# Analysis of the Beam-Beam Interaction 

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#### Abstract

I developed PENELOPE, a companion program to the electron storage ring simulation program ODYSSEUS. PENELOPE is a text-driven user interface that formats storage ring parameters, entered by the user, so that ODYSSEUS can read them. Once PENELOPE was completed and operational, I turned my attention to the study of electron/positron bunch instabilities. At the moment, we are using ODYSSEUS to investigate the damping rates of these instabilities, in both colliding and non-colliding beams.


## Introduction

Electron storage rings are particle accelerators in which clusters (or bunches) of electrons and positrons are repeatedly collided. Customarily these collisions are designed to occur within a limited portion of the ring which is surrounded by the large-scale equipment necessary to detect any particles created. This makes it imperative, through the use of bending magnets along the storage ring's length, to ensure that the electron and positron bunches avoid colliding with each other outside of the detection region. When oppositely charged bunches pass through each other in collision, each exerts electromagnetic forces upon the other. This effect is termed the beam-beam interaction. ODYSSEUS is an electron storage ring simulation program, written by Edwin Anderson of Cornell, that takes a dynamic beam-beam interaction into account in its calculations. ODYSSEUS is designed to model different storage rings, and therefore it takes numerous storage ring descriptive parameters as input, rather than having a particular ring's parameters hard-wired into the code. These storage ring parameters include the ring's length, its maximum energy, and various other numbers that describe the bending magnets, radiofrequency cavities, and similar relevant factors.

## Project Description

My objectives in this project have been twofold: first, to write a "front-end" user interface for ODYSSEUS; and second, to attempt to reproduce, using ODYSSEUS, the head-tail bunch instabilities observed at high currents in electron storage rings, and thereby gain a better understanding of this phenomenon.

## The ODYSSEUS User Interface

ODYSSEUS reads in the electron storage ring parameters, as well as execution specifications, from several input files. These files are commented and can be altered, but it was felt that it would be less awkward and confusing if the manipulation of the files were handled by an interactive "front-end" user program (which I have chosen to call PENELOPE). PENELOPE would allow the user to enter values in a straightforward manner, and then it would format the parameters and specifications so that ODYSSEUS could read them. However, the primary motivation for PENELOPE was to create an environment that would ensure that the user entered a self-consistent set of parameters. Since many of the storage ring parameters are interrelated (see

Fig. 1 and Table 1), and since there are many different ways to describe a storage ring's characteristics, PENELOPE would need to have the capability to calculate certain parameters from previously entered values, and to notify the user if a newly-entered parameter was inconsistent with previous ones.

There were several additional requirements incumbent upon PENELOPE. It should be written in either C or FORTRAN for portability (I chose C; ODYSSEUS, incidentally, is in FORTRAN), and it should be text-driven for the same reason. Additionally, PENELOPE should allow the user to "tweak" the parameters, if desired, in physically unrealizable ways. Although the major goal of the program was to help the user create a consistent set of parameters, in some cases it might be very useful and/or necessary for the user to enter inconsistent values, and PENELOPE should grant the user that freedom.

## The Structure of PENELOPE

To accomplish these goals, PENELOPE contains four multidimensional data arrays. The first, MainArray, contains the majority of the relevant storage ring parameters; RunArray contains all of the execution specifications for ODYSSEUS; and the arrays Cavities and Resonators are employed if the user decides to specify the characteristics of the individual radiofrequency cavities and resonators (which describe the wakefields), respectively.

Upon running PENELOPE, the user first encounters the main menu (Fig. 2) and is there presented with three options: create a new data file, modify an old one, or exit the program. If the user chooses to compose a new file from scratch, the program's data arrays are initialized. If instead the user chooses to modify an existing data file, he or she is prompted to enter in the desired file's name, and PENELOPE attempts to open that file and read its contents into the data arrays. At this point, in both instances, the user will be shown the first of several tables (Fig. 3) displaying the parameter values and the status of each. The status flag, which is contained within the data array alongside its value, has five possible settings-'No Entry', 'Input', 'Derived', 'Read In', and 'Default'-that indicate the manner in which that parameter's value appeared in the array. The user now has the options of viewing more parameters in a different table, entering a value, clearing a value, or exiting this portion of the program. The user selects a parameter to modify by typing in its corresponding number, as indicated in the table. He or she is then prompted to enter in the desired value; if the value is inconsistent with previous values, PENELOPE will indicate so, and ask whether the user would like to keep the newly-entered value or retain the original one. After a parameter has been selected and then either modified or left alone, a subroutine is called within PENELOPE that checks whether any other parameters can now be derived. As long as some parameter is derived during a pass through this subroutine, the subroutine will keep executing. After it finishes looping, the user is returned to the table he or she was previously viewing, and the above process can be repeated indefinitely. When the user chooses to quit, he or she is shown a menu with choices concerning the saving of the entered parameters. The parameters can be saved to the ODYSSEUS input files only after a sufficient number of them have been entered. They can, however, be saved to a file of the user's choosing at any point, after which the user will be returned to the main menu.

## Bunch Instabilities

In many electron storage rings it has been observed that the electron and positron bunches exhibit instabilities in their collective motion at high currents. These instabilities are labeled by their modes: $\mathrm{m}=0, \pm 1, \pm 2, \ldots$ and occur at frequencies $\mathrm{f}_{\mathrm{v}}+\mathrm{mf}_{\mathrm{s}}$. In the $\mathrm{m}=0$ mode, the bunch oscillates translationally up and down; in the $m= \pm 1$ mode, the bunch rocks back and forth; and so forth. The $\mathrm{m}=0$ mode is stabilized by wake fields when the chromaticity is greater than 0 , but
unfortunately the $\mathrm{m}= \pm 1$ mode is destabilized when the chromaticity is positive. During normal storage ring operation, the $m= \pm 1$ mode is stable due to the effects of synchrotron radiation damping, but when the beams are in collision the $m= \pm 1$ mode is less stable (that is, instability occurs at a lower current). At the time of this writing, we are just beginning the task of using ODYSSEUS to look at the damping rates of the bunch oscillation modes with and without collisions.

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TABLE 1. Variables used in PENELOPE.

| Symbol | Parameter |
| :---: | :---: |
| L | accelerator length |
| E | beam energy |
| $\mathrm{I}_{i}$ | synchrotron radiation integral $i$ |
| $\mathrm{U}_{0}$ | synchrotron energy loss per turn |
| $\mathrm{Q}_{\mathrm{S}}$ | synchrotron tune |
| $f_{\text {RF }}$ | RF frequency |
| $\mathrm{V}_{\text {RF }}$ | RF voltage |
| $\alpha$ | momentum compaction |
| $\left(\sigma_{\mathrm{p}} / \mathrm{p}\right)^{2}$ | momentum spread |
| h | Planck's constant |
| e | electron charge |
| $\mathrm{r}_{\text {e }}$ | electron radius |
| $\mathrm{m}_{\mathrm{e}}$ | electron mass |
| c | speed of light |
| $\varepsilon_{0}$ | permitivity of free space |
| $\tau_{\text {transverse, longitudinal }}$ | transverse and longitudinal damping times |
| $\beta_{x, y}$ | horizontal and vertical beta (at the interaction point) |
| $\sigma_{x, y, z}$ | initial size in $\mathrm{x}, \mathrm{y}$, and z |
| $\Delta_{\text {x, y }}$ | horizontal and vertical radiation excitation |
| $\varepsilon_{x, y}$ | horizontal and vertical emittance |
| $\gamma$ | $\mathrm{E} /\left(\mathrm{mc}^{2}\right)$ |

$$
\begin{aligned}
& \alpha=\frac{I_{1}}{L} \\
& U_{0}=\left[\frac{2 r_{e}}{3\left(m_{e} c^{2}\right)^{3}}\right] \times E^{4} I_{2} \\
& \left(\frac{\sigma_{p}}{p}\right)^{2}=\left(\frac{\sigma_{E}}{E}\right)^{2}=\left[\frac{55 \mathrm{~h} \gamma^{2}}{32 \sqrt{3} m_{e} c}\right] \times \frac{I_{3}}{2 I_{2}+I_{4}} \\
& \varepsilon_{x}=\left[\frac{55 \mathrm{~h} \gamma^{2}}{32 \sqrt{3} m_{e} c}\right] \times \frac{I_{5}}{I_{2}-I_{4}}=\frac{\sigma_{x}^{2}}{\beta_{x}} \\
& \varepsilon_{y}=\frac{\sigma_{y}^{2}}{\beta_{y}} \\
& \varepsilon_{z}=\frac{2 \pi^{2} \sigma_{z}^{2} Q_{S}}{\alpha L} \\
& Q_{S}=\sqrt{\frac{\alpha L f_{R F} V_{R F}}{2 \pi c E}} \\
& \tau_{\text {transverse }}=\frac{12 \pi \varepsilon_{0} m_{e} c L}{e^{2} \gamma^{3}\left(I_{2}-I_{4}\right)} \\
& \tau_{\text {longitudinal }}=\frac{12 \pi \varepsilon_{0} m_{e} c L}{e^{2} \gamma^{3}\left(2 I_{2}+I_{4}\right)} \\
& \Delta_{x}=\sqrt{\frac{55 h e^{2} \gamma^{5} I_{5} \beta_{x}}{96 \sqrt{3} \pi m_{e}{ }^{2} c^{2} \varepsilon_{0}}} \\
& \Delta_{y}=\sqrt{\varepsilon_{y} \beta_{y}\left(1-e^{\left.-L / c_{\text {transverse }}\right)}\right.}
\end{aligned}
$$

FIGURE 1. Equations implemented in PENELOPE.


Figure 2. PENELOPE Program flow chart.

Choose one of the following parameters to modify:


Figure 3. First Parameter Table

