# Electron Spin Tracking Studies for the EIC

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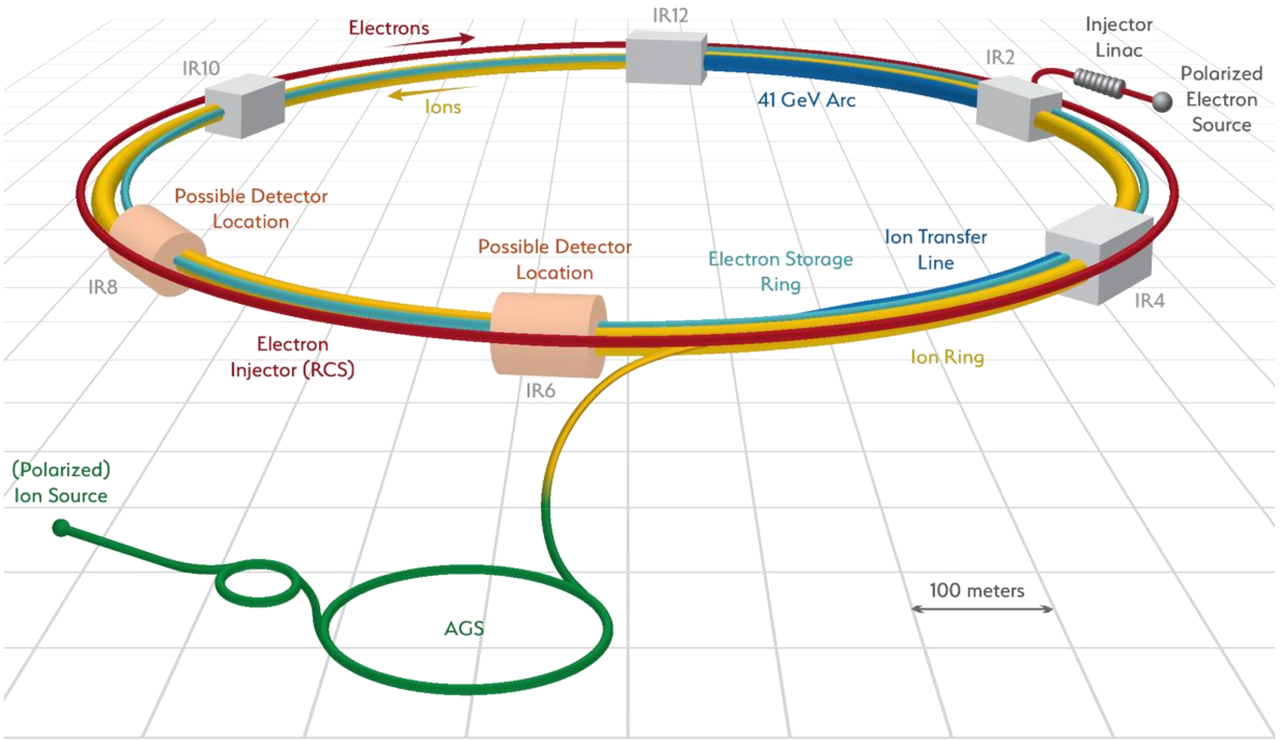
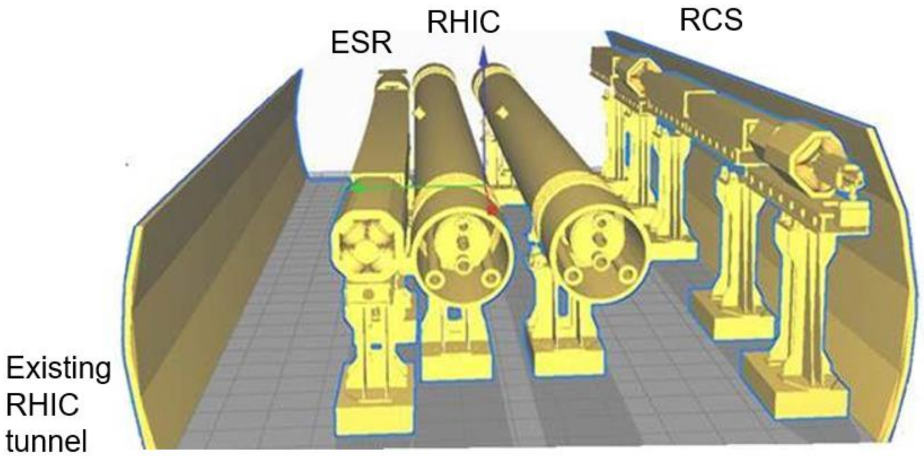
# Overview

- **Background**
    - What is the EIC?
    - Electron Polarization in a Storage Ring
- **Motivation/Methods for Tracking Studies**
- **Results**
    - ESR v5.3 1IP and a “mystery” effect
    - ESR v5.6
        - 1IP, vertical emittance creator study
    - Resolution of the mystery

# Background

## What is the Electron-Ion Collider (EIC)?





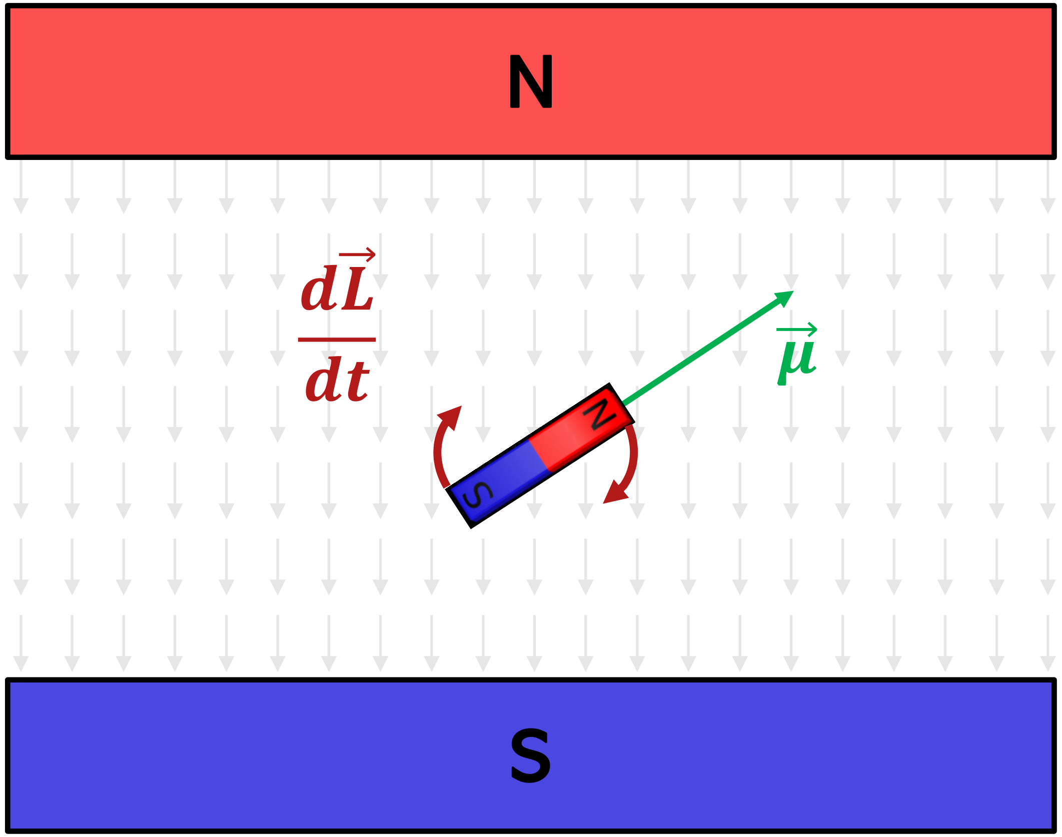
- Hadron Storage Ring (HSR)
- Rapid Cycling Synchrotron (RCS)
- **Electron Storage Ring (ESR)**

| Species | proton | electron | proton | electron | proton | electron | proton | electron | proton | electron |
|---|---|---|---|---|---|---|---|---|---|---|
| Energy [GeV] | 275 | 18 | 275 | 10 | 100 | 10 | 100 | 5 | 41 | 5 |
| CM energy [GeV] | 140.7 | | 104.9 | | 63.2 | | 44.7 | | 28.6 | |

# Background

## Electron Polarization in a Storage Ring

$$\frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B}$$



# Background

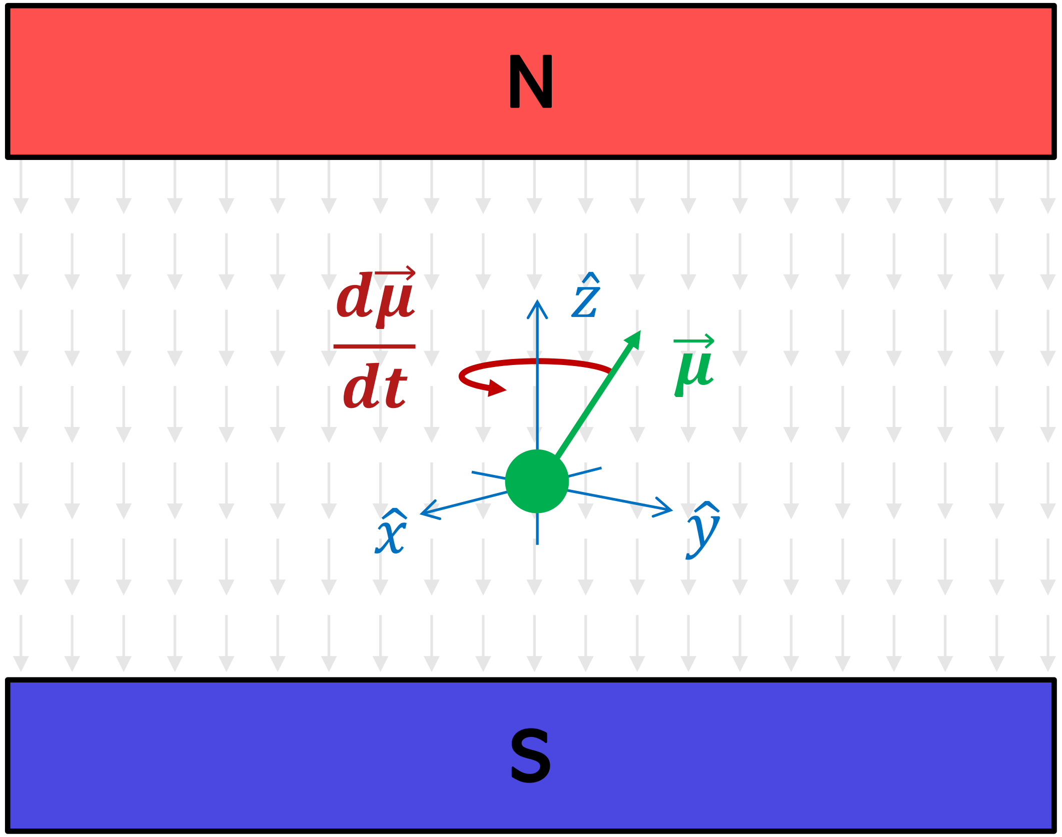
## Electron Polarization in a Storage Ring

$$\frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B}$$

For single particles,

$$\vec{L} = \gamma\vec{\mu}$$

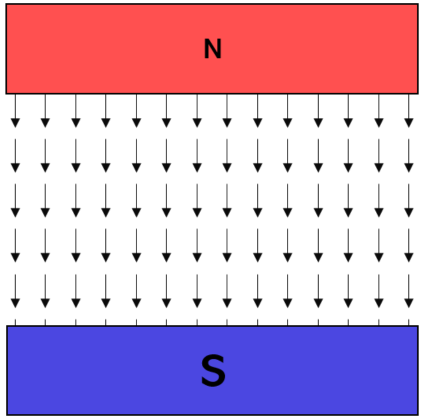
$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma\vec{B}$$



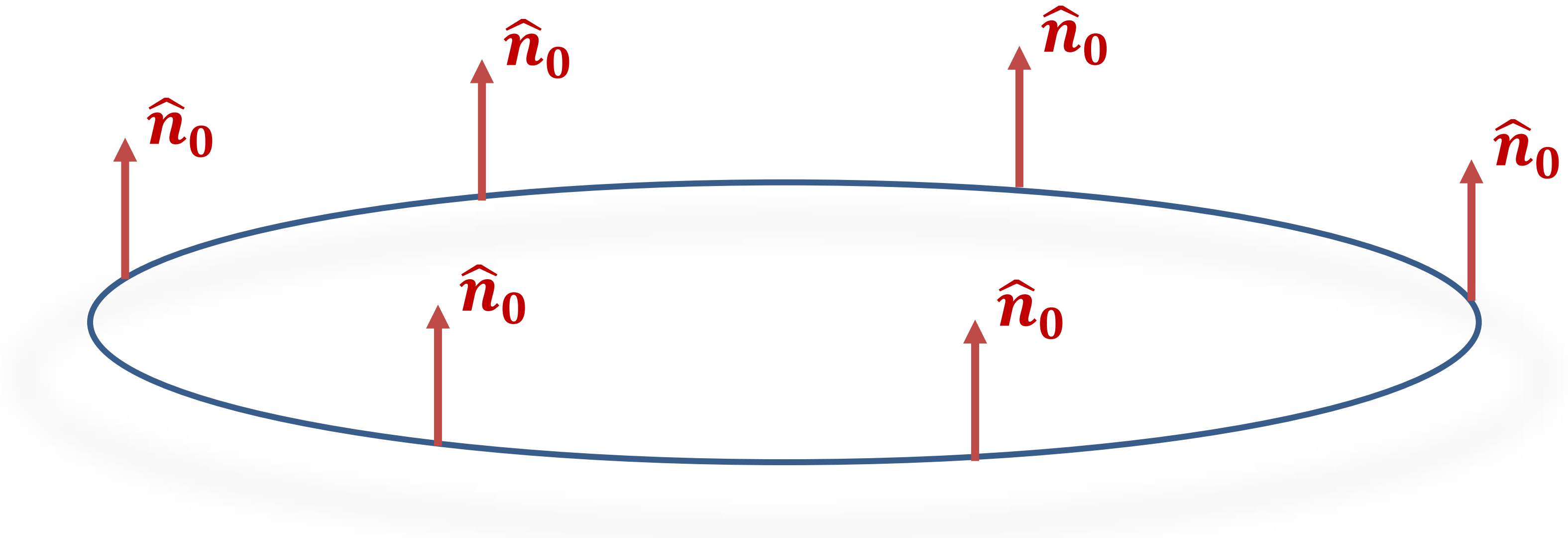
Spin precesses around magnetic fields

# Background

## Electron Polarization in a Storage Ring



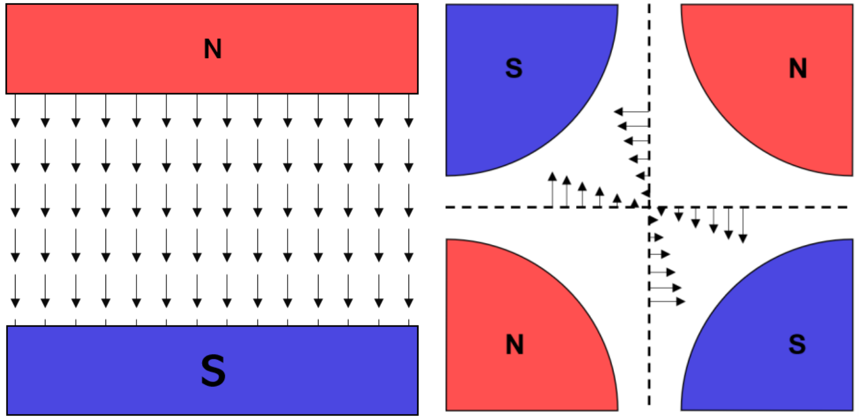
- Ring with only dipoles



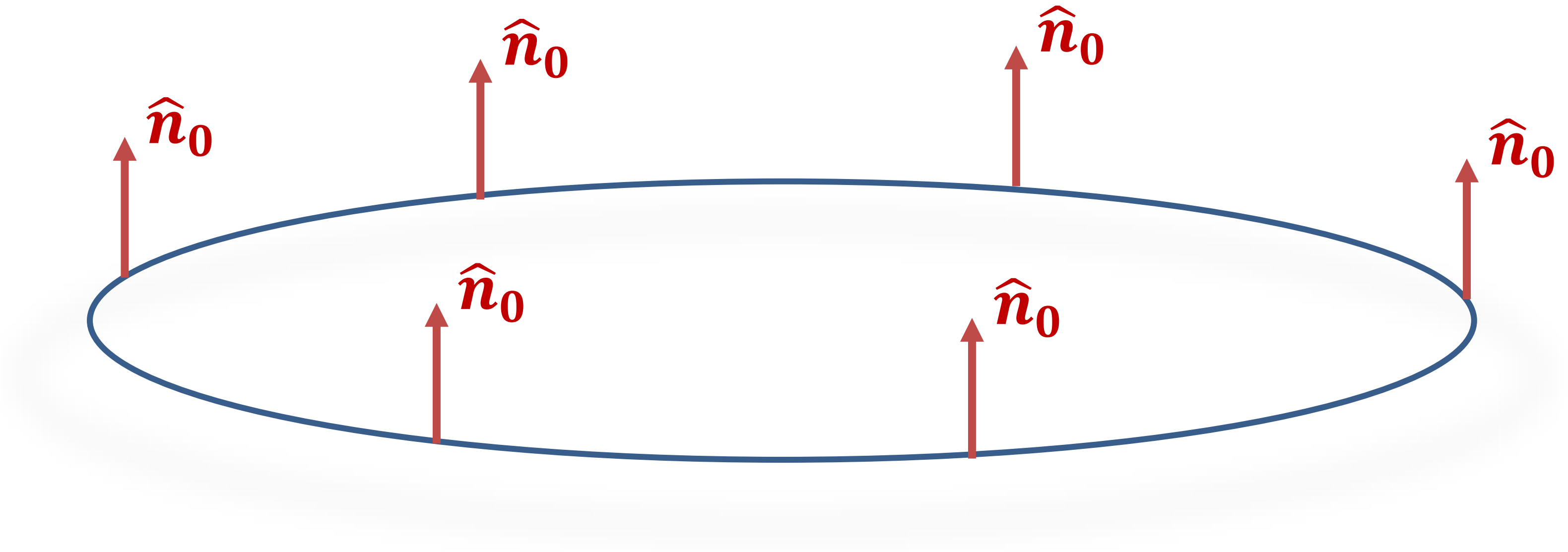
- Periodic spin direction $\hat{n}_0$

# Background

## Electron Polarization in a Storage Ring



- Ring with only dipoles and quadrupoles

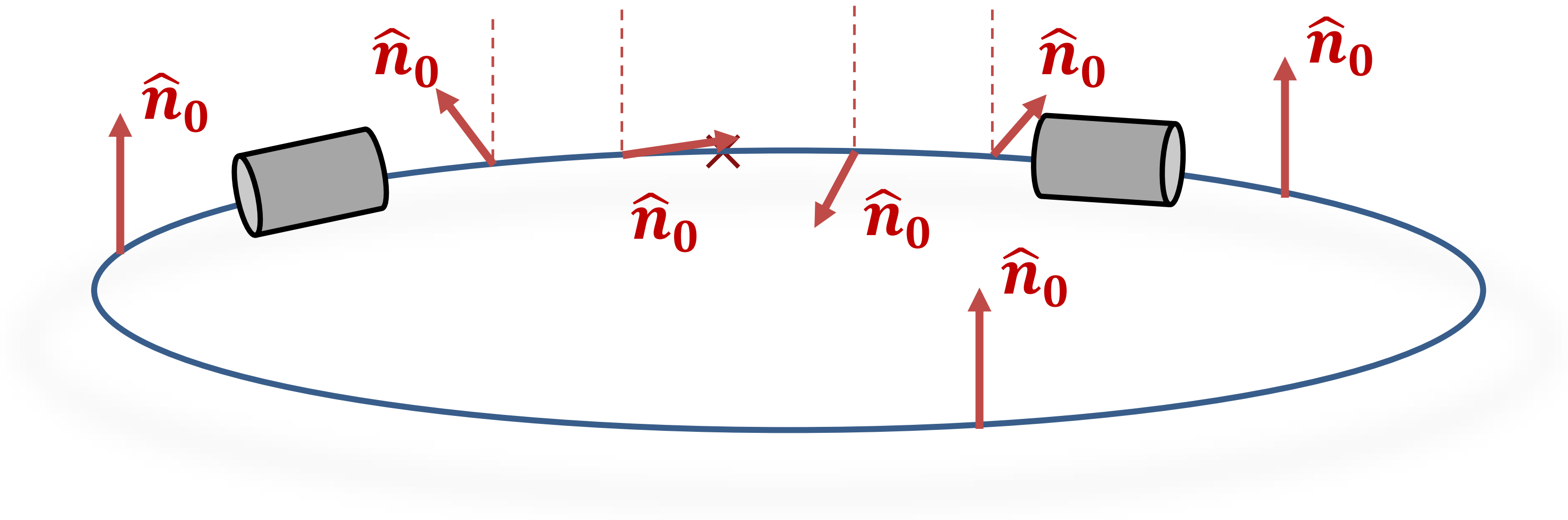


- Periodic spin direction $\hat{n}_0$

# Background

## Electron Polarization in a Storage Ring

- What if we want longitudinal spin at the interaction point?



# Background

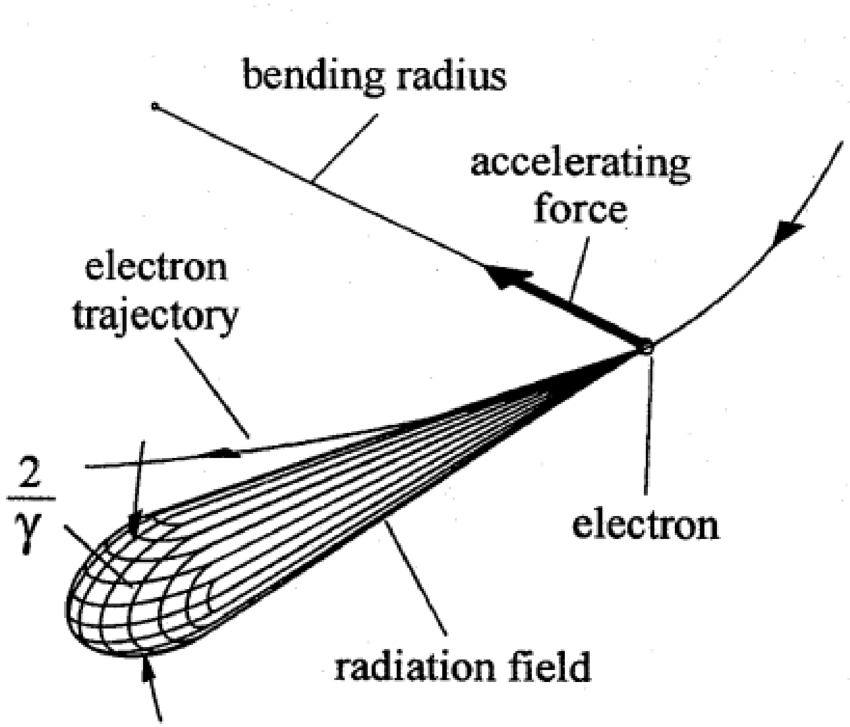
## Electron Polarization in a Storage Ring

Is that it?

For hadrons, mostly. But…

**Electrons emit synchrotron radiation!**

1. Sokolov-Ternov (spin flip) effect
2. Spin diffusion
3. Kinetic polarization (usually small)



# Background

## Electron Polarization in a Storage Ring

### Sokolov-Ternov Effect

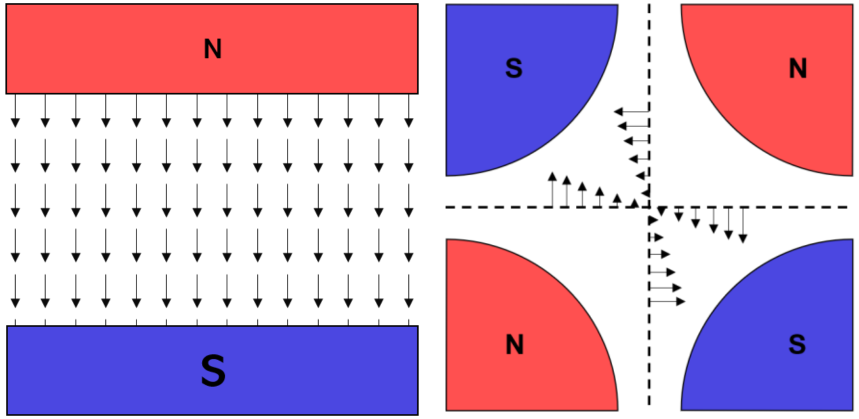
- Derivable from single-particle Dirac theory
- Spin may flip during synchrotron radiation emission in homogenous field
- Asymmetry $A$: higher rate to flip antiparallel to $\vec{B}$-field than parallel to

$$A = \frac{w_- - w_+}{w_- + w_+} = \frac{8}{5\sqrt{3}} = 0.9238$$

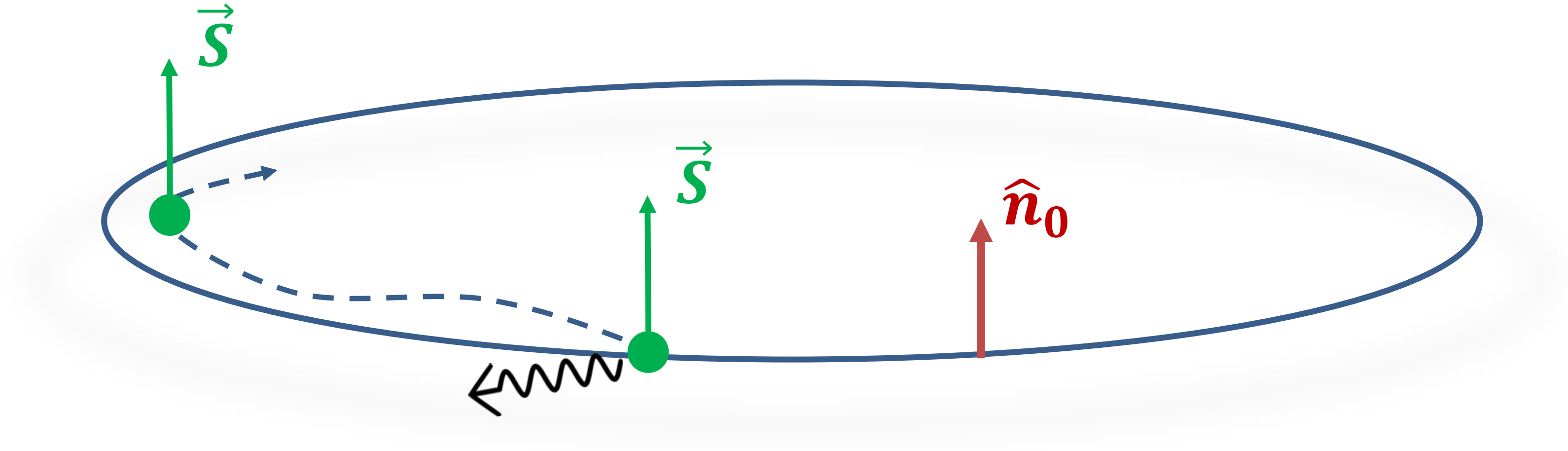
- Builds up polarization in a storage ring!

# Background

## Electron Polarization in a Storage Ring



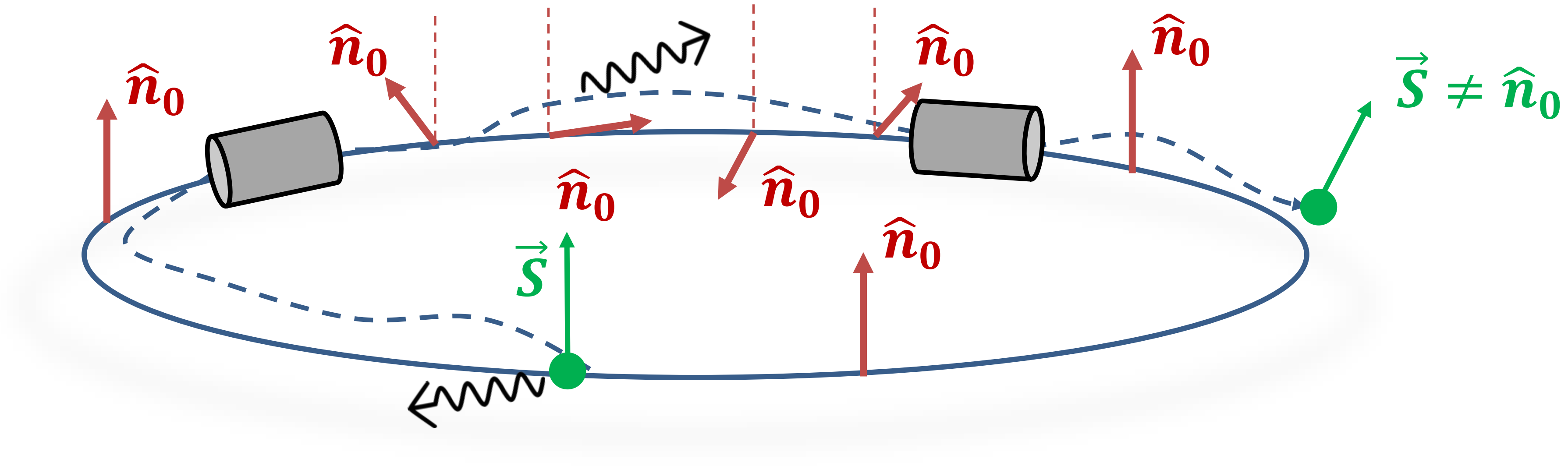
- Ring with only dipoles and quadrupoles



- No depolarizing effects of radiation in perfectly planar ring

# Background

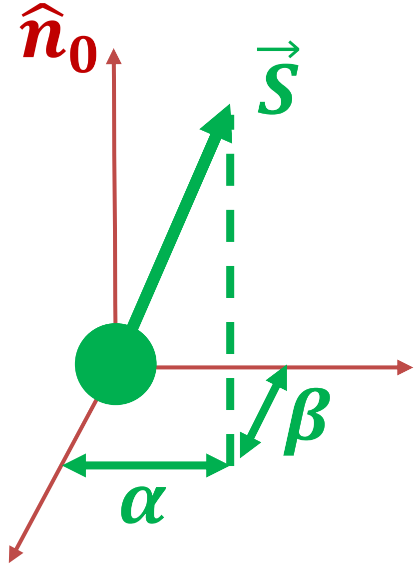
## Electron Polarization in a Storage Ring



- **Spin diffusion**

# Background

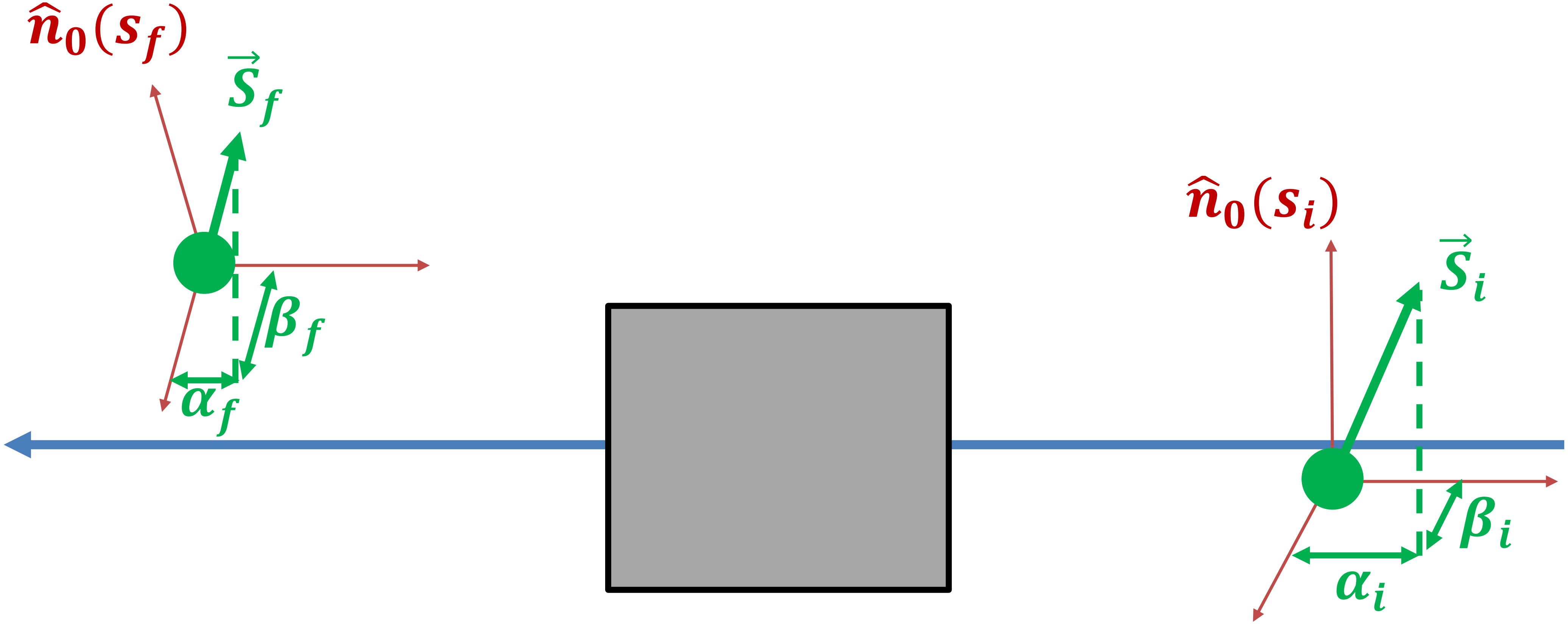
## Electron Polarization in a Storage Ring



From [1-8]

# Background

## Electron Polarization in a Storage Ring



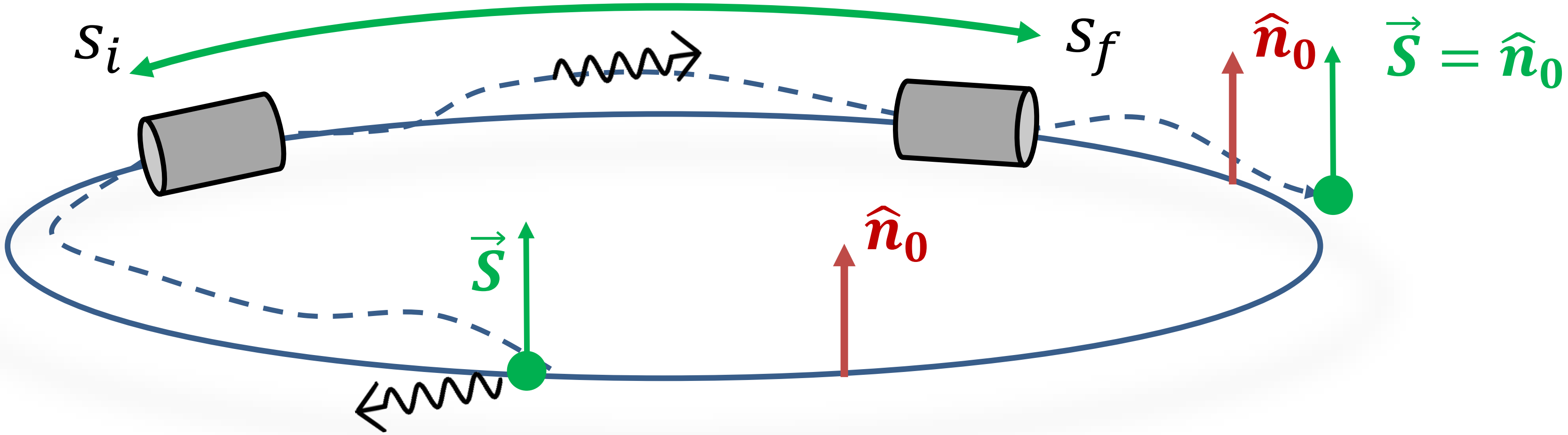
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_f = \boldsymbol{G_x}\begin{pmatrix}x\\ p_x\end{pmatrix}_i + \boldsymbol{G_y}\begin{pmatrix}y\\ p_y\end{pmatrix}_i + \boldsymbol{G_z}\begin{pmatrix}z\\ p_z\end{pmatrix}_i + \boldsymbol{D}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_i$$

From [1-8]

# Background

## Electron Polarization in a Storage Ring

$$\boldsymbol{G_{x,y,z}}(s_i \rightarrow s_f) = \boldsymbol{0}$$



- “Spin matching”

See [9] for more details.

From [1-8]

# Background

## Electron Polarization in a Storage Ring

$$P(t) = P_\infty\left(1 - e^{-t/\tau_{eq}}\right) + P_0 e^{-t/\,\tau_{eq}}$$

$$\tau_{eq}^{-1} = \tau_{pol}^{-1} + \tau_{dep}^{-1}$$

✓ Can be accurately approximated from the closed orbit with analytical formulas

× Hard to estimate analytically. May be affected significantly by nonlinearities

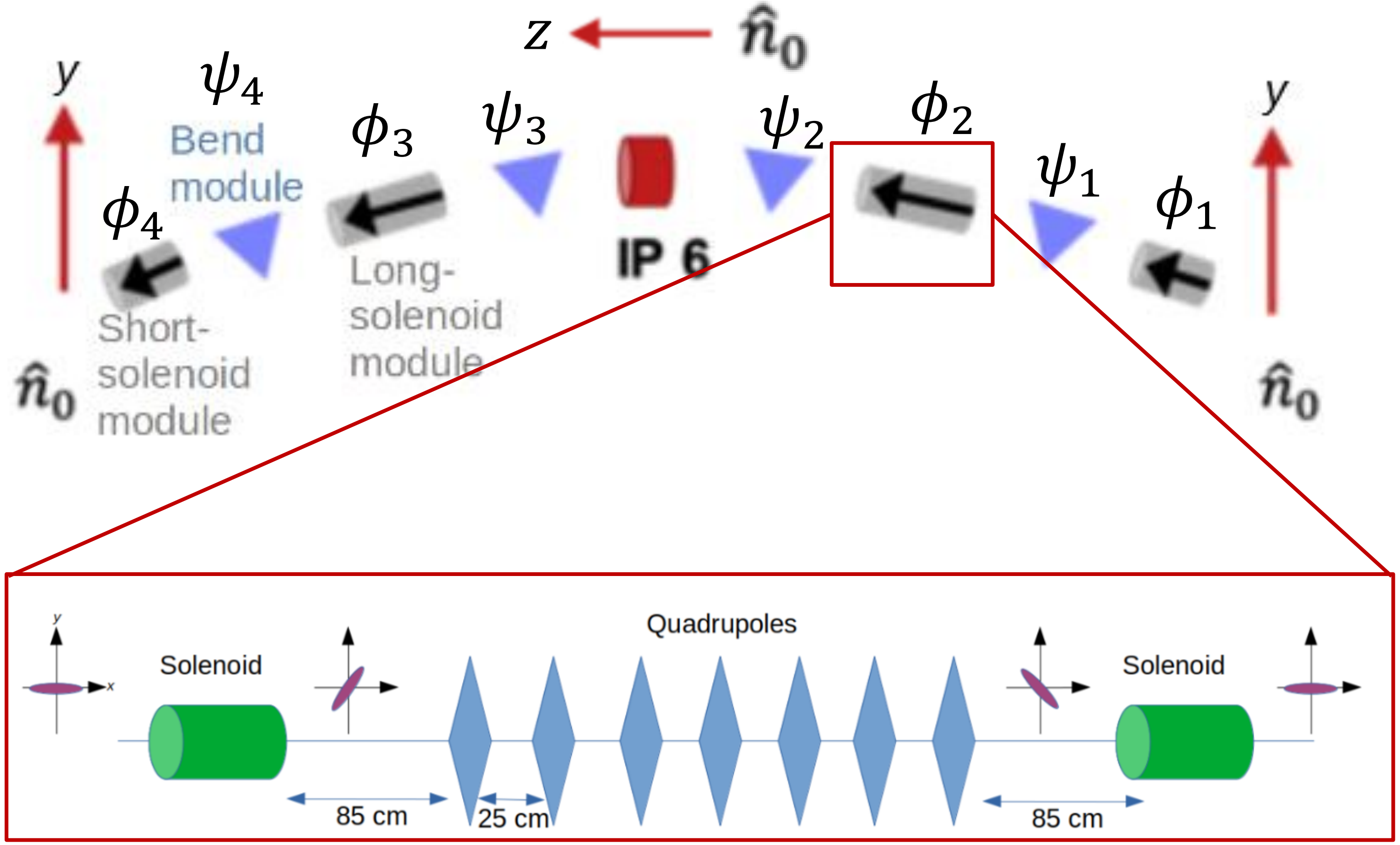
To estimate $\tau_{dep}^{-1}$ , do Monte Carlo tracking with *only* spin diffusion effects

$$P_{tr}(t) = P_0 e^{-t/\tau_{dep}} \approx P_0 - t/\tau_{dep}$$

From [1-8]

# Background

## EIC-ESR Spin Rotator System

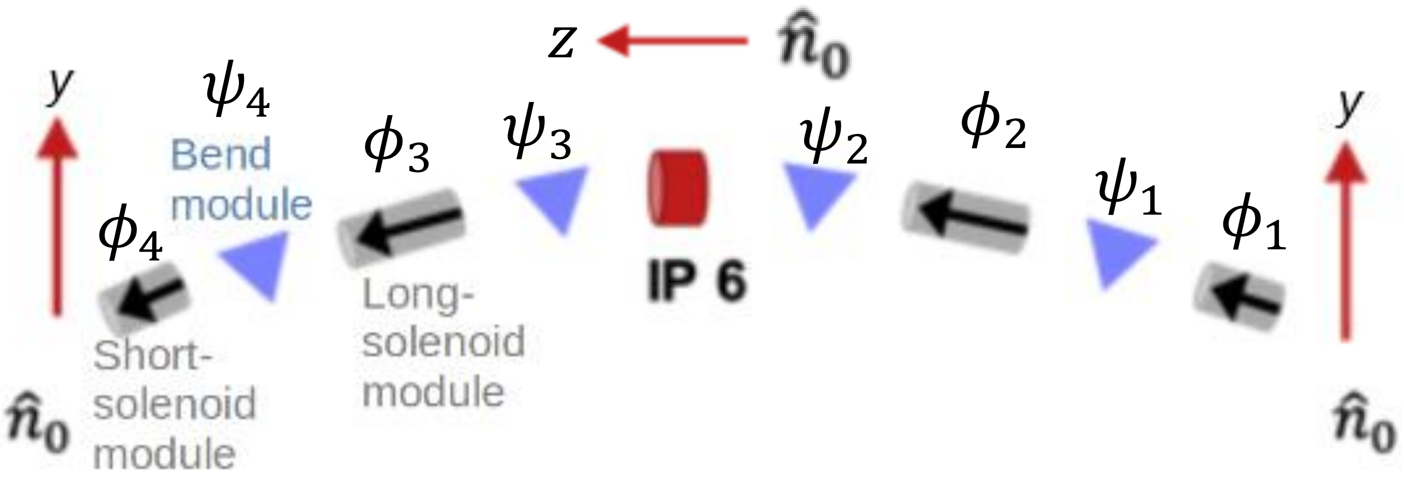


- Rotates $\hat{n}_0$ to longitudinal at the IP for a wide range of e-beam energies (5-18 GeV)

Images from [10]

# Background

## EIC-ESR Spin Matching Conditions



$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_f = \boldsymbol{G_x}\begin{pmatrix}x\\ p_x\end{pmatrix}_i + \boldsymbol{G_y}\cancelto{0}{\begin{pmatrix}y\\ p_y\end{pmatrix}_i} + \boldsymbol{G_z}\begin{pmatrix}z\\ p_z\end{pmatrix}_i + \boldsymbol{D}\cancelto{0}{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_i}$$

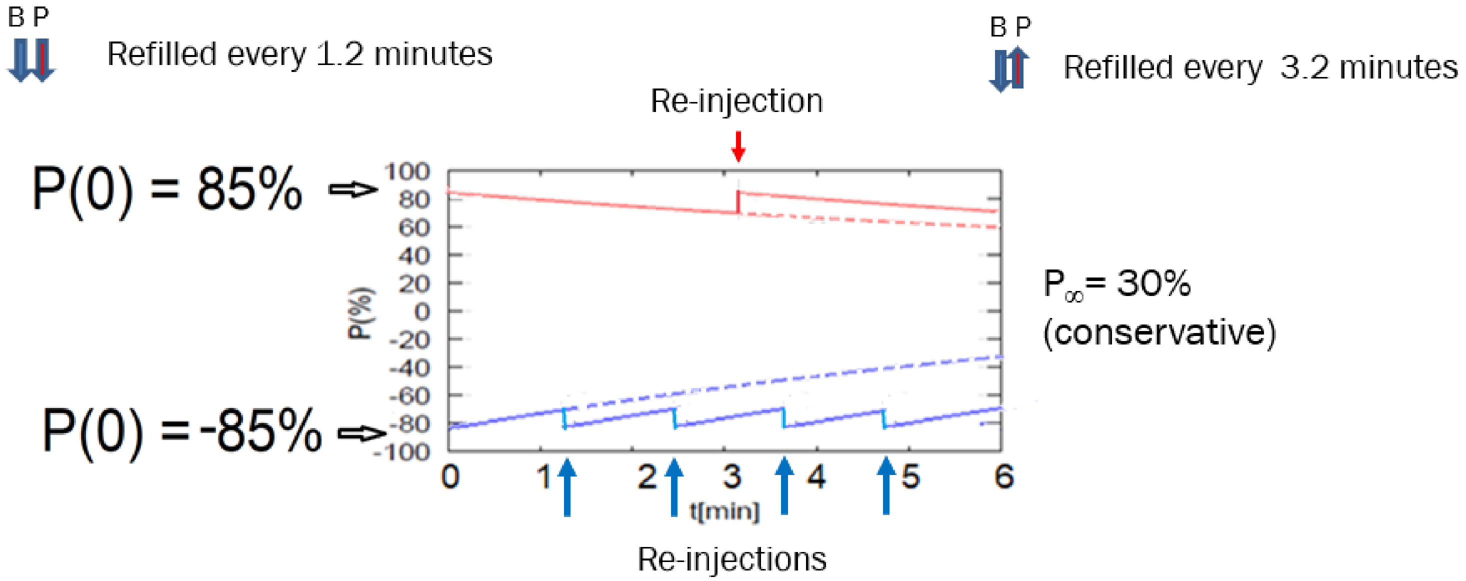
$\boldsymbol{G_x} = \boldsymbol{0}$ by conditions on quads between solenoids

$\boldsymbol{G_z} = \boldsymbol{0}$ by conditions on periodic dispersion $\eta, \eta'$

Images from [10], ESR spin matching conditions in [11] or [9].

# Background

## EIC-ESR Polarization Requirements



- Maintain average polarization of at least 70%
- Bunches should be replaced on average every 2.2 minutes
- For $P_\infty = 30\%$, need $\tau_{eq} > 4.6$ min

Images from [10]

# Motivation for Tracking Studies

- Most accurate statistics including all nonlinearities
- Verify effects/significance of first-order spin matching
- Must cross-check between various modern codes
- Verify polarization robustness (i.e. with misalignments, $\epsilon_y$-creator)

# Methods

## Monte-Carlo Spin Tracking Methods with Radiation

- **Map Tracking** – damped maps generated between each bend center (radiation points*) by PTC w/ user-specified order

- **Bmad Tracking** – element-by-element damped nonlinear maps w/ radiation points after each element

- **PTC Tracking** – element-by-element symplectic integration w/ radiation points at each step within the element

- Bmad toolkit conveniently implements all the above tracking methods and can be run in parallel on a GPU cluster

**bend-splitting for radiation – “SLICKTRACK” – is necessary for accurate spin tracking*

# Methods

## ESR 18 GeV Lattice Tracking Studies

- **v5.3: $\boldsymbol{G_x = 0,\ \ G_z = 0}$**
  - 1IP

- **v5.6: $\boldsymbol{G_x = 0,\ \ G_z \neq 0}$**
  - 1IP
  - $\epsilon_y$-creator

- **v5.3 with varying $\boldsymbol{Q_s}$**

*All trackings started with 5,000–10,000 particle distribution generated from analytical equilibrium* $\epsilon_a, \epsilon_b, \epsilon_c$

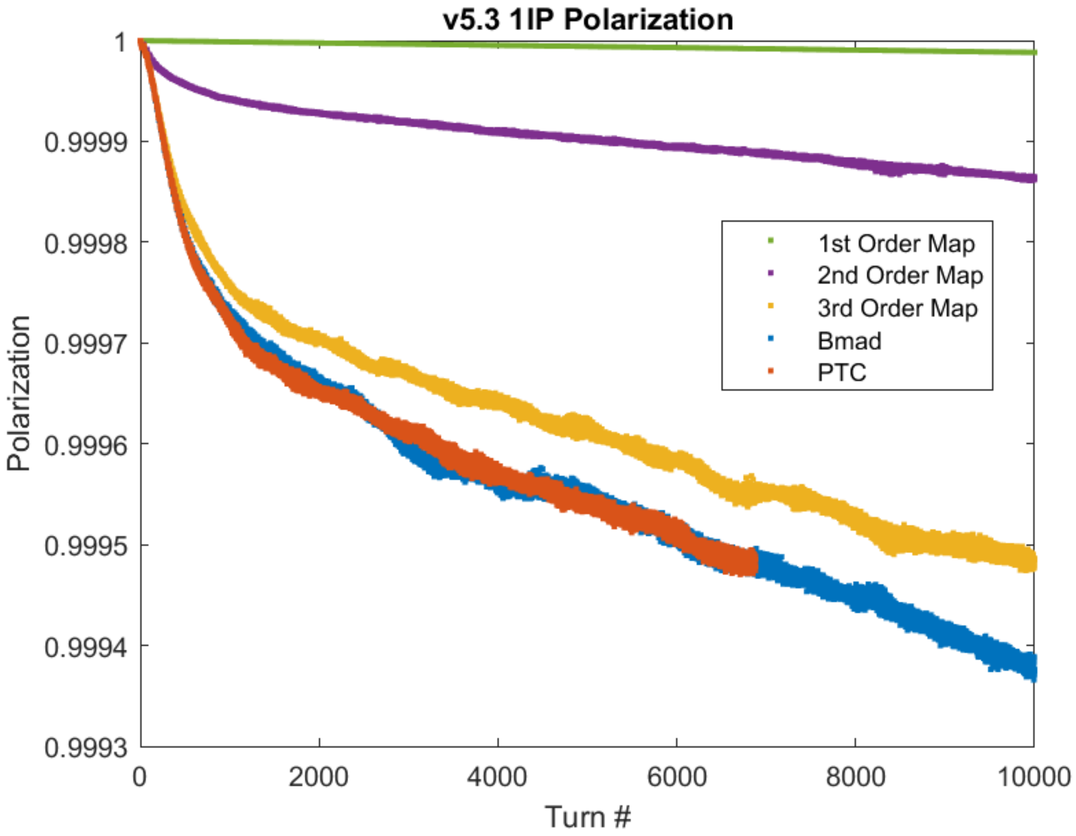
# v5.3 Results

$$G_x = 0, \;\; G_z = 0$$

# Results – v5.3

## Polarization

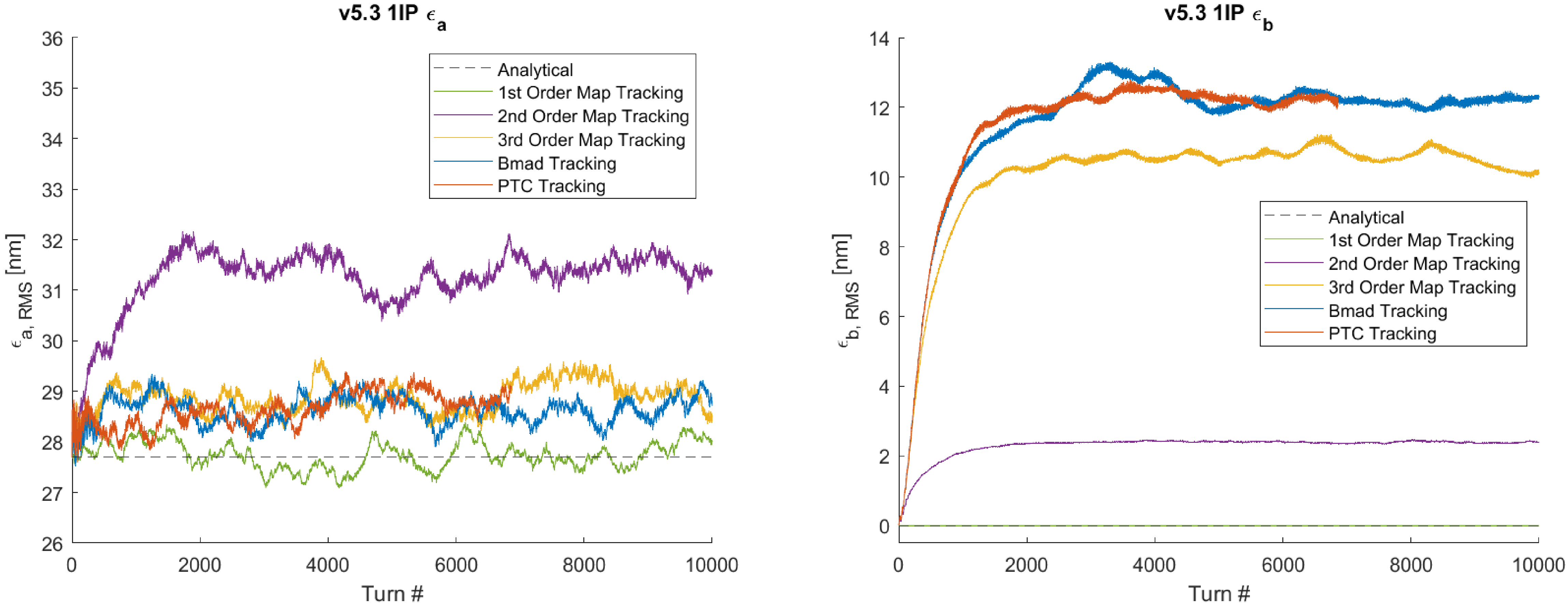
| | v5.3 1IP | |
|---|---|---|
| | $\tau_{eq}$ [min] | $P_\infty$ |
| **Analytical** | 31.1 | 61.3% |
| **1st Order Map Tracking** | 30.7 | 66.4% |
| **2nd Order Map Tracking** | 15.7 | 33.8% |
| **3rd Order Map Tracking** | 6.5 | 14.0% |
| **Bmad Tracking** | 5.6 | 12.1% |
| **PTC Tracking** | 5.7 | 12.3% |



- Polarization significantly worse in nonlinear case
- Such significant damping should not occur if starting w/ equilibrium distribution. Is this a clue on what's happening?

# Results – v5.3

## Turn-by-turn RMS emittances



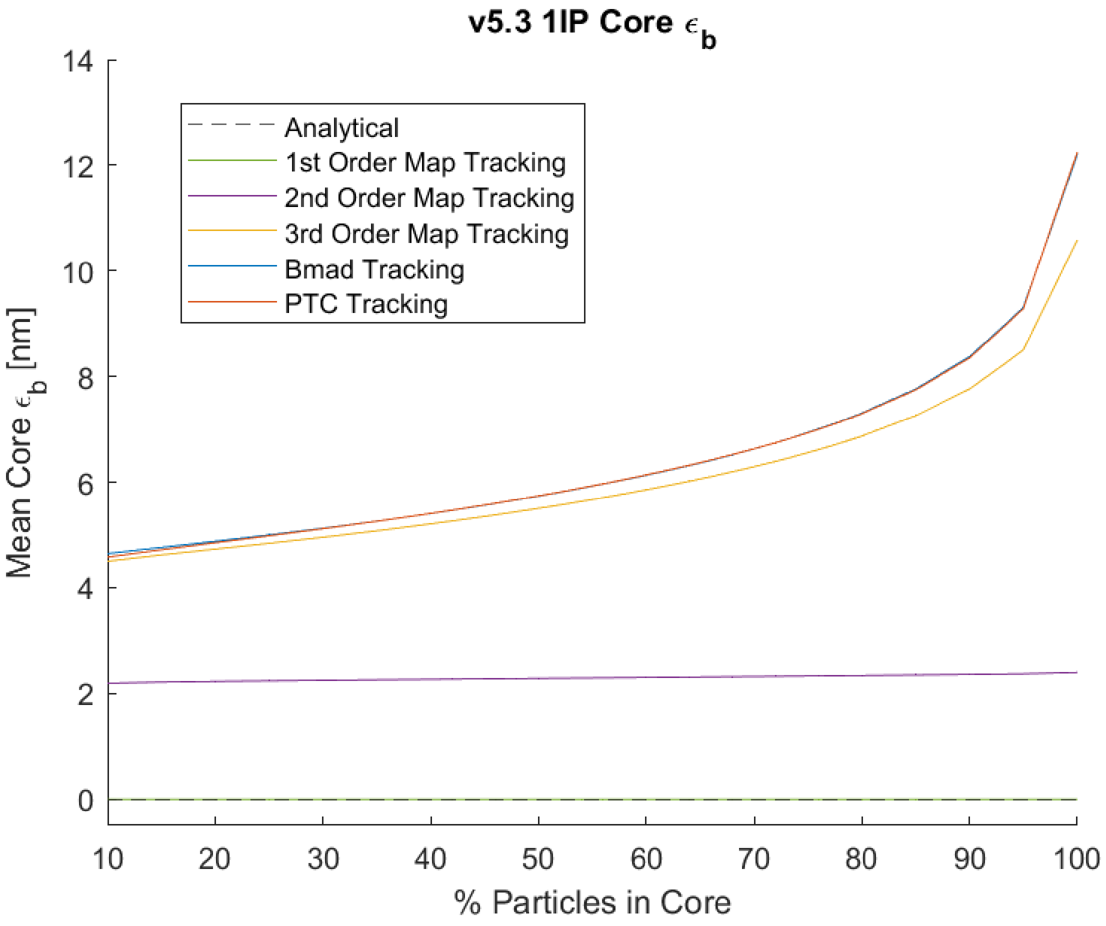
Nonlinearities might be driving tails of y-distribution to high amplitudes

→ **Core emittance** likely a better measure…

# Results – v5.3

## Core emittance

- Emittance obtained by fitting a Gaussian to particles within some cutoff amplitude
- If perfectly Gaussian distribution, $\epsilon_{core} = \epsilon_{RMS}$ for all cutoff amplitudes



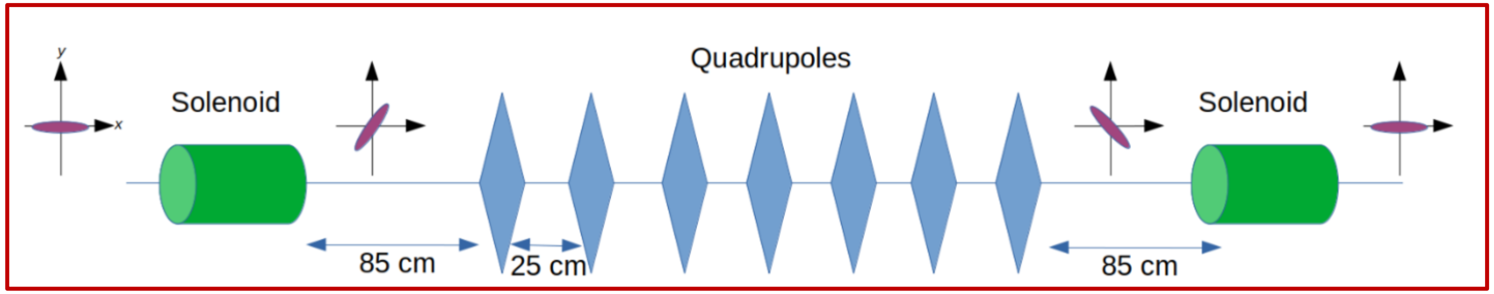
- Core emittances calculated as means of core emittance over turns 4,000 to end

- **In nonlinear case, obtaining ~5 nm of vertical emittance even in the core**

# Results – v5.3

- **There is some nonlinear effect present that:**
  → Creates 5nm $\epsilon_b$ even in the core
  → Reduces DK polarization from 60% to 12%

- Only regions in ring where $\epsilon_b$ might be created is where there is coupling

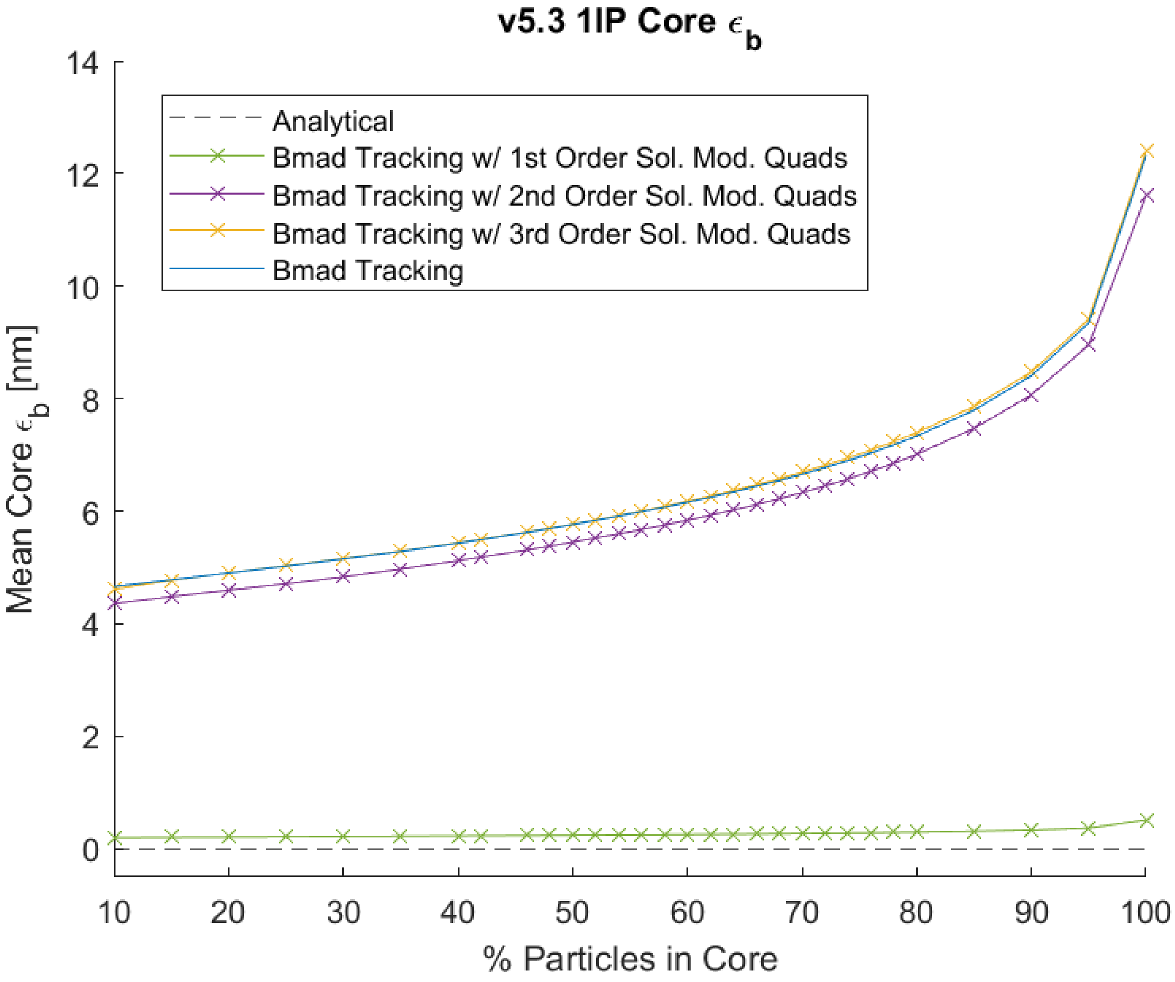


- Try fully nonlinear trackings (including nonlinear solenoids) but with 1st, 2nd and 3rd order quadrupoles in between solenoids (settable in Bmad)

# Results – v5.3

## Core Emittance

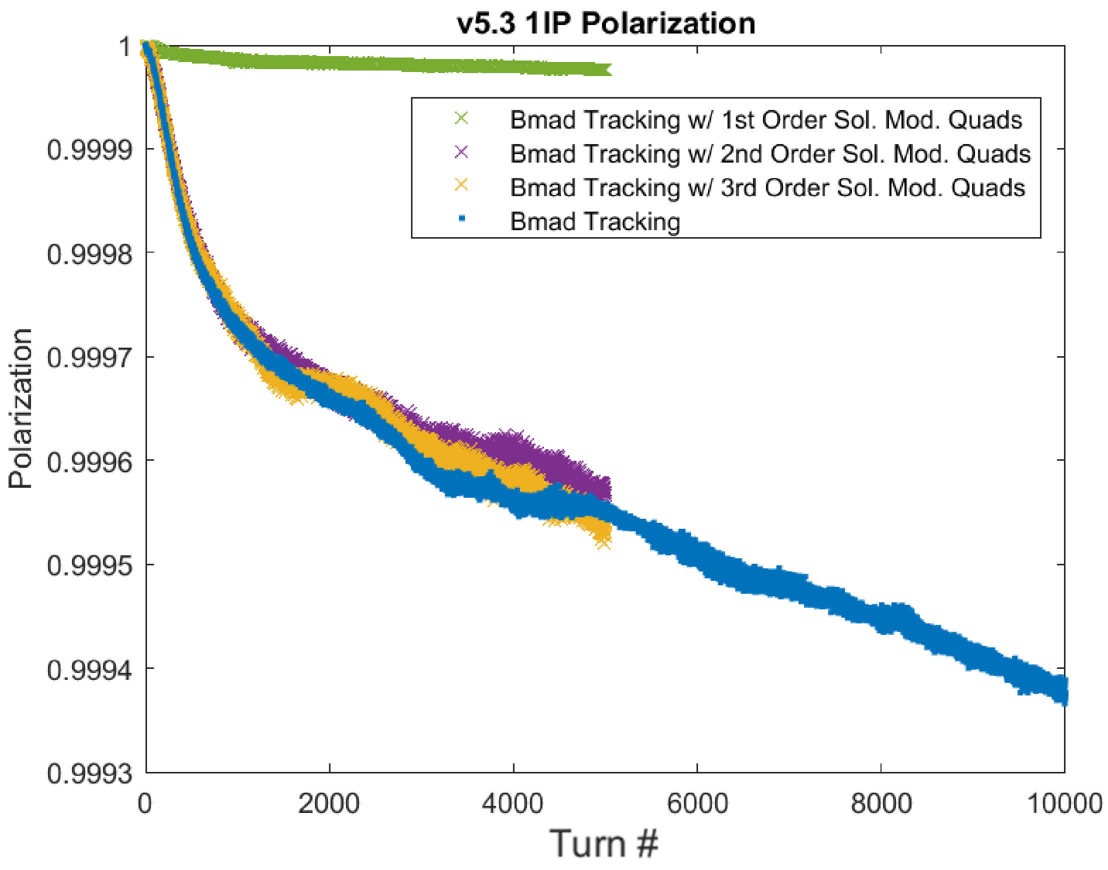
- **~5 nm vertical emittance created by 2^nd order effects of quadrupoles in between solenoids**
- **Polarization?**



# Results – v5.3

## Polarization

| | v5.3 1IP | |
|---|---|---|
| | $\tau_{eq}$ [min] | $P_\infty$ |
| Analytical | 31.1 | 61.3% |
| Bmad w/ 1st Order S.M. Quads | 28.3 | 61.1% |
| Bmad w/ 2nd Order S.M. Quads | 7.0 | 15.1% |
| Bmad w/ 3rd Order S.M. Quads | 5.0 | 10.8% |
| Bmad Tracking | 5.6 | 12.1% |



- $2^{nd}$ order effects in quadrupoles between solenoids the primary culprit

# Conclusions – v5.3

- **Mystery $2^{nd}$ order effect…**
  → Creates ~5 nm of core vertical emittance
  → Reduces $P_\infty$ to 12%

- **DA studies suggest having $\boldsymbol{\eta, \eta' = 0}$ in solenoids removes the effect**
  → Must turn off the short solenoid & lose the longitudinal spin match

- **Leads to the v5.6: $\boldsymbol{\eta, \eta' = 0}$ in solenoids but $\boldsymbol{G_z \neq 0}$**

# v5.6 Results

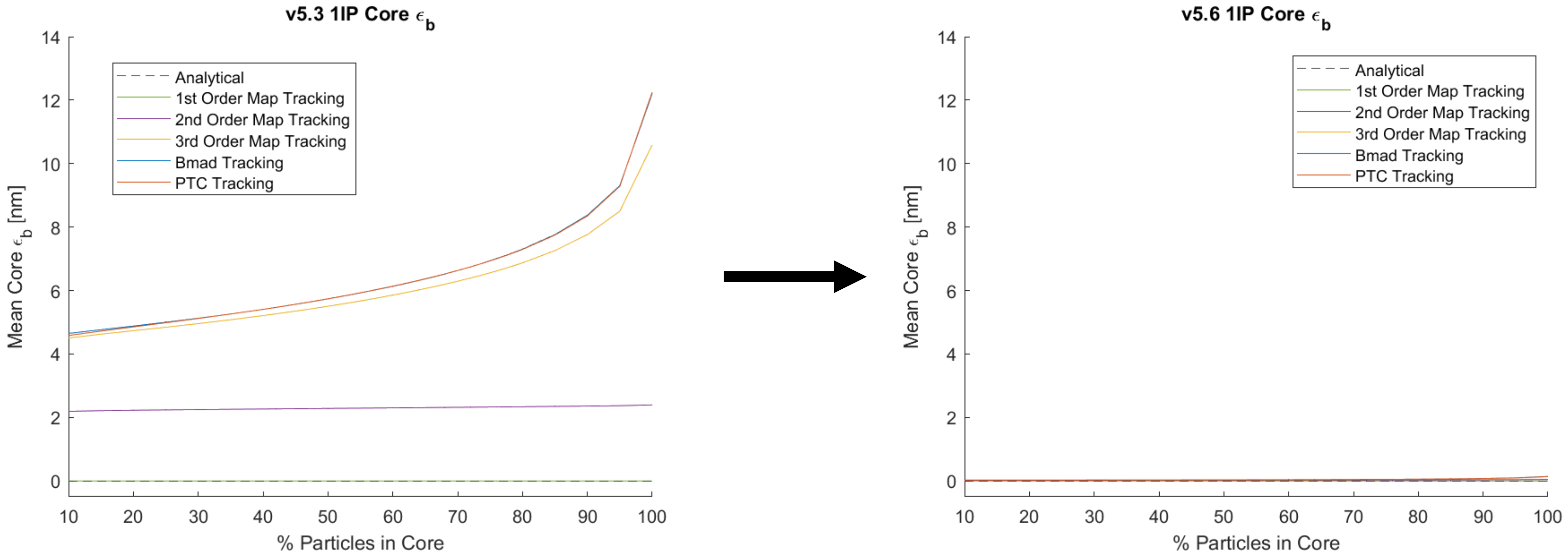
$G_x = 0, \;\; G_z \neq 0$

$\eta, \eta' = 0$ in solenoid modules

# Results – v5.6 1IP

## Does having $\eta, \eta' = 0$ in the solenoids fix mystery effect? ✓

Vertical core emittances:



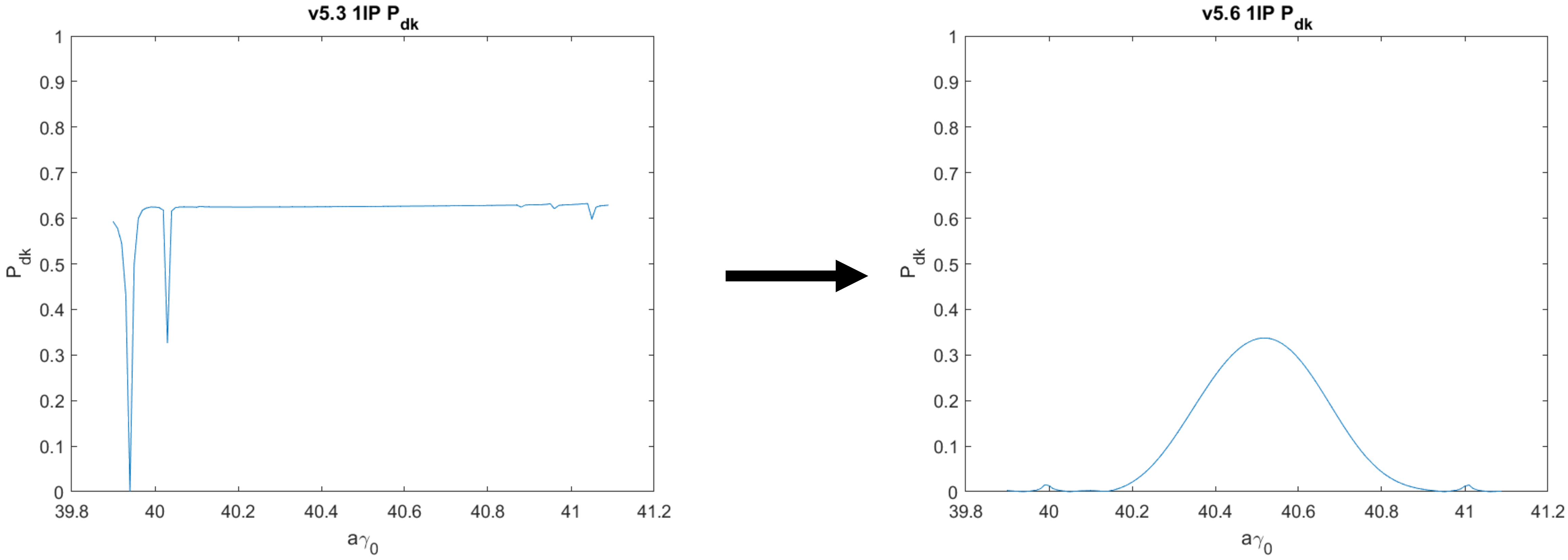
$\eta, \eta' \neq 0$

$\eta, \eta' = 0$

# Results – v5.6 1IP

**Can we live without a longitudinal spin match at 18 GeV?** Maybe – need to check nonlinear tracking

*Linear* $P_\infty$:



$$G_z = 0$$

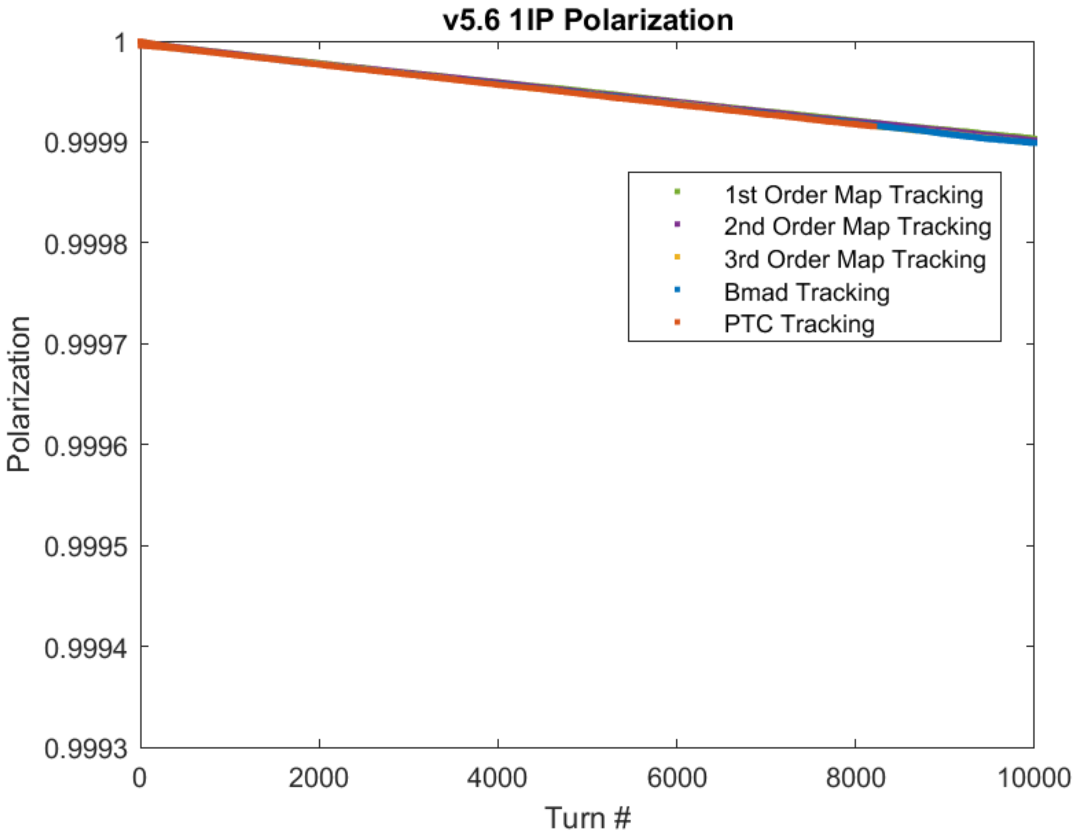
$$G_z \neq 0$$

**nonlinearities give much lower actual* $P_\infty$

# Results – v5.6 1IP

## Polarization

| | v5.6 1IP | |
|---|---|---|
| | $\tau_{eq}$ [min] | $P_\infty$ |
| Analytical | 15.0 | 33.4% |
| 1st Order Map Tracking | 14.0 | 32.9% |
| 2nd Order Map Tracking | 13.9 | 32.7% |
| 3rd Order Map Tracking | 13.7 | 32.1% |
| Bmad Tracking | 13.7 | 32.1% |
| PTC Tracking | 13.6 | 31.9% |



- Polarization is robust for baseline 1IP v5.6 in fully nonlinear case
- But what about misalignments, beam-beam force, and vertical emittance creation?

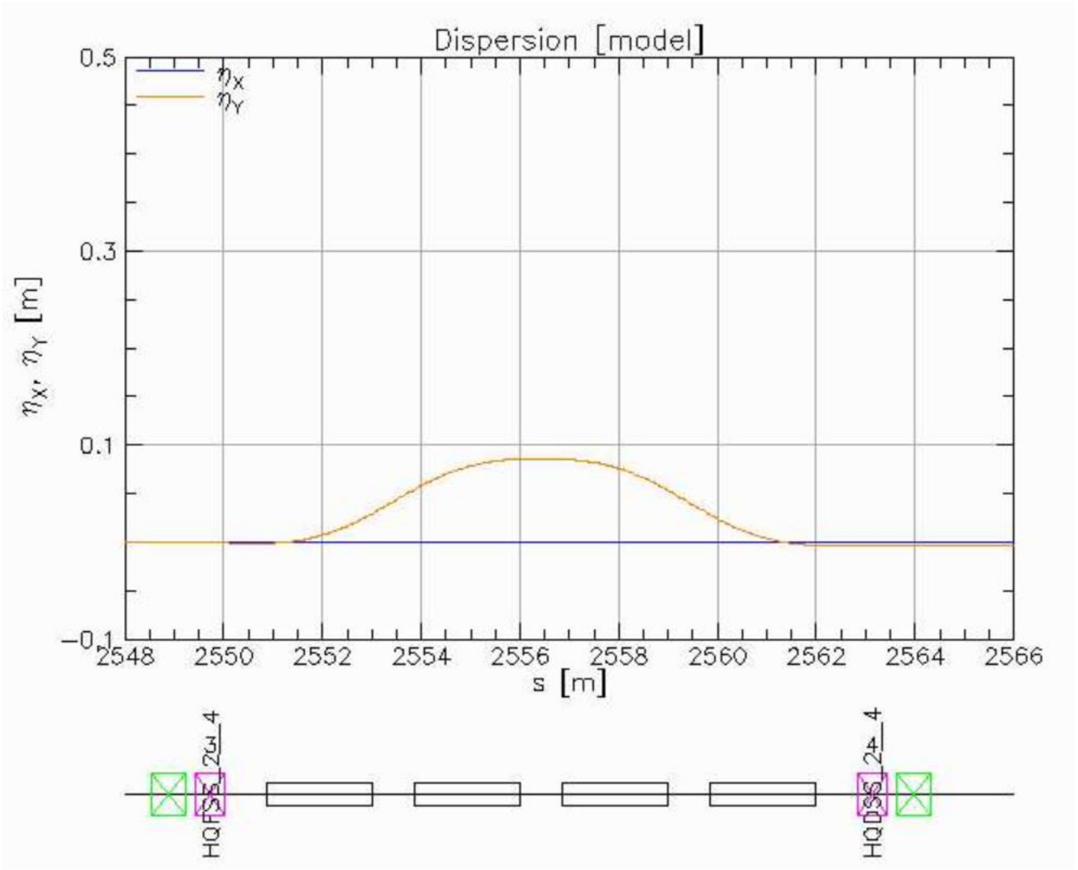
# Results – v5.6 $\epsilon_y$-creator

**Several methods to create $\boldsymbol{\epsilon_y}$**

1. Localized closed $\eta_y$ bump ⟵
2. Delocalized $\eta_y$
3. Localized coupling near the IR

- Inserted closed $\eta_y$ bump in IP2 drift space that creates 2.5 nm $\epsilon_y$

- Optimized so $\boldsymbol{G_y} = 0$ for 1-turn from center of chicane



- Spin match was tricky: $\epsilon_y$ grew to ~ 5 nm

# Results – v5.6 $\epsilon_y$-creator

## Polarization

| | v5.6 1IP | | | v5.6 1IP<br>+ $\eta_y$ bump | | | v5.6 1IP<br>+ $\eta_y$ bump<br>+ $G_y = 0$ | |
|---|---|---|---|---|---|---|---|---|
| | $\tau_{eq}$ [min] | $P_\infty$ | | $\tau_{eq}$ [min] | $P_\infty$ | | $\tau_{eq}$ [min] | $P_\infty$ |
| Analytical | 15.0 | 33.4% | | 6.8 | 29.3% | | 12.2 | 31.9% |
| 1st Order Map Tracking | 14.0 | 32.9% | | 6.4 | 14.5% | | 8.9 | 24.5% |
| 2nd Order Map Tracking | 13.9 | 32.7% | | 5.8 | 13.4% | | 6.2 | 17.1% |
| 3rd Order Map Tracking | 13.7 | 32.1% | | 5.6 | 13.0% | | 6.6 | 18.0% |
| PTC Tracking | 13.6 | 31.9% | | 5.4 | 12.5% | | 6.4 | 17.5% |

- **Polarization drops below acceptable levels!**
- **Careful implementation and vertical spin matching will be necessary if closed $\eta_y$-bump used as vertical emittance creator**

# Conclusions – v5.6

- **Zero dispersion in the solenoid modules fixes mystery effect**

- **However, doing so loses longitudinal spin match**
  ⟶ Polarization drops significantly, even in linear case
  ⟶ $\epsilon_y$ creation drops polarization below acceptable levels

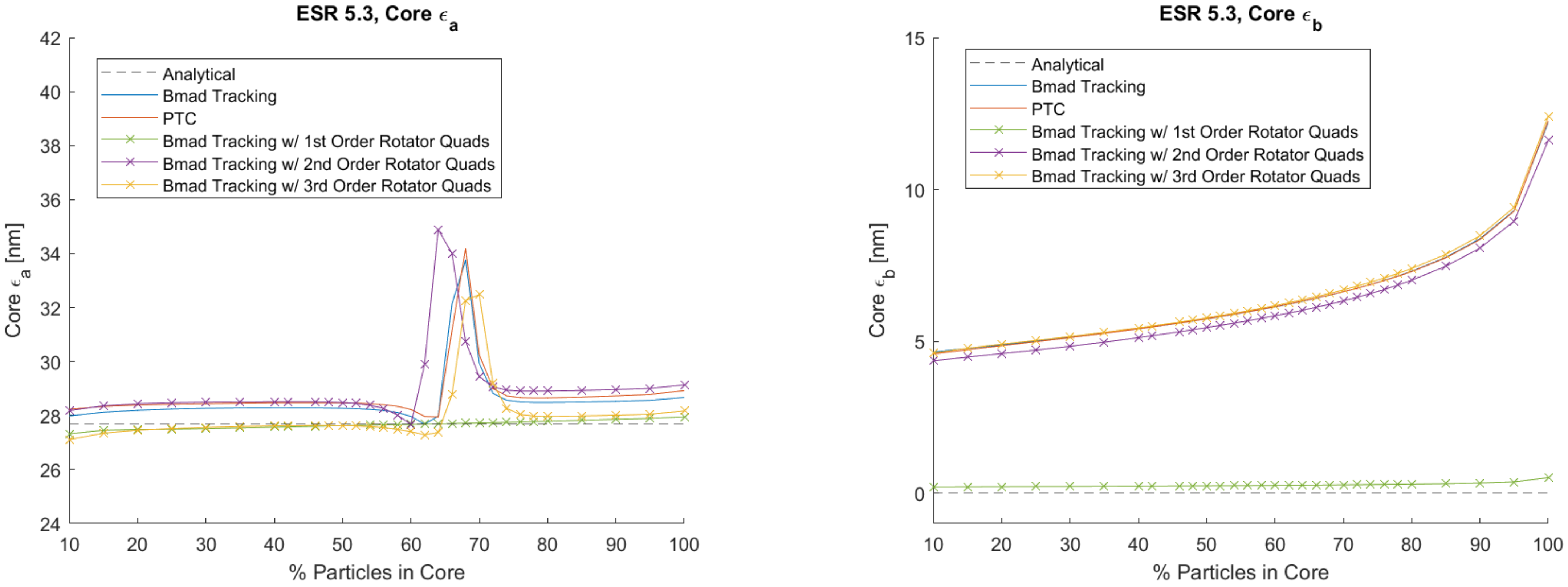
- **Dispersion in the solenoids still inevitable**
  - Misalignments, beam-beam force worrisome

**What was the mystery effect??**

# Resolution of the v5.3 “mystery”

# Overview

## Recall, for v5.3, 2nd order effects of quads between solenoids:



- **Resolved with $\eta = 0$ in solenoids, but important to understand these effects**
    - Tolerance on dispersion in solenoids in 5GeV, 10GeV lattices
    - Robustness against misalignments, uncontrolled vertical emittance increase

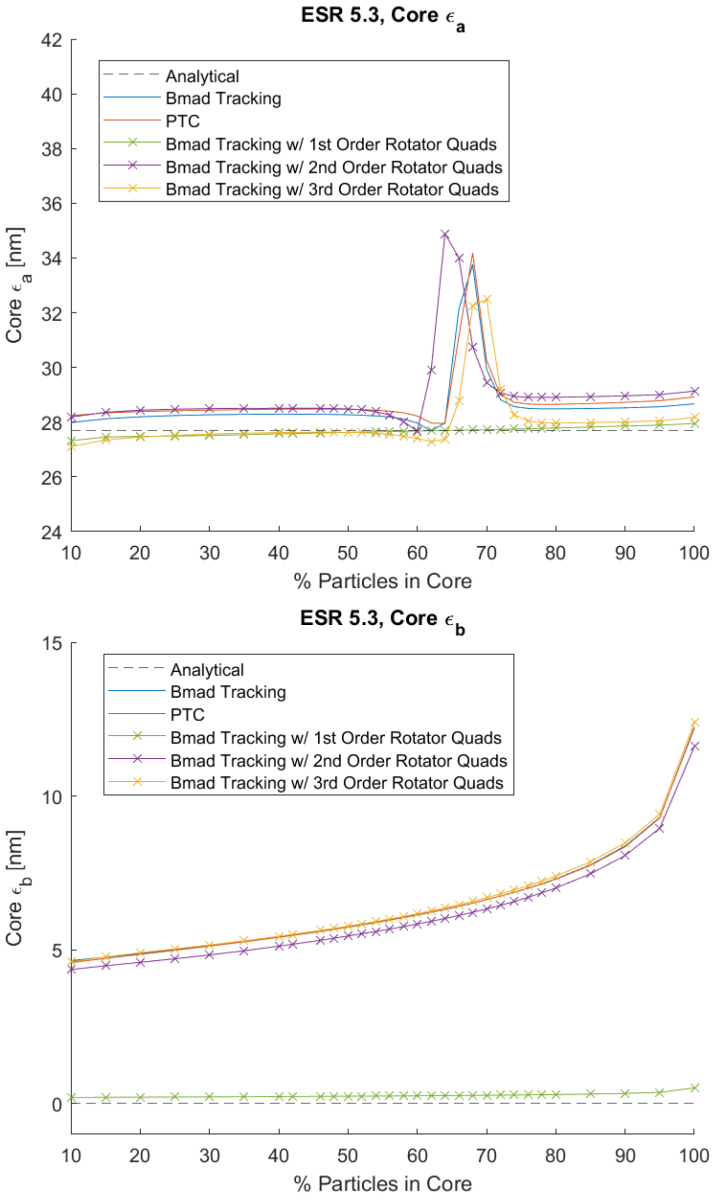
# Overview

*I will show that*

- Blowup in vertical emittance, very low polarizations, and “spike” in horizontal core emittance are caused by the $Q_y - 2Q_s$ resonance

*and that by changing* $Q_s$

- You can have dispersion in the solenoids
- Longitudinal spin match is achievable, and holds up well in nonlinear tracking



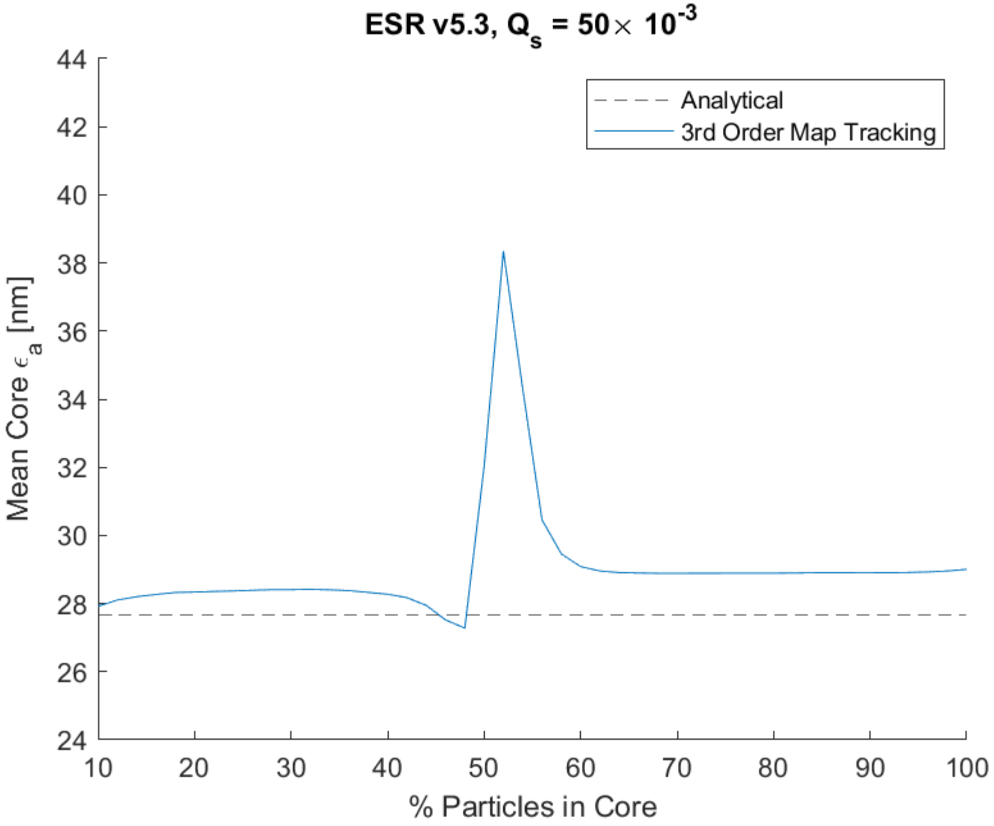
# Methods

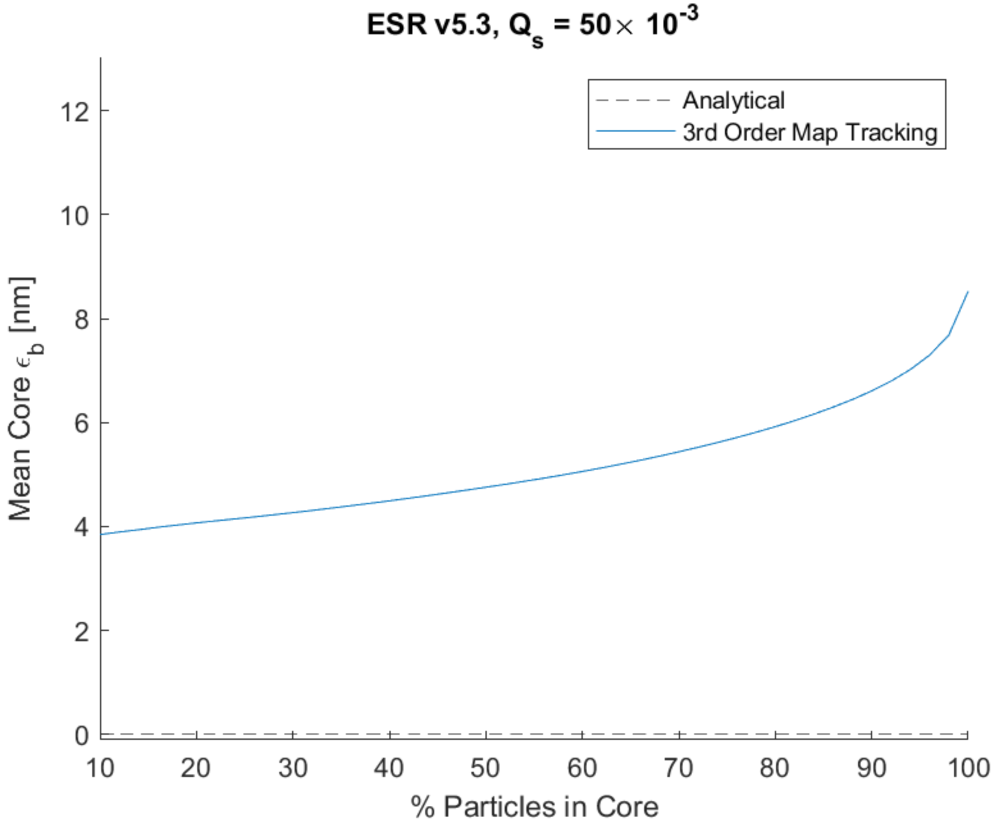
- v5.3 Baseline working point: $Q_x = 48.12, Q_y = 43.10, Q_s = 0.05$

**Nonlinear tracking study:** vary $Q_s$, keep $Q_x, Q_y$ constant

- 5,000 particles, 5,000 turns. Emittances are means of turns 4,000-5,000
- Radiation damping + fluctuations, bends split for stochastic radiation
- Synchrotron tune set *after* turning on radiation and tapering
- All 3$^{\text{rd}}$ order map tracking for speed
  - Agrees very well with fully nonlinear Bmad, PTC tracking

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$Q_s =$

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | $\boxed{50}$ | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

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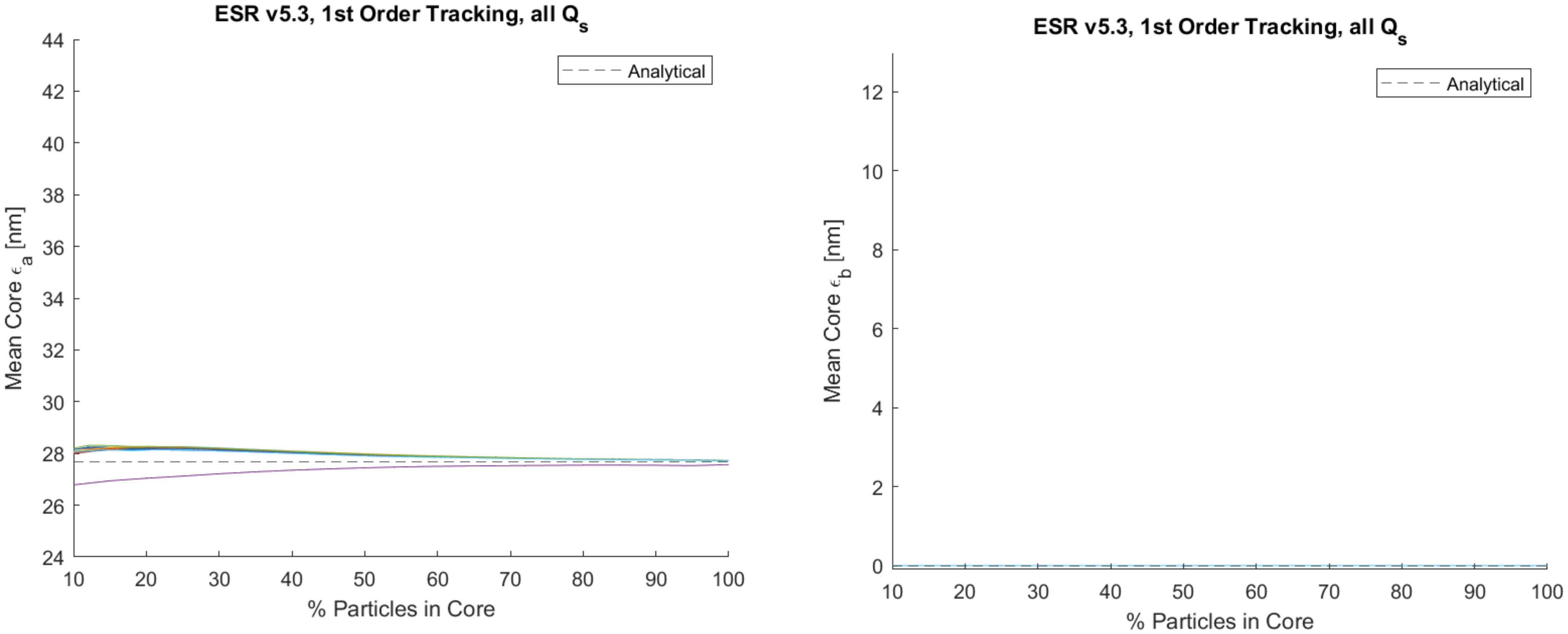
# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$

## Control: 1st order map tracking for all $Q_s$



$Q_s =$

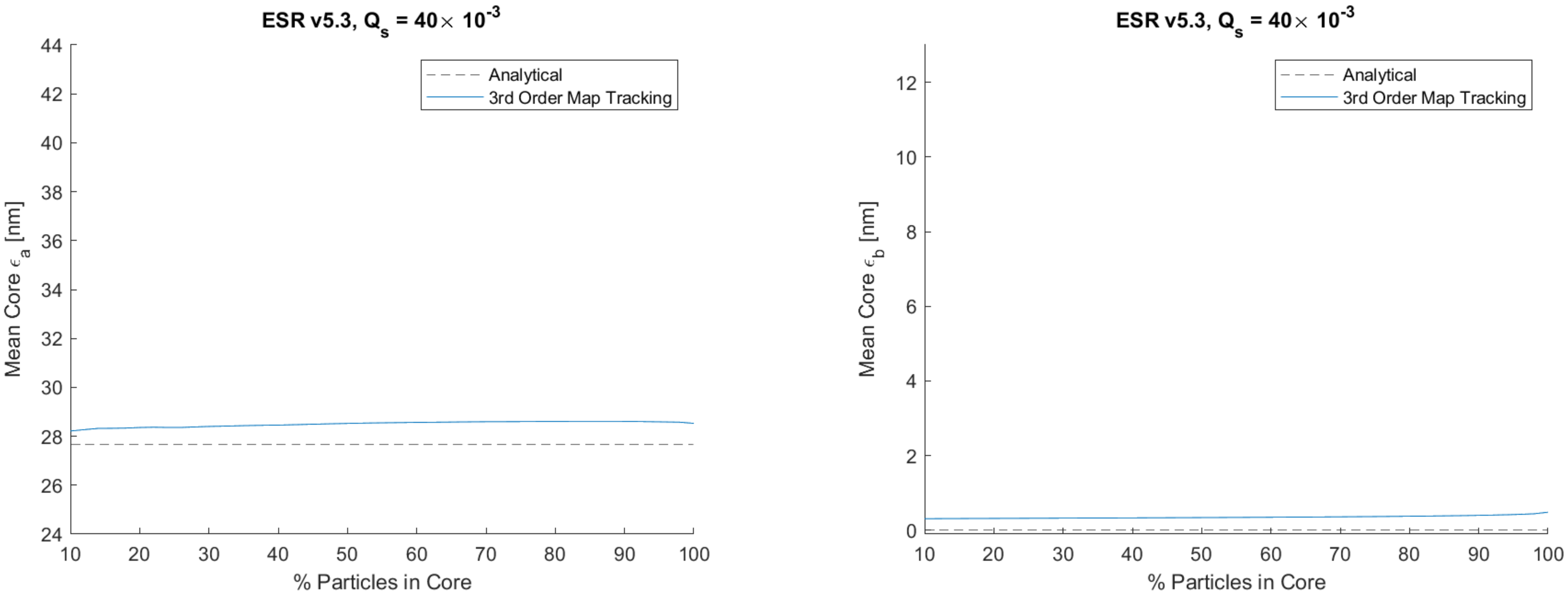
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$

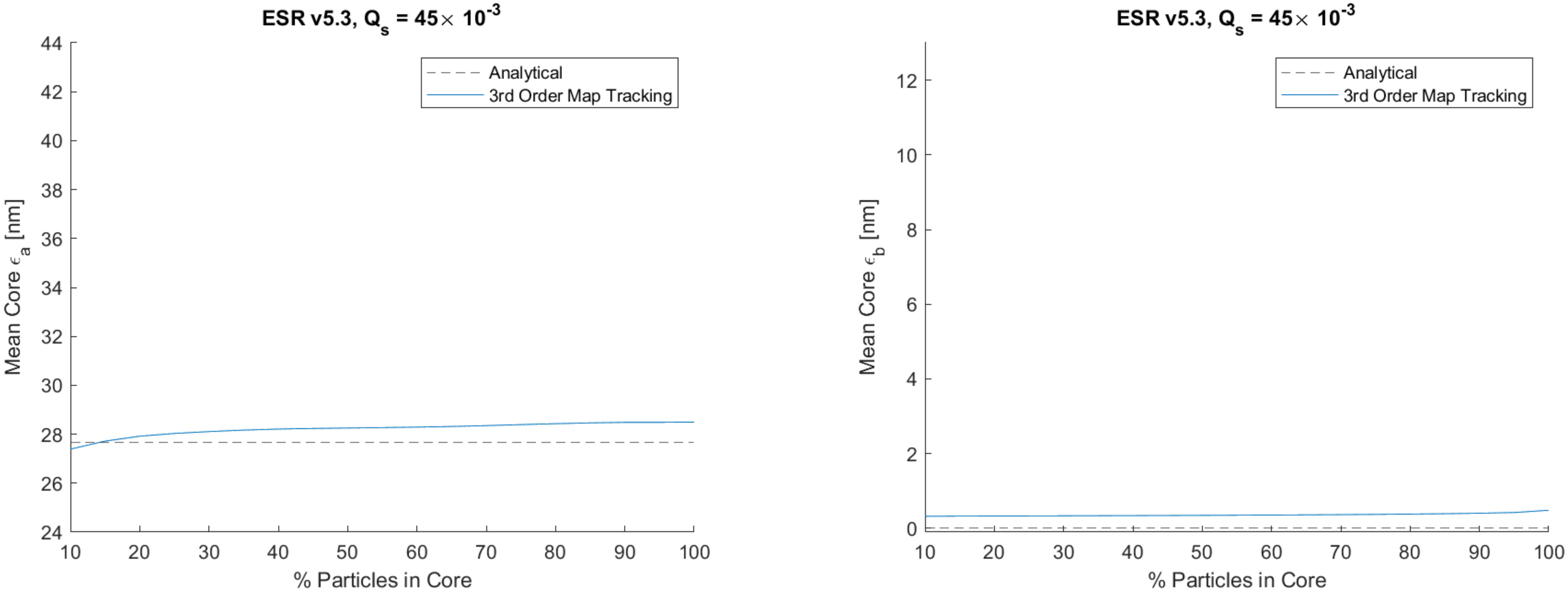


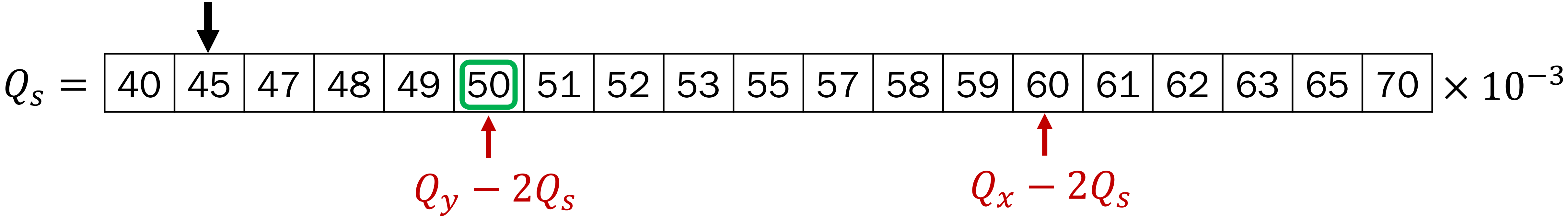
| $Q_s =$ | 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 | $\times 10^{-3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$Q_s =$

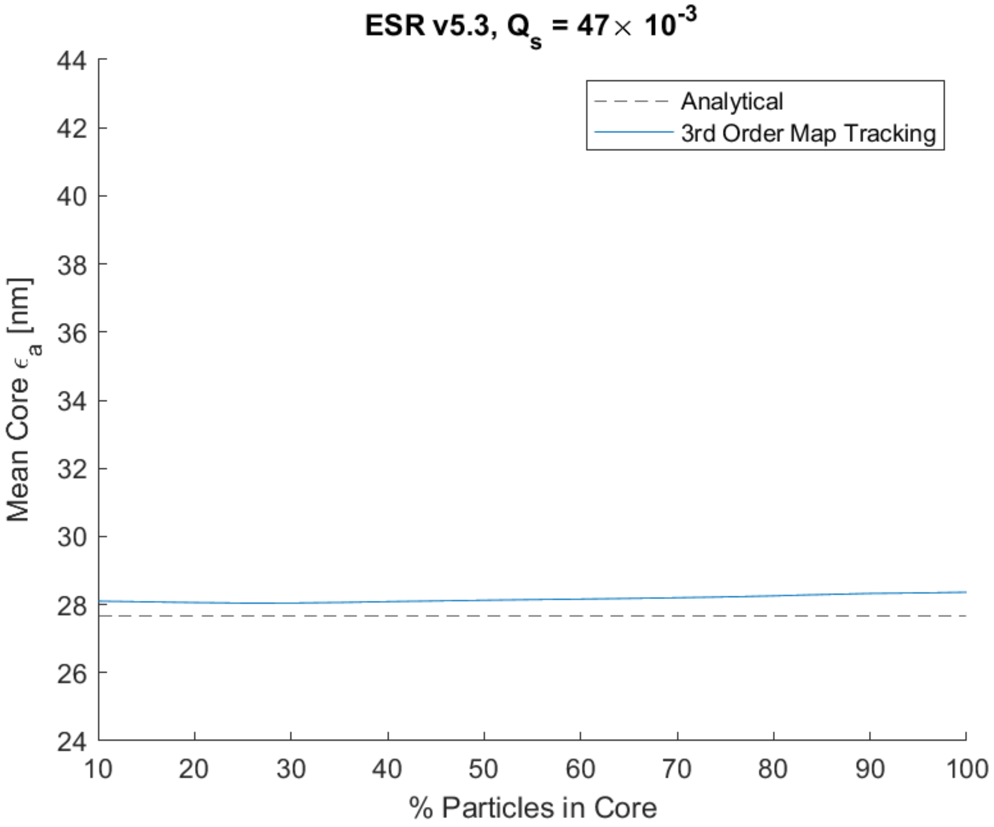
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

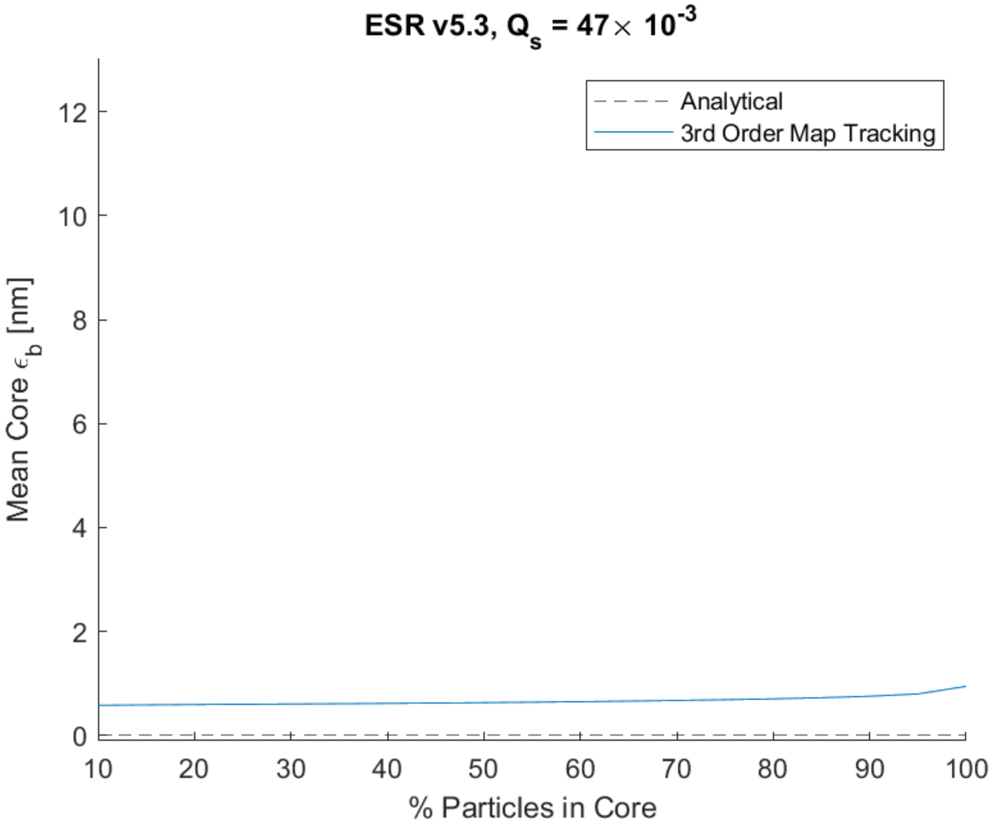
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



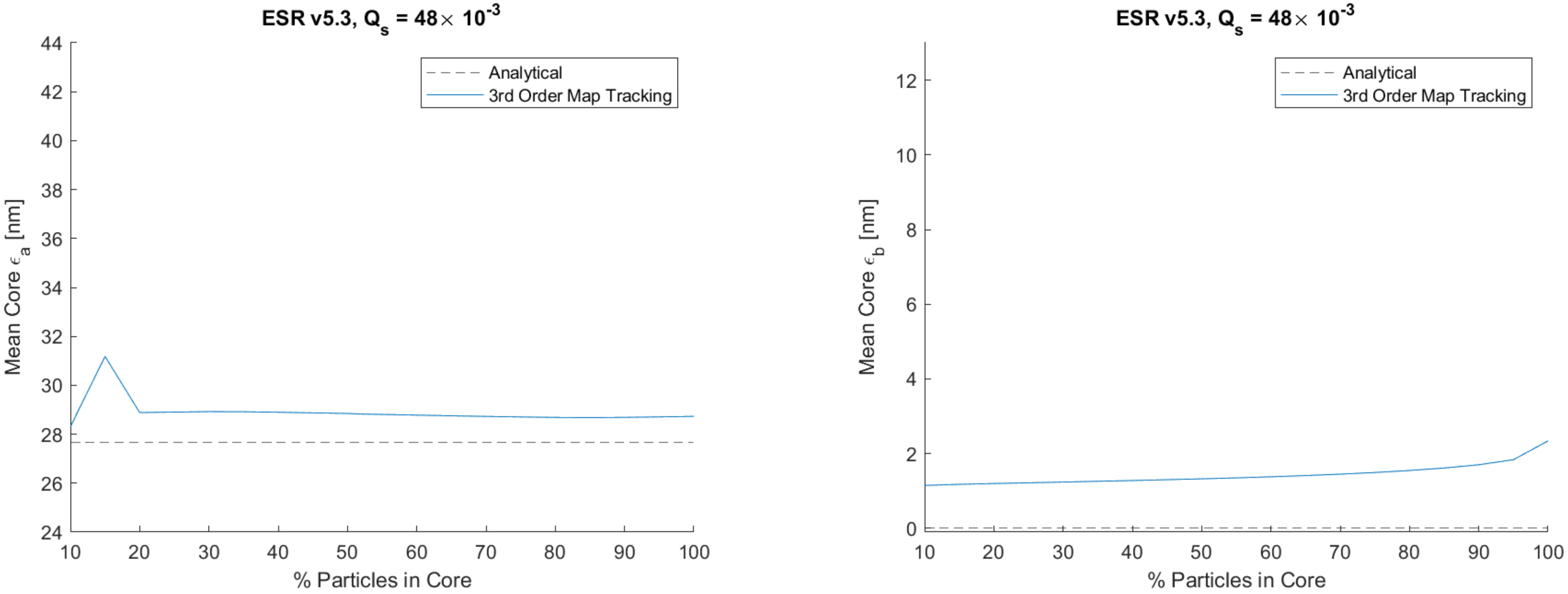


$Q_s =$

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ↓ | | | | | | | | | | | | | | | | |
| 40 | 45 | 47 | 48 | 49 | $\boxed{50}$ | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |
| | | | | | ↑ | | | | | | | | ↑ | | | | | |
| | | | | | $Q_y - 2Q_s$ | | | | | | | | $Q_x - 2Q_s$ | | | | | |

$\times 10^{-3}$

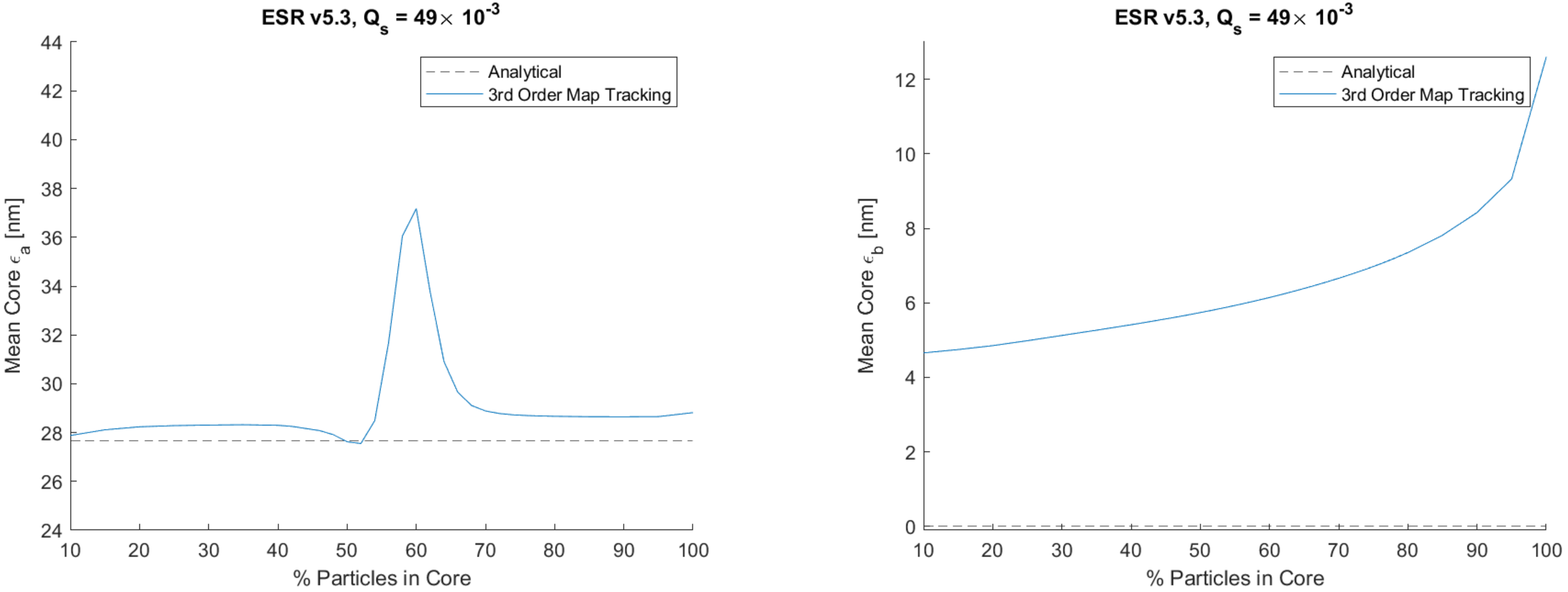
# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



| $Q_s =$ | 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 | $\times 10^{-3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$Q_y - 2Q_s$ $Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



$Q_s =$

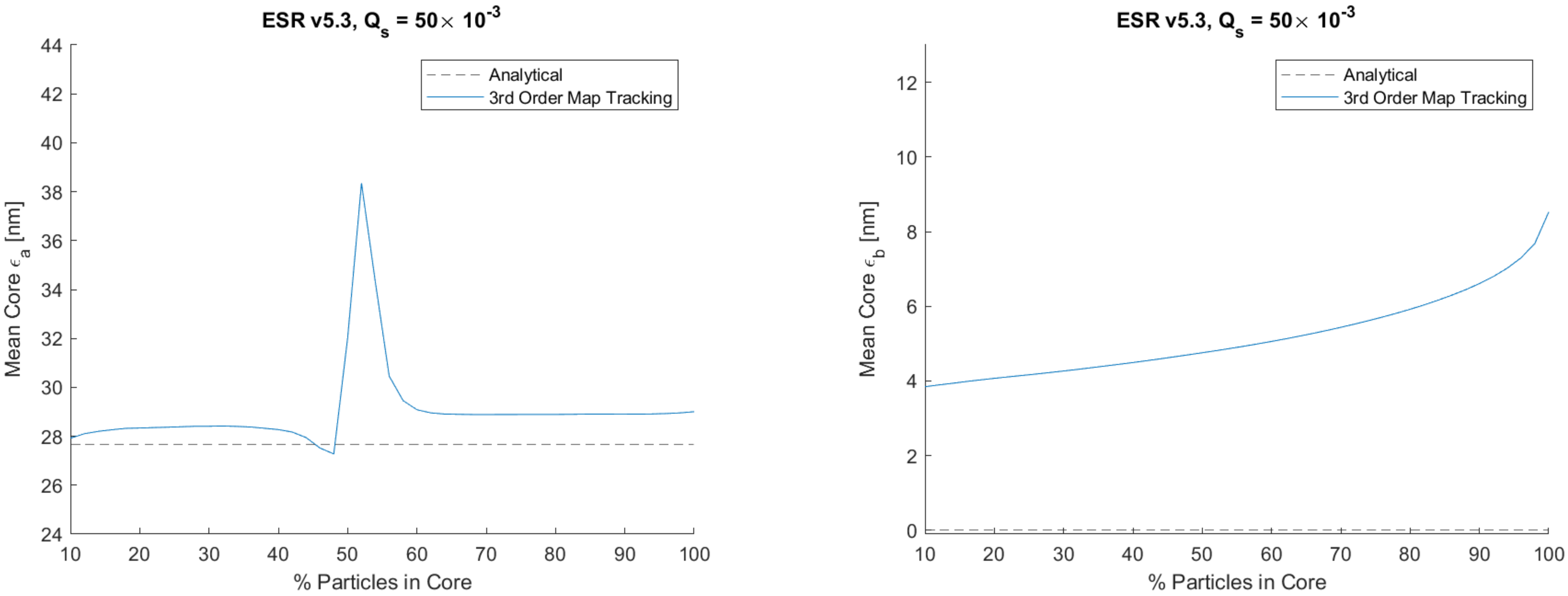
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



$Q_s =$

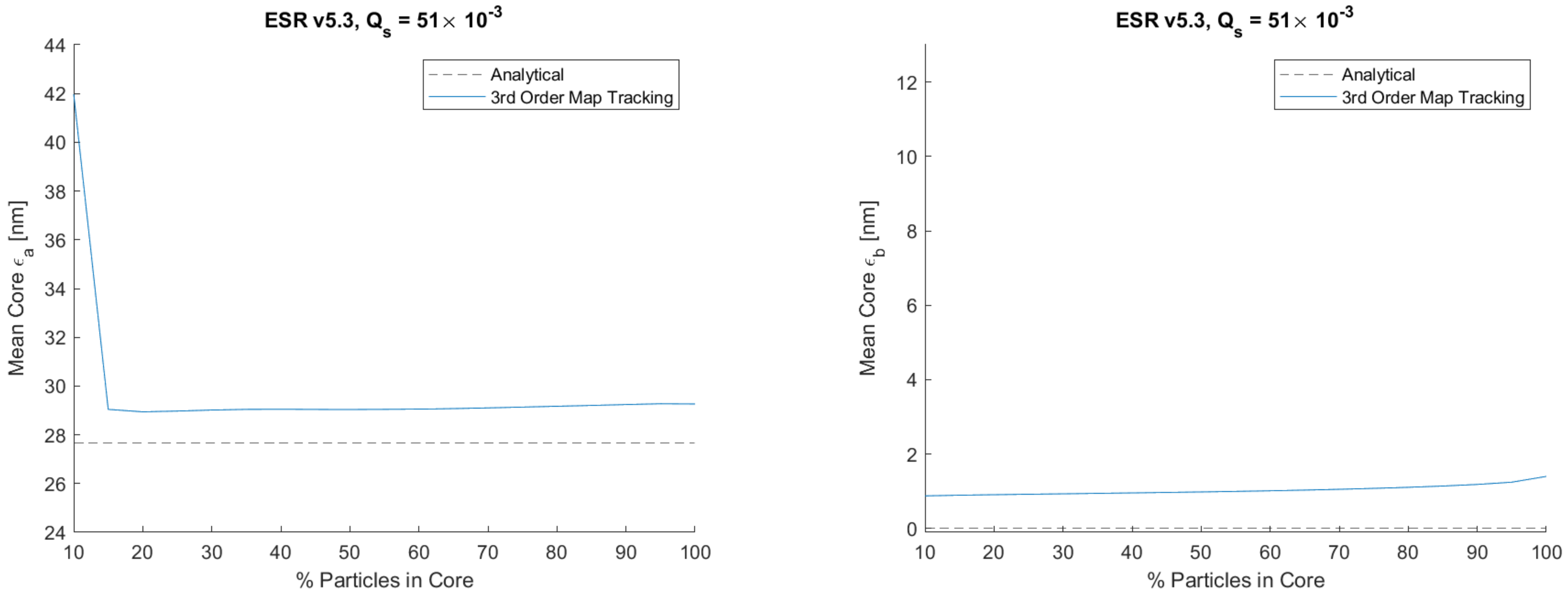
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

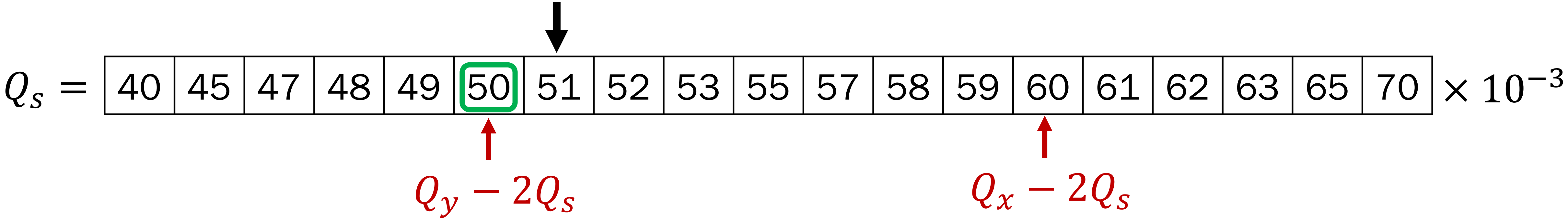
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$Q_s =$

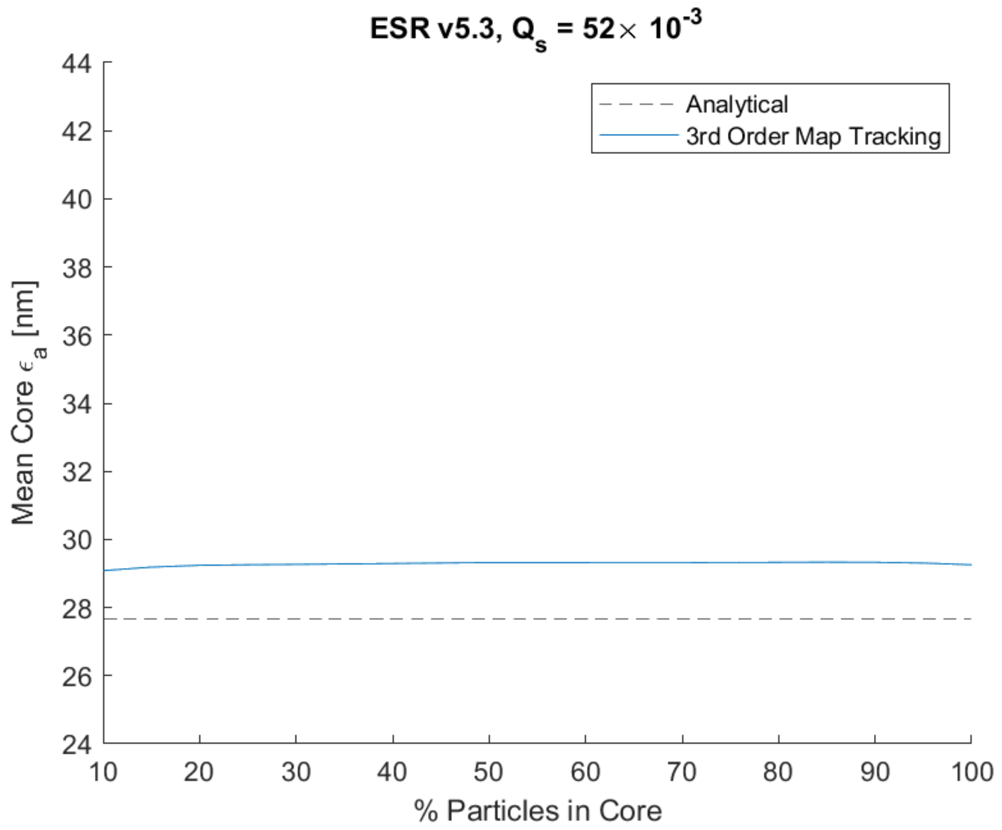
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

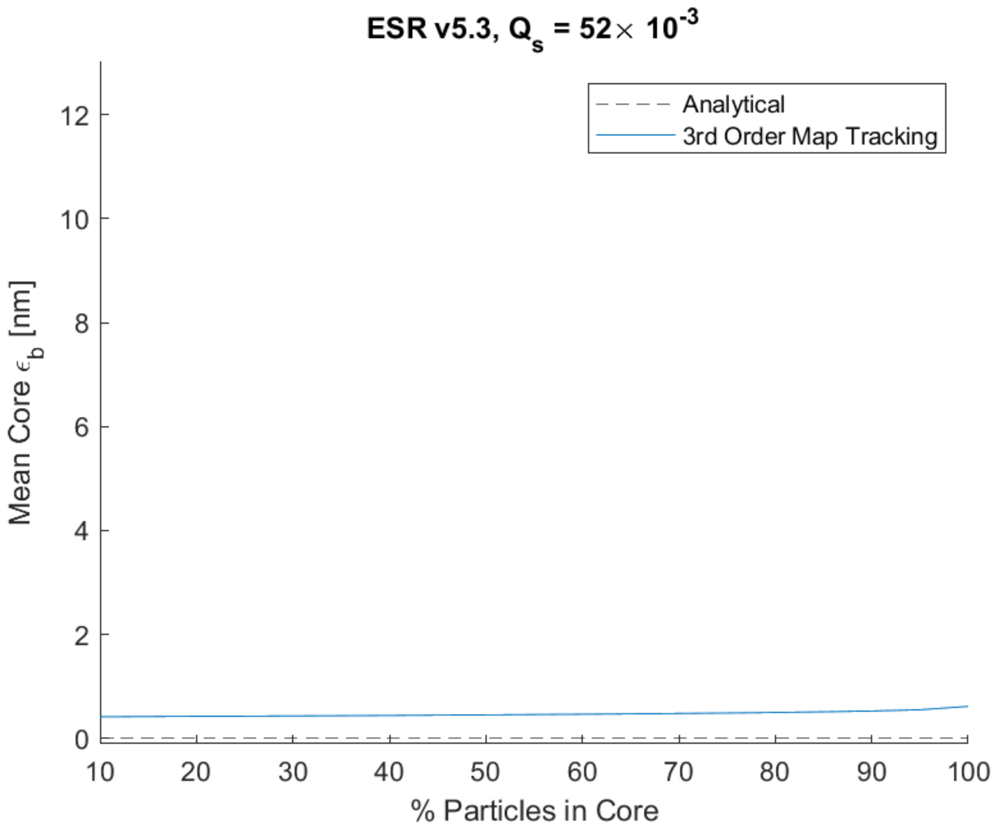
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$Q_s =$

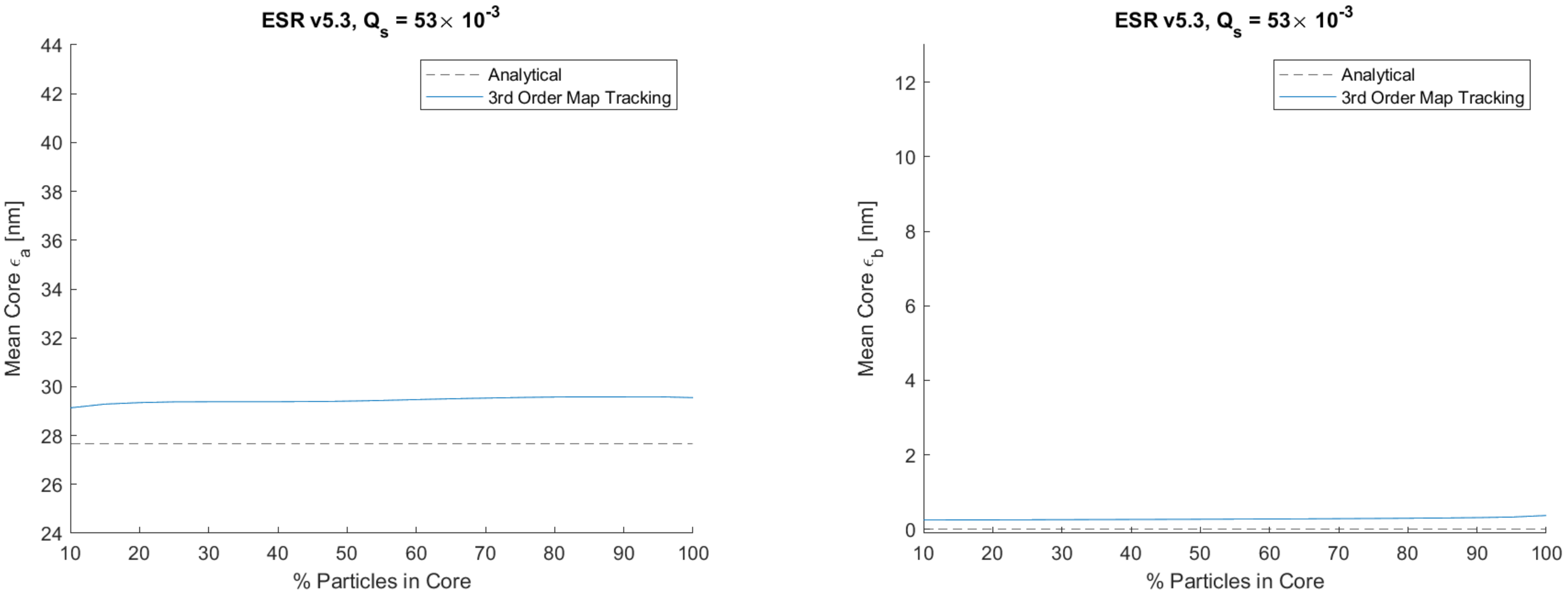
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

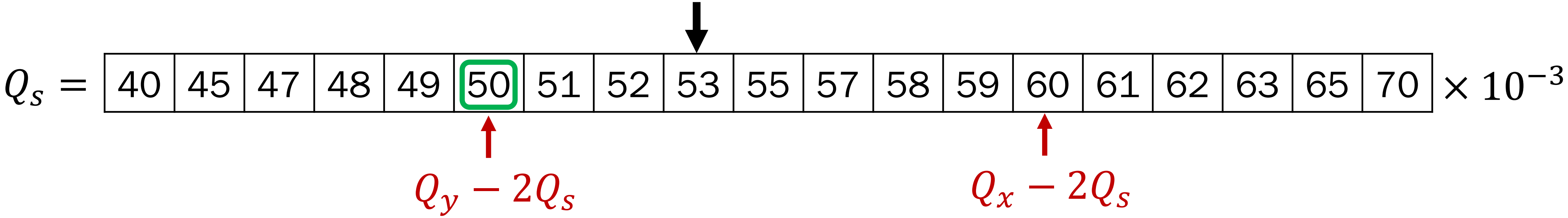
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



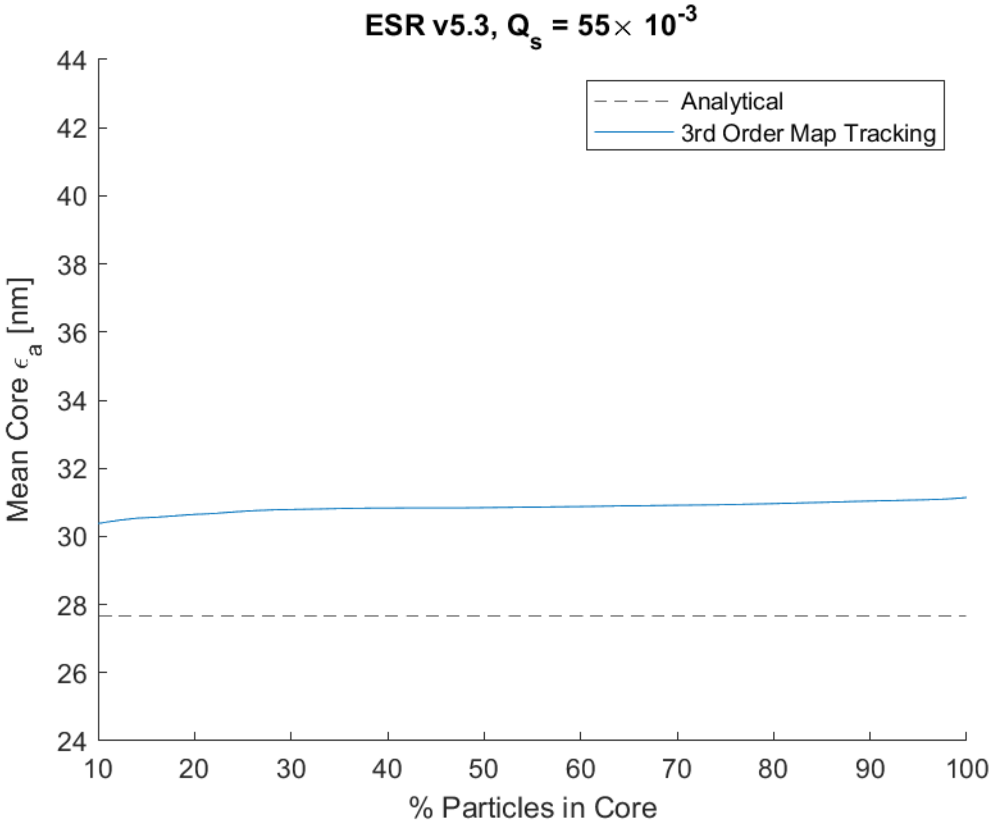


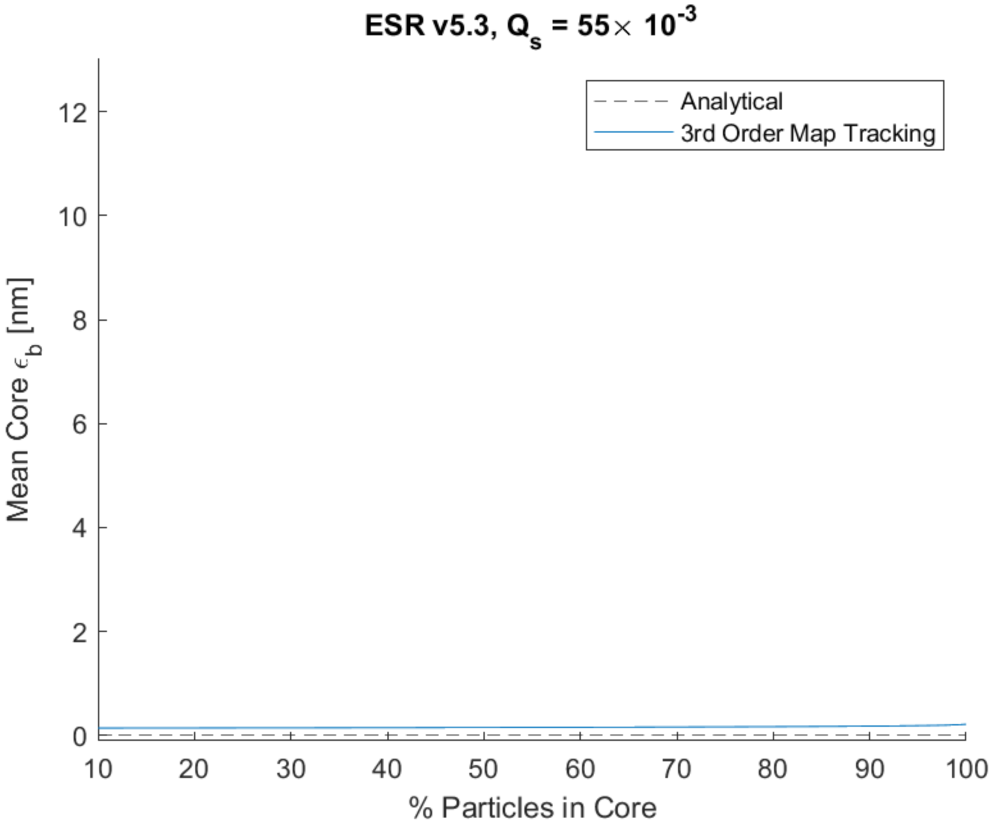
| $Q_s =$ | 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 | $\times 10^{-3}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$Q_s =$

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

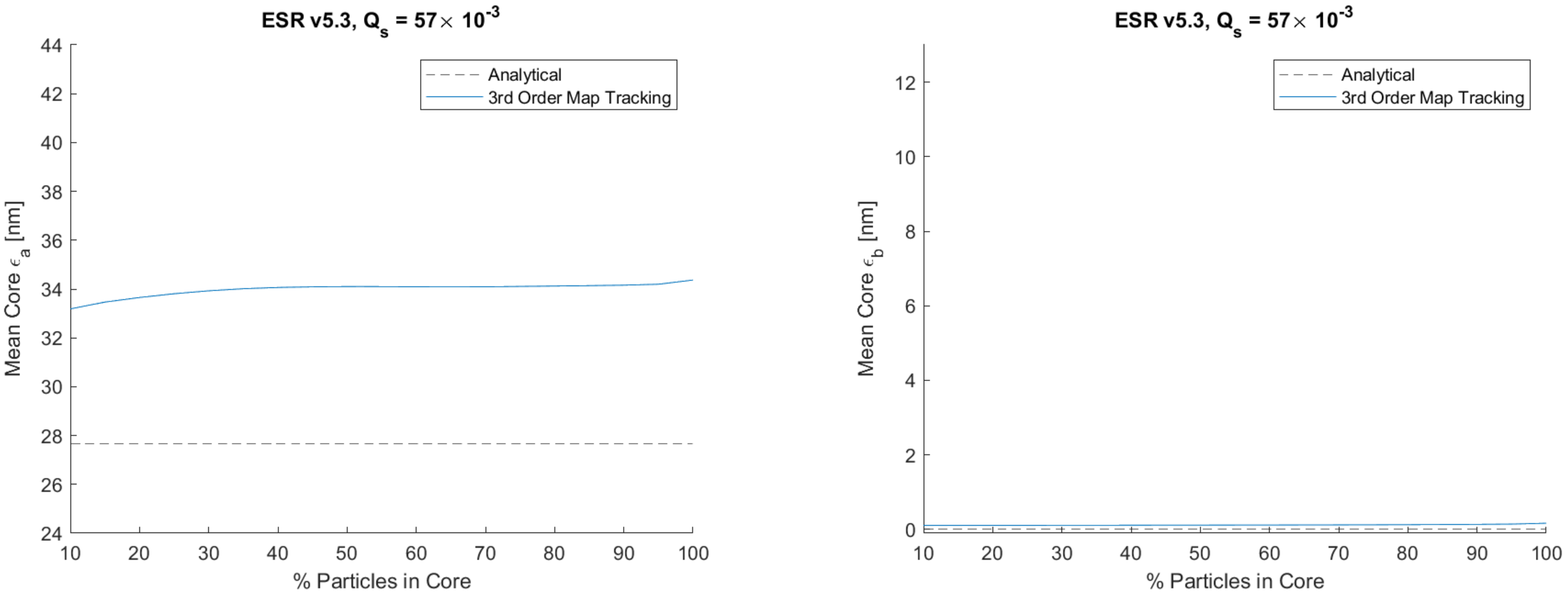
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

Matt Signorelli (mgs255@cornell.edu)

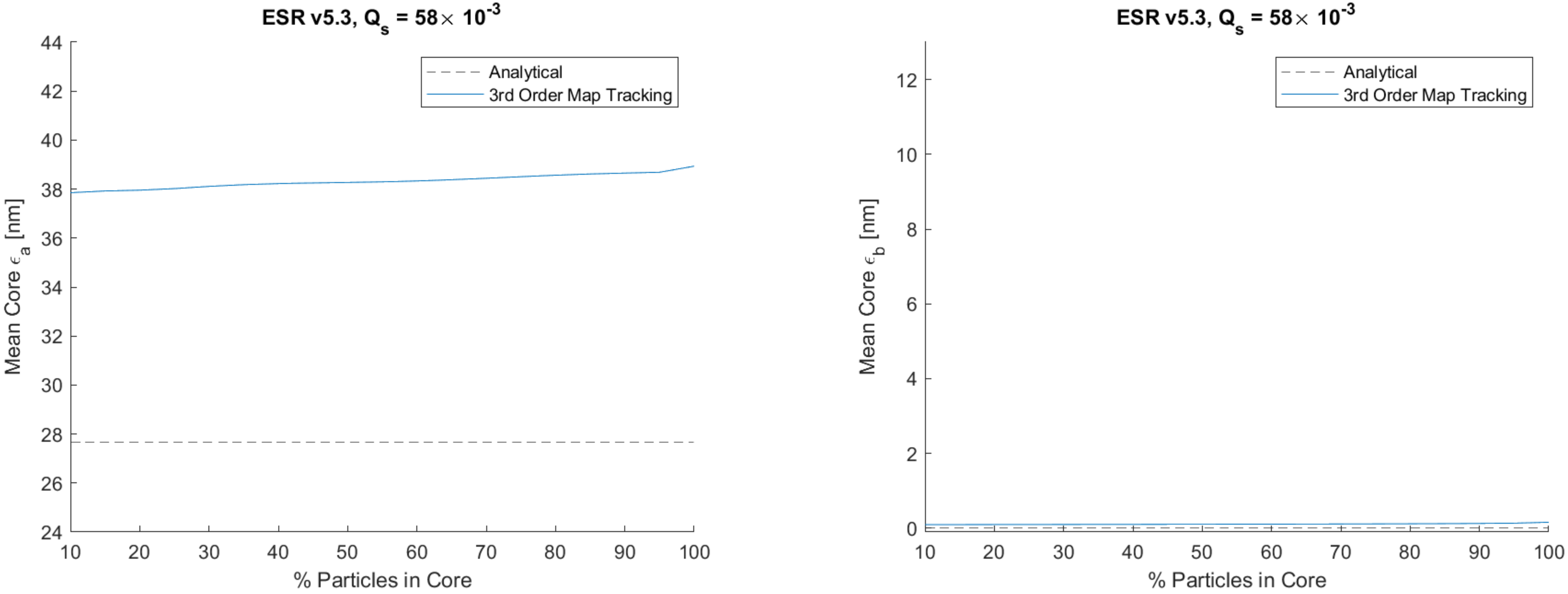
# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



$$Q_s = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline 40 & 45 & 47 & 48 & 49 & \boxed{50} & 51 & 52 & 53 & 55 & \overset{\downarrow}{57} & 58 & 59 & 60 & 61 & 62 & 63 & 65 & 70 \\ \hline \end{array} \times 10^{-3}$$

$$\underset{Q_y - 2Q_s}{\uparrow} \qquad \underset{Q_x - 2Q_s}{\uparrow}$$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$

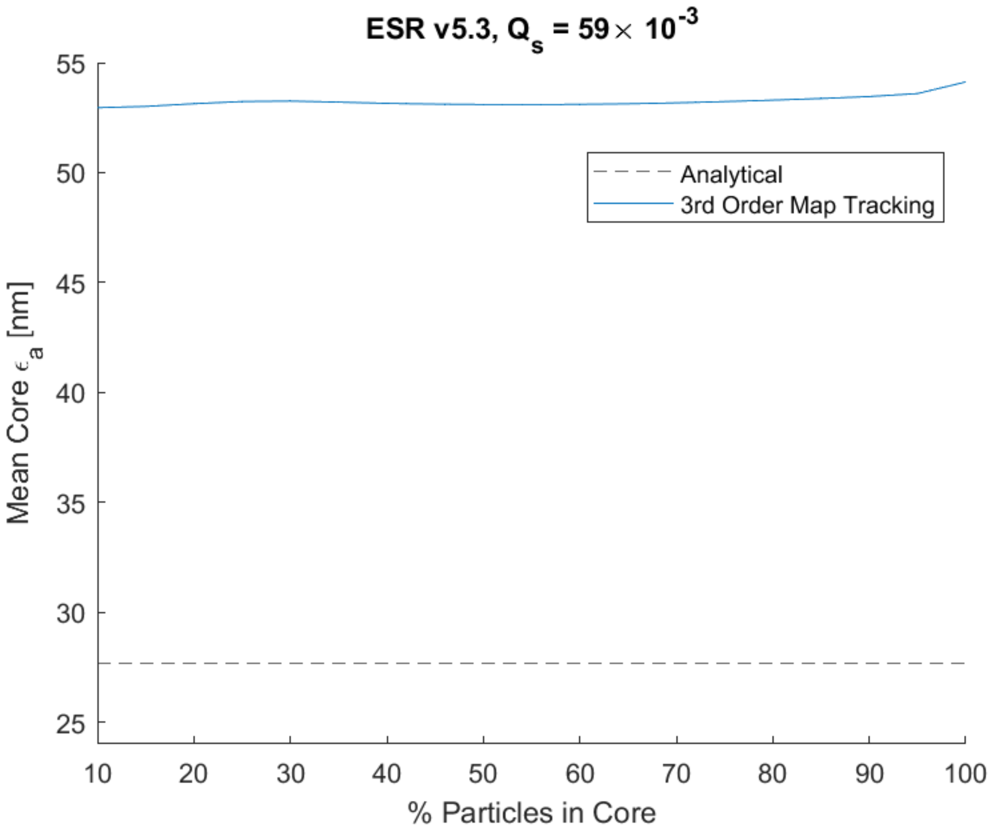


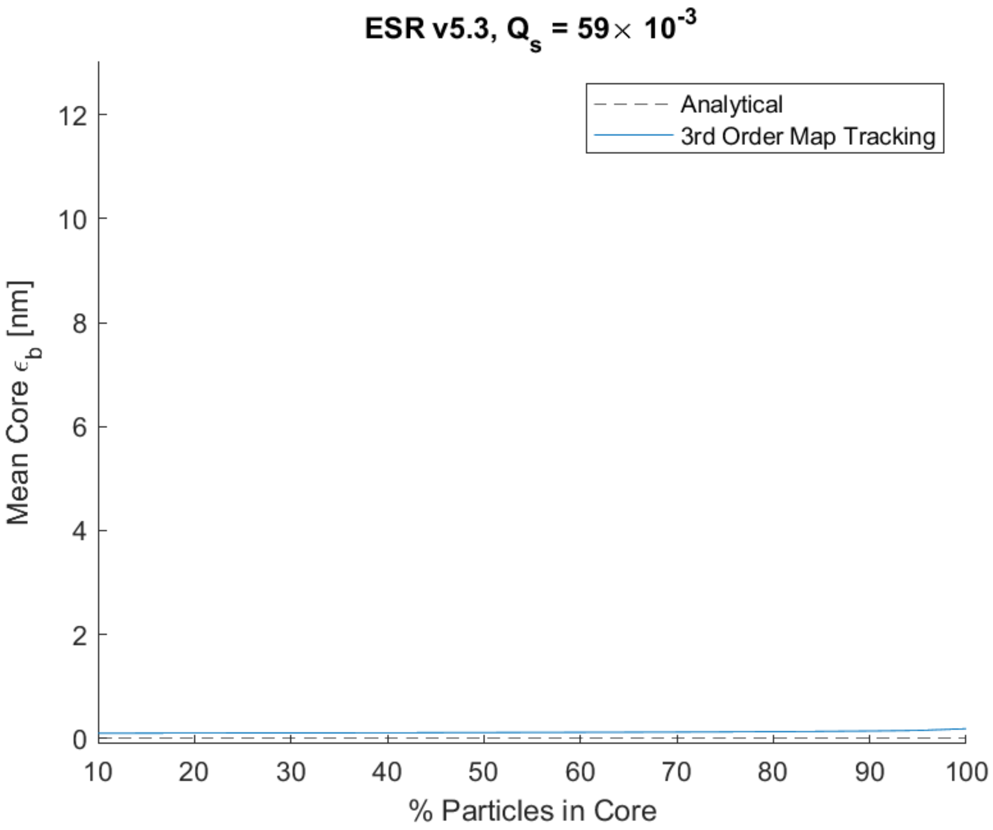
$$Q_s = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline 40 & 45 & 47 & 48 & 49 & \boxed{50} & 51 & 52 & 53 & 55 & 57 & 58 & 59 & 60 & 61 & 62 & 63 & 65 & 70 \\ \hline \end{array} \times 10^{-3}$$

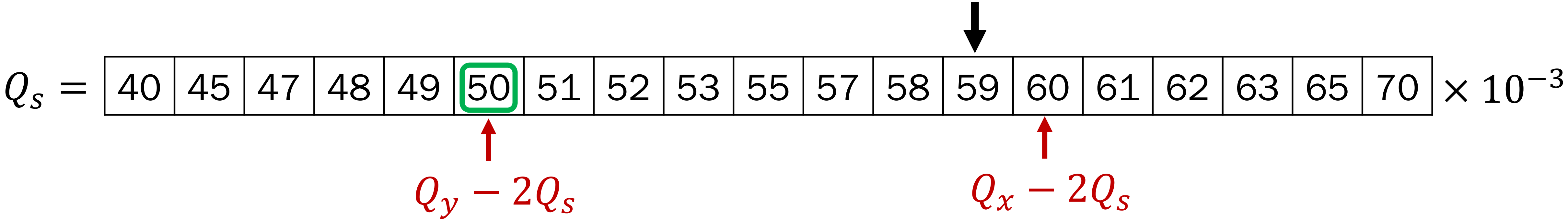
$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$







$Q_s =$

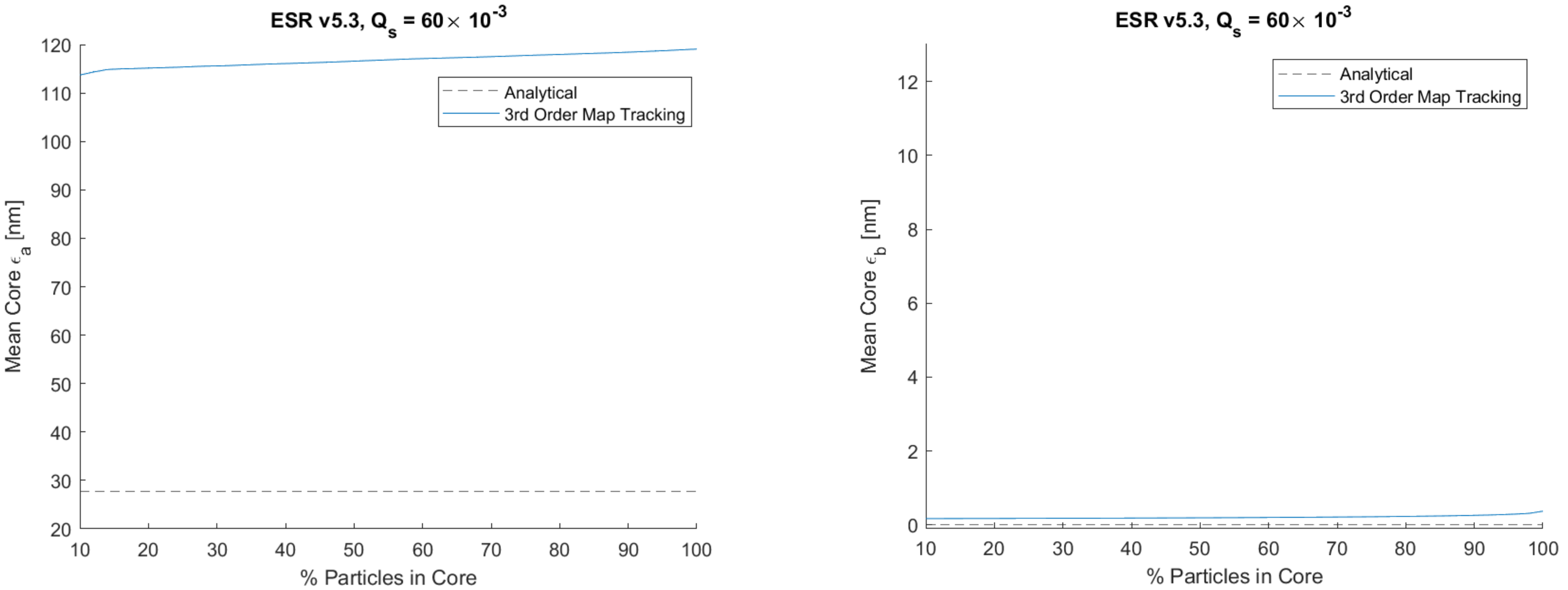
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



$Q_s =$

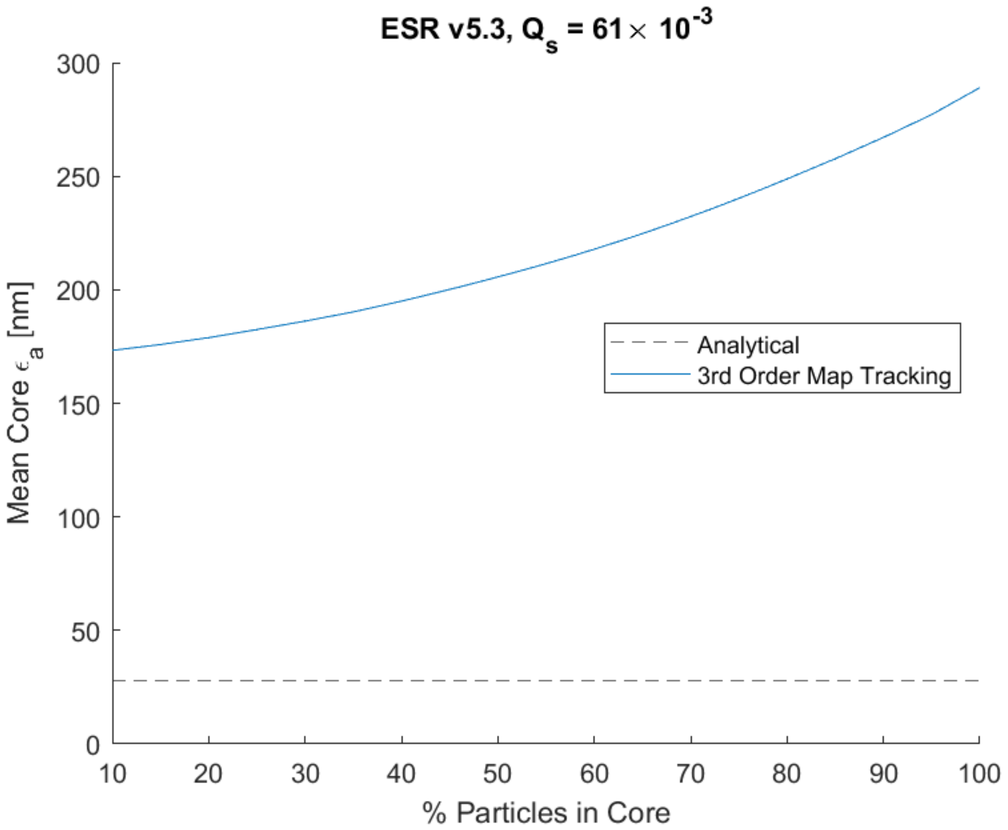
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

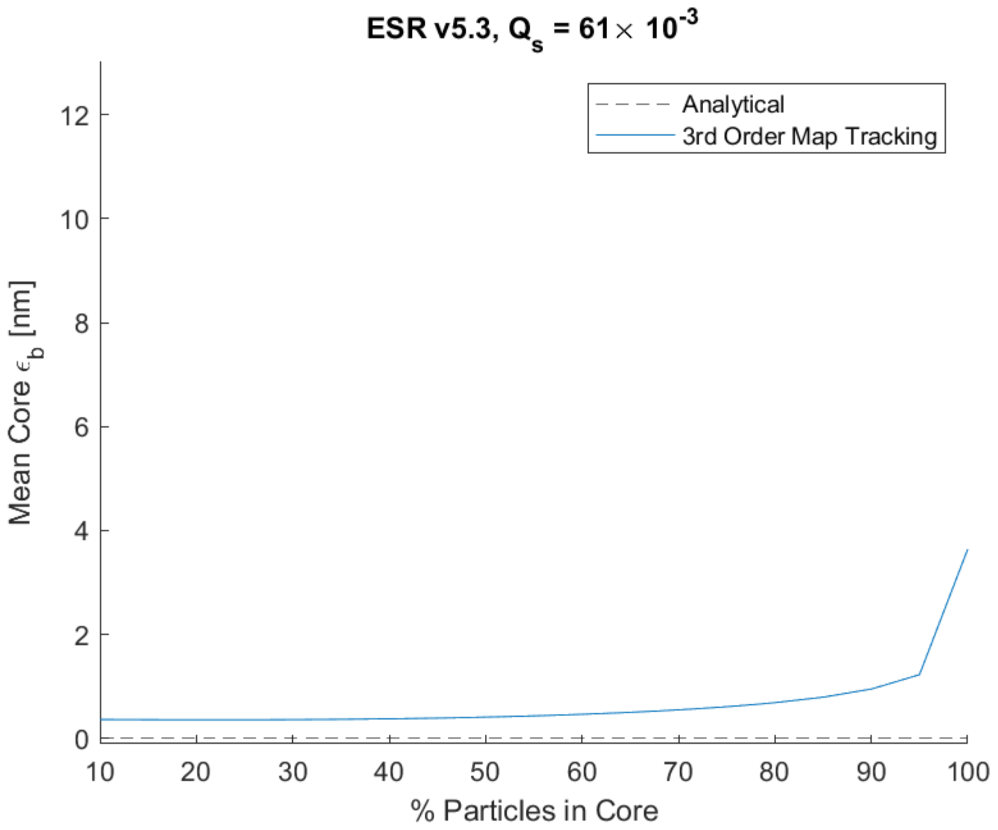
$\times 10^{-3}$

$Q_y - 2Q_s$

$Q_x - 2Q_s$

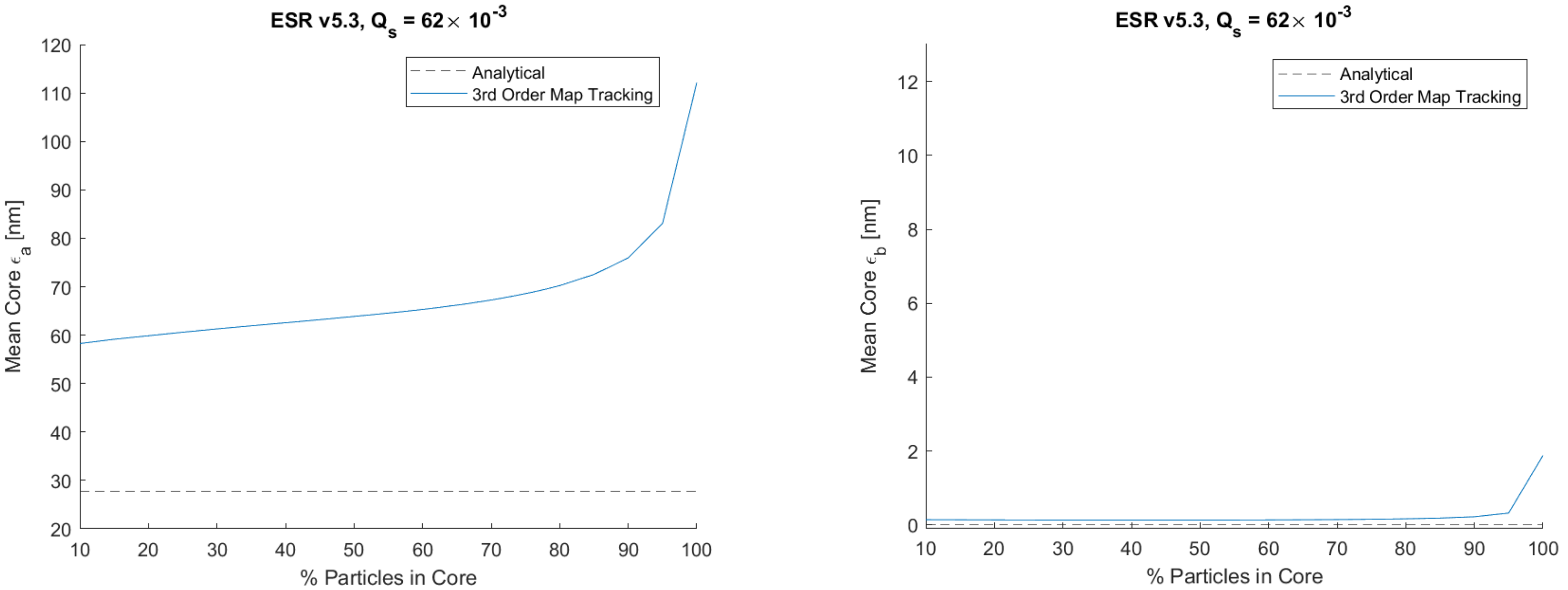
# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$





$$Q_s = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline 40 & 45 & 47 & 48 & 49 & \underset{\underset{Q_y - 2Q_s}{\uparrow}}{\boxed{50}} & 51 & 52 & 53 & 55 & 57 & 58 & 59 & \underset{\underset{Q_x - 2Q_s}{\uparrow}}{60} & \overset{\downarrow}{61} & 62 & 63 & 65 & 70 \\ \hline \end{array} \times 10^{-3}$$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$

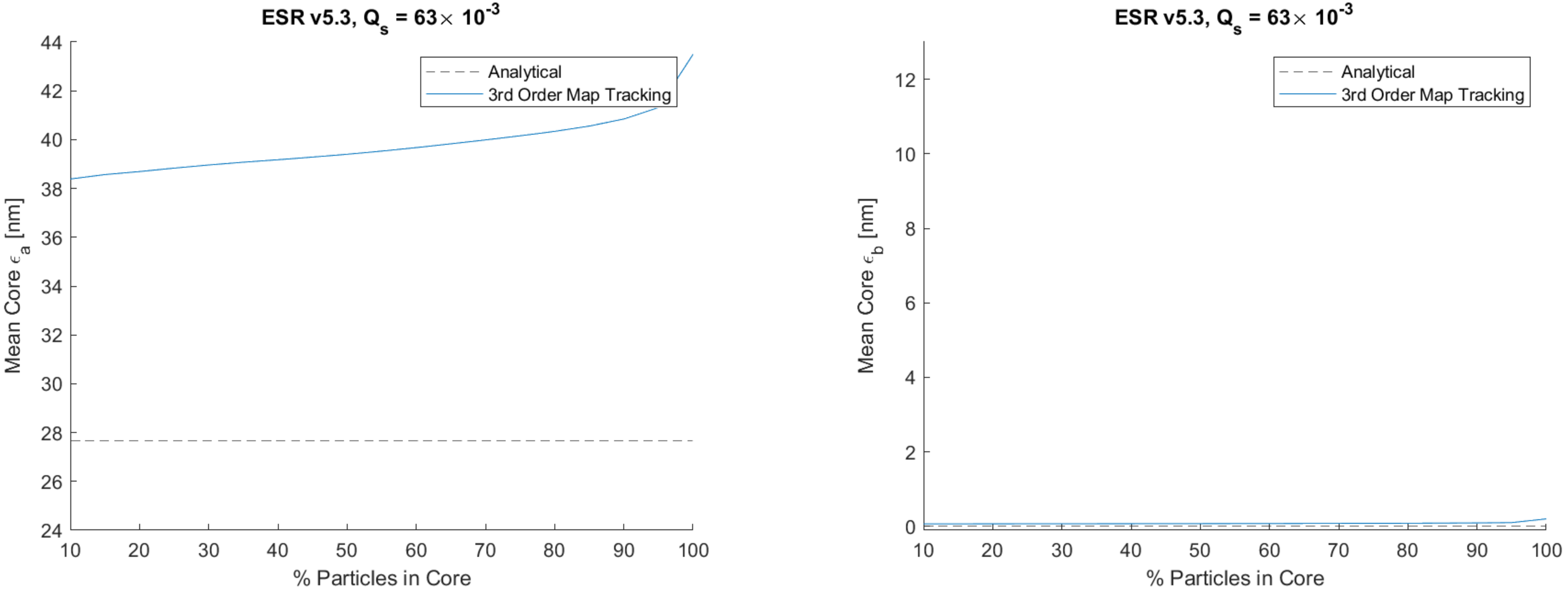


| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_s =$ | 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 $\times 10^{-3}$ |

$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$



$Q_s$ =

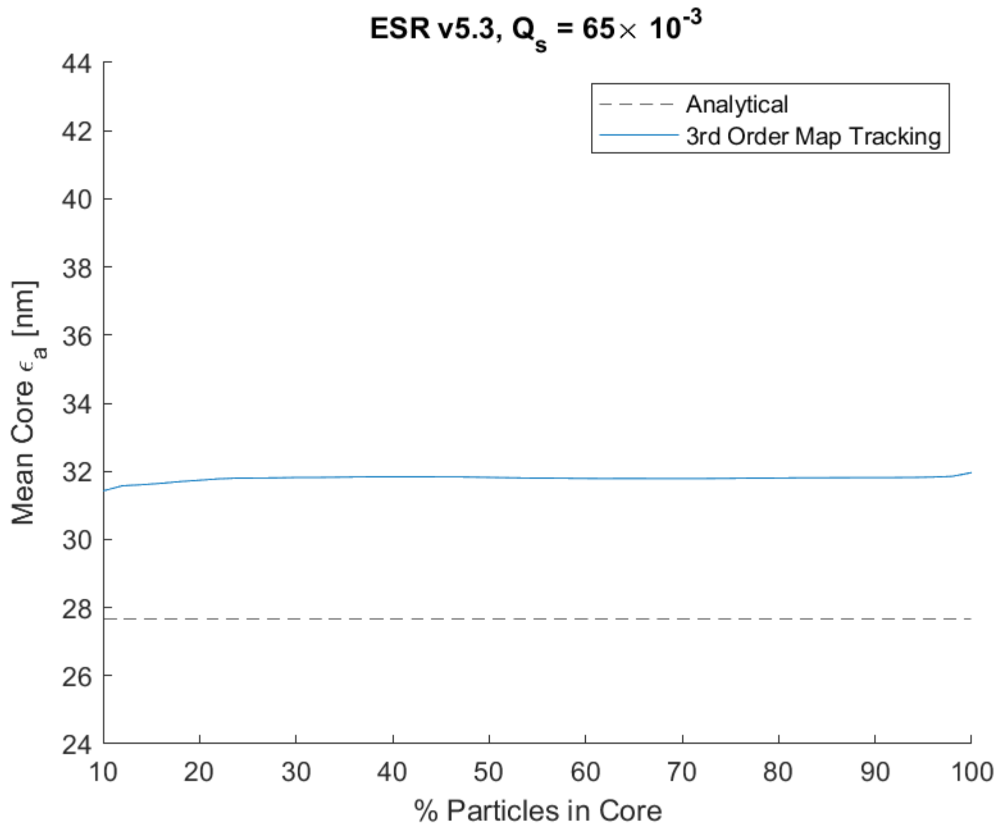
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

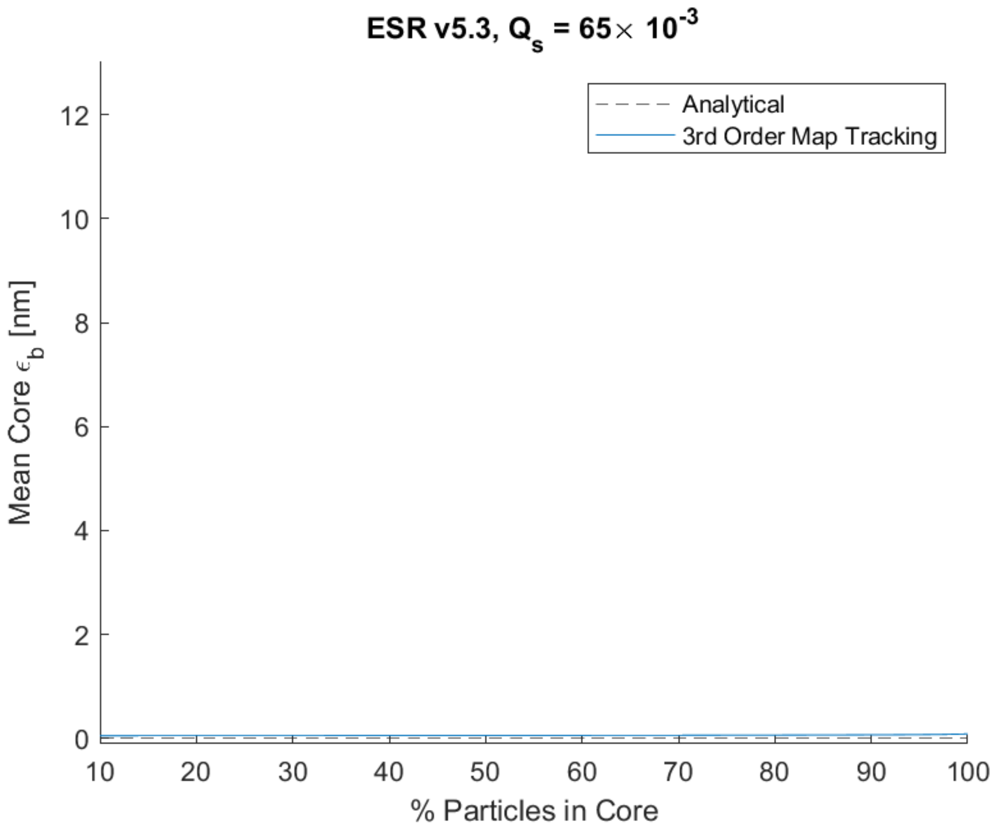
$\times 10^{-3}$

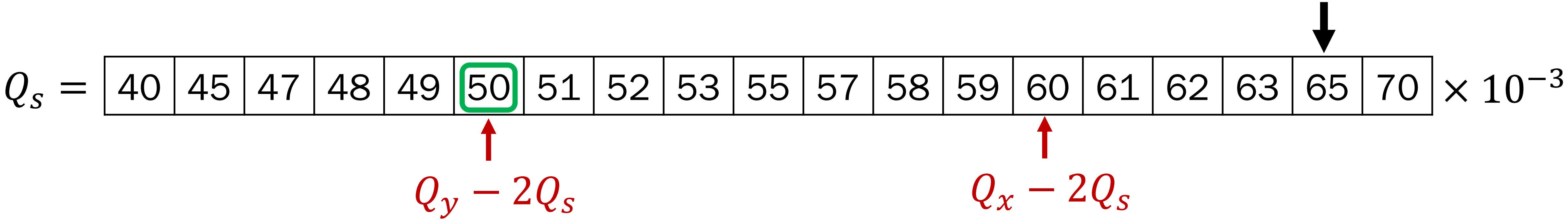
$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$







$Q_s =$

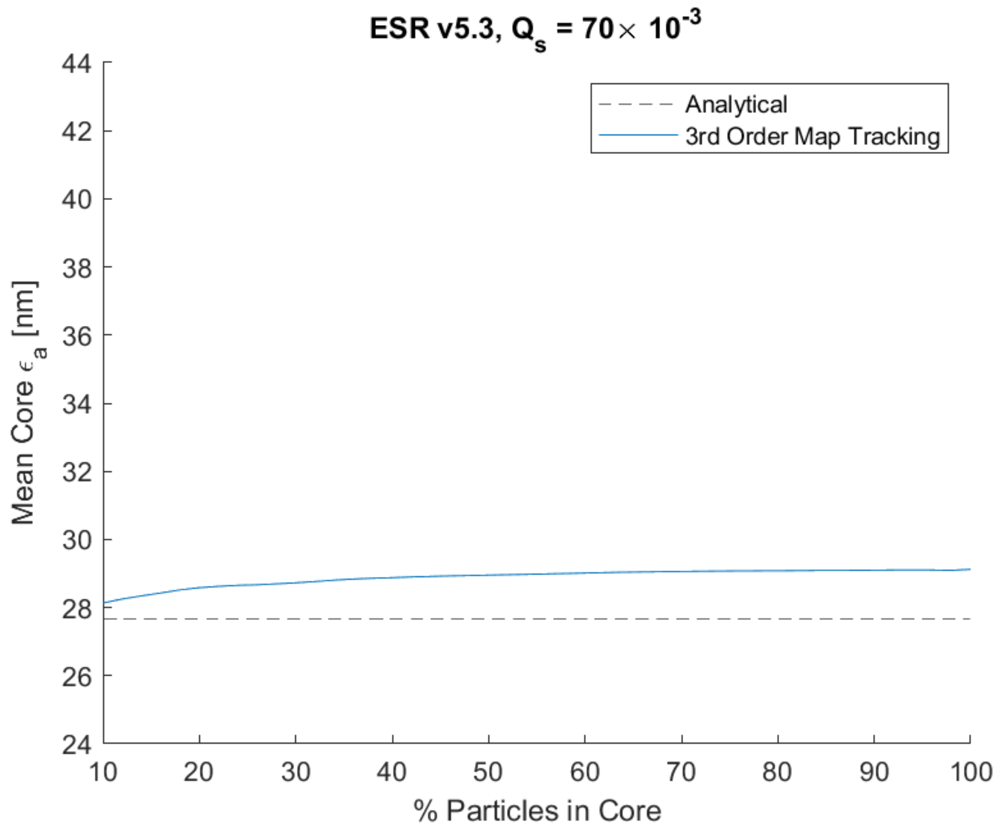
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 |

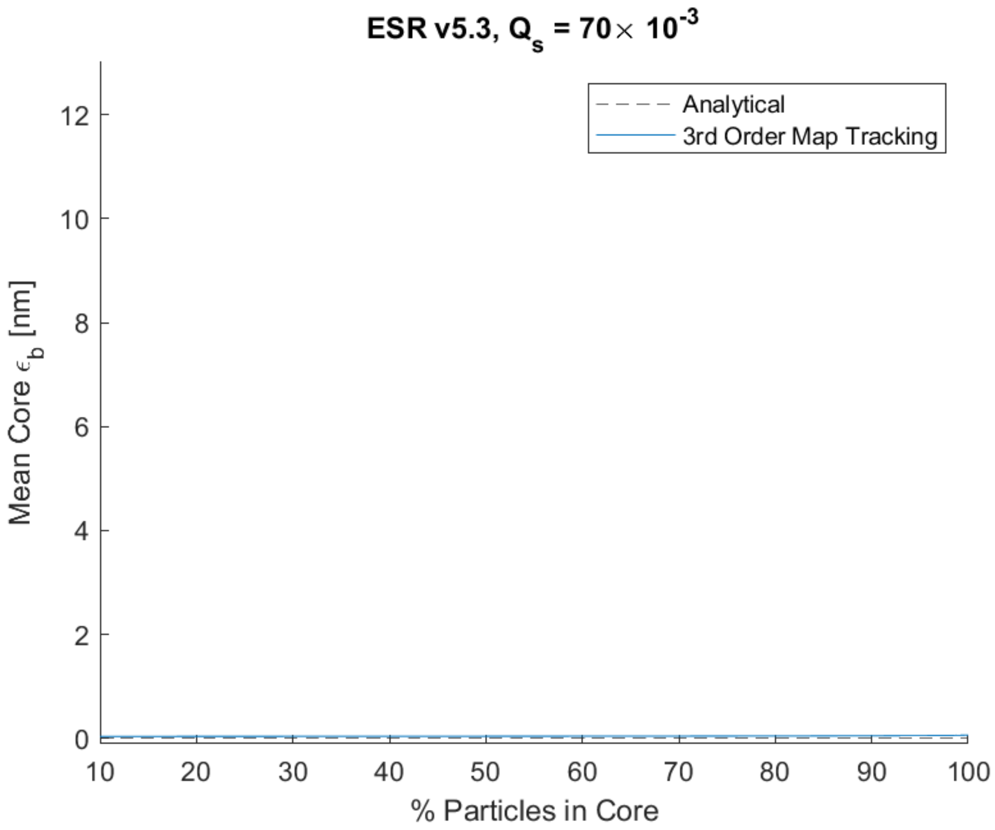
$\times 10^{-3}$

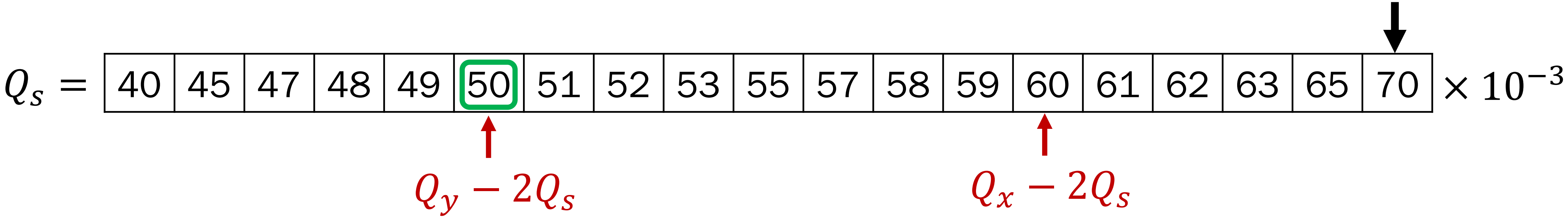
$Q_y - 2Q_s$

$Q_x - 2Q_s$

# v5.3 $Q_x = 48.12, Q_y = 43.10$, Vary $Q_s$







| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_s =$ | 40 | 45 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 55 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 65 | 70 | $\times 10^{-3}$ |

$Q_y - 2Q_s$

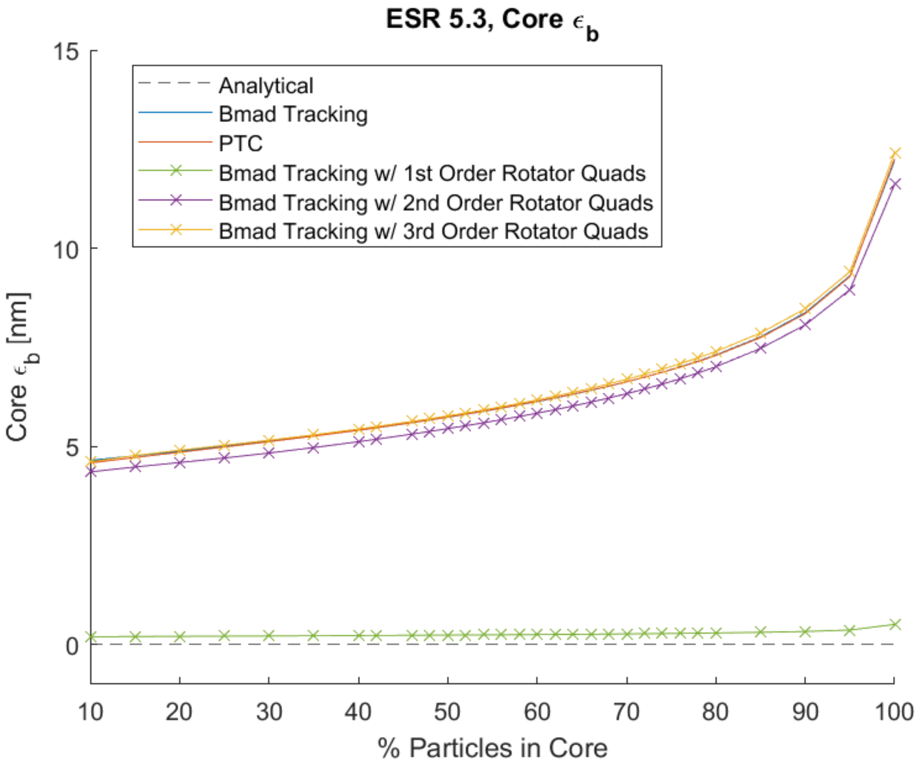
$Q_x - 2Q_s$

# Results

- Horizontal core emittances blow up on $Q_x - 2Q_s$ resonance

- Likewise, vertical core emittance blows up on $Q_y - 2Q_s$, which is the baseline
  - Considering the previous results →

→ **$Q_y - 2Q_s$ resonance is excited by 2nd order effects in quadrupoles between solenoids**



# Results

## Theory for $\boldsymbol{Q_y - 2Q_s}$ resonance, observed in quadrupoles

Looking for e$^{i(Q_y-2Q_s+p)\theta}$ → $y\delta^2$ term in Hamiltonian:

$$\begin{aligned}
H &= \frac{1}{2}K(x^2 - y^2) + \ \ldots \\
&= \frac{1}{2}\frac{K_0}{(1+\delta)}\left(\left(x_\beta + \eta_x\delta\right)^2 - \left(y_\beta + \eta_y\delta\right)^2\right) + \ldots \\
&= \frac{1}{2}K_0\left(2x_\beta\eta_x\delta - 2y_\beta\eta_y\delta\right)(1-\delta) + \ldots \\
&= \boldsymbol{K_0\eta_y y_\beta\delta^2} + \ldots
\end{aligned}$$

$$\downarrow$$

Excited by nonzero vertical dispersion in quadrupoles, which we have in between the solenoids for v5.3!

# Results

## Recall polarization results for v5.3:

| v5.3 Baseline ($Q_s = 0.05$) | $P_{dk}$ |
|---|---|
| Analytical | 61.3% |
| 1st Order Map Tracking | 66.4% |
| 2nd Order Map Tracking | 33.9% |
| 3rd Order Map Tracking | 14.0% |
| Bmad Tracking | 12.1% |
| PTC Tracking | 12.3% |
| Bmad Tracking w/ 1st Order Rotator Quads | 61.1% |
| Bmad Tracking w/ 2nd Order Rotator Quads | 15.1% |
| Bmad Tracking w/ 3rd Order Rotator Quads | 10.9% |

| v5.3 $Q_s = 0.04$ | $P_{dk}$ |
|---|---|
| Analytical | 61.3% |
| 1st Order Map Tracking | 66.6% |
| | |
| 3rd Order Map Tracking | 56.1% |

- Very large increase in polarization
- Longitudinal spin match highly beneficial
    - v5.6 baseline has ~30%, drops below acceptable values with vertical emittance creation

# Conclusions

- In the 18GeV lattice, vertical emittance blow-up and “spike” in horizontal core emittance is caused by the $Q_y - 2Q_s$ resonance

- The $Q_y - 2Q_s$ resonance is excited by vertical dispersion in quadrupoles

- Implications:
  - Misalignments in v5.6 will lead to vertical dispersion in solenoids, and excitation of this resonance
  - Dispersion in the solenoids is not bad unless on the resonance
  - Longitudinal spin match may not need to be dropped. Nonlinear ~56% asymptotic polarization is possible in v5.3 with different choice of $Q_s$
  - Reconsider choice of $Q_s$

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**Thank you!**

Questions?