

# The Wayne State Plot

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The goal of this project is to study the properties of statistical quantities in order to get maximal information from certain experiments. Simulations of old experiments and the use of their results show that a “good” goodness of fit parameter ( $\chi^2$ ) is an inadequate method of determining good fits from bad fits. Because there exists an expectation value for  $\chi^2$ ,  $\sigma$  (the uncertainty),  $M_3$  (the skewness) and  $\rho$  (the global correlation coefficient), we study their spaces.

## I. INTRODUCTION

$\chi^2$  has been calculated numerous times in the past to determine acceptable experiments. Although  $\chi^2$  has been the only tool used in getting a good result, the repeated findings of big  $\sigma$ s caused by misfits over the years has instigated a closer analysis of  $\chi^2$ . By using the MINUIT code from CERN, a standard package (HBOOK) and our own minimization program that extracts the higher moments of the extended likelihood, we simulated and fitted experiments done by different scientists. Among those experiments, we show discrepancies in the final results of Hetherington for the mixing of the 17 keV neutrino [1], Apalikov, in his search for heavy neutrinos [2] and Hime and Jelly for their evidence for the 17keV neutrino [3].

## II. BIASES IN HETHERINGTON'S RESULTS

Because Hetherington's data was good, we used some of the statistics of his experiment [1] to demonstrate the incorrect assumptions that arise from finding a good  $\chi^2$ . Fig.1 shows a simulation of an experiment with a linear fit and a linear distortion (“good fit”) in comparison with a linear fit with a quadratic distortion (“bad fit”). Both plots have a good  $\chi^2$  and the bias is only shown by the  $\sigma$  but the error we observed is tremendously large with a standard deviations of over three.

In order to make the “kurie plots” [1] of both wide scan and narrow scan spectrum, Hetherington had to introduce a fourth linear parameter (the shape factor). This fourth parameter that was chosen arbitrarily was necessary in obtaining an acceptable  $\chi^2$ . Using his calculated parameters, we simulated and fitted his experiment for his best found fits changing only the number of fits to 300,000 for the region 26-67 keV in which the neutrino mass was fixed at 17 keV with the rest of the parameters floating [1]. In our following graphs, because we are working with very large statistics, we did not find the need to use the  $M_3$  (which tends to get smaller as the statistics gets larger). Fig.2 shows the generated ( $\chi^2$ ,  $\sigma$ ) and ( $\sigma$ ,  $\rho$ ) plots. The correlation coefficient was obtained by the utilization of our own minimization program that uses brutal force to get the minimum values. Although  $\rho$  gives important information, it isn't customary to always provide it. Hence, we have no data to compare it with. Fig.3 shows the ( $\chi^2$ ,  $\sigma$ ) plot of the narrow scan spectrum which is slightly different from Fig.2. The difference observed is due to the number of bins used which is 64

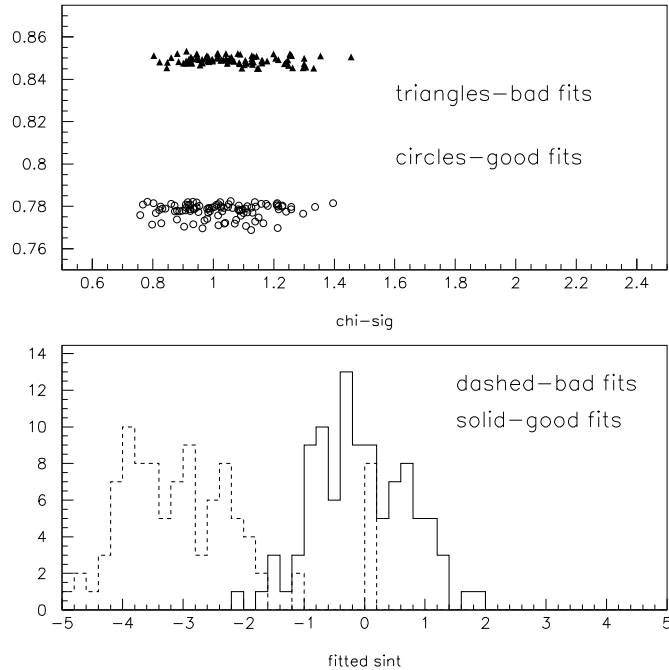


FIG. 1: Biased and unbiased plots generated using 100 fits. The  $(\chi^2, \sigma)$  plots show results of experiments of a linear shape factor and a linear fit producing a “good fit” and, another one using a quadratic shape factor with a linear fit producing a “bad fit”. The second plot shows the fitted  $\sin^2\theta$  parameter with a true value that is equal to 0 demonstrating the error incurred by the choice of the fitting function.

for the narrow scan spectrum and 144 for the wide scan spectrum.

### III. COMPARISON OF HETHERINGTON’S RESULT WITH APALIKOV’S AND HIME’S

The “Wayne State Plot” (contour plot in Fig.4) shows the level of generalized goodness of fit. Typically, experiments with good fits fall on that area while experiments with bad fits fall off of it. The level at which an experiment actually falls on it determines how good the result is. Moving from the inner most level which has the best fit to the outer most level, the “goodness” of the fit diminishes until it crosses the boundary and falls off of it producing a bad fit. As demonstrated in Fig.4, Apalikov’s, Hime’s and Hetherington’s results fall completely off the Wayne State plot.

### IV. CONCLUSIONS

For every experiment done over the years, the production of graphs generated by careful analysis of data plotted against proper fitting functions have been very important. Finding a good “ $\chi^2$ ” was considered victory in finding a good fit. Our research this summer shows indeed the necessity of getting a good  $\chi^2$  but as demonstrated in the previous sections, studying the spaces for which we have expectation values for is the most important element.

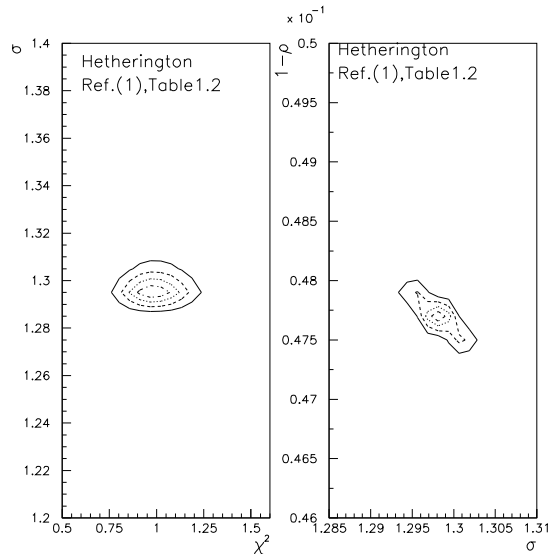


FIG. 2:  $(\chi^2, \sigma)$  and  $(\sigma, \rho)$  plots for the best fit of the wide scan spectrum using the unconstrained fits with data ranging from 26 keV to 67 keV.

Although we have two more ongoing analysis, at this stage of this project, it is safe to deduce that the testing of the “goodness of fit” using both  $\chi^2$  and  $\sigma$  is a crucial step in finding and interpreting final results. If certain data are used for limits, the  $\sigma$  should be close to it’s expectation value.

## V. ACKNOWLEDGMENTS

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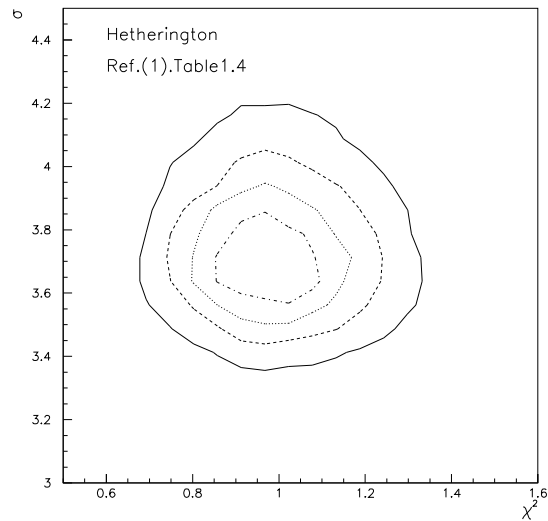


FIG. 3: Two dimensional space of  $(\chi^2, \sigma)$  for the narrow scan spectrum using data ranging between 46-54 keV with all parameters allowed to float.

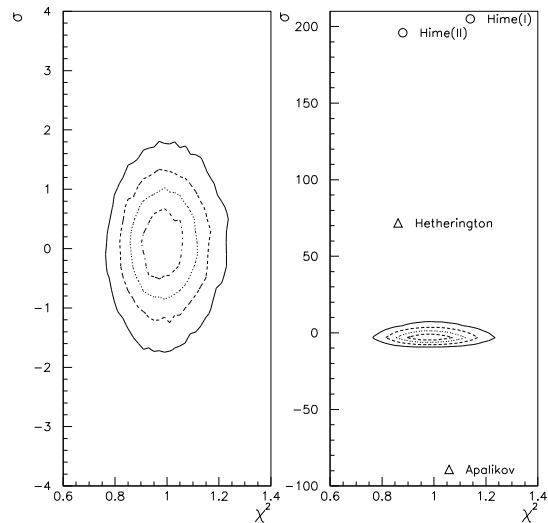


FIG. 4: Comparison of a “good fit” with results obtained from Hetherington, Apalikov and Hime. The Wayne State Plot, the first  $(\chi^2, \sigma)$  plot delimits the area in which experiments have good fits. The second  $(\chi^2, \sigma)$  contour plot shows the results of Hetherington, Apalikov and Hime in comparison with the Wayne State Plot.