Theoretical advances in SRF

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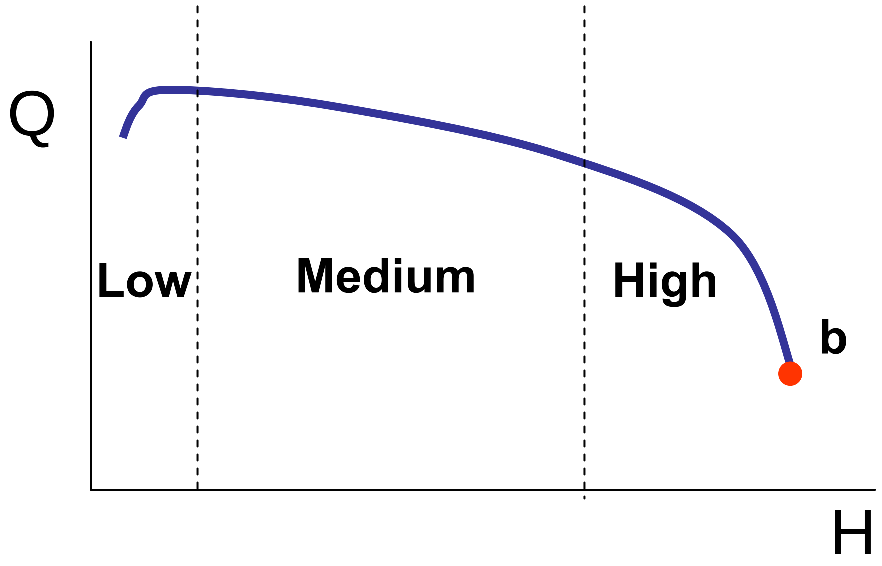
INTRODUCTION

Possible mechanisms behind the low-field, medium-field and high-field parts of Q(H) curve?



- Low - H slope:

Linear BCS + residual resistance R_i. Hypersound generation and acoustic resonances

- Medium – H slope

Nonlinear BCS resistance. Heating and nonequilibrium effects

- High - H slope

Vortex penetration, grain boundaries and flux focusing. Hotspots and thermal breakdown

BCS and residual surface resistance

$$R_s = \frac{A\omega^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) + R_i$$

$R_i \sim$ 1-20 nΩ

Constant R_i at $T \to 0$ for small H_0
Is inconsistent with the BCS theory

Mechanisms of R_i are likely unrelated to superconductivity

Field, temperature and frequency dependences of R_i are poorly understood

Effect of surface oxides (hydrides) or more fundamental mechanisms?

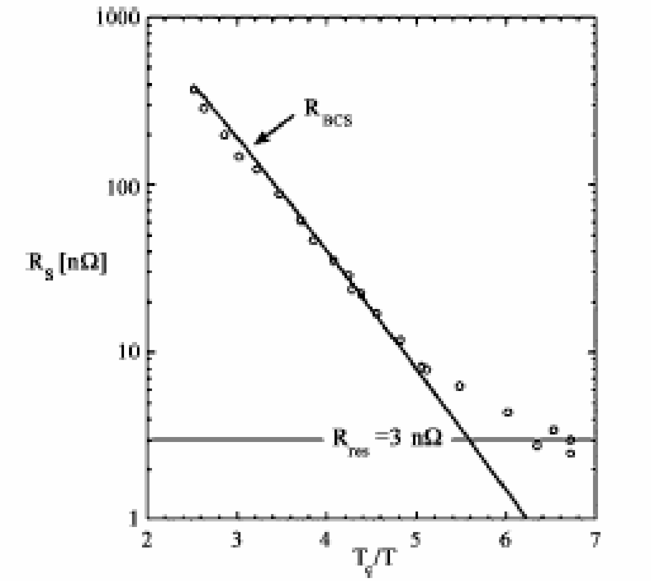
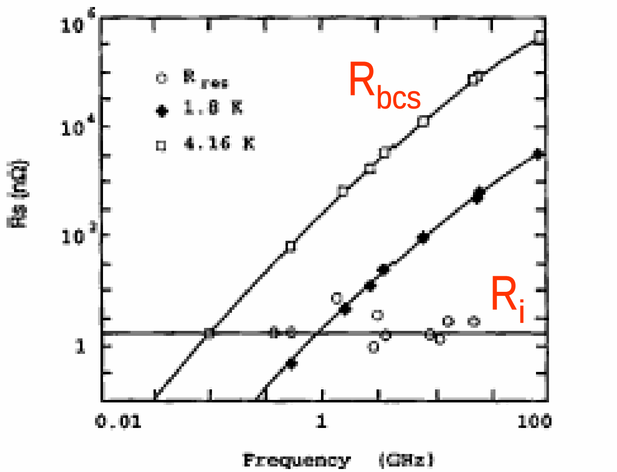
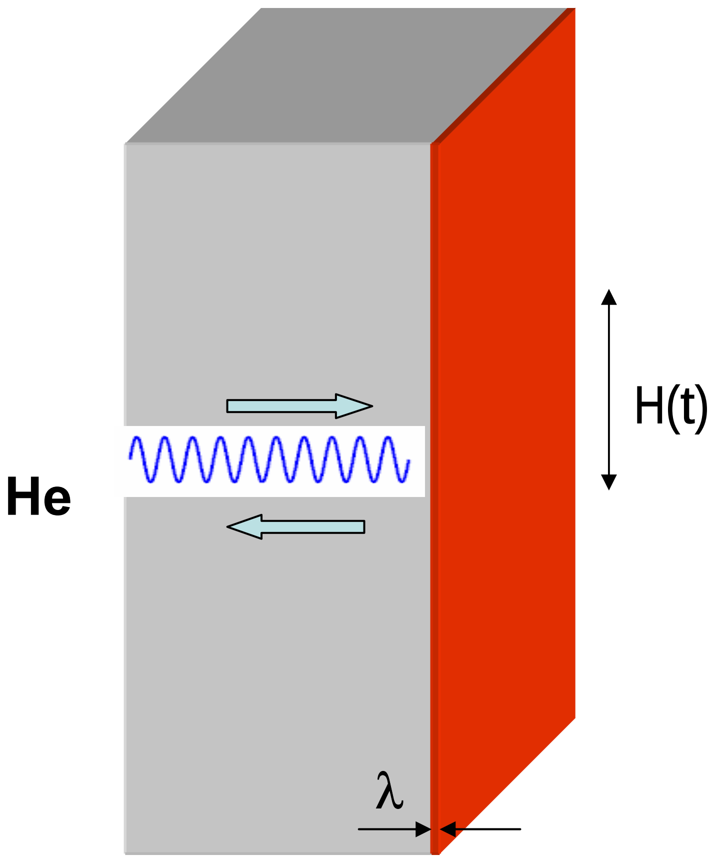


Figure 16. Measured temperature dependence of the surface resistance of a Nb cavity at 1.3 GHz. In this semi-log plot, the linear region gives an energy gap of $\Delta = 1.9kT_c$. The residual resistance is 3 nΩ.



Padamsee, SUST 14, R28 (2001)

Sound generation by rf field



Rf oscillating force generates a hypersound wave with the wavelength $\Lambda = s/f = 1.75\ \mu m$ for f = 2 GHz, c_s = 3.5 km/s

Λ is much greater than the London penetration depth $\lambda = 40$ nm, but much smaller than the wall thickness d = 2-3 mm

Nearly ideal reflection ($\Re \approx 1$) due to large acoustic mismatch between Nb and He

Standing sound wave unlike traveling wave does not cause rf dissipation … ?

$$\Re = \left(\frac{s_{Nb}\rho_{Nb} - s_{He}\rho_{He}}{s_{Nb}\rho_{Nb} + s_{He}\rho_{He}} \right)^2 \approx 0.996$$

Generation of transverse sound by rf electric field

Halbritter, JAP 42, 82 (1971); Passow, PRL 28, 427 (1972); Kartheuser and Rodriguez, JAP, 47, 700 (1967); Scharnberg, JAP 48, 3462 (1977)

$$R_i = \frac{\mu_0^2 n e^2 \lambda^4 \omega^2 s^3}{M(s^2+\omega^2\lambda^2)^2}\left(\frac{l}{l+\xi_0}\right)^2,$$

$$\frac{R_i}{R_{BSC}} \propto \frac{p_F}{Ms}\frac{T}{\Delta}\exp\left(\frac{\Delta}{T}\right) \quad (clean) \qquad \frac{R_i}{R_{BSC}} \propto \frac{p_F}{Ms}\frac{T}{\Delta}\frac{l\lambda}{(l+\xi_0)^2}\exp\left(\frac{\Delta}{T}\right) \quad (dirty)$$

Taking $p_F/Ms \sim 2\times10^{-3}$ for Nb, we get $R_i/R_{bcs} \sim 1$ at 2K in the clean limit, the ratio $\mathbf{R_i/R_{bcs}}$ decreasing as the rf surface layer gets dirtier.

Pros:

1. Right order of magnitude
2. Right temperature dependence

Cons:

1. Ignores that only $1-\mathfrak{R} = 0.4\%$ of sound energy contributes to R_i

Experiment: Kneisel et al (1971) + later works

Generation of longitudinal sound by rf magnetic pressure

$$s^2 u'' - \ddot{u} = \frac{BH}{2\lambda\rho} e^{-2x/\lambda} \cos 2\omega t, \qquad u'(0) = 0$$

Propagating wave:

$$u = \frac{iBH\lambda s}{16\rho(s^2 + \omega^2\lambda^2)\omega} \exp\left(\frac{2i\omega}{s} x\right)$$

Rf dissipation: $Q = R_i H^2/2 = 4\omega^2 s\rho|u|^2/2$

$$R_i = \frac{B^2 s^3}{64\rho(s^2 + \omega^2\lambda^2)^2}$$

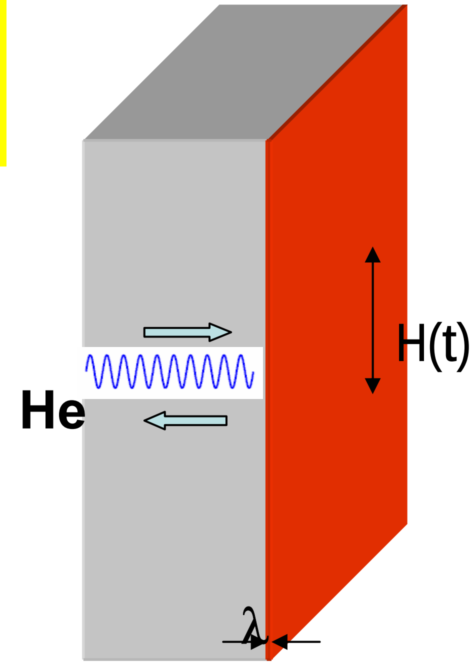
1. Independent of ω and T for $\Lambda \gg \lambda$
2. Quadratic in rf field

For B = 100 mT, s = 3.5 km/s, ρ = 8.5 g/cm^3, we get R_i = 0.08nΩ

Effect of sound attenuation and reflection

$$s^2u'' + \omega^2 u + i\gamma\omega u = -\frac{eE_0}{\rho}\Theta(q,\omega)e^{-x/\lambda + i\omega t}$$

$$\gamma \cong \omega \frac{k_B T}{Ms^2}\left(\frac{T}{\theta_D}\right)^3$$



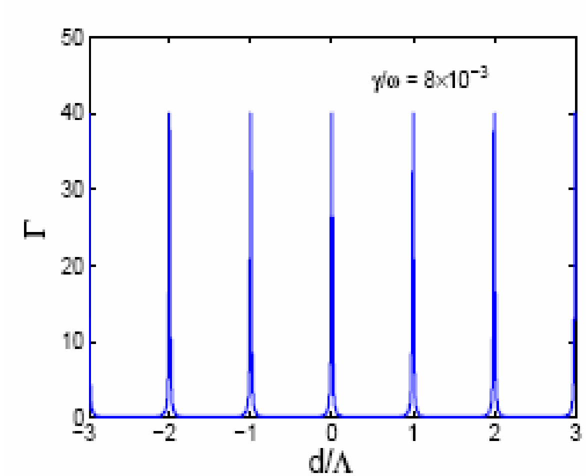
Acoustic Q factor $\omega/\gamma \sim 10^7$ at 2K provides very weak attenuation $\gamma d \ll s$

$$R_i = \frac{R_{i0}\sinh(\gamma d/s)}{\cosh(\gamma d/s) - \cos(2\omega d/s)}$$

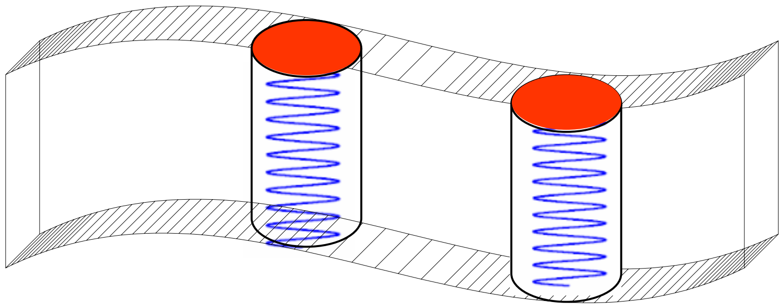
Sound reflection makes R_i negligible unless the resonance condition

$\omega d = \pi s n$, n = 1, 2, 3 ... is satisfied:

$$n\Lambda = d$$



Acoustic hotspots



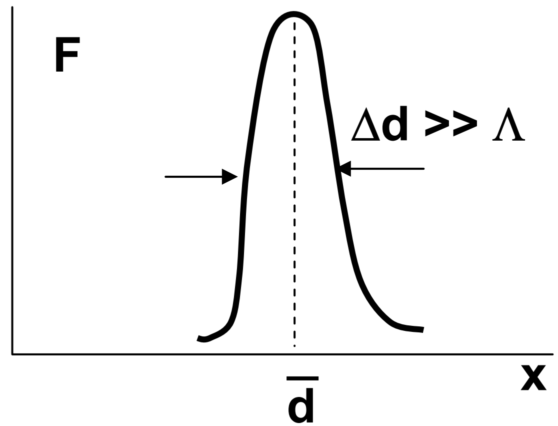
Distribution function of acoustic resonance frequencies due to:

1. Smooth thickness wall variation by
$\Delta d >> \Lambda \sim 1\ \mu m$ on the scale $\sim d = 2\text{-}3$ mm
2. Spectrum of rf frequencies in coupled cavities

Hotsport in the regions where the local thickness d(x,y) satisfies the resonance condition $n\Lambda = d$

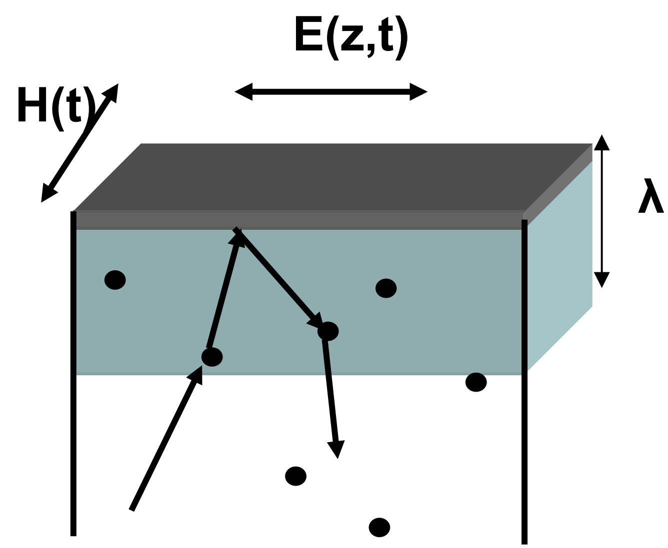
Averaging with the thickness distribution function F(x):

$$\overline{R_i} = R_{i0}\int_0^\infty \frac{\sinh(\gamma x/s)F(x)dx}{\cosh(\gamma x/s)-\cos(2\omega x/s)} \cong \sqrt{2}R_{i0}$$



Averaged R_i is of the order of R_{i0} and depends neither on the small γ nor the shape of F(x). Effect of sound scattering and generation of Rayleigh surface waves

BCS rf dissipation



- Thermal activation of normal electrons

  $n_r = n_0(\pi T/2\Delta)^{1/2}\exp(-\Delta/T)$

- Accelerating electric field

  $E(z,t) = \mu_0\omega\lambda H_\omega e^{-\lambda|z|}\sin\omega t$

- Scattering mechanisms and normal state conductivity: $\sigma_n = e^2 n_0 \mathbf{l}/p_F$, $p_F = \hbar(3\pi^2 n_0)^{1/3}$

- Surface: from specular to diffusive

- Normal skin effect ($\mathbf{l} \ll \lambda$): multiple impurity scattering in the λ - belt:

  $R_s \sim (\mu_0^2\omega^2\lambda^3\sigma_n\Delta/T)\exp(-\Delta/T)$

- Anomalous skin effect ($\mathbf{l} \gg \lambda$): scattering by the gradient of the ac field E(z):

  Effective $\sigma_{eff} \sim e^2 n_0\lambda/p_F;$ $\mathbf{l} \to \lambda$

R_s is independent of bulk impurities

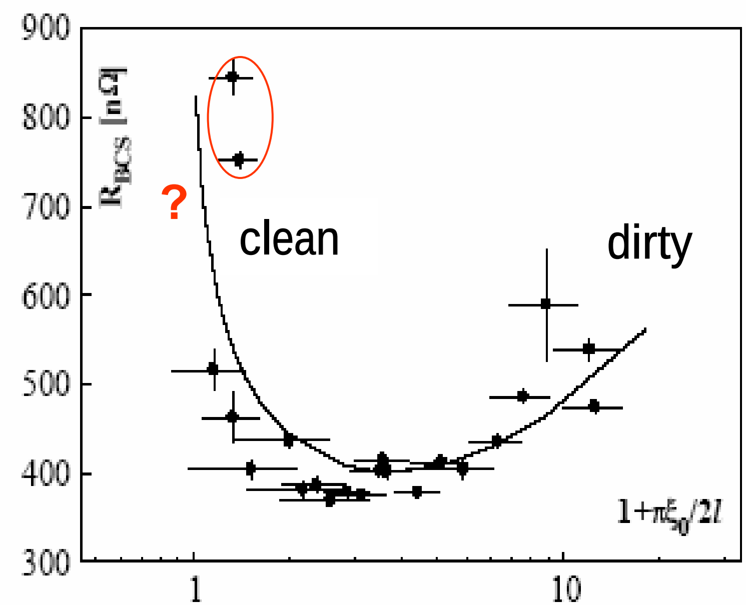


Figure 3. The BCS resistance at T=4.2K.

Low-frequency R_s for a clean ($l \gg \lambda$) type II superconductor

Linear BCS surface resistance for small-amplitude rf field $H \ll H_c T/T_c$

$$R_s \propto \frac{\mu_0^2 \omega^2 \lambda^4 \Delta n_0}{k_B T p_F}\left[\ln\left(\frac{\Delta}{\hbar\omega}\right) + C_0\right]\exp\left(-\frac{\Delta}{k_B T}\right)$$

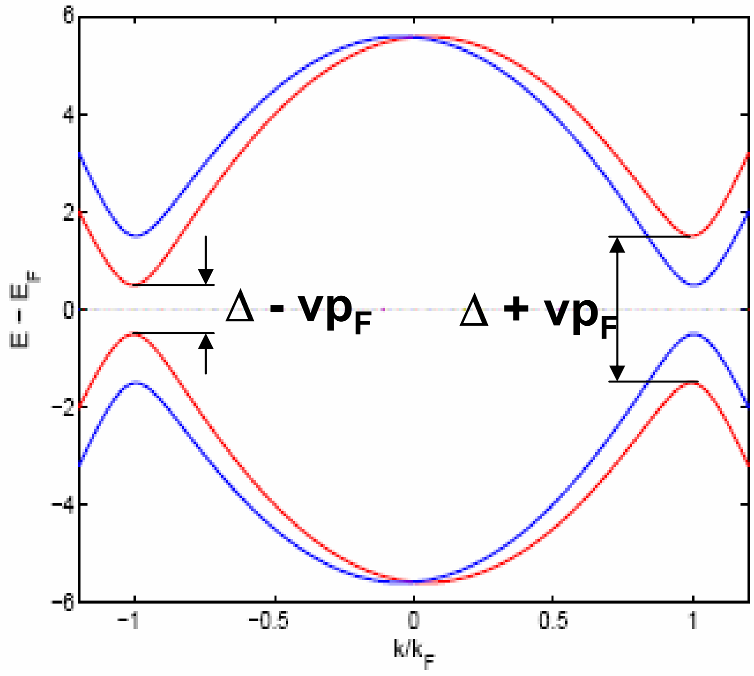
Logarithmic term $\ln\omega$ comes from the BCS coherence factors

Density of thermally-activated electrons:

$$n_r = n_0 \left(\frac{\pi k_B T}{2\Delta}\right)^{1/2} \exp\left(-\frac{\Delta}{k_B T}\right)$$

The main R_s nonlinearity in strong rf fields comes from the dependence of $n_r(J)$ on J

Effect of current on thermal activation



Rocking "tilted" electron spectrum in the current-carrying state $J = J_0 \cos\omega t$

$$E(p) = \pm\sqrt{\Delta^2 + (p^2/2m - E_F)^2} \pm \vec{p}_F \vec{v}_s(t)$$

Superfluid velocity $v_s(t)$

$$v_s(t) = \frac{J(t)}{n_s e}$$

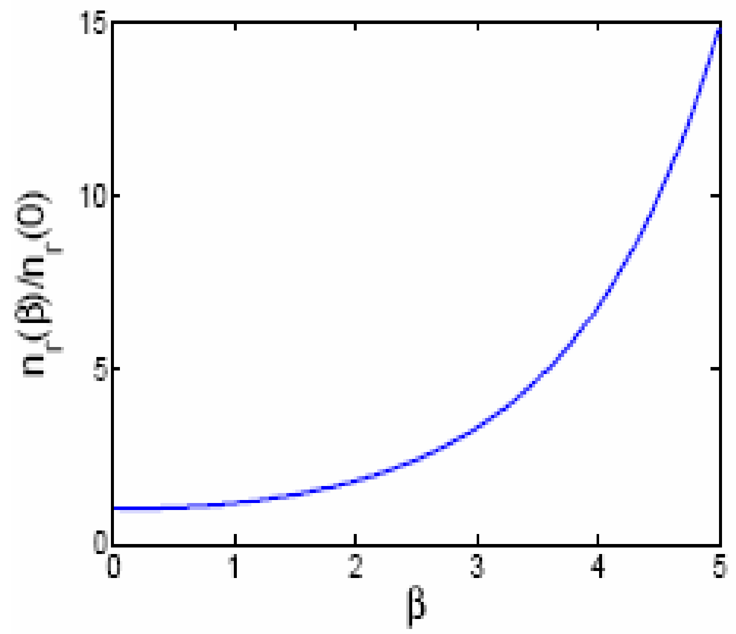
- Reduction of the gap from Δ to $\Delta - p_F|v_s|$ increases density of thermally-activated normal electrons $n_r(J)$, thus increasing R_s

- General theory requires solving kinetic equation for the electron distribution function taking into the account impurity and electron-phonon scattering

Simple model: density of normal electrons

If J(x) varies weakly over the coherence length ξ, then:

$$n_r(J) = N(0)\int_0^\infty e^{-\frac{\sqrt{\Delta^2+\eta^2}}{k_B T}} d\eta \int_0^\pi e^{-\frac{p_F v_s \cos\theta}{k_B T}} \sin\theta d\theta = n_r(0)\frac{\sinh\beta(t)}{\beta(t)}$$



Current driving parameter:

$$\beta = \frac{p_F v_s(t)}{k_B T} = \frac{\pi}{2^{3/2}}\frac{\Delta}{k_B T}\frac{H(t)}{H_c}$$

- Thermodynamic critical field $H_c = \phi_0/2^{3/2}\pi\lambda\xi$.
- The nonlinearity becomes more pronounced at lower T for $H > H_c T/T_c \ll H_c$

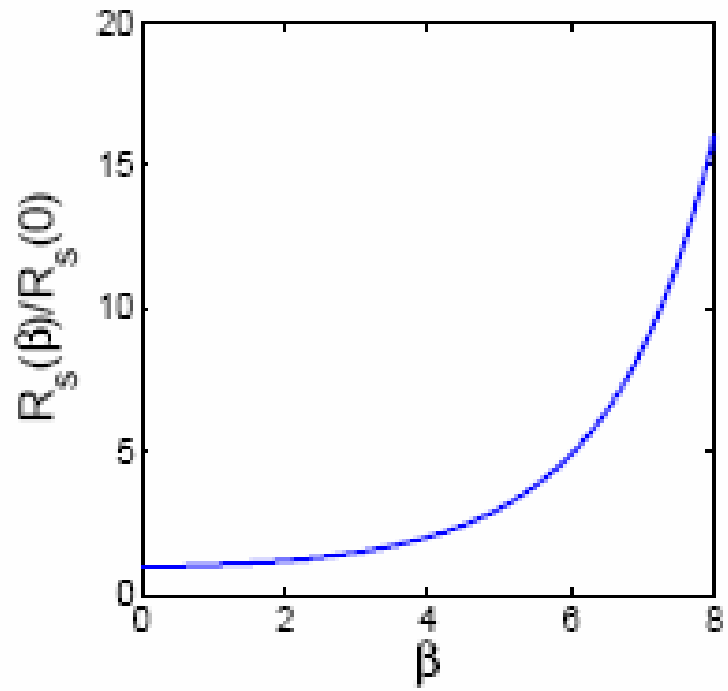
Simple model: nonlinear rf surface resistance

Time-averaged dissipated power:

$$Q = \frac{R_s H^2}{\pi} \int_0^{\pi} \sin^2 t \, \frac{\sinh(\beta_0 \cos t)}{\beta_0 \cos t} dt$$

$$Q \cong \frac{R_s H^2}{2} \left(1 + \frac{\pi^2}{192} \left(\frac{\Delta}{k_B T} \right)^2 \left(\frac{H}{H_c} \right)^2 \right), \qquad \beta_0 = \frac{\pi}{2^{3/2}} \frac{\Delta}{k_B T} \frac{H}{H_c} << 1$$

$$Q \cong R_s H_c^2 \frac{8 k_B^2 T^2}{\pi^2 \Delta^2} \frac{\exp(\beta_0)}{\sqrt{2\pi\beta_0}}, \qquad \beta_0 = \frac{\pi}{2^{3/2}} \frac{\Delta}{k_B T} \frac{H}{H_c} > 1$$



At small fields $R_s(H)$ gets a quadratic correction in H, but for $\beta_0 > 1$, the surface resistance increases exponentially with the rf field amplitude.

For T = 2K and T_c = 9.2K, the parameter β_0 varies from 0 at H = 0 to 9.7 at H = H_c

Theory of nonlinear R_s for a clean type-II superconductor, $\lambda \gg \xi$, $\omega\tau_r < 1$

Solving a kinetic equation for the electron distribution function with the account of the BCS coherence factors. Superimposed dc and ac field: $H(t) = H_{dc} + H_0 \cos\omega t$:

$$R_s(\beta) = \frac{\omega^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right)\sum_{n=0}^{\infty} g_n(\beta)\left[A(\omega) + \frac{1}{2n+1}A\left(\frac{\omega}{2n+1}\right)\right],$$

$$g_n = \frac{(2n+1)!!}{2^{n-1}(2n)!(n+2)!(n+1)}\int_0^{\pi} \sin^2 t(\beta_{dc} + \beta_0 \cos t)^{2n}\frac{dt}{\pi},$$

$$R_{bcs} = \omega^2 \frac{A(\omega)}{T}\exp\left(-\frac{\Delta}{k_B T}\right), \qquad A \propto \frac{\Delta \mu_0^2 n_0 e^2 \lambda^4}{p_F}\ln\frac{k_B T \Delta \xi^2}{\hbar^2 \omega^2 \lambda^2}$$

The simple model gives very similar temperature, field and frequency dependencies of $R_s(T,H,\omega)$

Dependence of the nonlinear **$R_s(T,H,\omega)$ on dc field unrelated to vortices**

Example: low rf amplitude

Theory (to the accuracy of small logarithmic terms in ω):

$$R_s(H) \cong \left[1 + \frac{\pi^2}{384}\left(\frac{\Delta}{k_B T}\right)^2 \left(4H_{dc}^2 + H_0^2\right)\right] R_{bcs}$$

The simple model captures the correct field and temperature dependence of the nonlinear R_s

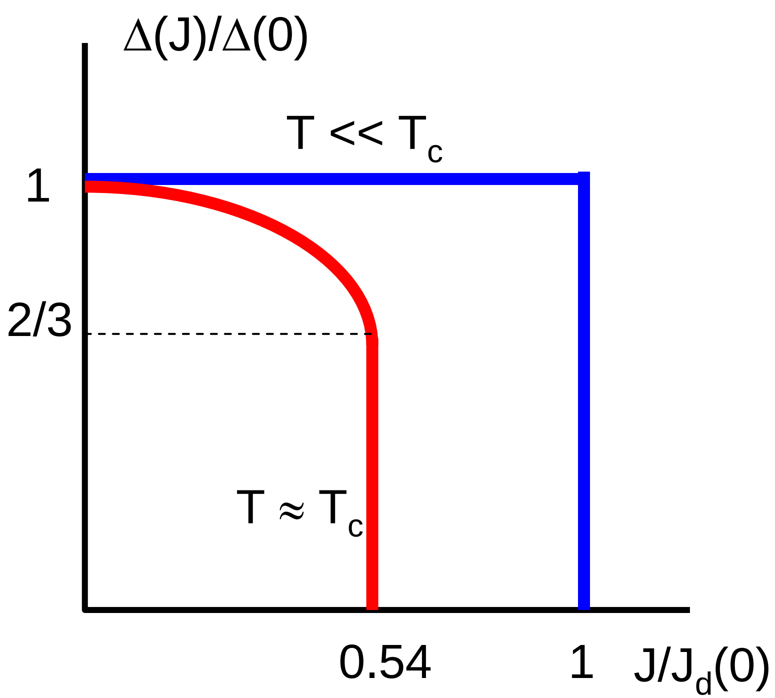
For Nb at T = 2K, the nonlinear contribution is essential even for $H_0 < H_c$

$$R_s(H_0, 2K) \cong \left(1 + 2\left(\frac{H_0}{H_c}\right)^2\right) R_{bcs}(2K)$$

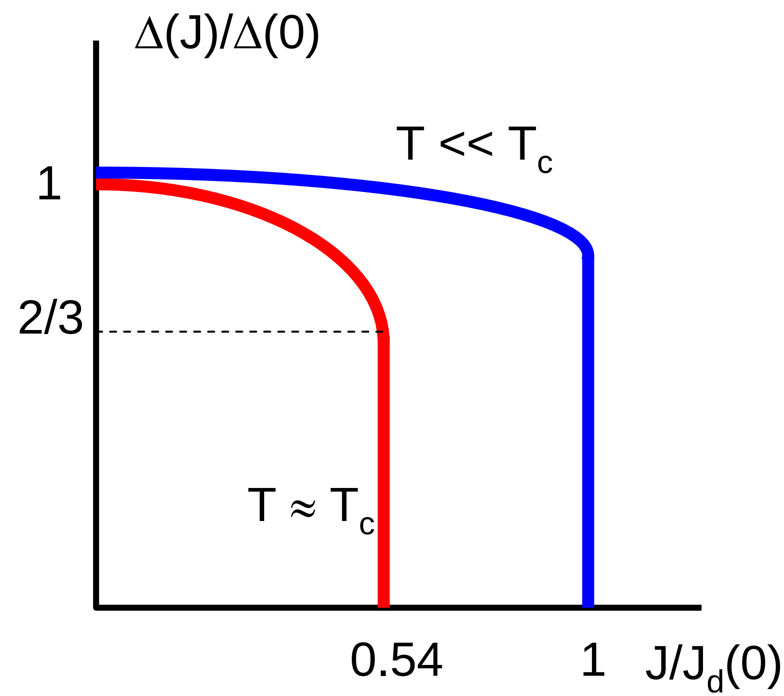
The BCS nonlinearity becomes more pronounced at lower temperatures

Effect of impurities on Δ(J)

- **Clean limit ($l \gg \xi_0$)**



- **Dirty limit ($l \ll \xi_0$)**



In the clean limit Δ(J) is independent of J at low T (J. Bardeen, Rev. Mod. Phys. 34, 667 (1962)).

Dirty 40 nm layer near the Nb surface can decrease the nonlinearity of R_s

Kinetics of normal electrons

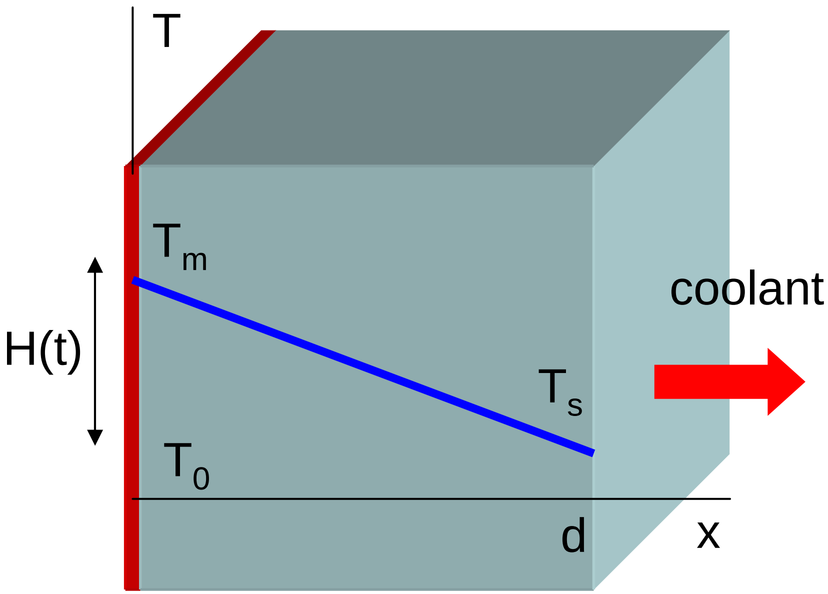
- Bulk of Nb cavities is usually clean enough to ensure **l** $>> \lambda \sim 40$ nm, but the dirtiness of the rf surface layer is unclear
- For **l** $>> \lambda$, the normal state resistivity is irrelevant to the rf surface resistance.
- Quasi-static rf resistance $\omega << \Delta$ (good approximation for SC cavities)
- Quasi-equilibrium Fermi-Dirac distribution function for normal electrons: $2\pi\tau_r f << 1$
- Recombination time due to electron-phonon collisions (Kaplan et al, PRB 14, 8454 (1976))

$$\tau_r^{-1} = \tau_0^{-1}\left(\frac{\pi T}{T_c}\right)^{1/2}\left(\frac{2\Delta}{T_c}\right)^{5/2}\exp\left(-\frac{\Delta}{T}\right)$$

TESLA single cell cavity (f = 1.3 GHz, T = 2K, τ_0^{-1} = 6.7 GHz), $\tau_r^{-1} \approx$ **0.03 GHz**

Nonequilibrium effects can be important for strong rf fields $H_a \sim H_c$

Analytical thermal breakdown model



$$\frac{\partial}{\partial x}\kappa(T)\frac{\partial T}{\partial x}+\frac{1}{2}H_\omega^2 R_s(T_m)\delta(x)=0$$

Instead of numerically solving this ODE, one can solve much simpler equations for T_m and T_s

$$\frac{1}{2}H_0^2 R_s(T_m)=\kappa(T_0)(T_m-T_s)/d,$$

$$\int_{T_0}^{T_s}\kappa(T)dT = d\alpha(T_s,T_0)(T_s-T_0)$$

Kapitza thermal flux: $q = \alpha(T,T_0)(T - T_0)$

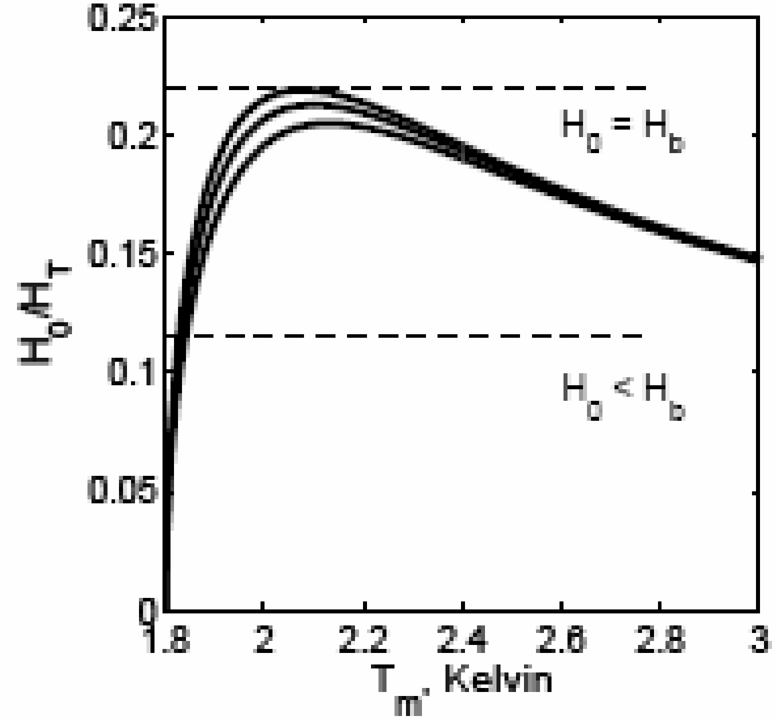
For a general case of thermal quench, see Gurevich and Mints, Reviews of Modern Physics 59, 941 (1987), & Argonne workshop, 2004.

Maximum temperature

BCS + residual surface resistance R_i

$$R_s = \frac{A\omega^2}{T}\exp\left(-\frac{\Delta}{T}\right) + R_i$$

Since $T_m - T_0 \ll T_0$ even H_b, we may take κ and h at $T = T_0$, and obtain the equation for $H(T_m)$:



$$H_0^2 = \frac{2T_m(T_m - T_0)\tilde{\alpha}}{[A\omega^2\exp(-\Delta/T_m) + T_m R_i]}, \qquad \tilde{\alpha} = \frac{\alpha}{1 + d\alpha/\kappa}$$

Breakdown rf field

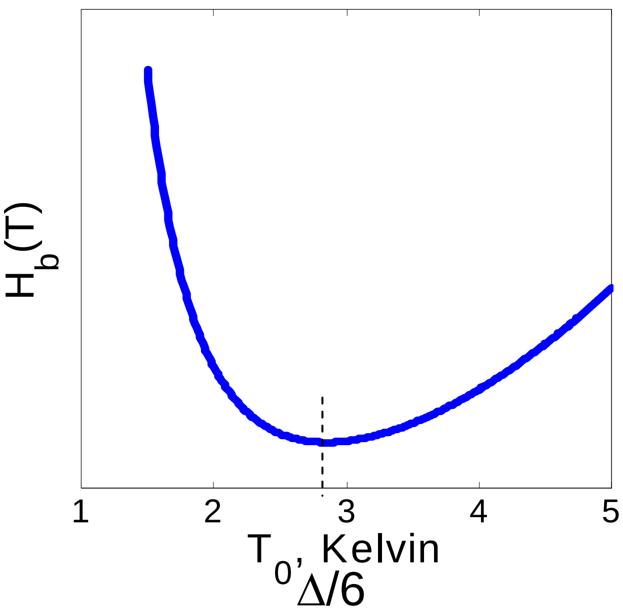
Thermal runaway occurs at a rather weak overheating:

$$T_m - T_0 \approx \frac{T_0^2}{\Delta} = \frac{T_0^2}{1.86\,T_c} = 0.23\,K,$$

$$H_b^2 = \frac{2h\kappa T_0^3}{(\kappa + dh)R_0 T_c \Delta e}\exp\left(\frac{\Delta}{T_0}\right)$$

For $\kappa \gg d\alpha$, the breakdown field is limited by the Kapitza resistance, $\alpha(T)=kT_0^3$. Thus,

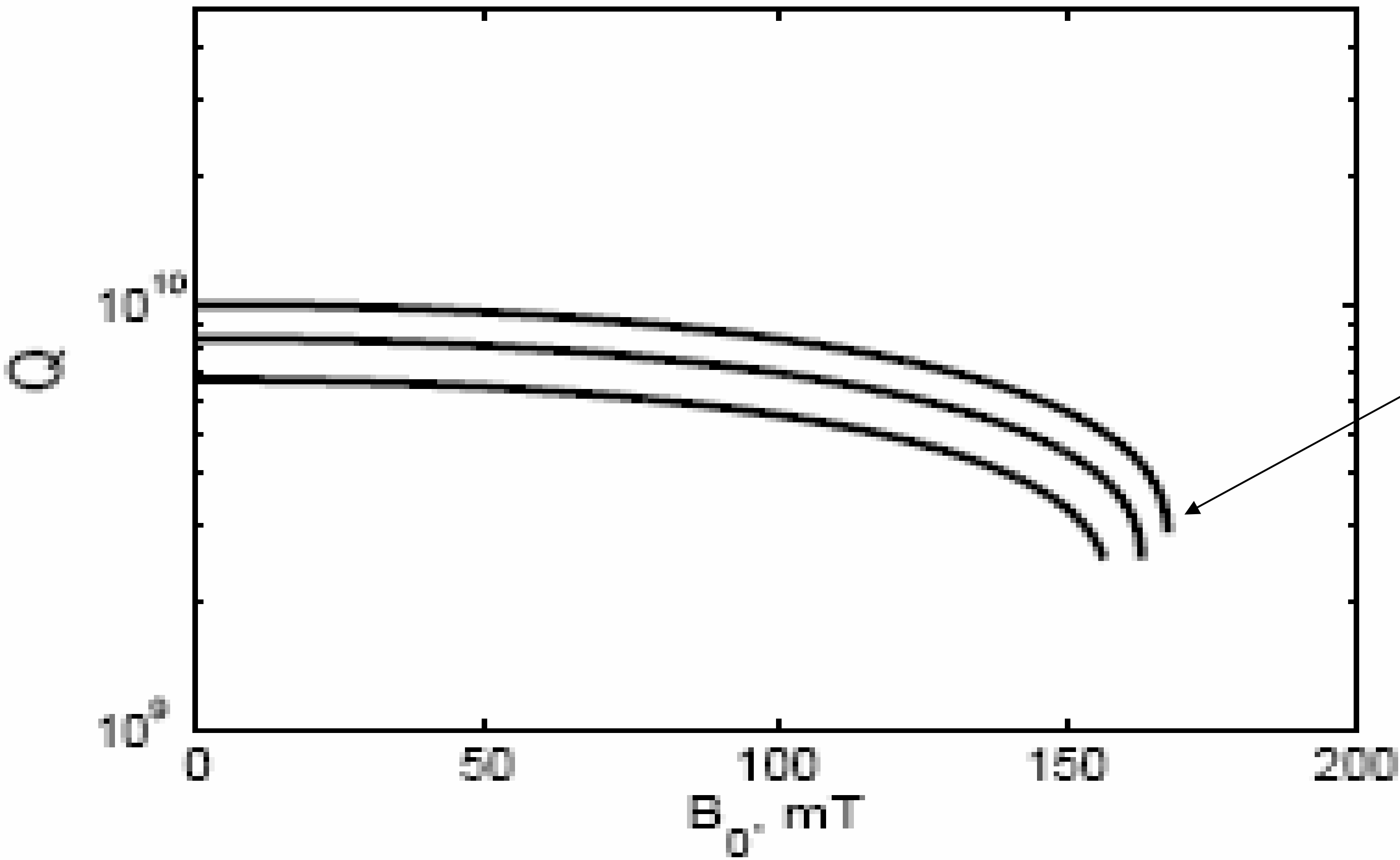
$$H_b = \left(\frac{2k}{R_0 T_c e \Delta}\right)^{1/2} T_0^3 \exp\left(\frac{\Delta}{2T_0}\right)$$



is minimum at $T_0 = \Delta/6$

For low T, the BCS nonlinearity becomes important

Q-factor (linear resistance)



$Q(H_b) \approx Q(0)/e = 0.37Q(0)$

Q versus H_0 for T_0 = 2.2K and different $R_i/R_{BCS}(T_0)$ = 0, 0.2 and 0.5 (top to bottom).

Thermal breakdown for nonlinear BCS resistance

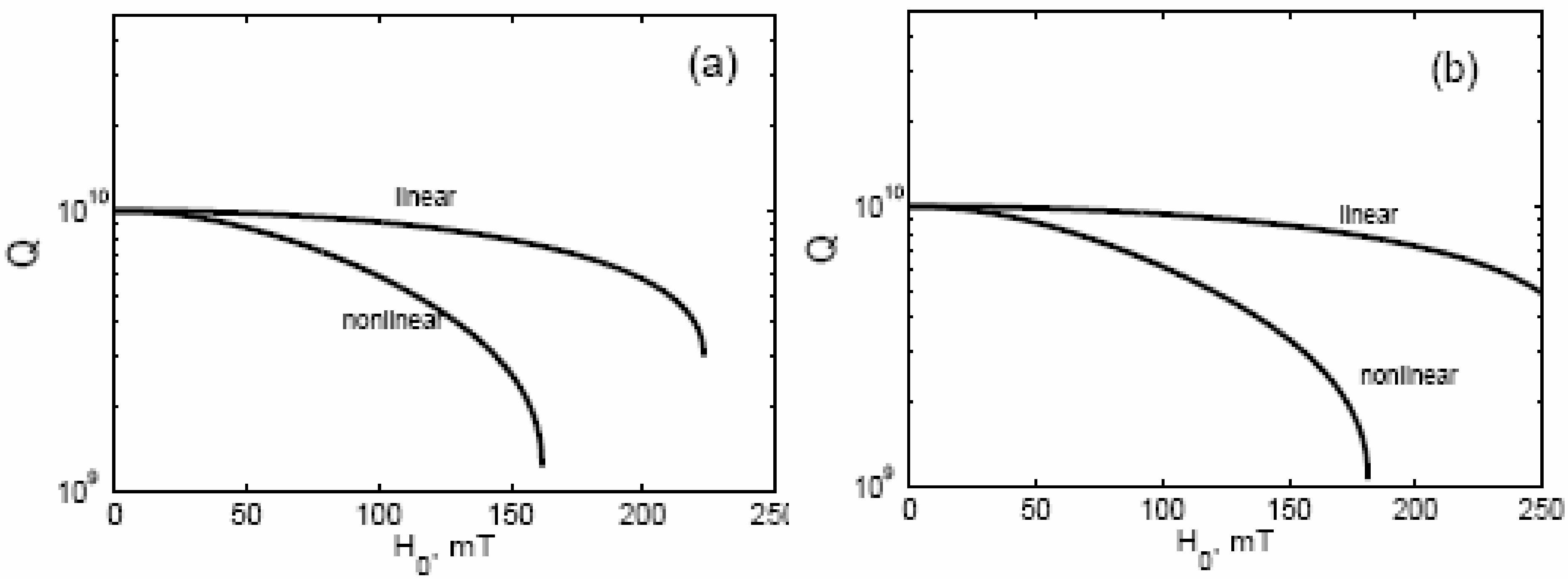
Bi-quadratic equation for $H_0(T_m)$:

$$\left[1+C\left(\frac{T_c}{T}\right)^2\left(\frac{H_0}{H_c}\right)^2\right]H_0^2=\frac{2\alpha\kappa T_m(T_m-T_0)}{A\omega^2(d\alpha+\kappa)}\exp\left(\frac{\Delta}{T_m}\right)$$

Breakdown field

$$H_b^2=\frac{T^2H_c^2}{2C\Delta^2}\left(\sqrt{1+\frac{4C\Delta^2H_{b0}^2}{T^2H_c^2}}-1\right)$$

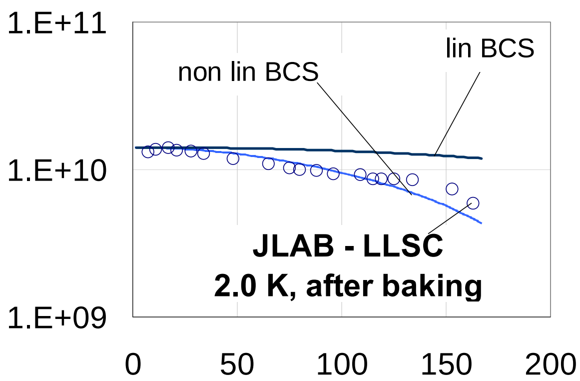
Q-factor (nonlinear resistance)

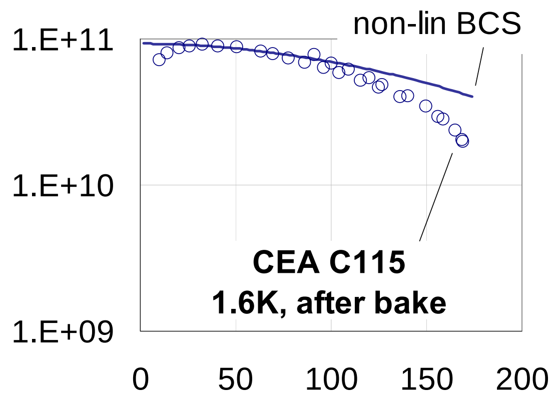


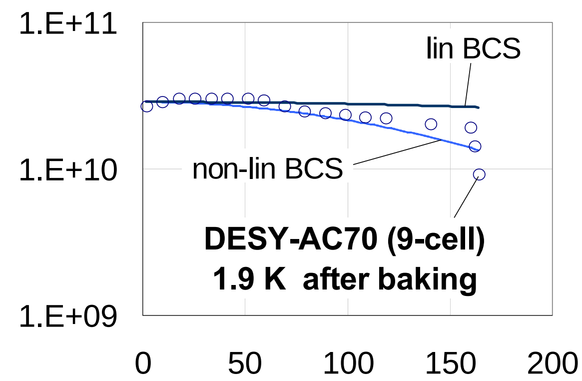
$Q(H_0)$ for linear and nonlinear models for $\kappa = 20$ W/mK at $T_0 = 2$K and $R_i = 0$. (b) Same as in (a), except that the Kapitza coefficient α is doubled, from 0.5 W/cm^{2}K to 1 W/cm^{2}K.

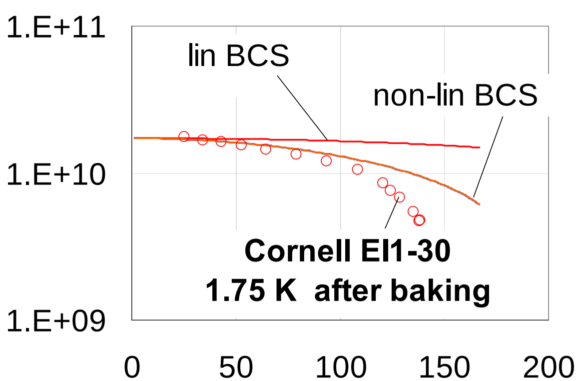
The BCS nonlinearity increases the medium and high field Q slope

P.Bauer et al. - Comparison of Cavity data with TFBM using lin or lin+non-lin BCS



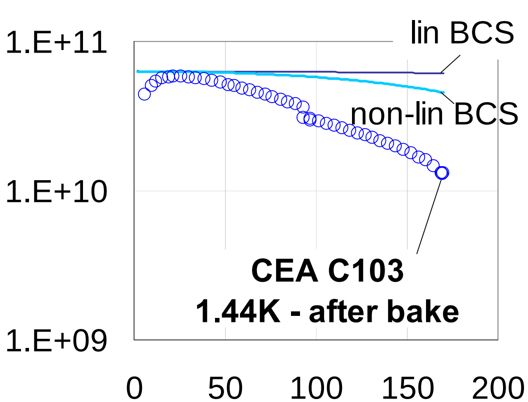


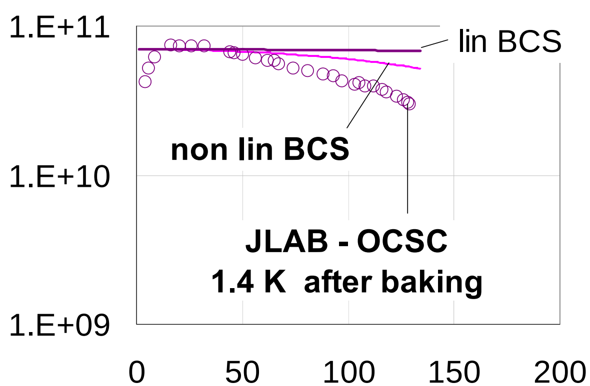


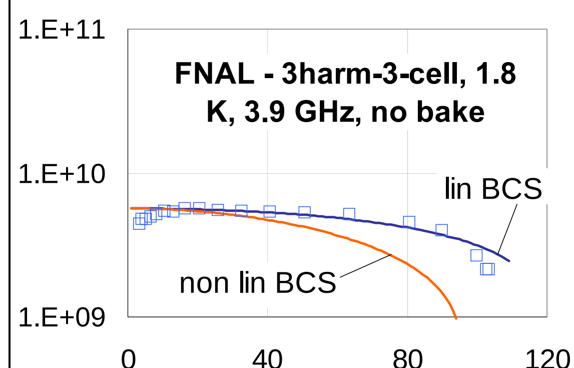


Q-data after baking for Jlab (1.5 GHz, BCP, single cell), CEA (1.3 GHz, EP, single cell), Cornell (1.3 GHz, BCP, single cell) and DESY AC70 (1.3 GHz, 9-cell, EP) are better fitted with the non-linear BCS.

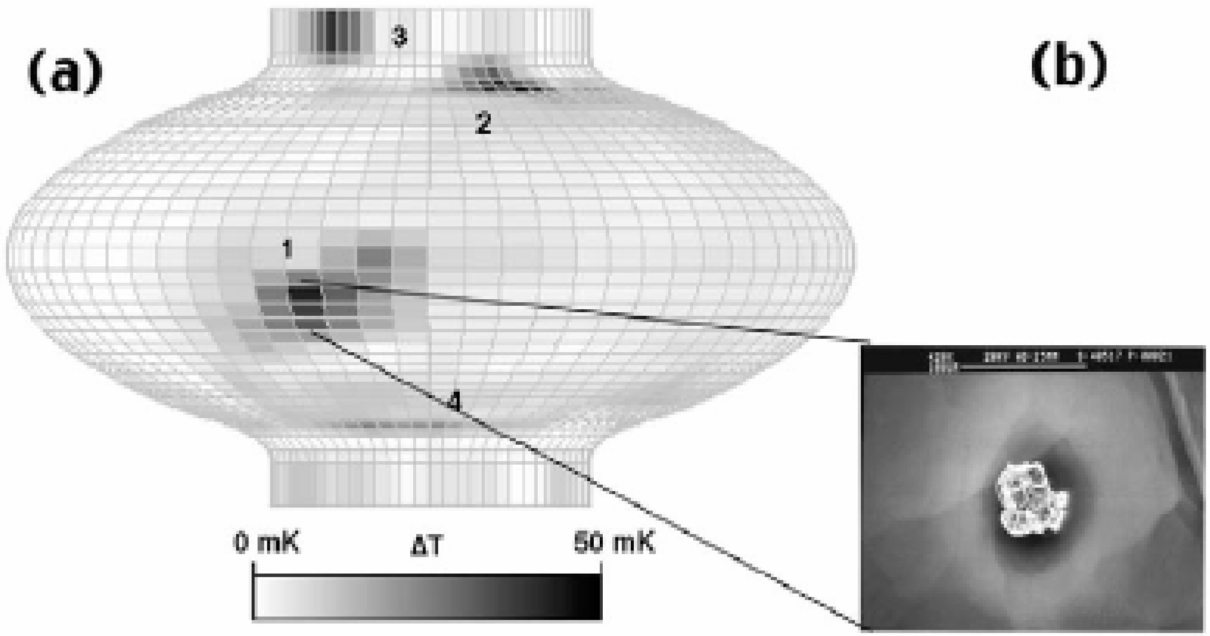
Exceptions: very low temp, high f Fnal (3.9 GHz, BCP)







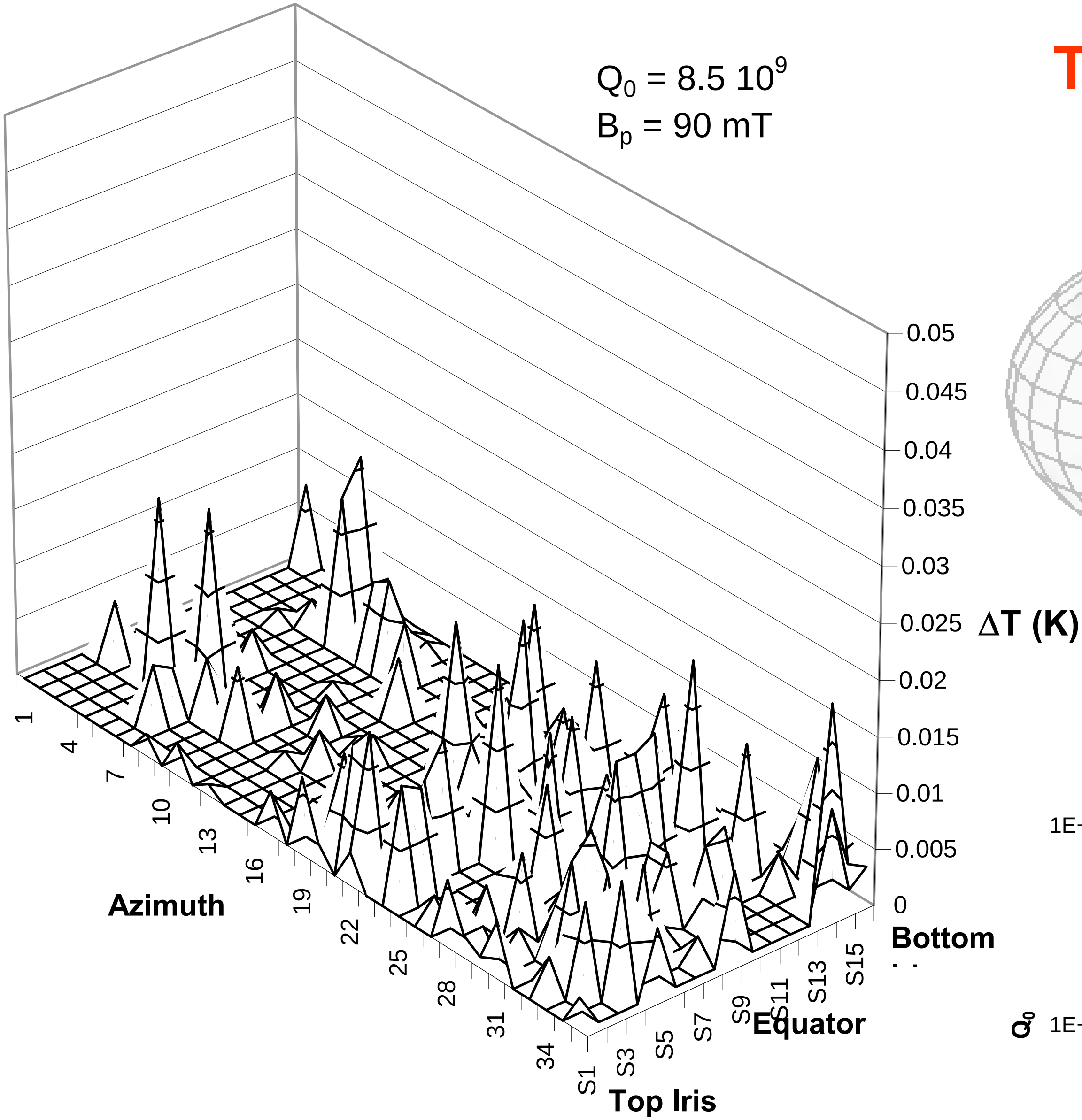
Is thermal breakdown uniform?

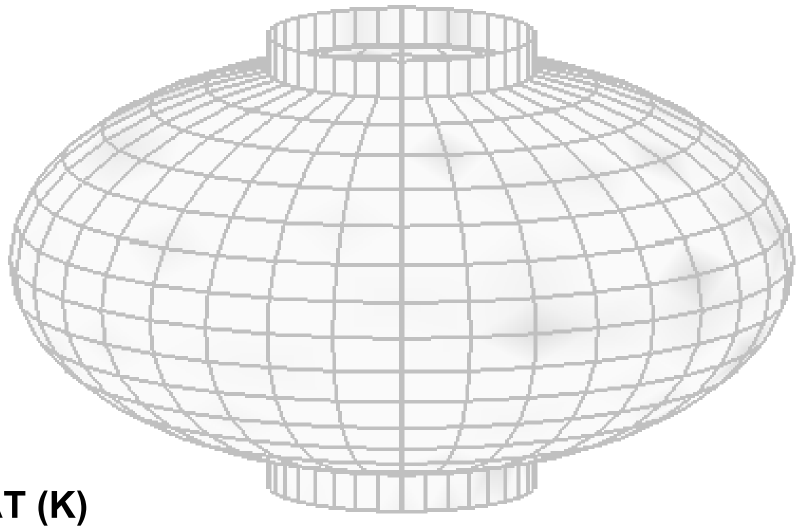


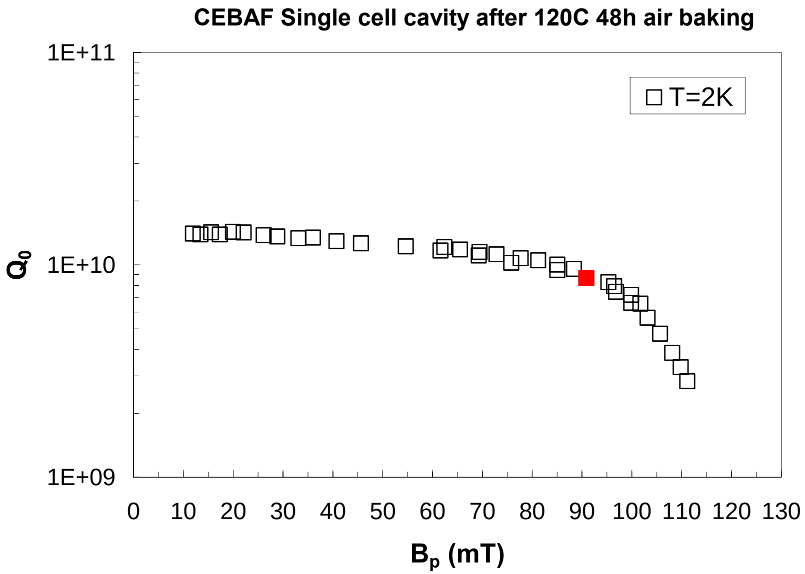
Defects can trigger thermal breakdown, Padamsee, Knobloch, Hays (1998), K. Saito, opening talk.

Different types of defects: 1. Local inhomogeneities in BCS resistance (oxide patches) 2. Normal inclusions, 3. Defects which facilitate vortex penetration (GBs, flux focusing)

Temperature map

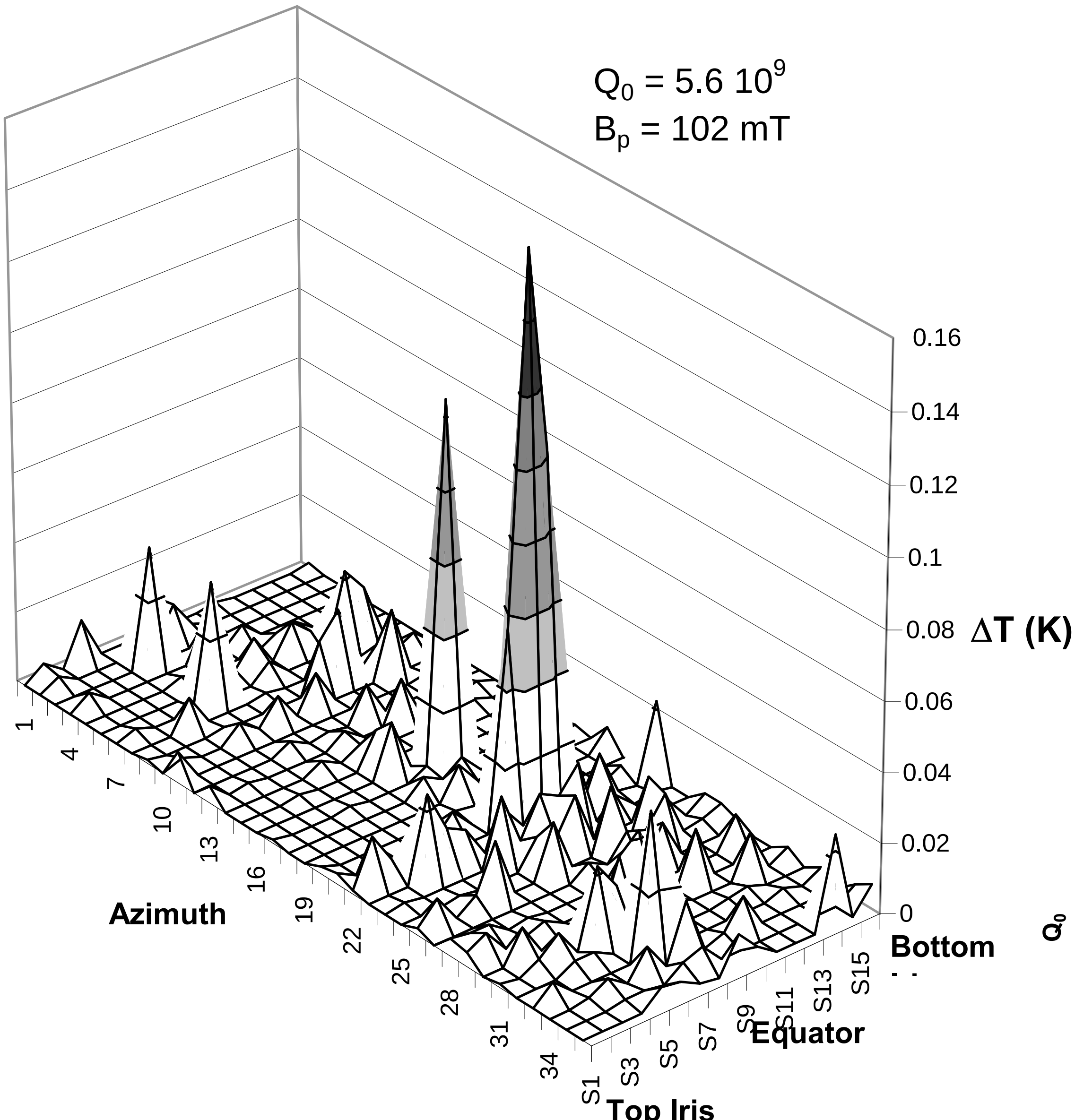


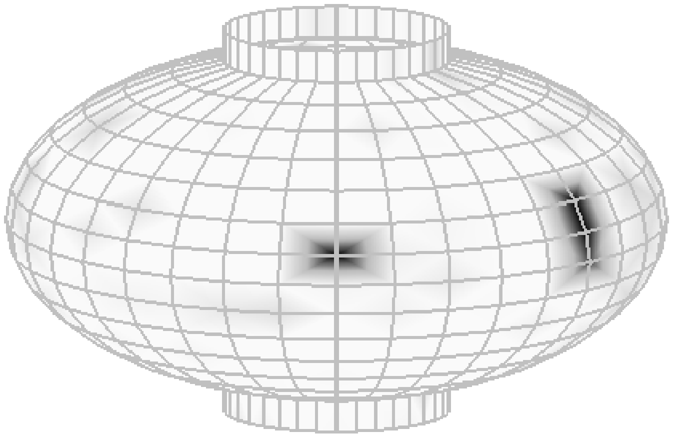


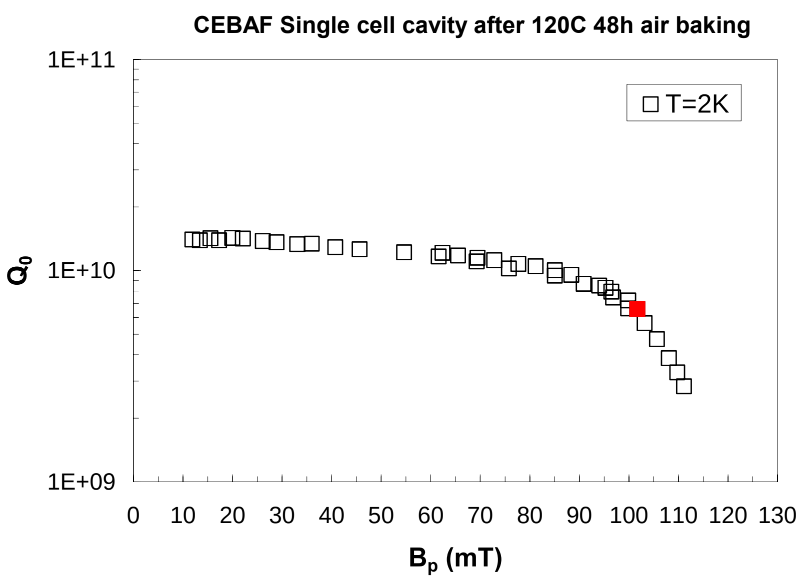


From Gigi Giovati, JLab (2005).

Temperature map







From Gigi Giovati, JLab (2005).

Temperature map

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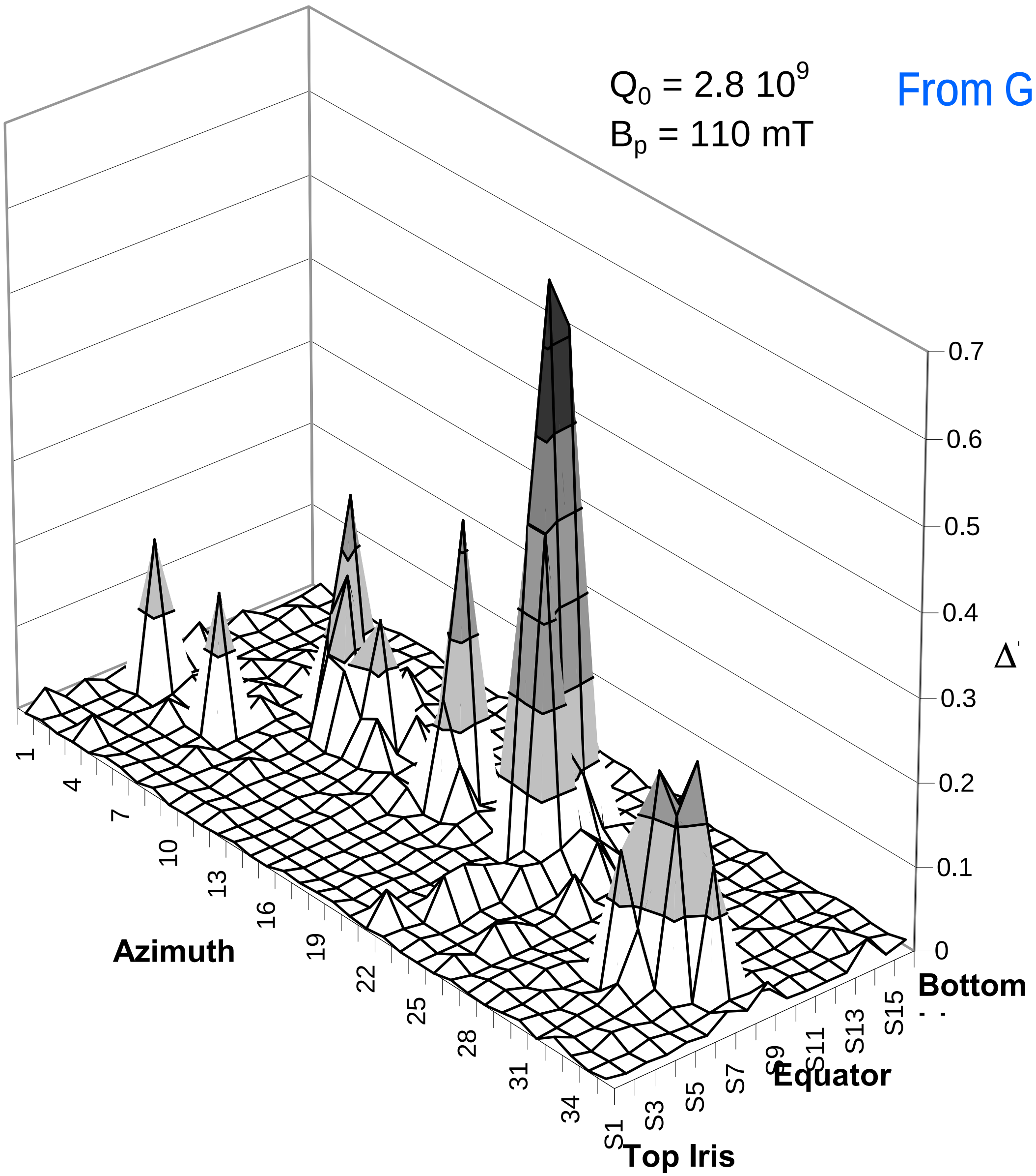
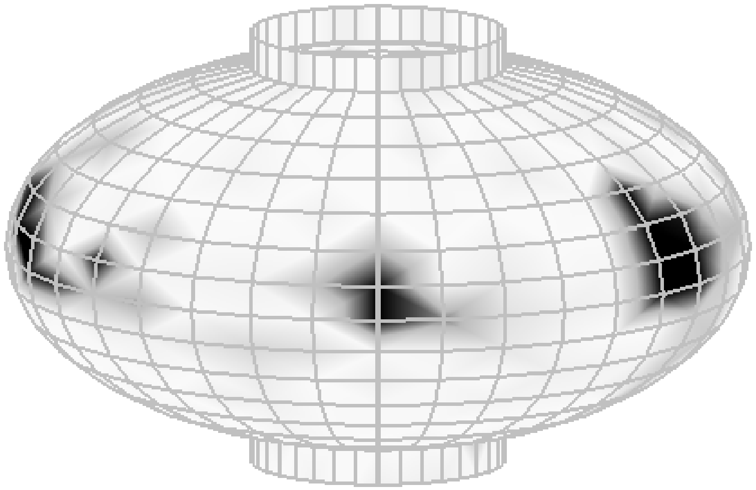
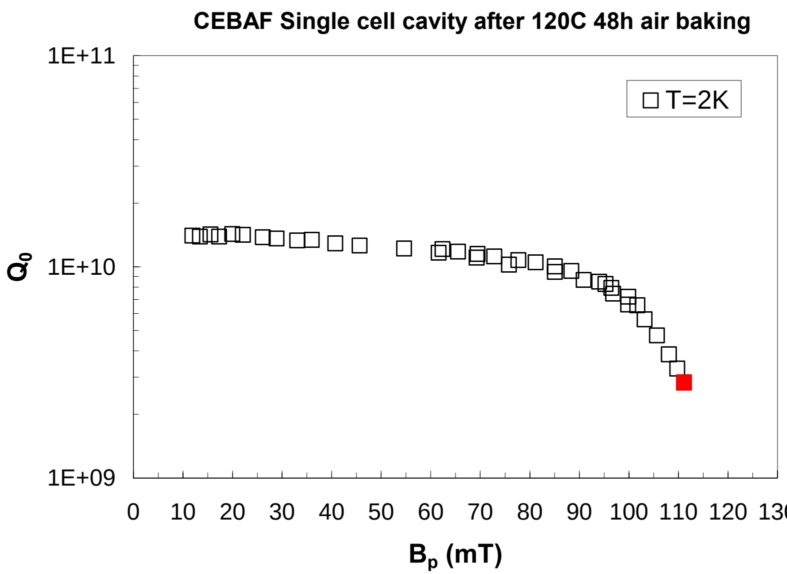
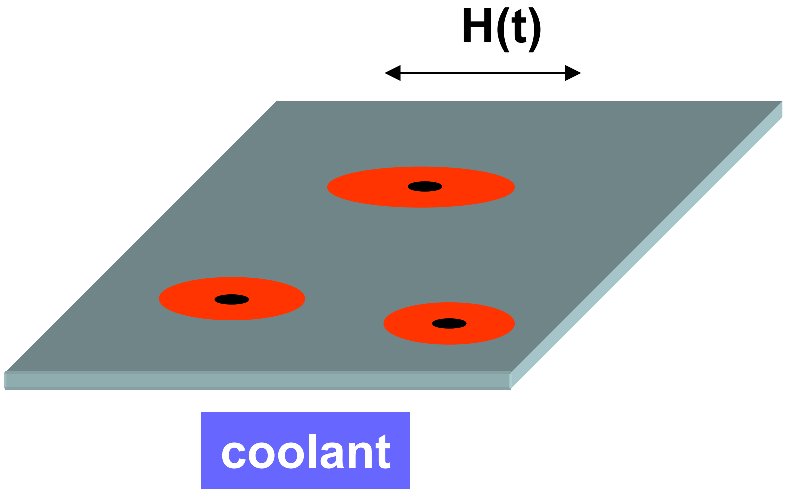






Hotspots expand as H approaches H_b

Effect of hotspots



Regions of radius r_0 where A(x,y) or H(x,y) is locally enhanced (impurities, GBs, thicker oxide patches, field focusing near surface defects, local vortex penetration, etc.)

$$\kappa\nabla^2 T - \tilde{\alpha}(T)(T - T_0) + q(T, H, r) = 0$$

T(x,y) = T_s+ δT(x,y), where **T_s** satisfies the uniform heat balance $\alpha(T_a)(T_a - T_0) = q_0(T_s, H)$, and δ**T(x,y)** is a disturbance due to defects:

$$\kappa\nabla^2 \delta T - \left(\tilde{\alpha} - \frac{\partial q}{\partial T}\right)\delta T + \delta q = 0$$

Excess heat generation $\delta q = H^2 \delta R/2 + R\delta H^2/2$ in the region of radius r_0

Temperature distribution

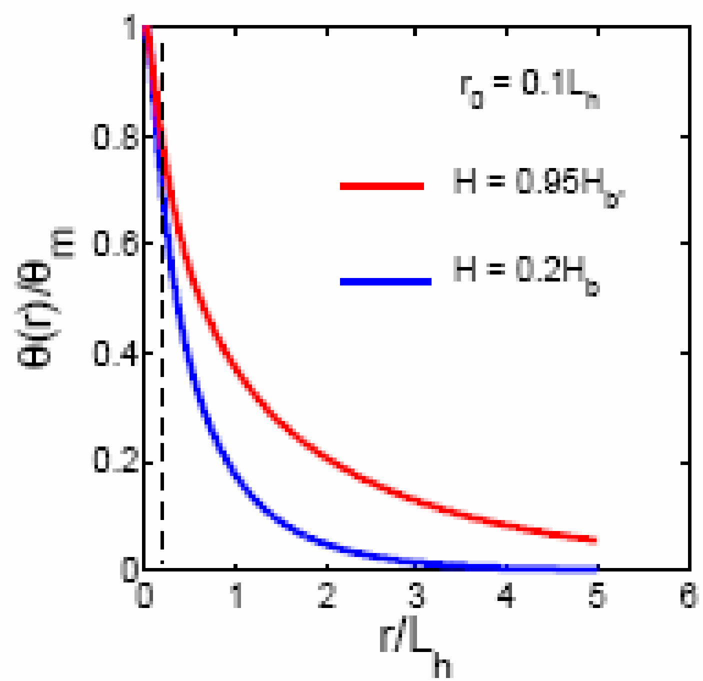
$$\delta T(r) = \frac{\Gamma}{2\pi\kappa} K_0\left(\frac{r}{L}\right), \qquad r < r_0, \qquad \Gamma = \int \delta q(x,y)\,dx\,dy$$

A hotspot produces a temperature disturbance $\delta T(r)$, which spreads along the cavity wall over the distance $L \gg r_0$ greater than the defect size

$$L = \frac{L_h}{\sqrt{1 - f(H/H_b)}}, \qquad L_h = \sqrt{\frac{d\kappa}{\tilde{\alpha}}}$$

Where $f(H/H_b) = (\partial q/\partial T)/\alpha \to 1$ at $H \to \mathbf{H_b}$

L increases with H and diverges at the uniform breakdown field, $H = H_b$

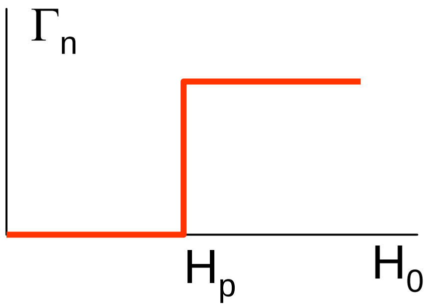


Weak hotspots

- Dimensionless SC defect strength (both R_s and field focusing)

$$\eta = \frac{r_0^2}{L_h^2}\left(\frac{\delta A}{A} + \frac{\delta H^2}{H^2}\right)$$

- Ohmic defects, $\Gamma_n(H) = H^2\Re_n/2$, or a vortex thermal switch:



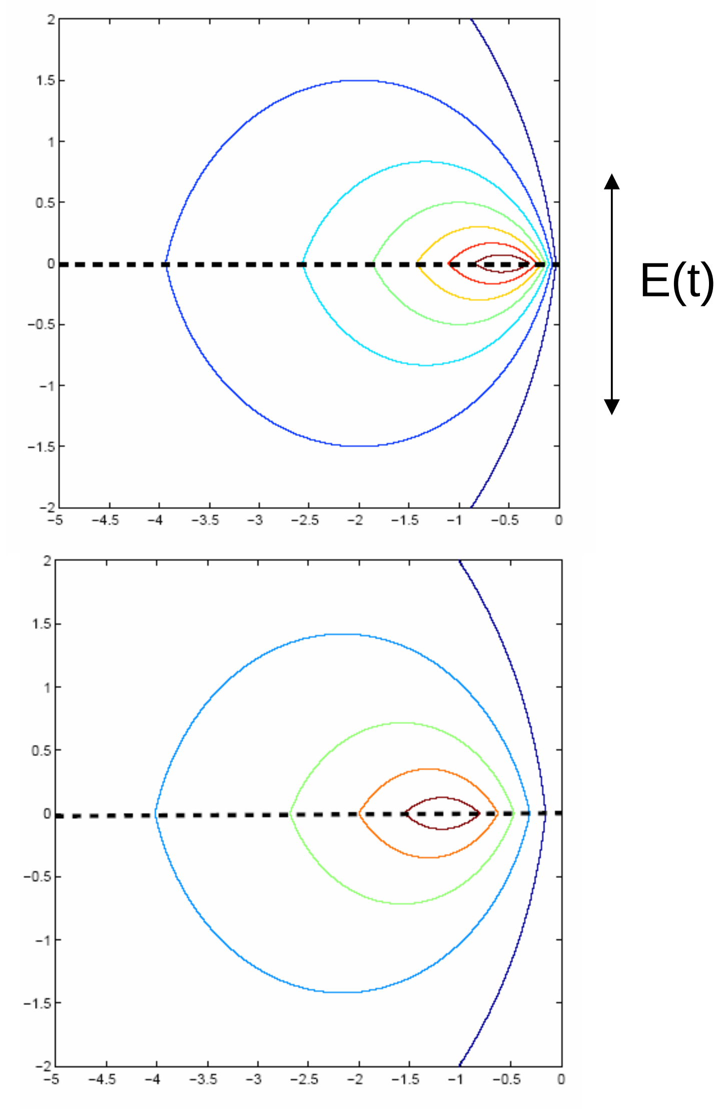
- Weak hotspots: $\eta \ll 1$. For κ **= 20 W/mK, T_0 = 2K and** α **= 0.5 W/cm²K,** $L_h \approx$ 3mm. Hotspots with $r_0 <$ 1mm are weak, even for strong inhomogeneity, $\delta A \sim A$ or δ**H² ~ H²**

- Maximum hotspot temperature T_m

$$T_m = T_s(H) + \frac{\eta}{2}(T_s - T_0)\ln\frac{1.12L}{r_0}$$

For $\eta = 0.3$, $L/r_0 = 10$, $T_0 = 2$K, we obtain $T_m - T_s = 0.08$K

Penetration of vortices along GBs through oscillating surface barrier



AJ vortex

GB as a hotspot site: reduced flux penetration field

Deformation of the vortex core during flux penetration along GBs.

Transformation of the Abrikosov to the Josephson and mixed Abrikosov-Josephson (AJ) vortices

Dissipation due to vortex oscillations in RF field

Averaged BCS surface resistance

Extra dissipation in a hotspot:

$$\tilde{\alpha}\int \delta T(x,y)dxdy = \frac{\pi}{2}L^2H^2\eta_s R_s(T_s) + \Gamma_n(H)\frac{L^2}{L_h^2}$$

$$L \cong \frac{L_h}{\sqrt{1-(H/H_{b0})^2}}$$

Global surface resistance with the account of non-overlapping hotspots:

$$\tilde{R}_s(T,H) = R_s(T,H)\left[1+\frac{g}{1-(H_0/H_{b0})^2}\right] + \frac{R_n(H_0)}{1-(H_0/H_{b0})^2}, \qquad g = <\eta>\frac{\pi L_h^2}{\ell_s^2}$$

$R_s(H)$ is the uniform surface resistance, ℓ_s is the mean spacing between hotspots, H_{b0} is the uniform breakdown field, $R_n = 2<\Gamma_n>/H^2\ell_n^2$

Nonlinear contribution to the global R_s due to expansion of hotspots with H.

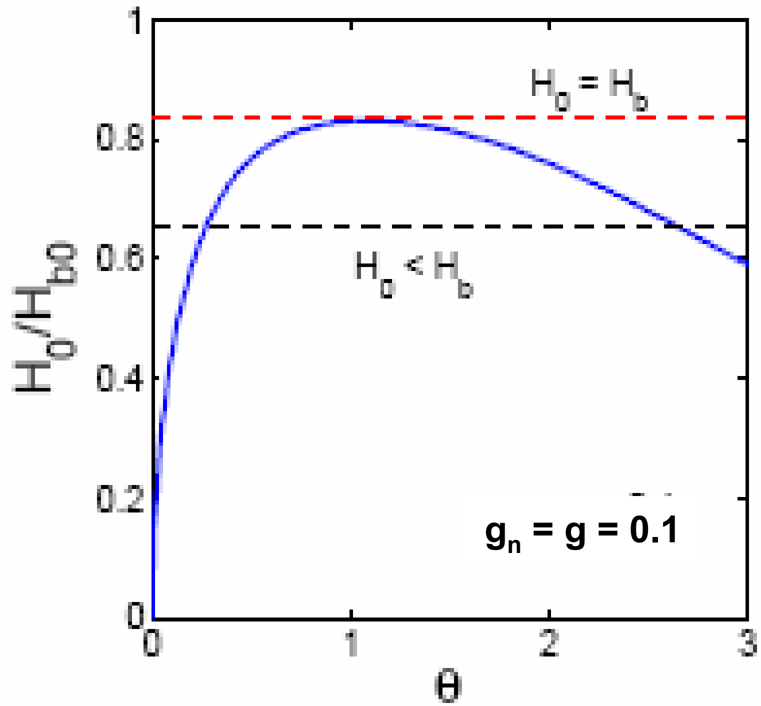
Example: linear BCS+hotspots (R_i = 0)

Thermal balance equation for the mean temperature T(H)

$$\frac{R_s(T,H)}{2}H^2 = \tilde{\alpha}(T - T_0)$$

Explicit dependence $H_0(T)$:

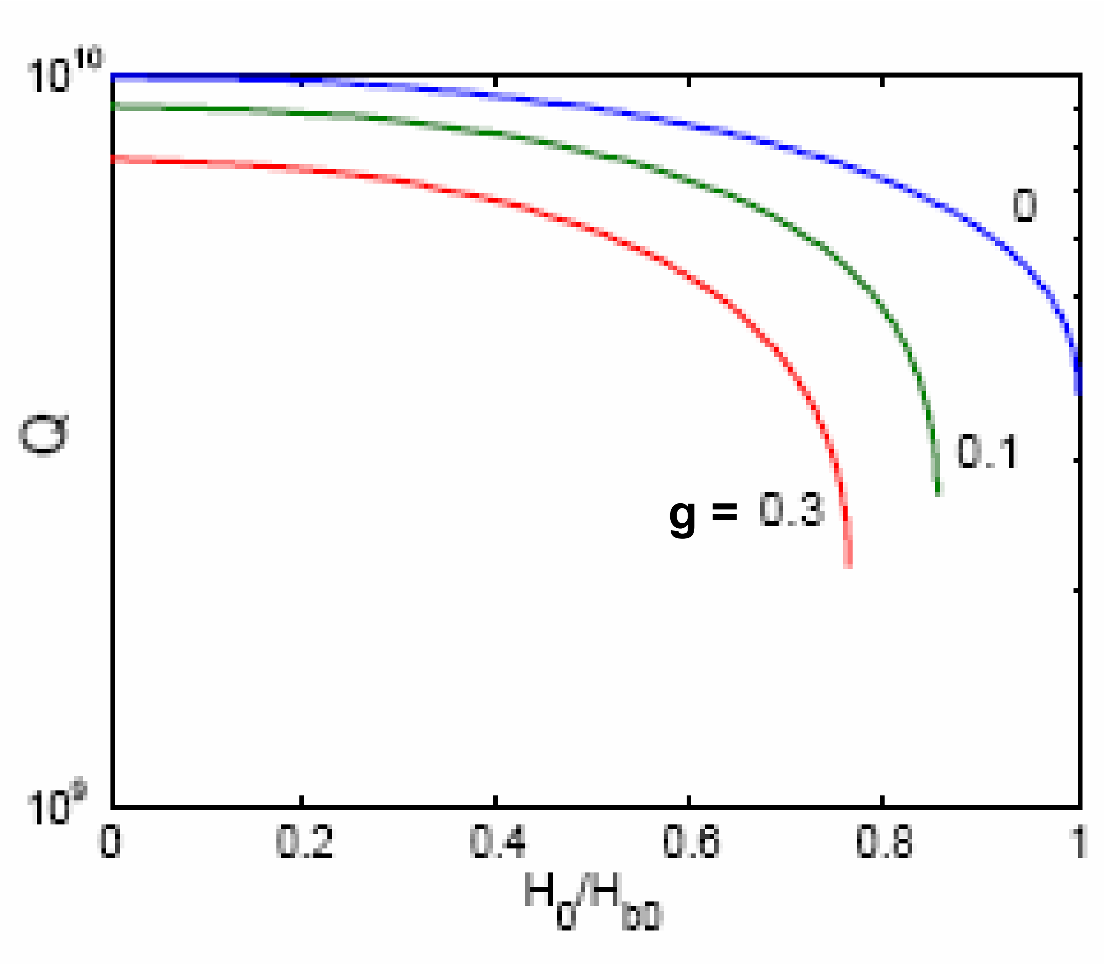
$$\frac{H_0^2}{H_{b0}^2} = \frac{1}{2}(1+g+u(\theta)) - \sqrt{\frac{1}{4}(1+g+u(\theta))^2 - f(\theta)}$$



$f = \theta\exp(1 - \theta)$, **$u = (g_n + e\theta)\exp(-\theta)$,**
$\theta = (T - T_0)\Delta/k_BT_0^2$, $g_n = R_n/R_s(T_0)$

Maximum in $H_0(T)$ at the breakdown field H_b above which stable thermal balance is impossible

Q(H) for the linear BCS+hotspots ($\Gamma_n = 0$)



Hotspots reduce the breakdown field:

$$H_b \cong \left(1 - \frac{\sqrt{g}}{2}\right) H_{b0},$$

Hotspots increase the high-field Q slope:

$$\frac{Q_0}{Q_b} = \frac{(1+\sqrt{g})e}{1+g} > e$$

Conclusions

- Ultimate cavity performance (in the absence of vortex penetration) is limited by nonlinear BCS pairbreaking and heating effects.

- Acoustic resonances and mechanisms of the residual resistance

- Hotspots limit the high-field cavity performance:
- New mechanism of nonlinearity, which can offset the BCS nonlinearity,
- Reduce the breakdown field
- Increase the high-field Q slope

- Mechanisms of hotspot formation
- Acoustic hotspots
- Vortex penetration along GBs
- Nonuniform surface oxide layers

Challenges

- Understanding nonequilibrium superconductivity and impurity surface scattering on nonlinear BCS resistance and rf breakdown
- Dynamics of vortex penetration and dissipation in rf field