Investigating Longitudinal Capture in the Synchrotron Using a Longitudinal Energy Oscillation Simulator

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Abstract

It is known that within the first few turns in the synchrotron, injected positrons are lost. We suspect there may be a problem with the longitudinal phase space matching and intend to investigate this possibility by developing a simulation program to model positron trajectories in phase space. Using the completed LEOS program, it was determined that the phase space could not be matched. We determined an optimum capture area by setting the synchrotron period to 9 turns. It was also shown that using the current synchrotron period of 15 turns, we are using less than 50% of the available energy aperature.

Introduction

The positrons stored in CESR are produced in the Linac by bombarding a tungsten target with electrons and then collecting as many positrons as possible from the ensuing electromagnetic shower. Having to produce positrons in this manner places a high priority in transporting as many of them into the synchrotron as possible. At present, it is known that a large number of positrons are lost in the first few turns of the synchrotron. We suspect there may be a problem with the longitudinal phase space matching and investigated this possibility with the aid of LEOS (**L**ongitudinal **E**nergy **O**scillation **S**imulator).

The Basics

Investigation of longitudinal energy oscillations is primarily done by visualizing the orbits of particle trajectories in phase space. Essentially, longitudinal energy oscillations are the variation of particle energy gain (and loss) with time relative to the beam center. That is, given an initial energy deviation, ϵ, and time displacement, τ, from the ideal orbit, we can get a plot of the particle's trajectory. In each phase space plot there is unique orbit which defines the boundary between stable and unstable orbits. This boundary is called the separatrix. An orbit within the separatrix is stable, anything outside of it is unstable and will be lost (Figure 1).

Using Approximate Models in LEOS

Initially, LEOS was written to model an overly simple situation. To model the separatrix we assumed the forces acting on the beam were continuous so we could use differential equations and also assumed that we could neglect any effects due to radiation loss. Effects due to acceleration were neglected as well.

To model the particle trajectories we correctly use difference equations but still neglect the affects of acceleration and radiation loss. These equations, derived by Sands [1] are

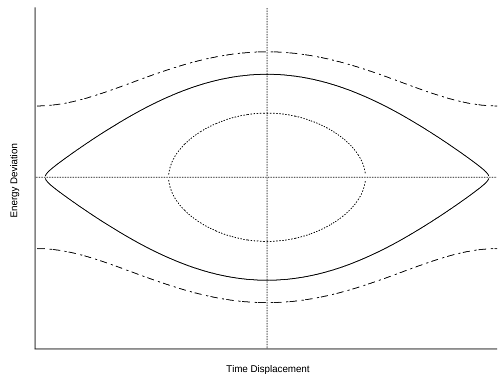

FIGURE 1. Separatrix indicated by solid line along with stable and unstable orbits

$$\Delta\tau = \frac{-\alpha T_0}{E_0}\epsilon \qquad (1)$$
$$\Delta\epsilon = e\hat{V}\sin(\omega_{rf}\tau_1)$$

where α is the momentum-compaction, T_0 is the period of one revolution in the synchrotron, E_0 is the nominal beam energy, ω_{rf} is the RF angular frequency and $\hat{V}$ is the peak voltage.

Before discussing the modifications made to include acceleration effects, it is helpful to get a sense what we have been modeling thus far. The non-accelerating case in Figure 2 shows three plots. The top graph is a simple sinusoid which represents the RF voltage function. One can think of this as the force the particle receives. The middle graph shows the potential well corresponding to the RF voltage (recall Potential = -∫Force). The bottom graph is of the resulting separatrix that is allowable with this particular potential well. Notice that it is symmetric about both the horizontal and vertical axes.

A More Realistic Approach

The primary difference between the approximations and the more realistic solution is that we take into account the effects of the particles accelerating with each turn in the synchrotron. By modeling LEOS after a non-accelerating beam, we were producing conditions analagous to those of a low energy storage ring.

To see what effects acceleration has on the simulation, it is advantageous to compare the non-accelerating case with the accelerating case in Figure 2. It is clear that for an RF voltage function that is accelerating, a phase shift and offset are introduced. Consequently, the potential well will take on a much different form. And as a result of the new potential well, the allowable separatrix will be quite distinct from the non-accelerated separatrix. Notice that the accelerated separatrix is only symmetrical about the horizontal axis and takes on the form of a "tear-drop". Notice also the reduction in separatrix area - there are now fewer stable orbits available in the accelerated scenario.

In addition to the acceleration effects, changes were made in order to include the effects of increasing energy and voltage. Previously, we had assumed the beam energy to be constant. Now we increment the energy by a fixed amount with each turn in the synchrotron,

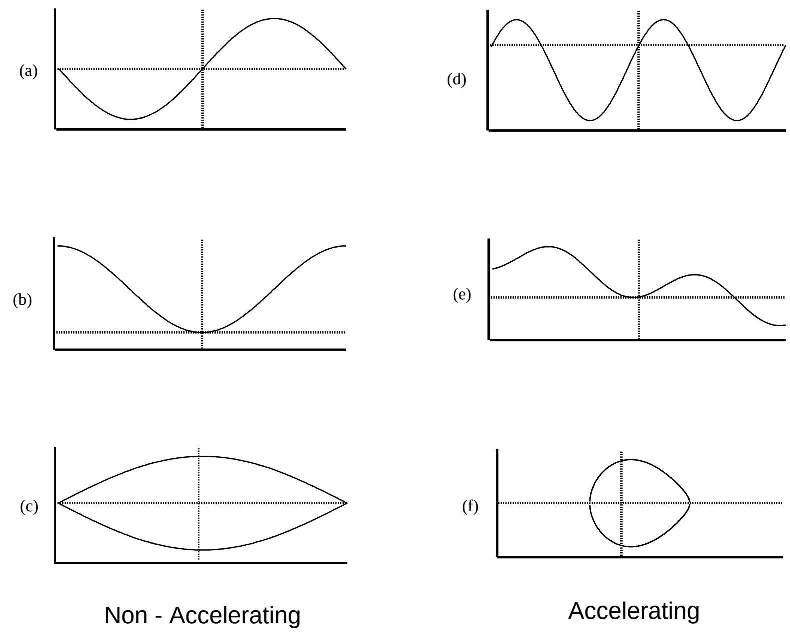

FIGURE 2. (a),(d) RF voltage (b),(e) potential well (c),(f) separatrix

corresponding to the increase in energy the particle feels after being accelerated through an RF cavity. To ramp up the voltage we used the simple case of making voltage directly proportional to beam energy. As a result of these changes, the difference equations used to simulate orbits in phase space are given by

$$\Delta\tau = \frac{-\alpha T_0}{E_0}\epsilon \tag{2}$$

$$\Delta\epsilon = e\hat{V}\sin(\omega_{rf}\tau_1 - \phi) + U_0$$

where

$$\phi = \arcsin\frac{U_0}{\hat{V}}$$

$$U_0 = \frac{dE_{inj}}{dt}.$$

The offset, U_0, is determined from the condition that $\epsilon(\tau = 0) = 0$.

While plots of ϵ versus τ are informative, for our purposes it is more convenient to look at plots of δ versus τ, where

$$\delta = \frac{\epsilon}{E_b}$$

in which E_b is the beam energy.

Figure 3 displays a plot of δ versus τ and also the corresponding ϵ versus τ plot. First notice the ϵ plot. This plot shows that the quantity energy $\times$ time remains invariant. That is, each complete orbit traced out has approximately the same area, conserving action - as we would expect. In the δ plot, the fractional energy changes with time displacement. This is a convenient plot since the 0.9% energy aperature limit, intrinsic to the machine, is fixed and allows us to readily see when a trajectory has exceeded those limits. Since the energy is increasing with each turn, and since the orbit areas in the ϵ plot are conserved, we expect that in the δ plot, as energy increases, the fractional energy will *decrease*. And this is the reason we see the spiraling inward of the orbits, a phenomenon known as adiabatic damping.

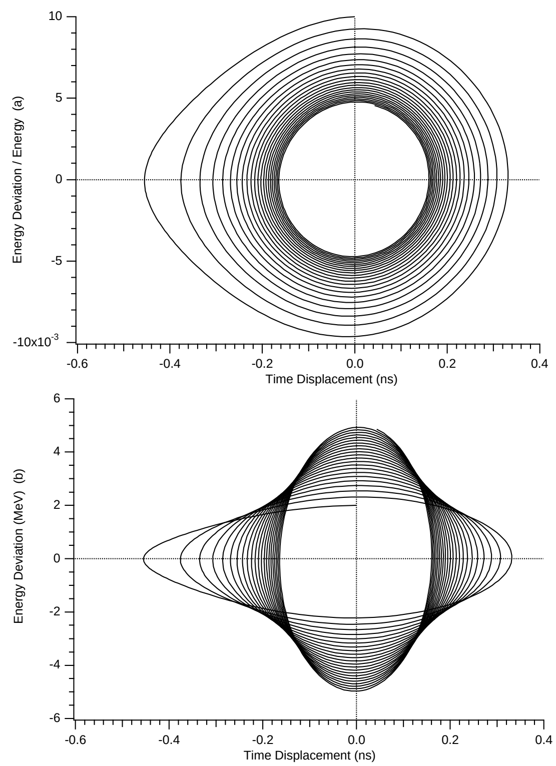

FIGURE 3. (a) δ vs τ (b) ϵ vs τ

Results

With the simulation running as it should, we can now attempt to match the phase space. Using results of beam simulations done by Fromowitz [2], we determine that an average particle bunch has length 3 cm and a 0.9% energy spread. The values of the parameters used in the simulation are given in Table 1.

It was found that the beam could be captured with a starting voltage of 0.91 MV (Figure 4) and remain captured for starting voltages up to 3.5 MV. Depending on the values of τ and ϵ used for the simulation, resonance effects occur at different starting voltages. But the general trend seems to indicate that for voltages greater than 3.5 MV, resonance effects become manifest. We were also able to determine that the beam could *not* be matched. We restricted ourselves to an arbitrary maximum starting voltage of 5.0 MV, and for voltages from 3.5 to 5.0 MV, the beam displayed the effects of nonlinear resonance (Figure 5). Due to these resonance effects, by increasing the starting voltage, matching the phase space becomes unattainable. It should be noted that in the case of 2 and 3 kicks per turn, resonance effects do not occur through 5.0 MV.

TABLE 1. Simulation Parameters

Parameter	Value
τ	0 ns
ϵ	1.8 MeV
$E_{injection}$	200 MeV
# Turns	3000
Slope at Injection	0.29 MV/turn
α	0.01
Kicks/Turn	1

It is interesting to note that when viewing a δ versus τ plot using points (Figure 4) rather than using lines to connect the data points (Figure 3a), an interesting feature emerges. As you can see in Figure 4, most notably towards the center, lie several sharply defined peaks pointing in a clockwise direction. The number of these peaks reveals the synchrotron period.

The synchrotron period, T_s, is the number of turns around the synchrotron it takes for a particle to complete an orbit. The synchrotron tune is given as $\frac{1}{T_s}$. At present, the synchrotron is running with a period of 15 turns. Using LEOS we adjusted the starting voltage to match the present synchrotron period. This gave us a good idea of what the machine was currently using as its starting voltage. The period was also determined for the starting voltage we found for optimum capture. The results are summarized in Table 2.

TABLE 2.

	Starting Voltage	Synchrotron Period
Current Settings	0.44 MV	15
Optimum Capture	0.91 MV	9

In the most significant findings of this research, we found that the separatrix corresponding to what we believe is currently being used as the starting voltage, uses less than 50% of the available energy aperature (Figures 6 and 7). This is crucial since according to Fromowitz's simulations [2], we need to be using the entire energy aperature to avoid losing a significant portion of the beam.

We also varied the slope at injection using the parameters to model optimum capture area at injection to see the effects, if any, it had on the phase space. Halving the slope we originally used, 0.145 MV/turn, we found that the separatrix stretched vertically, beyond the energy aperature. Yet the synchrotron period remained essentially the same at 9 turns (Figure 8).

Doubling the slope, 0.58 MV/turn, caused the separatrix to reduce in size considerably, having a maximum energy deviation of only ± 1.2 MeV. The synchrotron period in this case increased by a small amount to 10 turns (Figure 9).

Conclusions

As a result of this research, we have found that producing an optimum capture area requires having a synchrotron period of 9 turns. We also discovered that there is a limited range for the initial RF voltage. The lower limit is set by requirements of maximum capture and the upper limits are constrained by nonlinear resonance effects. It was determined that within this range of voltages, it is not possible to match phase space. We also simulated the present longitudinal dynamics at injection and found that less than 50% of the available energy aperature is used.

Acknowledgements

I would like to thank Dr. Robert Meller, of Cornell University, for proposing this Research Experience for Undergraduates project. Dr. Meller has been a pleasure to work for, having the ability to explain things in an easy to follow and often colorful manner. I am grateful for his guidance on this project. This work was supported by the National Science Foundaton REU grant PHY-9731882 and research grant PHY-9809799.

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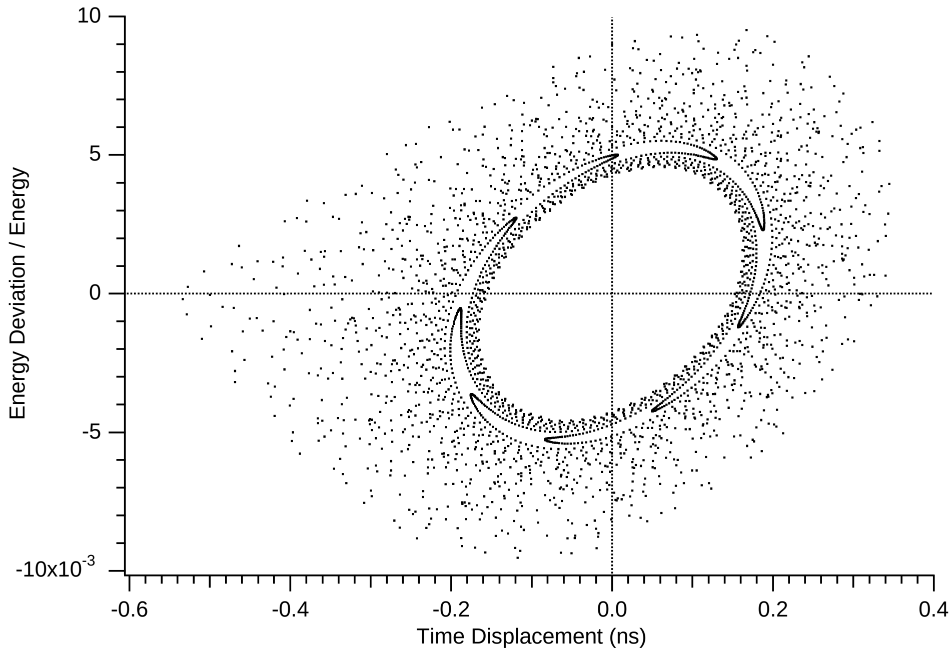

FIGURE 4. δ vs τ plot of beam captured at 0.91 MV

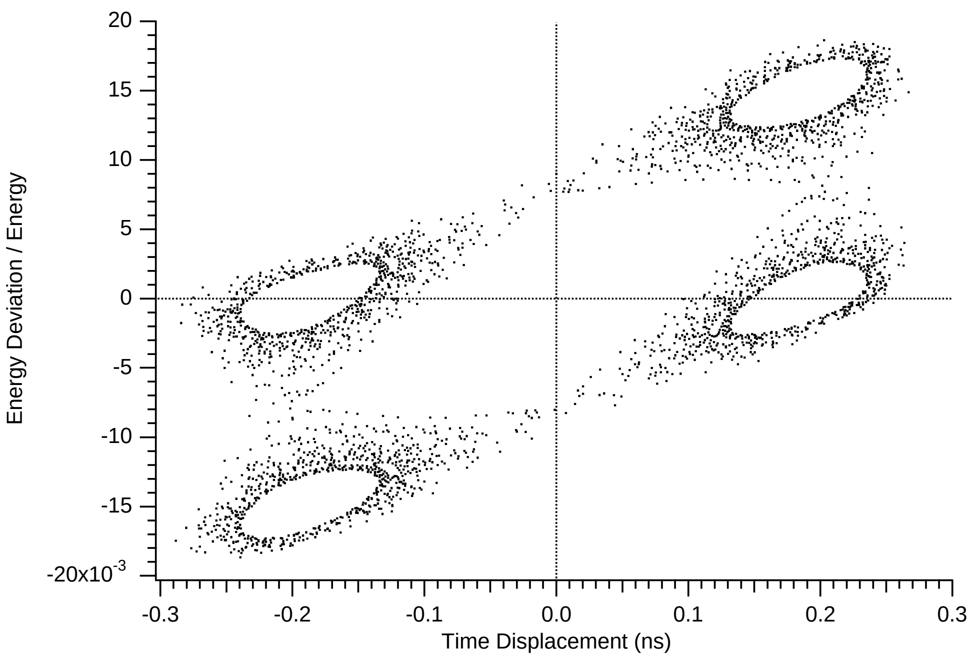

FIGURE 5. Nonlinear resonance effects due to a starting voltage of 4.0 MV

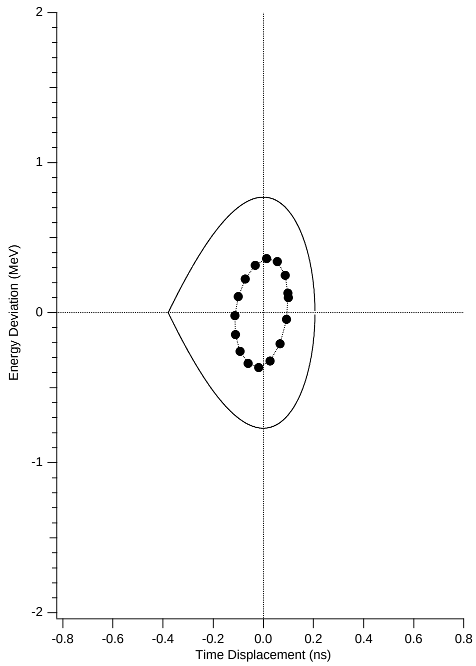

FIGURE 6. Separatrix at injection using the current synchrotron period of 15

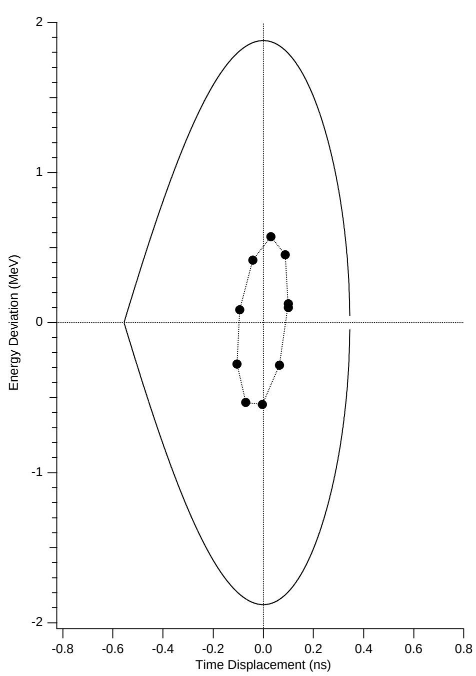

FIGURE 7. Separatrix at injection for optimum capture using a synchrotron period of 9

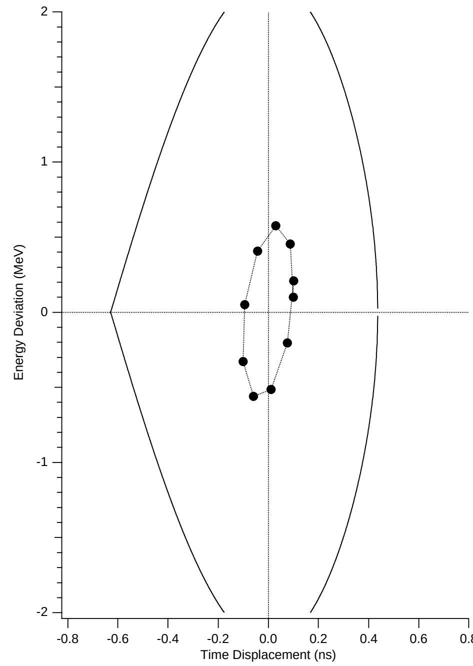

FIGURE 8. Separatrix using a slope of 0.145 Mev/turn, sychrotron period is 9

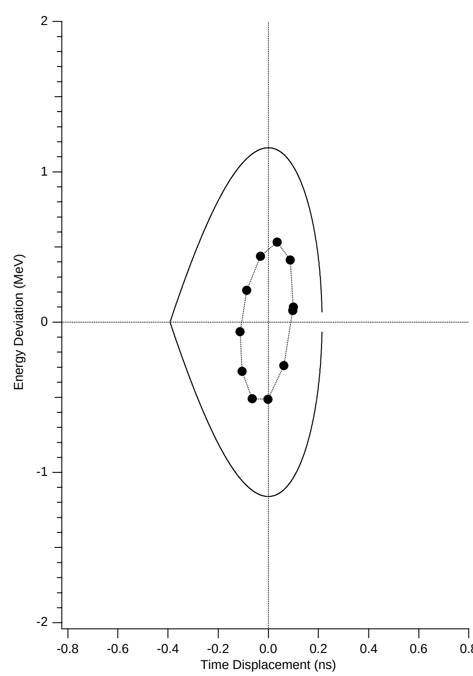

FIGURE 9. Separatrix using a slope of 0.58 MeV/turn, synchrotron period is 10