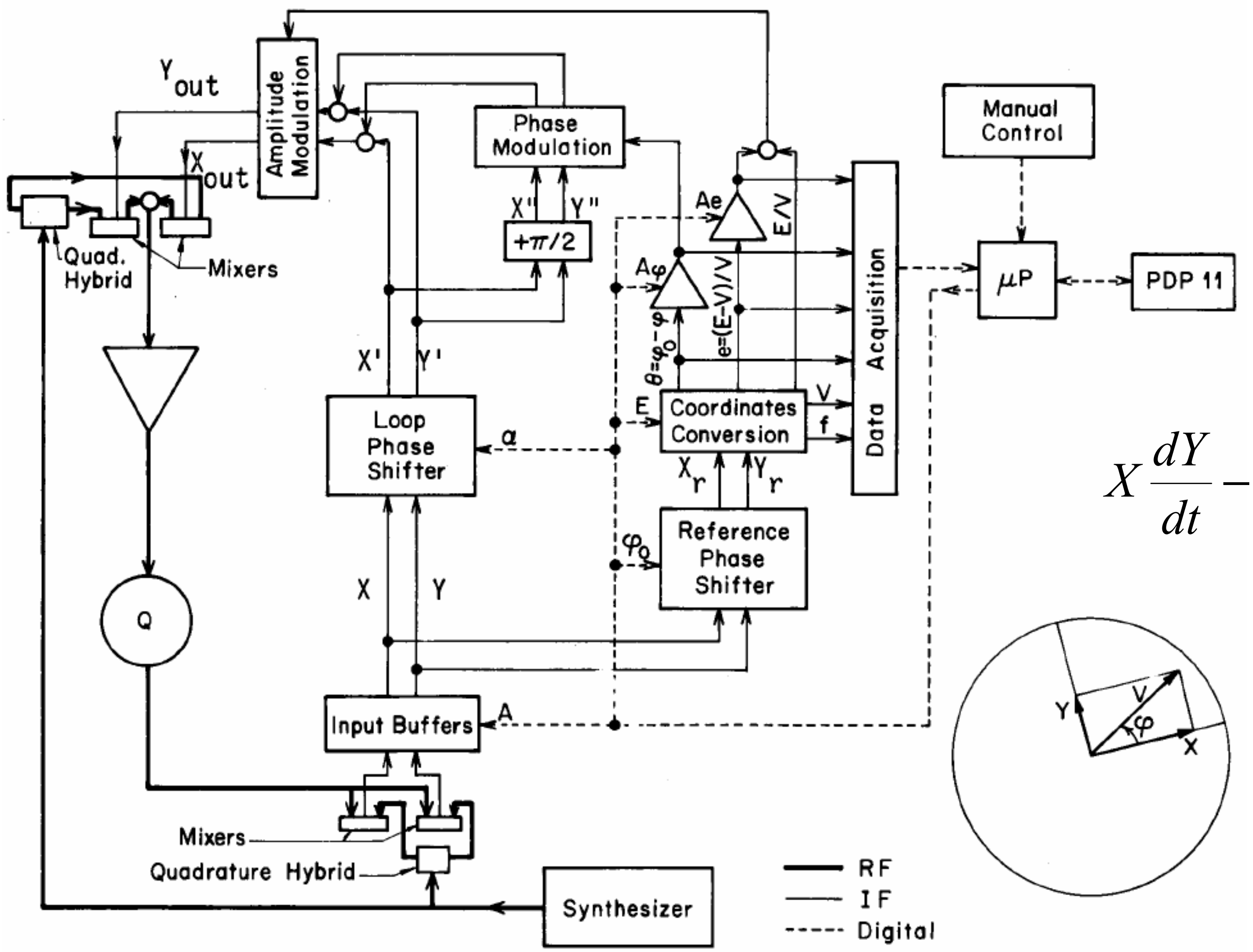


Bild 6.14: Effektive mechanische Abklingzeit als Funktion der Hf-Phase Θ und der Signalphase σ

$$\omega_0 / 2\pi = 72 \text{ MHz}, \quad \tau = 9 \text{ msec}, \quad \Delta\omega = 18 \text{ Hz}$$

$$\Omega_\mu / 2\pi = 89 \text{ Hz}, \quad \tau_\mu = 85 \text{ sec}, \quad k_\mu / 2\pi = 500 \text{ kHz}$$

Ponderomotive Instabilities in SEL



$$X\frac{dY}{dt}-Y\frac{dX}{dt}=V^2\frac{d\varphi}{dt}=V^2\delta\omega$$

Ponderomotive Instabilities in SEL

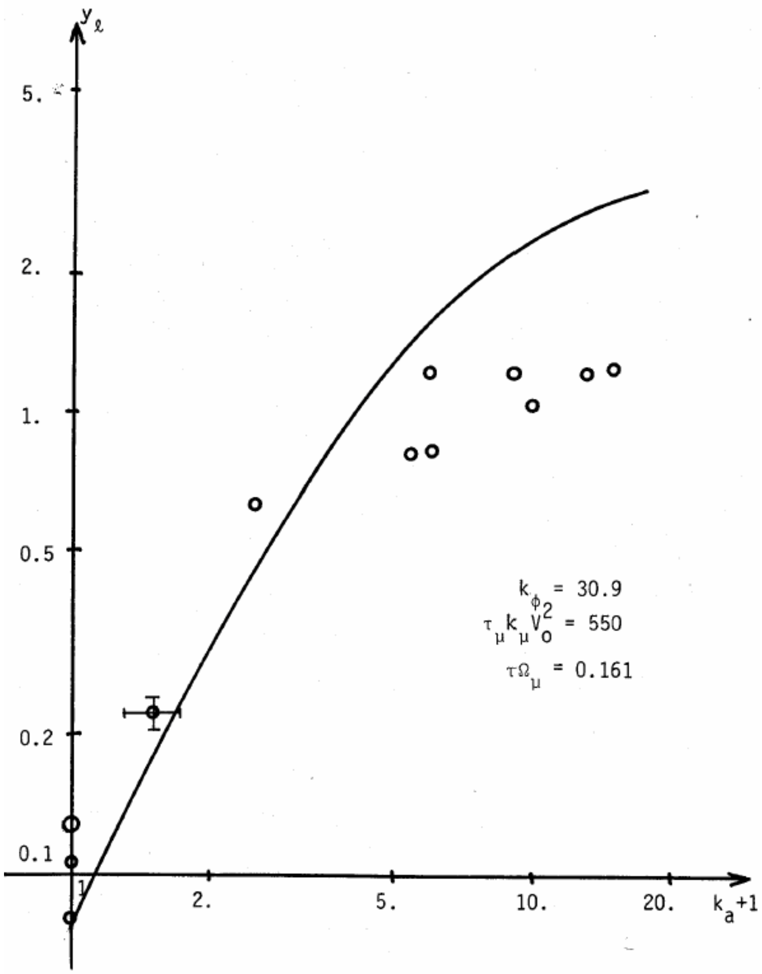


Fig. 5.5. Oscillatory stability boundary vs. amplitude feedback gain.

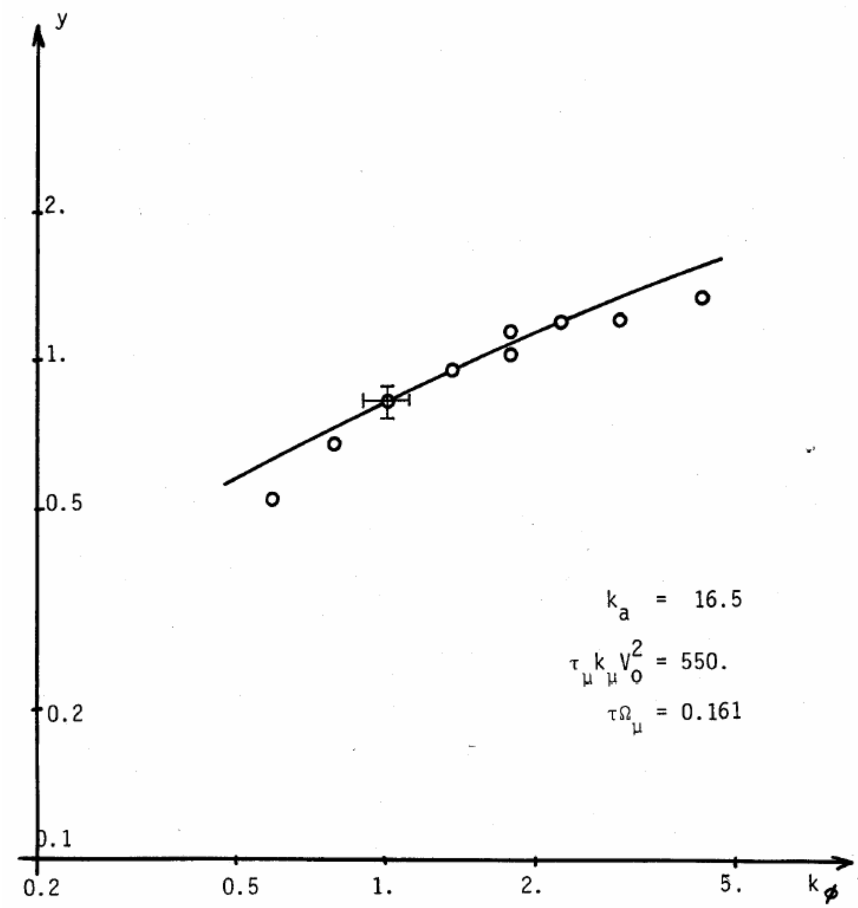


Fig. 5.6. Oscillatory stability boundary vs. phase feedback gain.

Microphonics

- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu{}^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o{}^2\delta v + n(t)$$

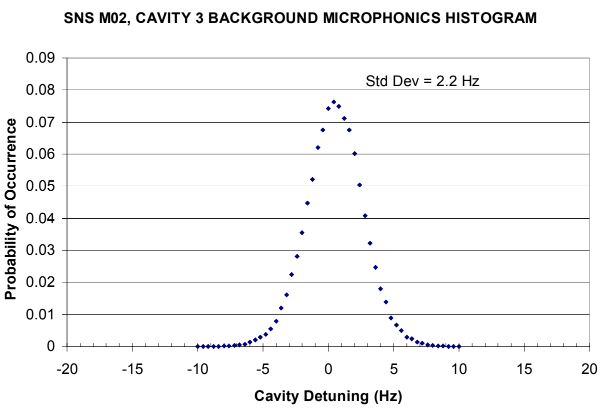
Microphonics

Two extreme classes of driving terms:

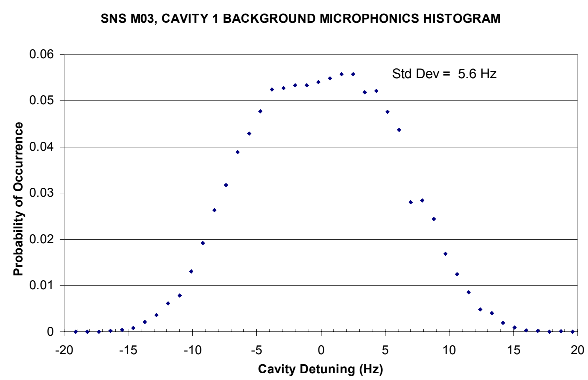
- **Deterministic, monochromatic**
  - Constant, well defined frequency
  - Constant amplitude
- **Stochastic**
  - Broadband (compared to bandwidth of mechanical mode)
  - Will be modeled by gaussian stationary white noise process

Microphonics (probability density)

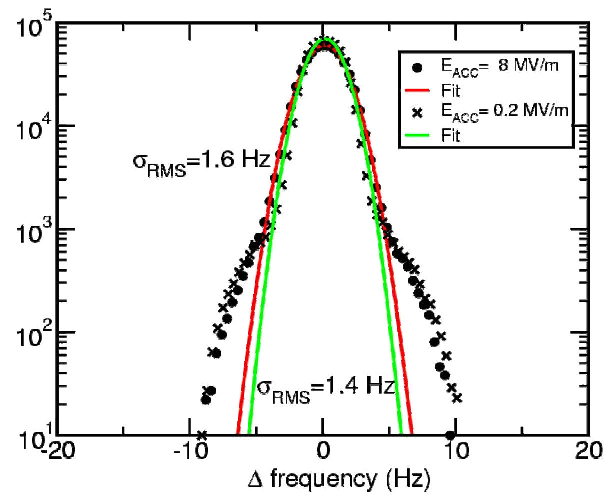
Single gaussian	Bimodal	Multi-gaussian
Noise driven	Single-frequency driven	Non-stationary noise



805 MHz TM



805 MHz TM

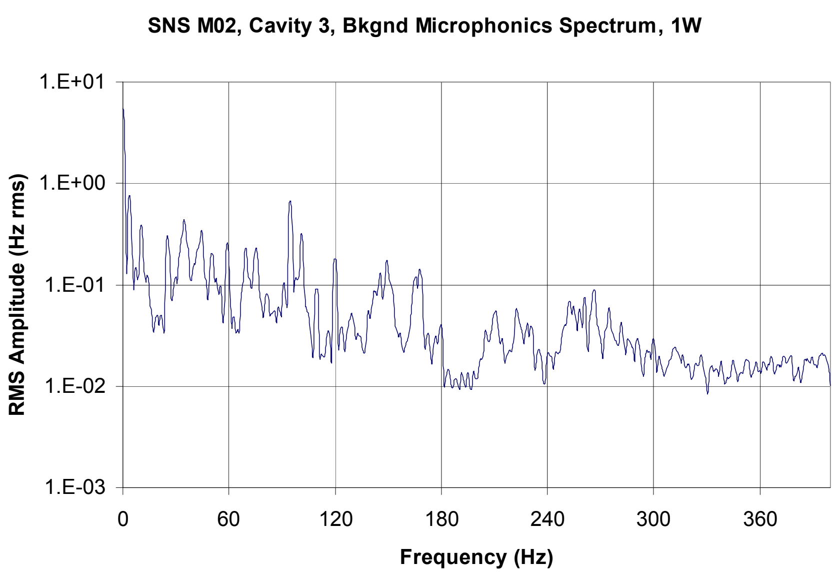


172 MHz TEM

Microphonics (frequency spectrum)

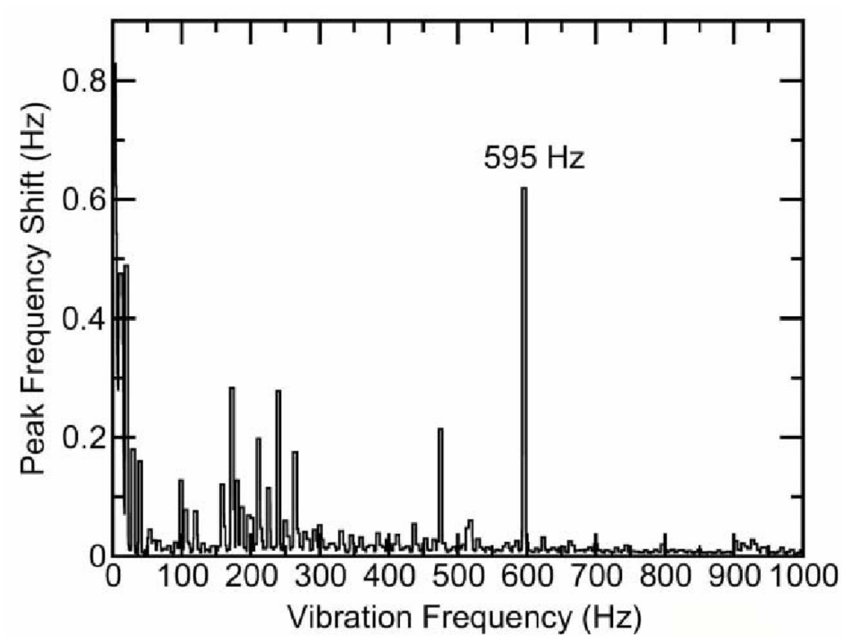
TM-class cavities (JLab, 6-cell elliptical, 805 MHz, β=0.61)

- Rich frequency spectrum from low to high frequencies
- Large variations between cavities

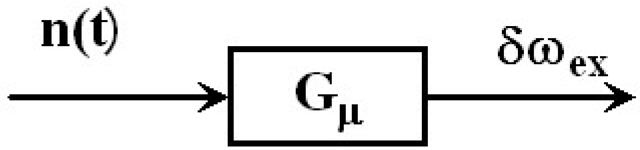


TEM-class cavities (ANL, single-spoke, 354 MHz, β=0.4)

- Dominated by low frequency (<10 Hz) from pressure fluctuations
- Few high frequency mechanical modes that contribute little to microphonics level.

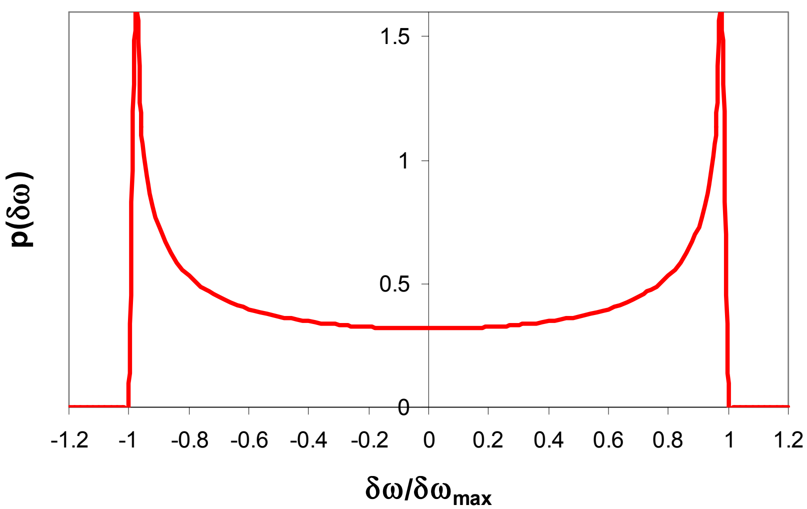


Probability Density (histogram)



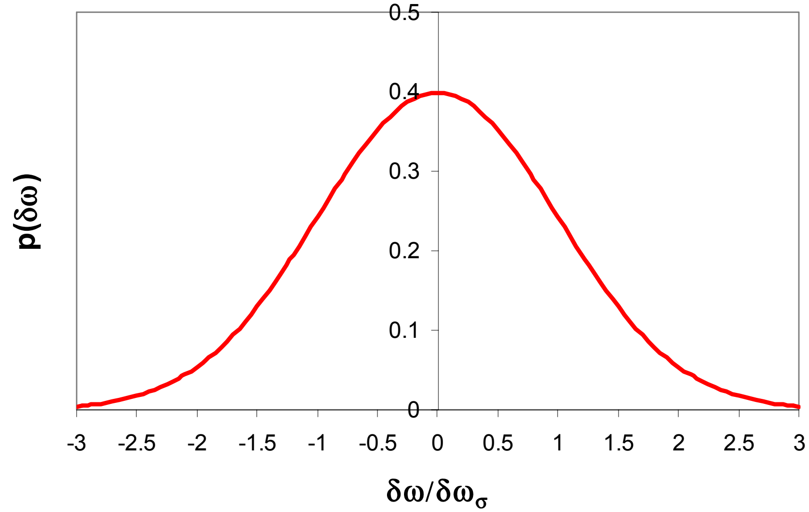
Harmonic oscillator (Ω_μ,τ_μ) driven by:

Single frequency, constant amplitude



$$p(\delta\omega) = \frac{1}{\pi\sqrt{\delta\omega_{\max}^2 - \delta\omega^2}}$$

White noise, gaussian



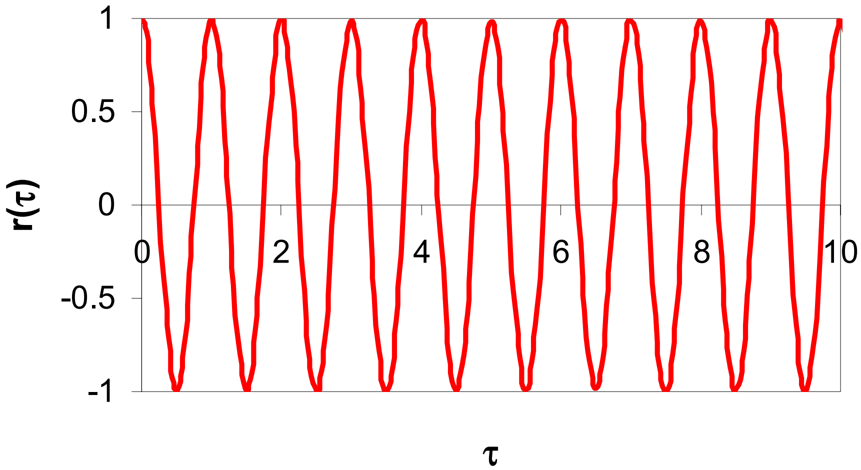
$$p(\delta\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\delta\omega}{\sigma_\omega}\right)^2\right]$$

Autocorrelation Function

$$R_x(\tau) = \langle x(t)\, x(t+\tau) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau)\, dt$$

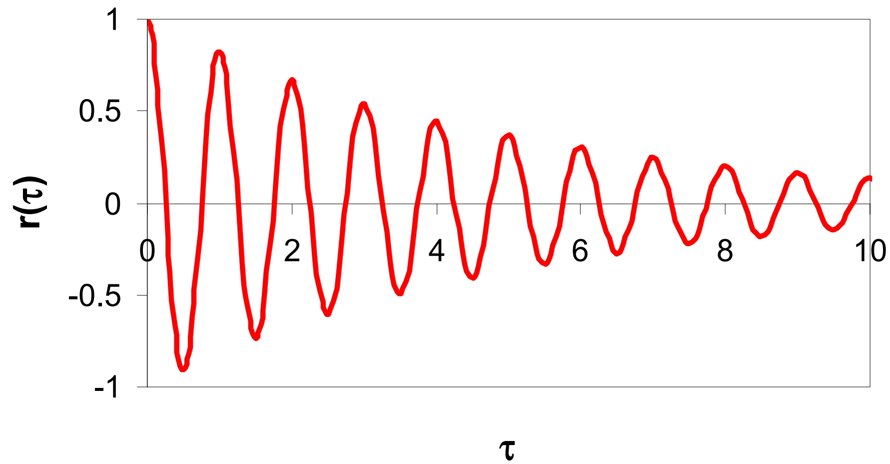
Harmonic oscillator (Ω_μ, τ_μ) driven by:

Single frequency, constant amplitude



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\omega_d \tau)$$

White noise, gaussian



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\Omega_\mu \tau)\, e^{-|\tau/\tau_\mu|}$$

Stationary Stochastic Processes

x(t): stationary random variable

Autocorrelation function: $$R_x(\tau) = \langle x(t)\, x(t+\tau) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau)\, dt$$

Spectral Density $S_x(\omega)$: Amount of power between ω and $d\omega$

$S_x(\omega)$ and $R_x(\tau)$ are related through the Fourier Transform (Wiener-Khintchine)

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau)\, e^{-i\omega\tau} d\tau \qquad R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega)\, e^{i\omega\tau} d\omega$$

Mean square value: $$\langle x^2 \rangle = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega)\, d\omega$$

Stationary Stochastic Processes

For a stationary random process driving a linear system

$$x(t) \longrightarrow \boxed{T(i\omega)} \longrightarrow y(t)$$

$$\left\langle y^2 \right\rangle = R_y(0) = \int_{-\infty}^{+\infty} S_y(\omega)\, d\omega \qquad \left\langle x^2 \right\rangle = R_x(0) = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega$$

$R_y(\tau)$ $\left[R_x(\tau)\right]$: auto correlation function of $y(t)$ $\left[x(t)\right]$

$S_y(\omega)$ $\left[S_x(\omega)\right]$: spectral density of $y(t)$ $\left[x(t)\right]$

$$\boxed{S_y(\omega) = S_x(\omega)\left|T(i\omega)\right|^2}$$

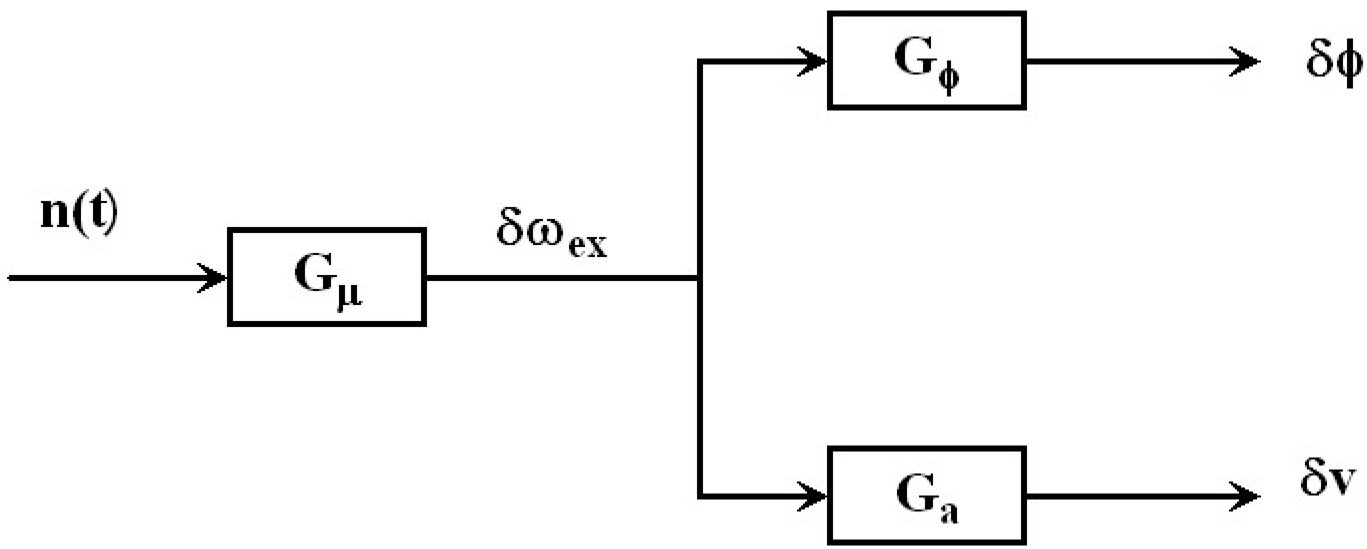
$$\boxed{\left\langle y^2 \right\rangle = \int_{-\infty}^{+\infty} S_x(\omega)\left|T(i\omega)\right|^2 d\omega}$$

Performance of Control System

Residual phase and amplitude errors caused by microphonics

Can also be done for beam current amplitude and phase fluctuations

Assume a single mechanical oscillator of frequency Ω_μ and decay time τ_μ excited by white noise of spectral density A^2



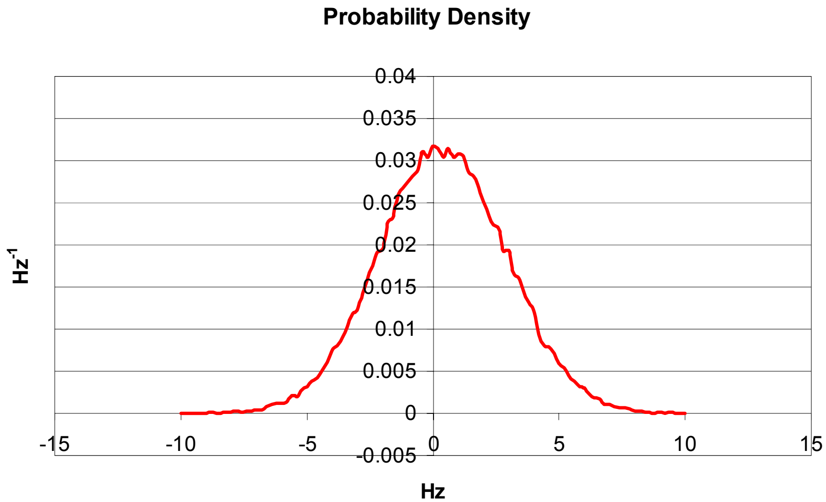
Performance of Control System

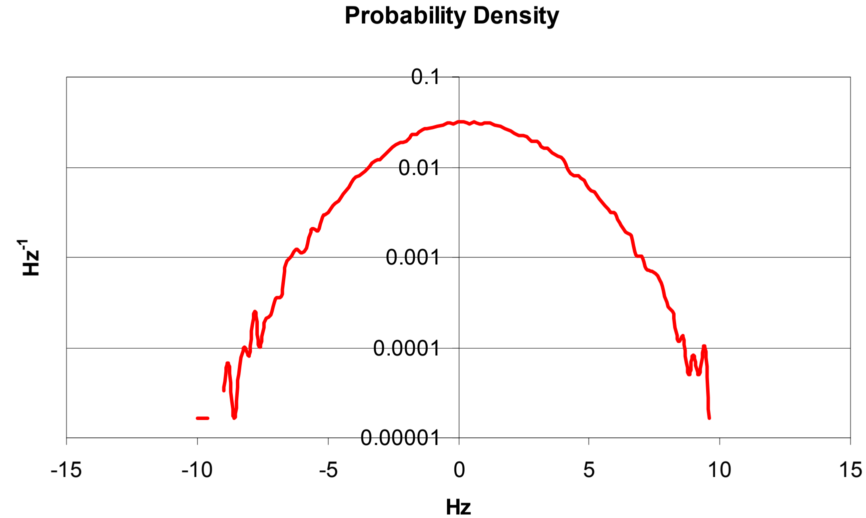
$$< \delta\omega_{ex}^2 > = A^2 \int_{-\infty}^{+\infty} \left|G_\mu(i\omega)\right|^2 d\omega = A^2 \int_{-\infty}^{+\infty} \frac{d\omega}{\left|-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2\right|^2} = A^2 \frac{\pi\tau_\mu}{2\Omega_\mu^2}$$

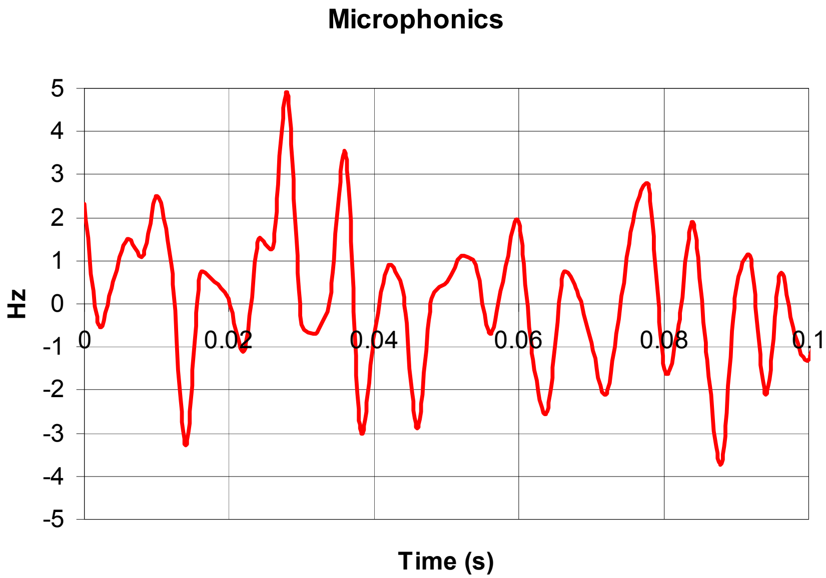
$$< \delta v^2 > = A^2 \int_{-\infty}^{+\infty} \left|G_\mu(i\omega) G_a(i\omega)\right|^2 d\omega = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{+\infty} \left|\frac{G_a(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2}\right|^2 d\omega$$

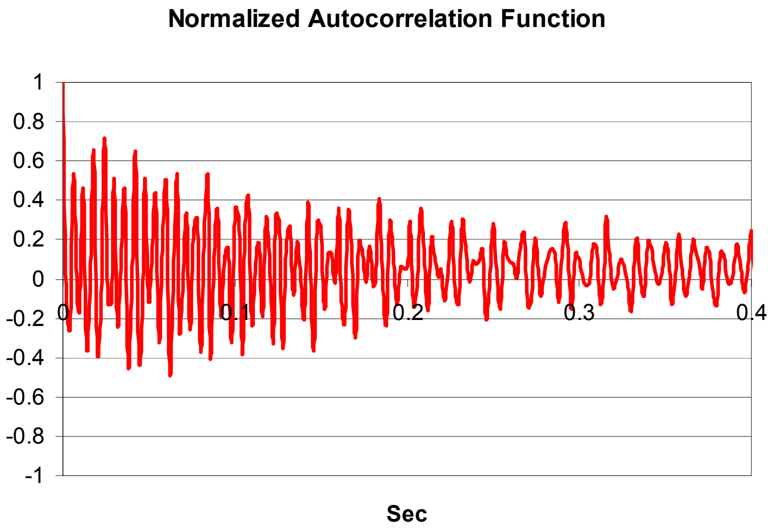
$$< \delta\varphi^2 > = A^2 \int_{-\infty}^{+\infty} \left|G_\mu(i\omega) G_\varphi(i\omega)\right|^2 d\omega = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{+\infty} \left|\frac{G_\varphi(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2}\right|^2 d\omega$$

The Real World

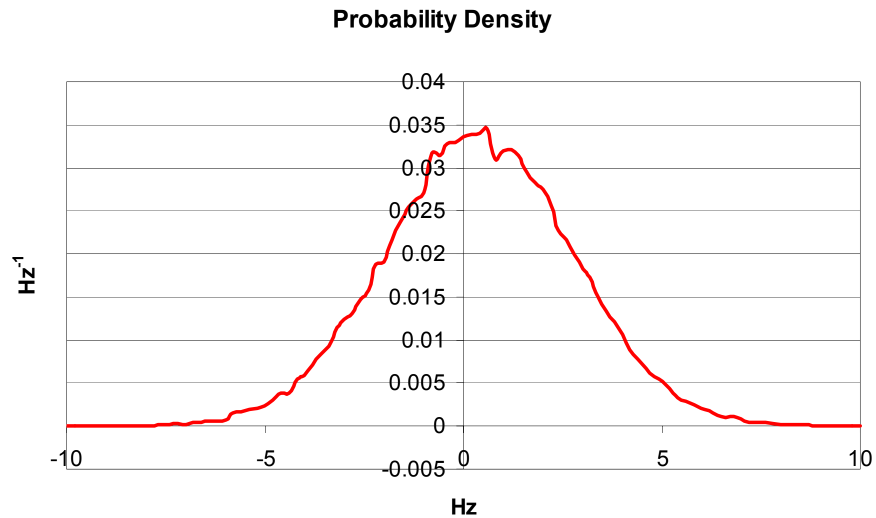


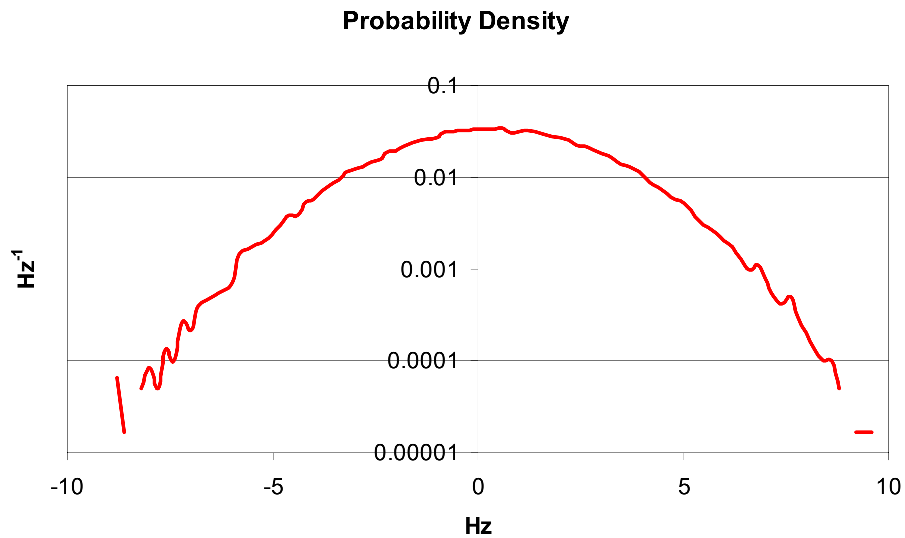


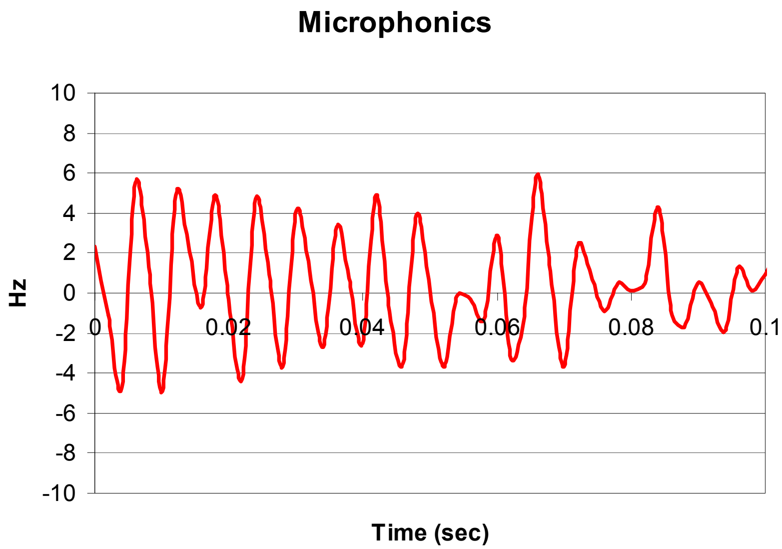


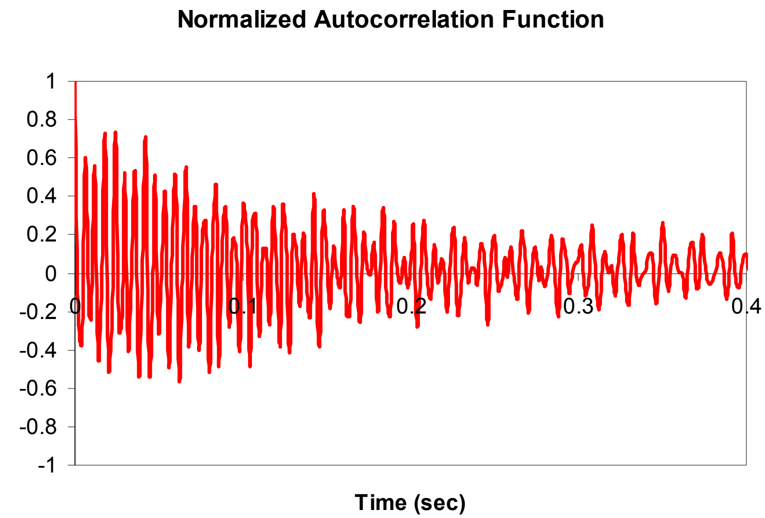


The Real World

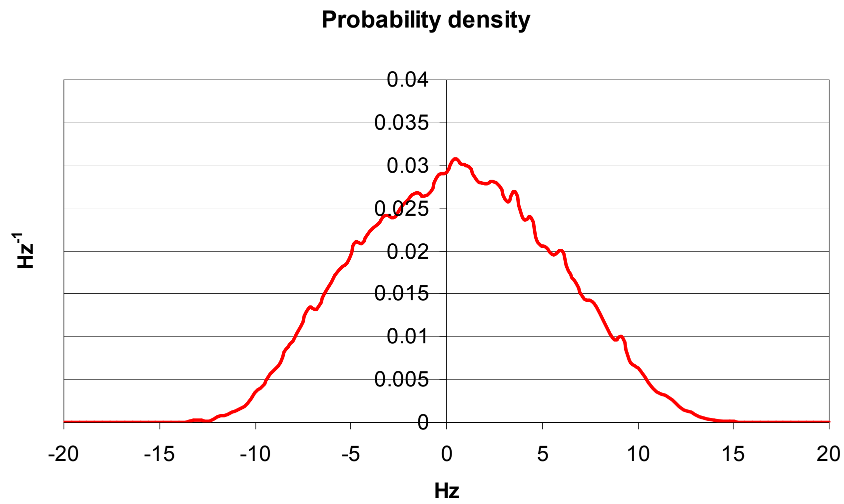


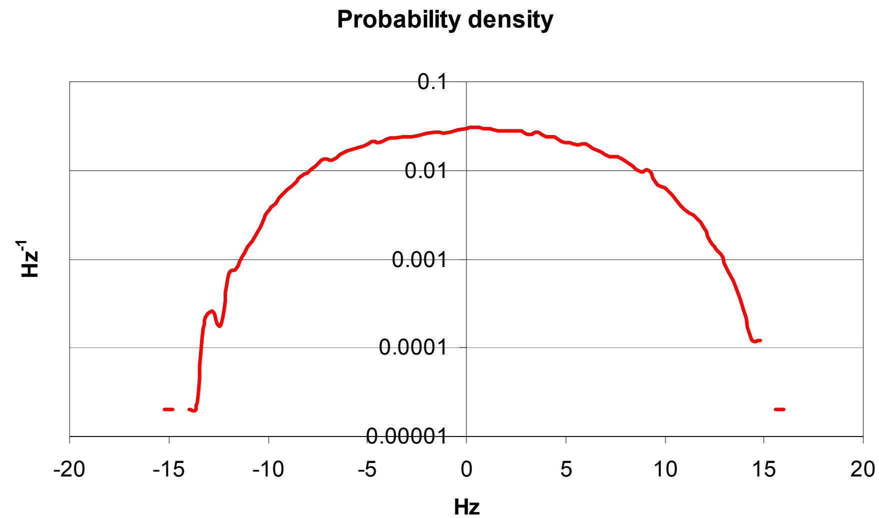


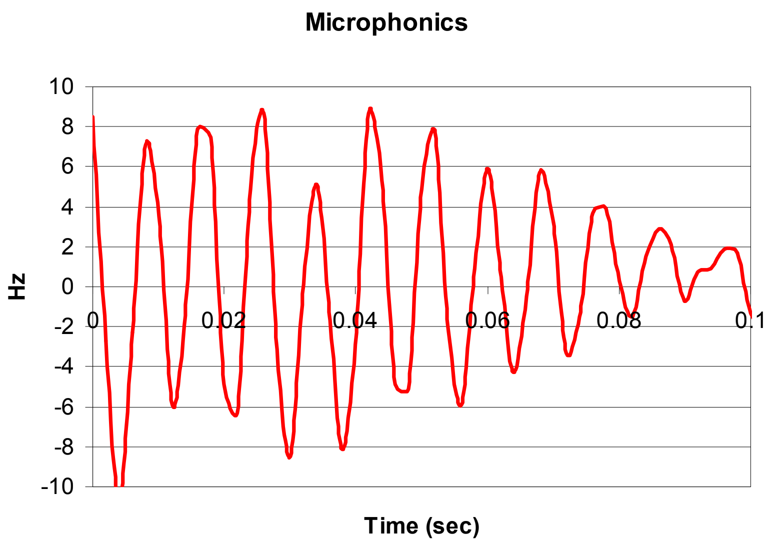


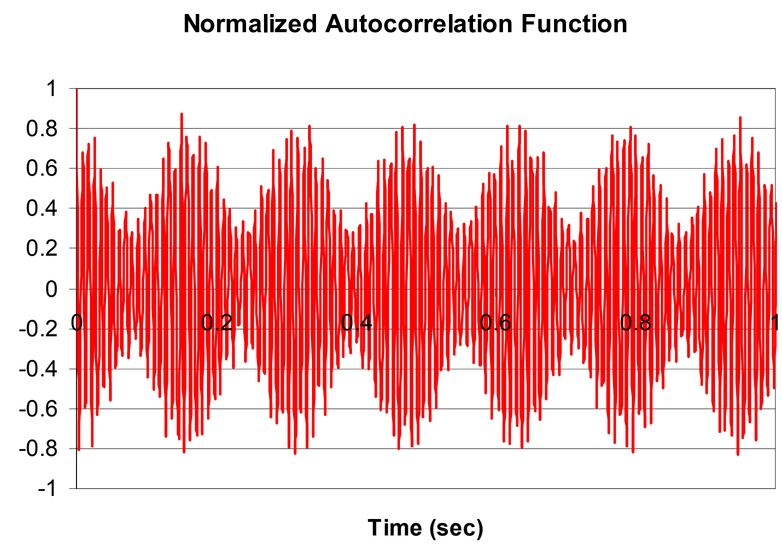


The Real World









Piezo control of microphonics

MSU, 6-cell elliptical 805 MHz, β=0.49

Adaptive feedforward compensation

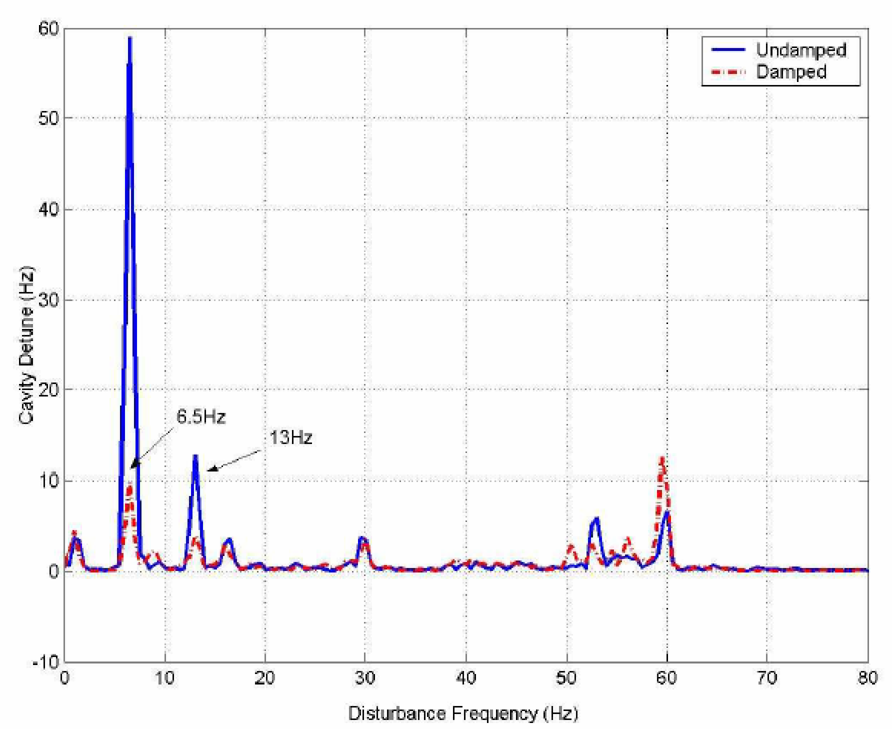


Figure 2. Active damping of helium oscillations at 2K.

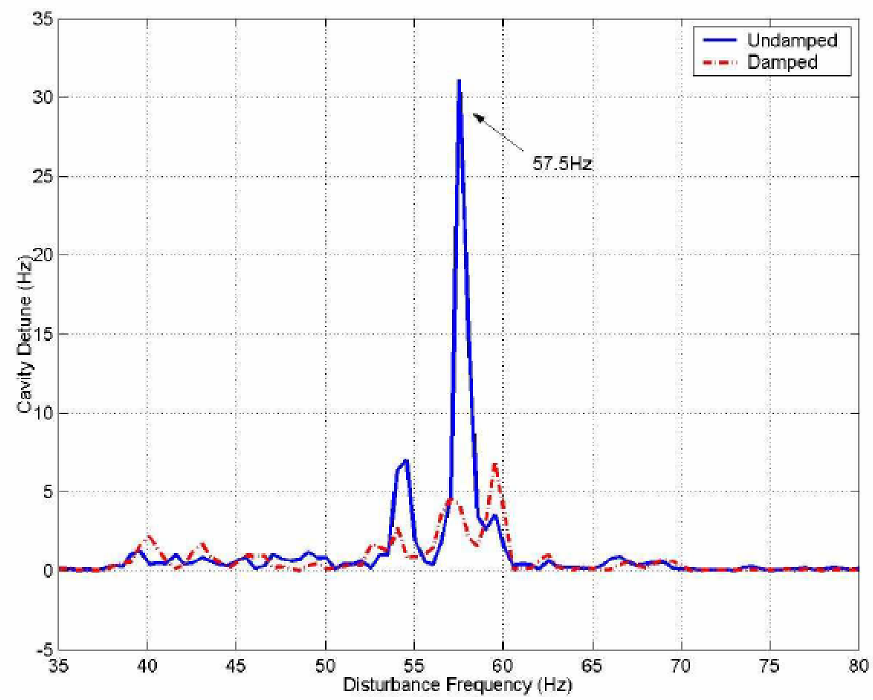
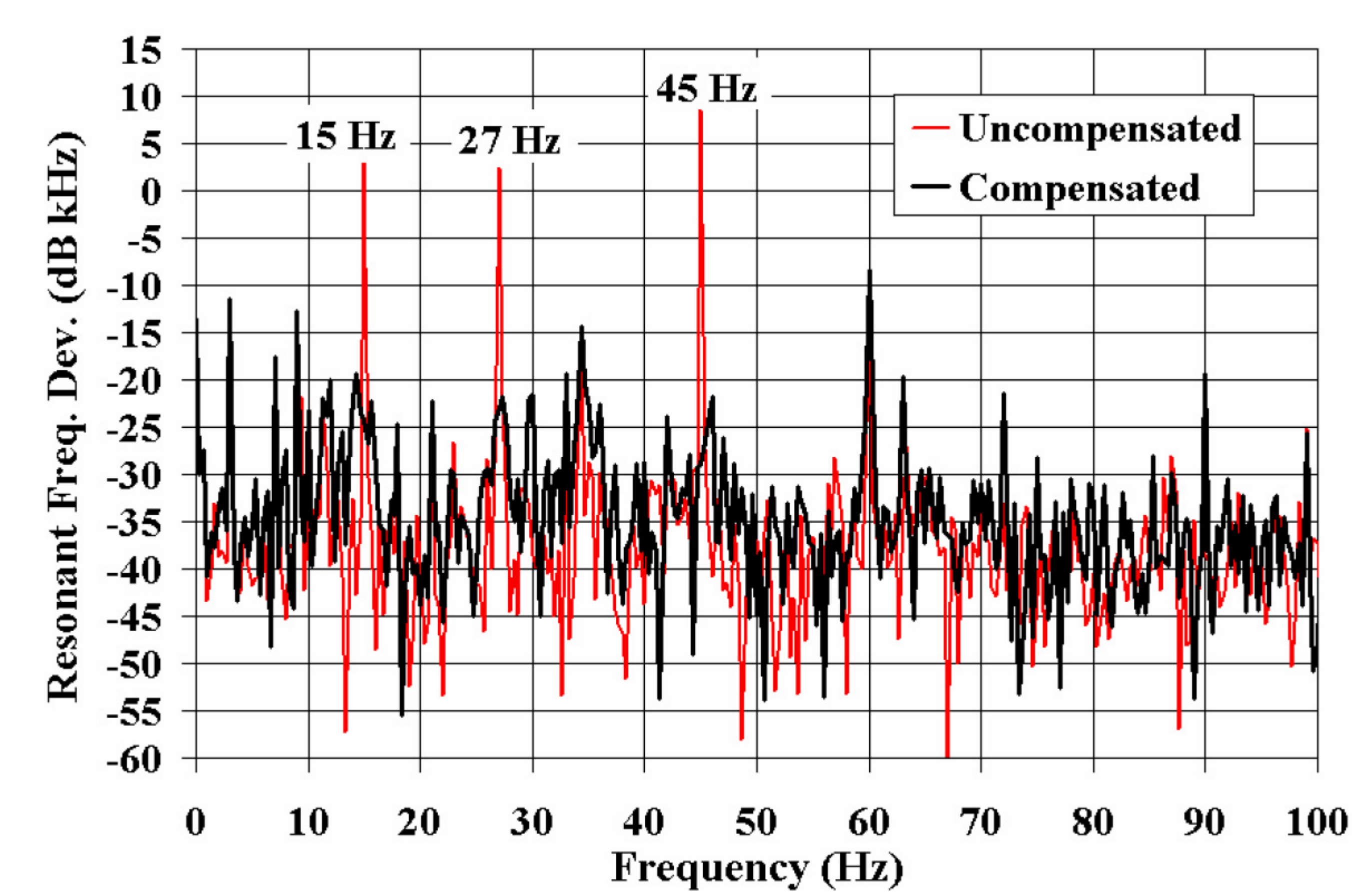


Figure 3. Active damping of external vibration at 2K.

Piezo Control of Microphonics

FNAL, 3-cell 3.9 GHz



Piezo Control of Microphonics

LANL 402.5 MHz scruncher cavity, $Q_L = 10^9$

Phase-locked with piezo only

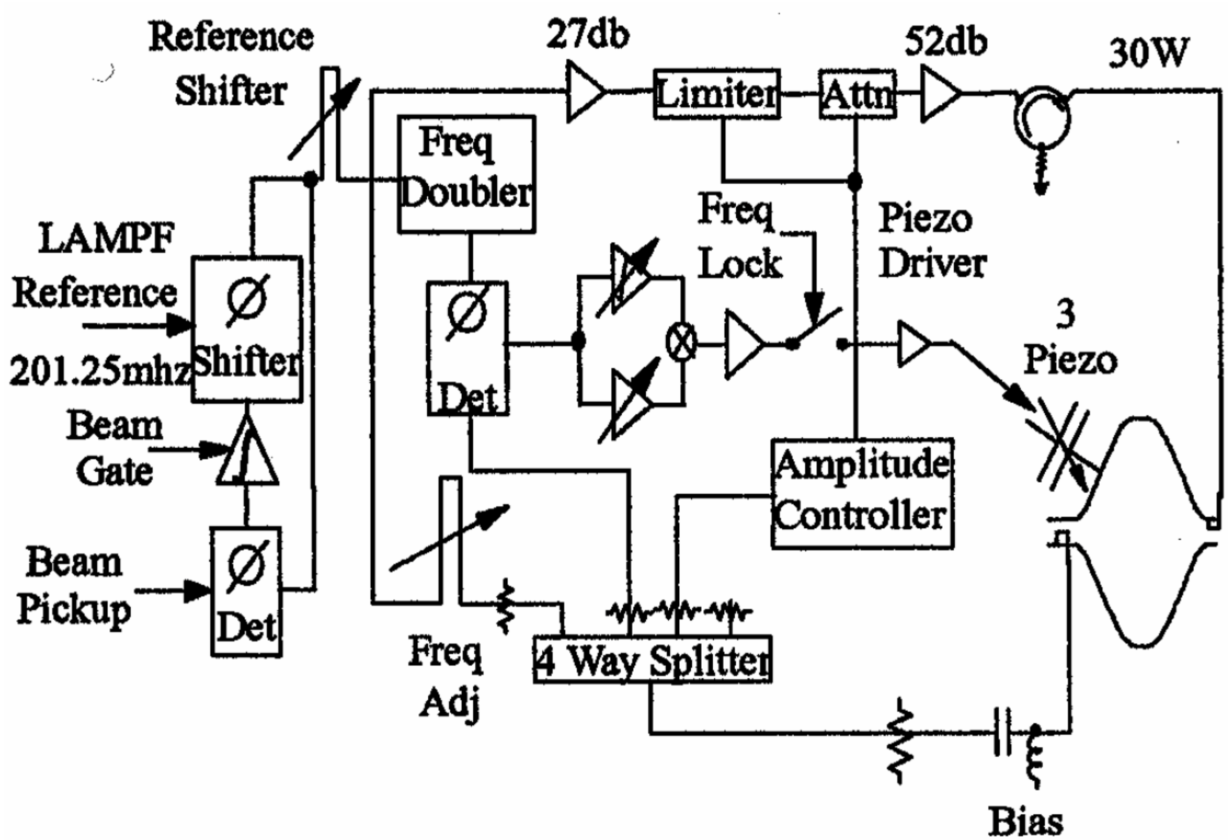


Fig 3. Control for the self oscillating mode.

Final Words

- Microphonics and ponderomotive instabilities issues in high-Q SRF cavities were “hot topics” in the early days (~70s), especially in low-β applications
- They were solved and are well understood
- They are being rediscovered in medium- to high-β applications
- Today’s challenges:
  - —Large scale (cavities and accelerators): need for optimization
  - —Finite beam loading
    - Small but non-negligible current (e.g. RIA)
    - Low current resulting from the not quite perfect cancellation of 2 large currents (ERLs)