

Study of D^0 - \bar{D}^0 mixing parameters using a time-dependent amplitude analysis of the decay $D^0 \rightarrow K_S^0 h^+ h^-$

Abstract

We present a measurement of the mixing parameters in the D meson system, using 473.9 fb^{-1} of data from the *BABAR* detector. A time-dependent fit to the Dalitz plot of the decays $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_S^0 K^+ K^-$, assuming no CP violation, finds for the $D^0 - \bar{D}^0$ mixing parameters the normalised mass and width differences x and y

$$x = (0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$$

$$y = (0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$$

where the uncertainties are statistical, instrumental, and model-related. For D^0 and \bar{D}^0 samples separately, we find

$$x_{D^0} = (0.00 \pm 0.33)\%$$

$$y_{D^0} = (0.55 \pm 0.28)\%$$

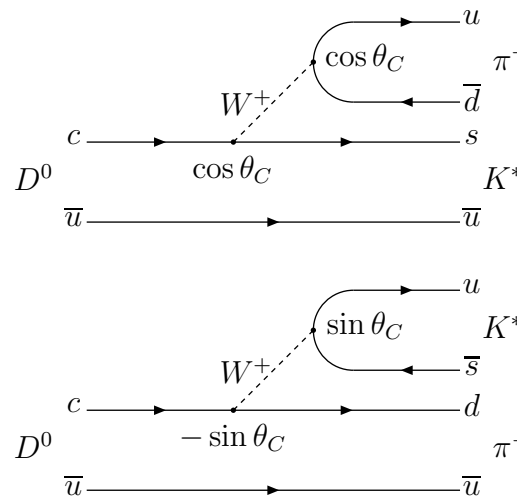
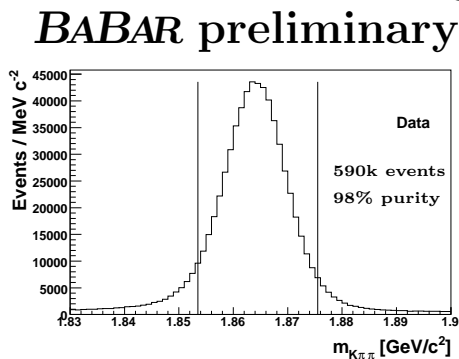
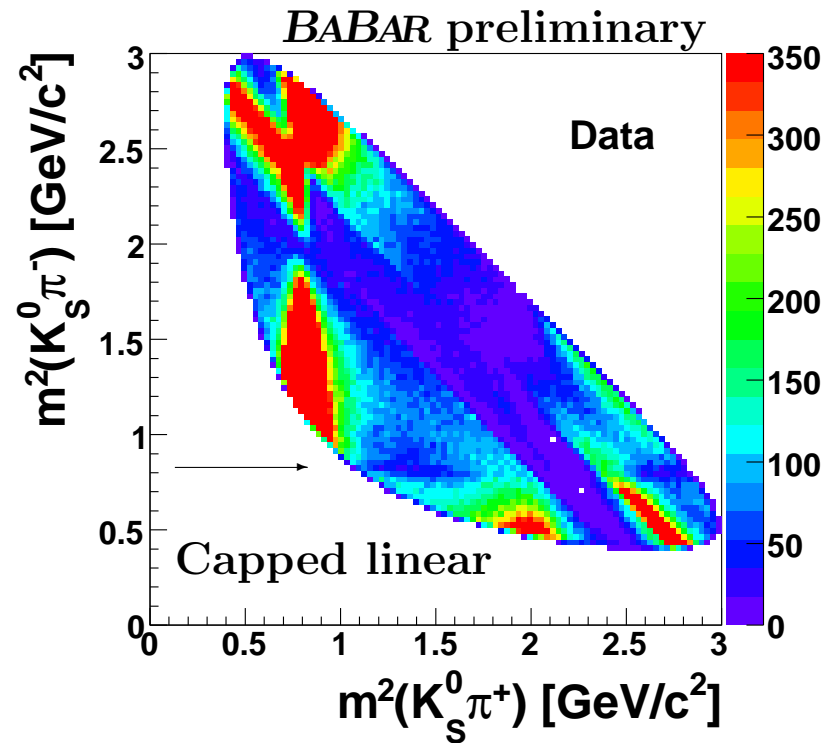
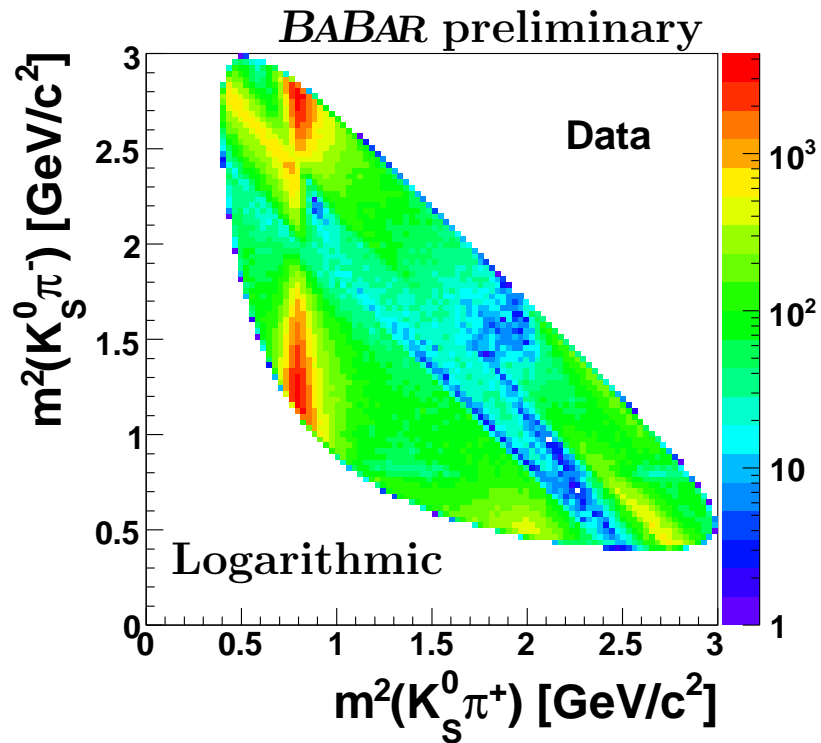
$$x_{\bar{D}^0} = (0.33 \pm 0.33)\%$$

$$y_{\bar{D}^0} = (0.59 \pm 0.28)\%.$$

Submitted to PRL

Preprint hep-ex/1004.5053

Dalitz plot



- Squares of two-daughter masses.
- Capped linear: Numbers above 350 are truncated.
- Visible interference!

Introduction to mixing I: Basics

- Mixing is a transition from a particle to its antiparticle.
- It occurs when the flavour eigenstates (D^0, \bar{D}^0) produced in decays are not the same as the mass eigenstates (D_1, D_2) which move through space.
- We parametrise mixing by the **normalised mass and width differences** of the mass eigenstates:

$$\Delta M = m_1 - m_2$$

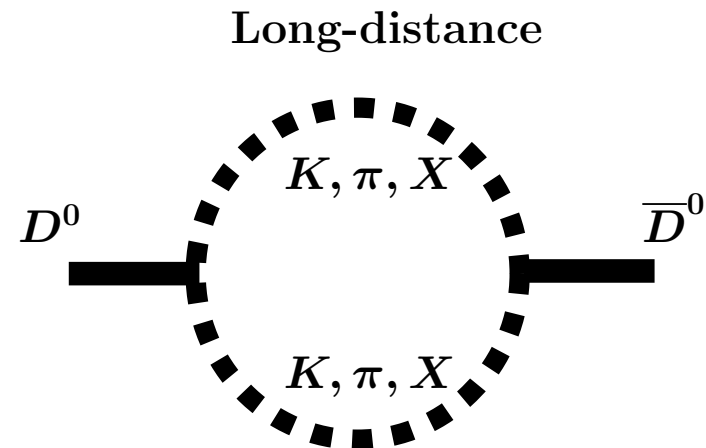
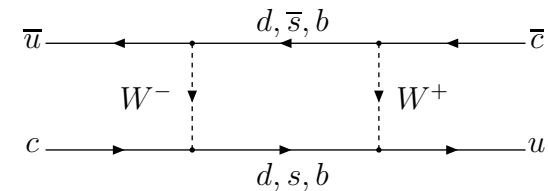
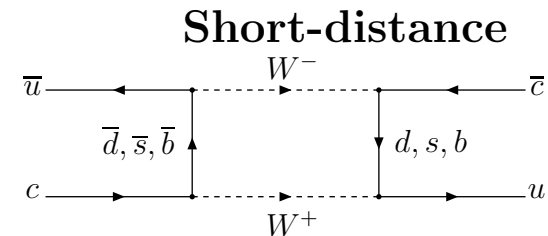
$$\Delta\Gamma = \Gamma_1 - \Gamma_2$$

$$\Gamma = (\Gamma_1 + \Gamma_2)/2$$

$$x = \Delta M/\Gamma$$

$$y = \Delta\Gamma/2\Gamma.$$

- Mixing is strongly suppressed in charmed mesons; the Standard Model predicts a **very tiny** ($x, y < 10^{-4}$) effect from calculable short-distance effects.



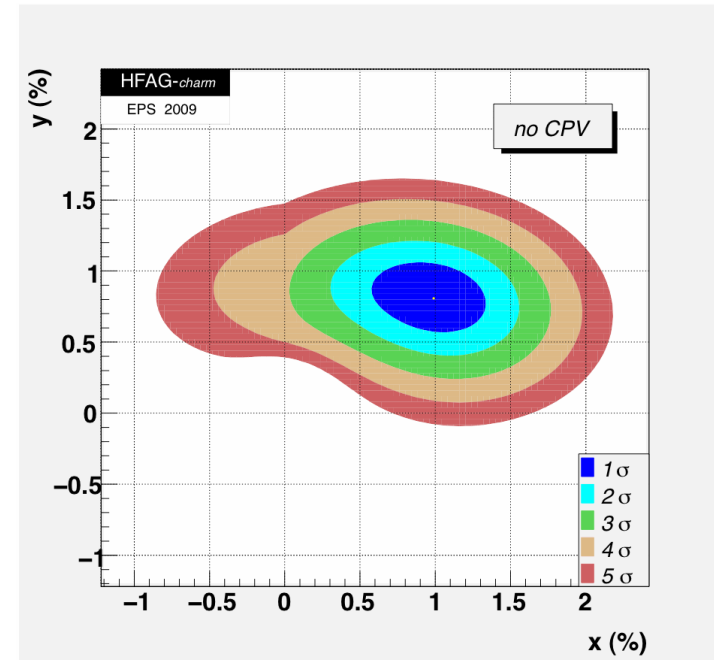
Introduction to mixing II: New Physics

- Long-distance effects are hard to calculate, but from considerations of phase-space SU(3) symmetry breaking we can get an upper bound of $y \sim 1\%$.
- Possible New Physics signatures:
 - CP violation (eg $(x, y)_{D^0} \neq (x, y)_{\bar{D}^0}$).
 - Large mixing - $x, y > 1\%$.
 - ‘Upside down’ mixing, $|x| > |y|$.
- Previous measurements using, e.g., $D^0 \rightarrow K\pi$, are not sensitive to the sign of x .
- Belle result from $K_S^0 \pi^+ \pi^-$:

$$x = (0.80 \pm 0.29 \pm 0.17)\%$$

$$y = (0.33 \pm 0.24 \pm 0.15)\%$$

Tantalising!

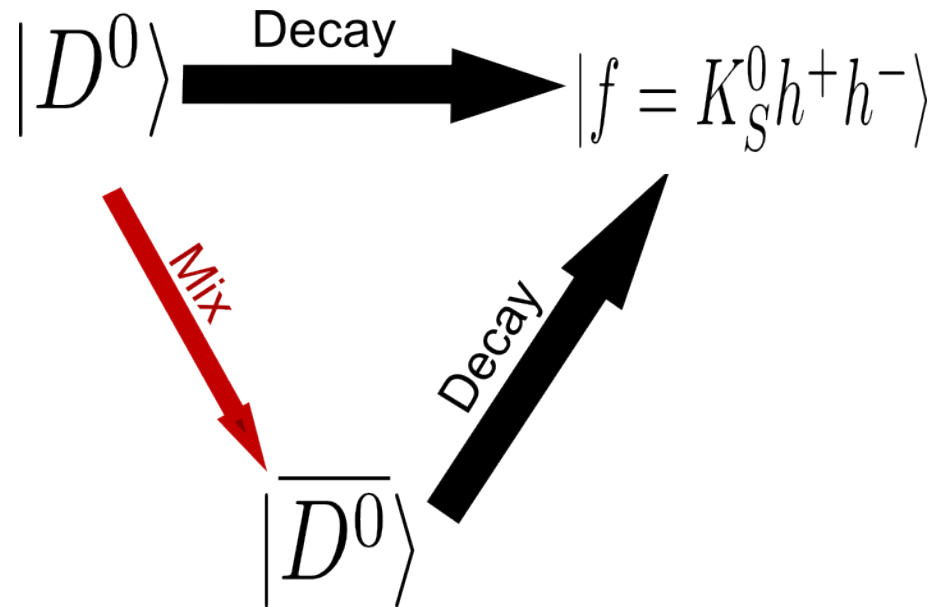


Falk *et. al.*, hep-ph/0110317
Falk *et. al.*, hep-ph/0402204

Mixing formalism I

- Reminder of basic time-development equations:

$$\begin{aligned}
 |D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\
 |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle \\
 |D_1(t)\rangle &= e_1(t)|D_1\rangle = e^{-i(m_1 - i\Gamma_1/2)t}|D_1\rangle \\
 |D_2(t)\rangle &= e_2(t)|D_2\rangle = e^{-i(m_2 - i\Gamma_2/2)t}|D_2\rangle \\
 |D^0(t)\rangle &= \frac{1}{2p} [p(e_1(t) + e_2(t))|D^0\rangle + q(e_1(t) - e_2(t))|\bar{D}^0\rangle] \\
 |\bar{D}^0(t)\rangle &= \frac{1}{2q} [p(e_1(t) - e_2(t))|D^0\rangle + q(e_1(t) + e_2(t))|\bar{D}^0\rangle].
 \end{aligned}$$



Mixing formalism II

- Describe amplitude for D^0 to decay to a point on the Dalitz plot at time t in terms of complex linear combinations A_x of intermediate states:

$$\begin{aligned}\mathcal{A}(\bar{D} \rightarrow \bar{f}) &= \frac{e_1(t)}{2q} (p(A_+ + A_- + A_{\bar{f}}) + q(\bar{A}_+ + \bar{A}_- + \bar{A}_{\bar{f}})) \\ &+ \frac{e_2(t)}{2p} (p(A_+ + A_- + A_{\bar{f}}) - q(\bar{A}_+ + \bar{A}_- + \bar{A}_{\bar{f}})) \\ &\equiv \bar{A}_1 e_1(t) + \bar{A}_2 e_2(t).\end{aligned}$$

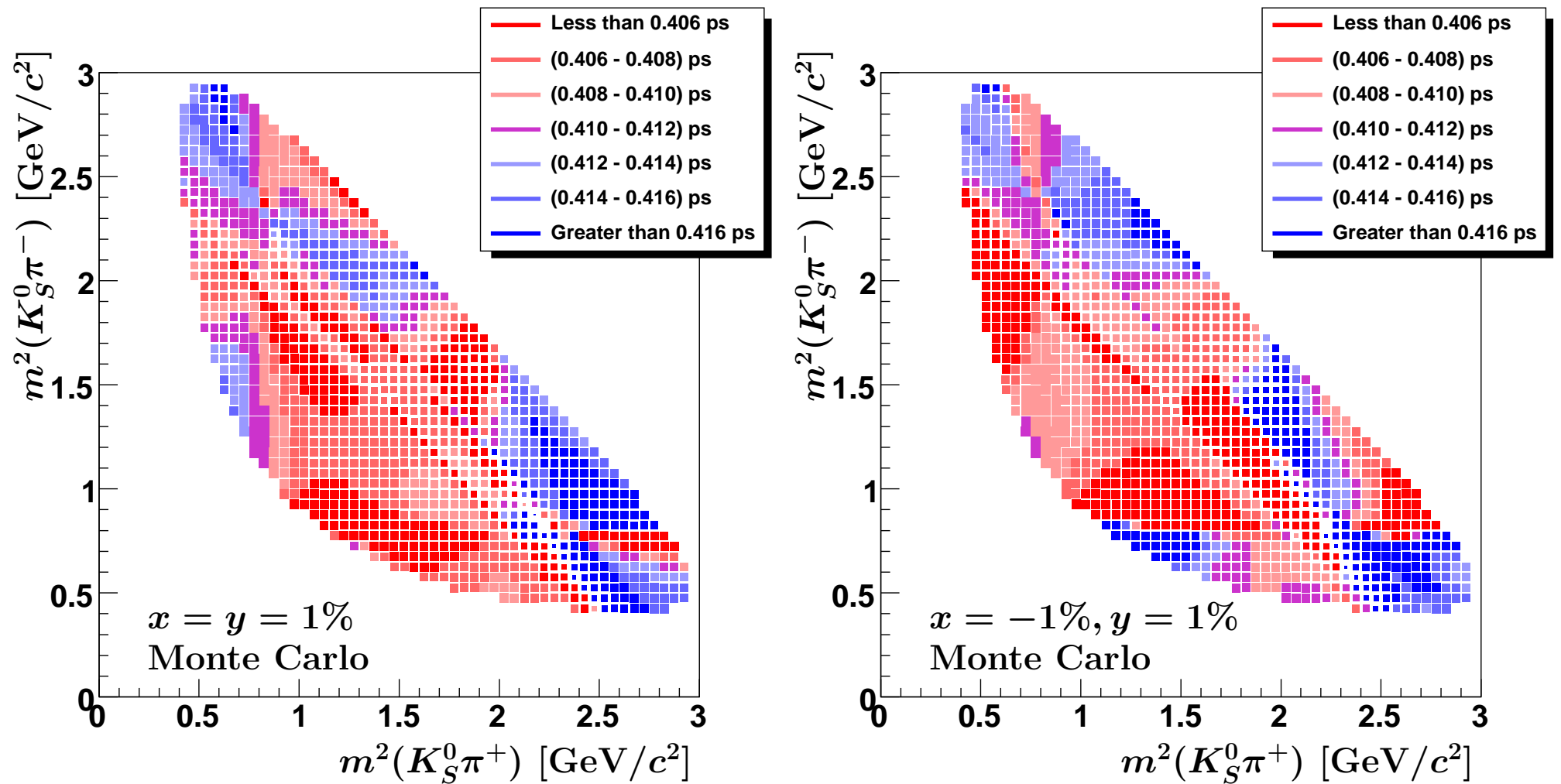
- A_+ , A_- , and A_f are sums over intermediate CP -even, CP -odd, and flavour eigenstates.
- With some algebra, and taking $p = q$, we get the ‘**Main Equation**’:

$$\begin{aligned}|\mathcal{M}(D \rightarrow f)|^2 e^{t/\tau} &= |\bar{A}_1|^2 e^{-yt/\tau} + |\bar{A}_2|^2 e^{yt/\tau} \\ &+ 2\Re(\bar{A}_1 \bar{A}_2^*) \cos(xt/\tau) + 2\Im(\bar{A}_1 \bar{A}_2^*) \sin(xt/\tau).\end{aligned}$$

This is the rate for a particle produced as a D^0 to decay to the state $|f\rangle = (m_{K_S^0 \pi^+}^2, m_{K_S^0 \pi^-}^2)$ at time t .

- $A_{1,2}$ depends very strongly on position in the Dalitz plot.

Average decay times



- Average lifetime in presence of mixing varies across Dalitz plot.
- Colour shows lifetime, box size shows log(number of events).

Isobar formalism

- Model $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decay as **series of two-body decays**, eg



- A common approach is to describe each decay mode using a **relativistic Breit-Wigner** function

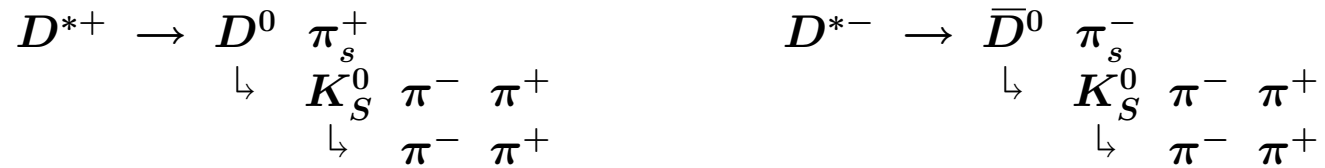
$$\mathcal{A}_k^j(m_+^2, m_-^2) = \frac{F_k F_D S_k}{(m_j^2 - m_{AB}^2) - i m_j \Gamma_j}.$$

- Some partial waves are better described with other models.
 - For the $\pi\pi$ S-wave, instead of a sum of $K_S^0 f_0(980)$, $K_S^0 f_0(1370)$, $K_S^0 \sigma$, $K_S^0 \sigma'$, and a uniform non-resonant term, we use a **K-matrix**.
 - For the two $K\pi$ S-waves, we use an **effective-range parametrisation** (from LASS, hep-ex/0307003) instead of $K_0^*(1430)^\pm \pi^\mp$ plus a uniform term.

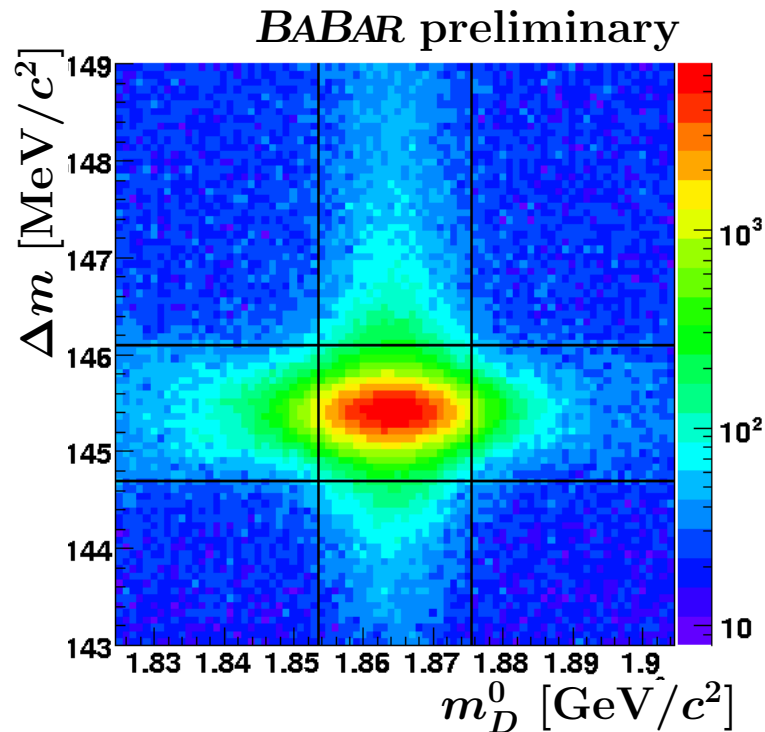
Model name	LASS	K-matrix	Comment
Pure BW			Bad fit
LASS-only	x		Systematic
K-matrix only		x	Systematic
K-matrix+LASS	x	x	Nominal

Analysis strategy

- Getting D^0 from decay of charged D^* allows ‘tagging’ the production flavour:



- This also improves rejection of non- D^0 backgrounds by introducing the variable $\Delta m = m_{D^*} - m_{D^0}$.
- Final data sample: Around 600k.



Digression: Comparison with $D^0 \rightarrow K\pi$

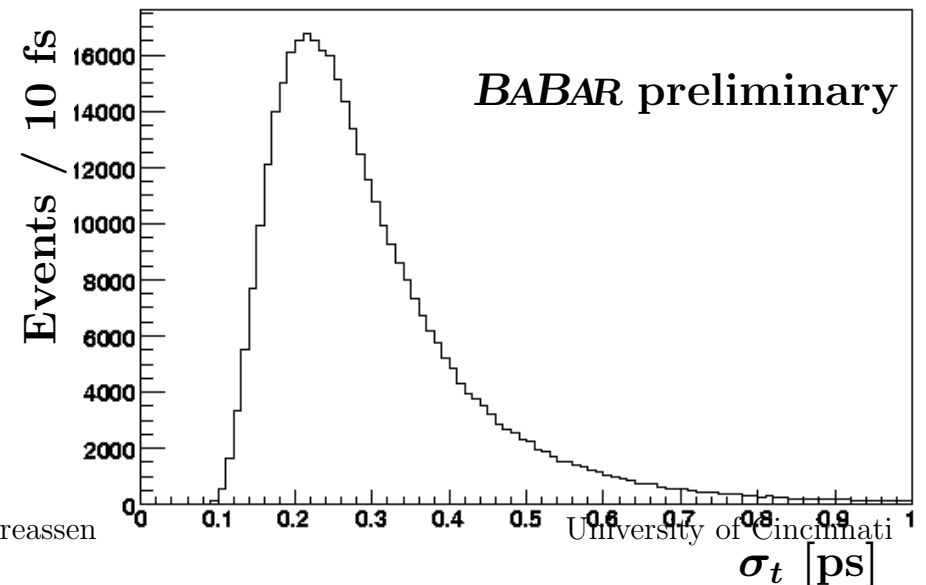
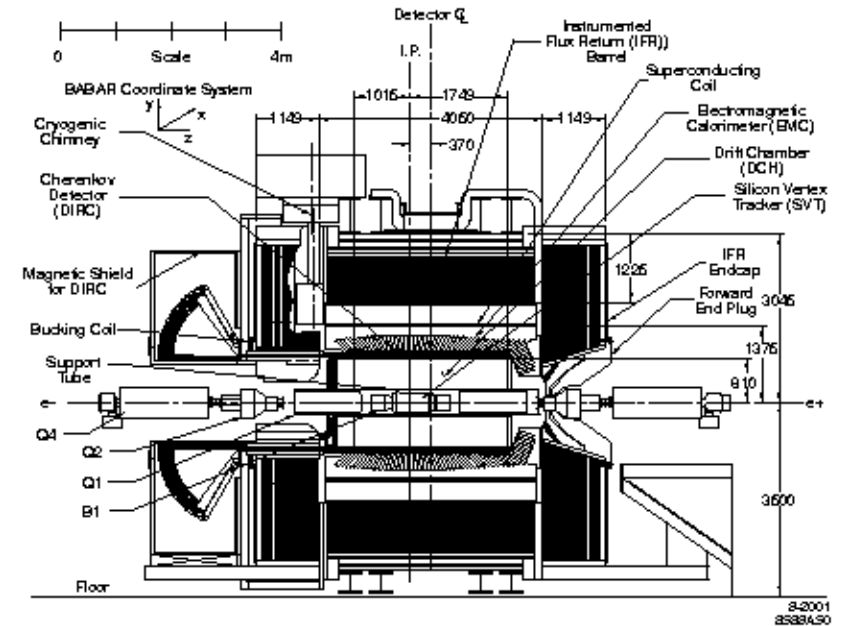
- Conceptually we can view the decay $D^0 \rightarrow K\pi$ as having a Dalitz plot with only one point. Analysis is much simpler!
- Branching fraction is 3.8%, compared to 2.9% for $D^0 \rightarrow K_S^0\pi^+\pi^-$; with 90% efficiency for charged tracks, $K\pi$ gets twice as much data.
- There is a textcolorredstrong phase δ between the DCS and CF decays, which we do not know; this enters into the main equation, and in $K\pi$ we can only measure

$$\begin{aligned}x' &= x \cos \delta_{K\pi} - y \sin \delta_{K\pi} \\y' &= y \cos \delta_{K\pi} + x \sin \delta_{K\pi}.\end{aligned}$$

- In the three-body case, there is an intermediate resonance that helps us out; the $K_S^0\rho$ is a CP eigenstate - the strong phase is known. And we can measure the phase of every other point on the Dalitz plot relative to this state!
- This allows us to extract **x and y directly**, with no rotation.

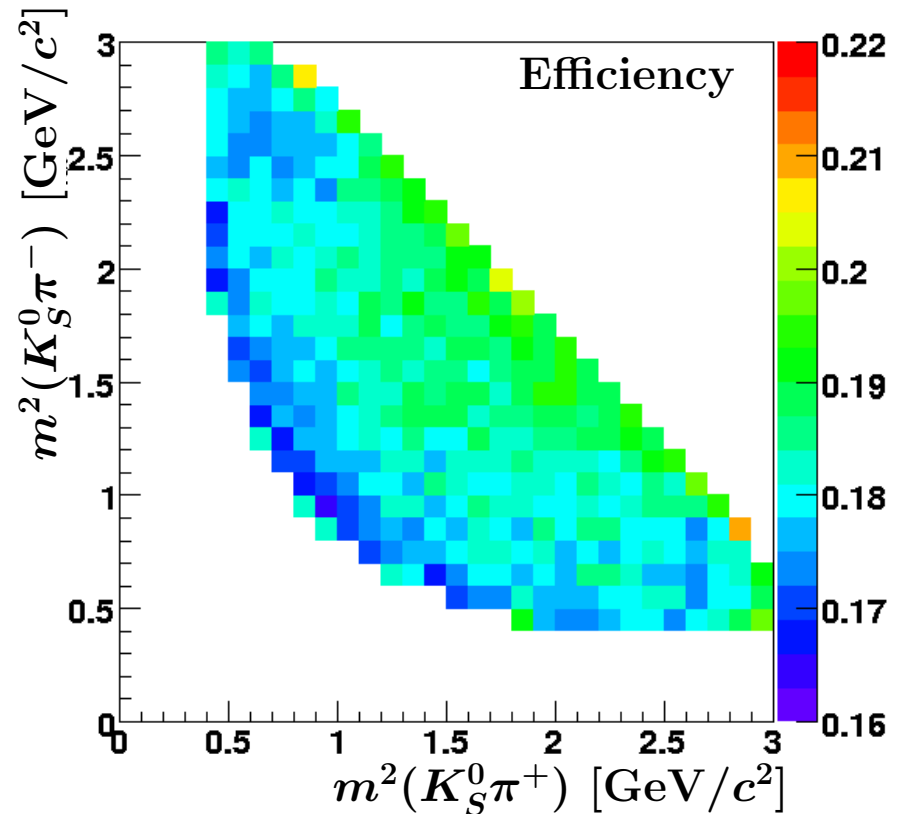
BABAR and event reconstruction

- *BABAR* has two tracking detectors: SVT (Silicon Vertex Tracker) and DCH (Drift Chamber), in a coaxial magnetic field of 1.5 Tesla.
- **Five charged tracks** are reconstructed into D^{*+} decay tree.
- **Kinematic fit** finds best overall momentum for each track, under the constraint that daughters of the same particle must share a production vertex, and that the D^{*+} must point back to the beam spot.
- Measure the **proper decay time t** of the D^0 with **uncertainty σ_t** .
- D^0 lifetime is 411 fs; average uncertainty is ~ 300 fs.



Event selection

- K_S^0 flight distance **at least 10 times its error** - suppresses $D^0 \rightarrow 4\pi$.
- D^0 candidate must have center-of-mass momentum at least $2.5 \text{ GeV}/c$.
- χ^2 -probability of kinematic fit at least 0.01% .
- Pion transverse momenta at least $100 \text{ MeV}/c$.
- D^0 daughters must have at least two hits in the inmost SVT layers, and the slow pion must have at least DCH hit.
- K_S^0 reconstructed mass within $9 \text{ MeV}/c^2$ of the world average.
- Cosine of angle between K_S^0 momentum, and vector between its production and decay vertex, greater than 0.99 .



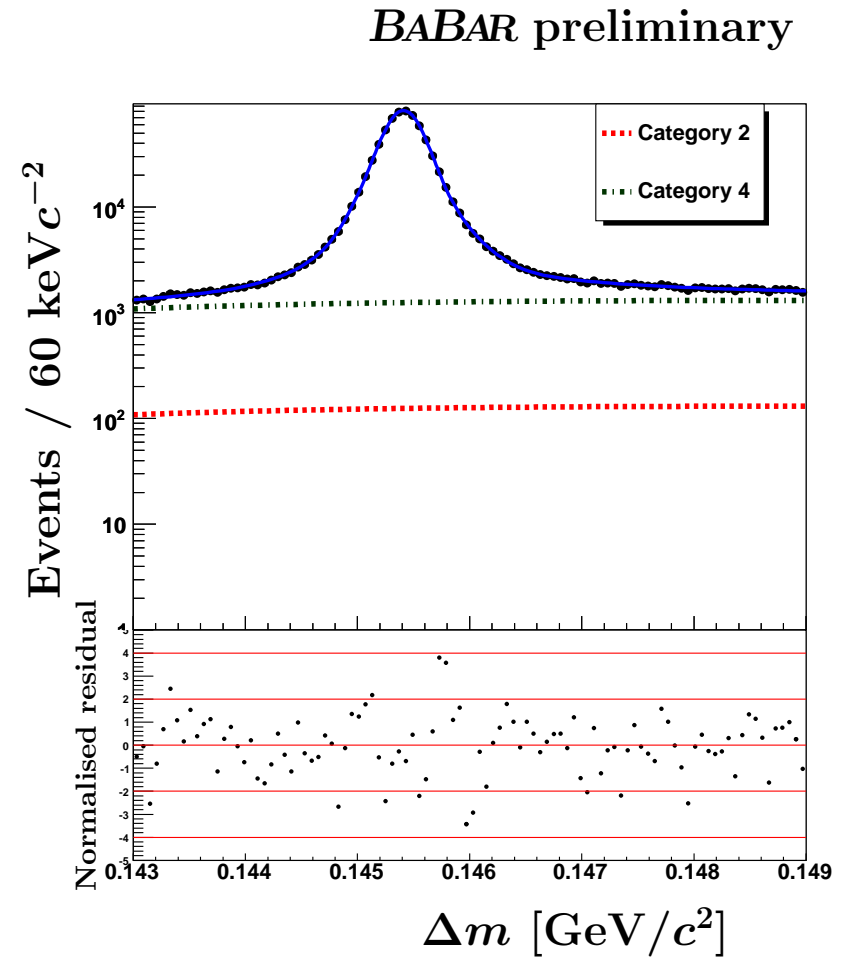
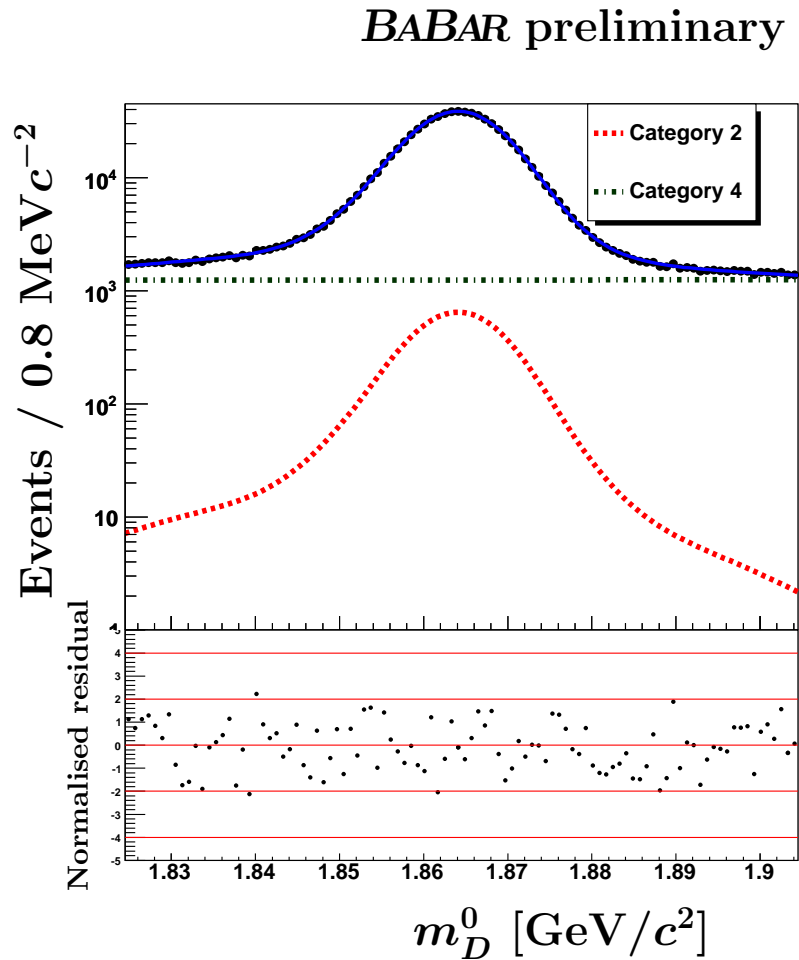
- Overall efficiency **roughly 18%** for signal, with little variation across the Dalitz plot.

Fitting strategy

- Fit (unbinned maximum-likelihood) in three steps:
 - Step 1 (mass fit): 2-D fit to $(m_{D^0}, \Delta m)$, where Δm is the difference between D^0 and D^* masses; this finds the amount of signal and background.
 - Step 2 (initialisation): Find initial values for the final step by doing a time-independent fit to the Dalitz plot and a “Dalitz-independent” fit to the time distribution.
 - Step 3 (mixing fit): Blinded 4-D fit to the **Dalitz plot variables** $(m_{K_S^0\pi^+}^2, m_{K_S^0\pi^-}^2)$, D^0 proper **lifetime** t , and its **uncertainty** σ_t . This extracts the mixing parameters, the D^0 lifetime τ , and the parameters of the amplitude model - the components of $A_{1,2}$. This step has 40 free parameters!
- In addition to signal (‘Category 1’), we have three kinds of background:
 - Category 2: Correctly reconstructed $D^0 \rightarrow K_S^0\pi^+\pi^-$, wrong slow pion.
 - Category 3: Slow pion from a $D^{*+} \rightarrow D^0\pi^+$ decay, bad D^0 .
 - Category 4: Combinatoric background - both levels of reconstruction are bad.

Category	Δm peak	m_D^0 peak
Signal	Yes	Yes
Bad π_s		Yes
Bad D^0	Kinda	
Combinatoric		

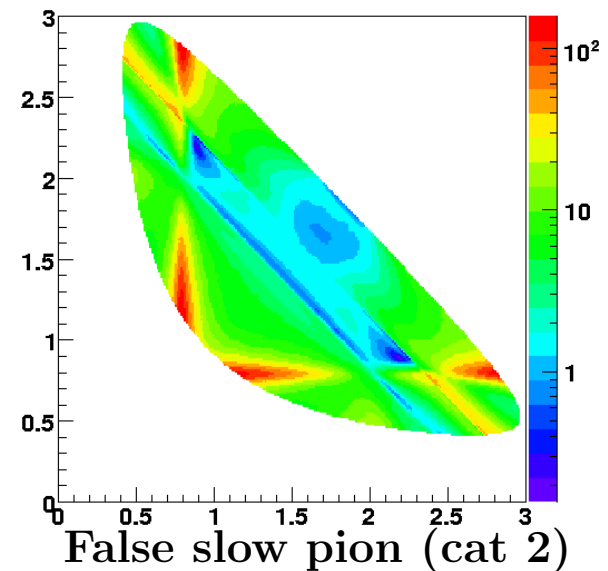
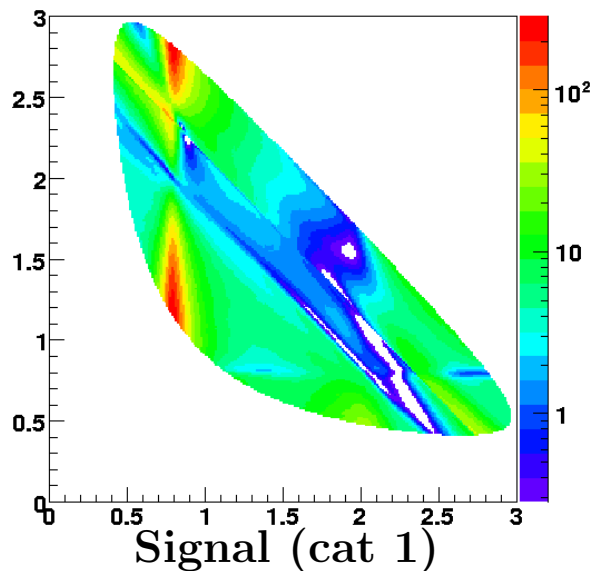
Step-1 fit projections



- Each plot integrates over the other variable.
- Category 2 is correctly reconstructed D^0 , category 4 is combinatoric background. Category 3 is very small!

Step-3 fit

- All development done with **blinded central values** - an unknown random number is added to the result.
- Signal is described by the **Main Equation** convolved with a **time resolution function**.
- The time resolution function is a sum of three Gaussians, representing core, tail, and outlier components. The width of the first two is proportional to the per-event error σ_t ; the last has a global width.
- Category-2 background (correctly reconstructed D^0) shares the signal PDF for the Dalitz plot, with about one-half ($f_l = 0.54$) flipped in the Dalitz plot to account for events where the slow pion has the wrong charge.



- Efficiency ($E(m_+^2, m_-^2)$) over Dalitz plot is described by a third-order polynomial obtained from fitting Monte Carlo events.
- Categories 3 and 4 use a histogram lookup for the Dalitz plot, and a sum of two Gaussians (from MC) for the decay time.
- All categories are multiplied by a separate distribution function for σ_t , which varies in bins across the Dalitz plot.
- Total PDF (**Signal**, **bad slow pion**, and **combinatoric**):

$$\begin{aligned}
P(m_+^2, m_-^2, t, \sigma_t) = & f_s(M(m_+^2, m_-^2, t) \otimes R(t, \sigma_t) \\
& \times S(\sigma_t; m_+^2, m_-^2) \times E(m_+^2, m_-^2)) \\
& + f_2[(f_l(M(m_+^2, m_-^2, t)) + (1 - f_l)M(m_-^2, m_+^2, t)) \\
& \otimes R(t, \sigma_t) \times S_2(\sigma_t; m_+^2, m_-^2) \times E(m_+^2, m_-^2)] \\
& + (1 - f_s - f_2)H_{34}(m_+^2, m_-^2) \\
& \times T_{34}(t, \sigma_t) \times S_{34}(\sigma_t; m_+^2, m_-^2).
\end{aligned}$$

- Fractions f_s and f_2 come from previous fit stage and are now fixed, later they will be varied for systematics.
- As this is very complex to implement, we have **two fitters**, developed separately, to check on each other.

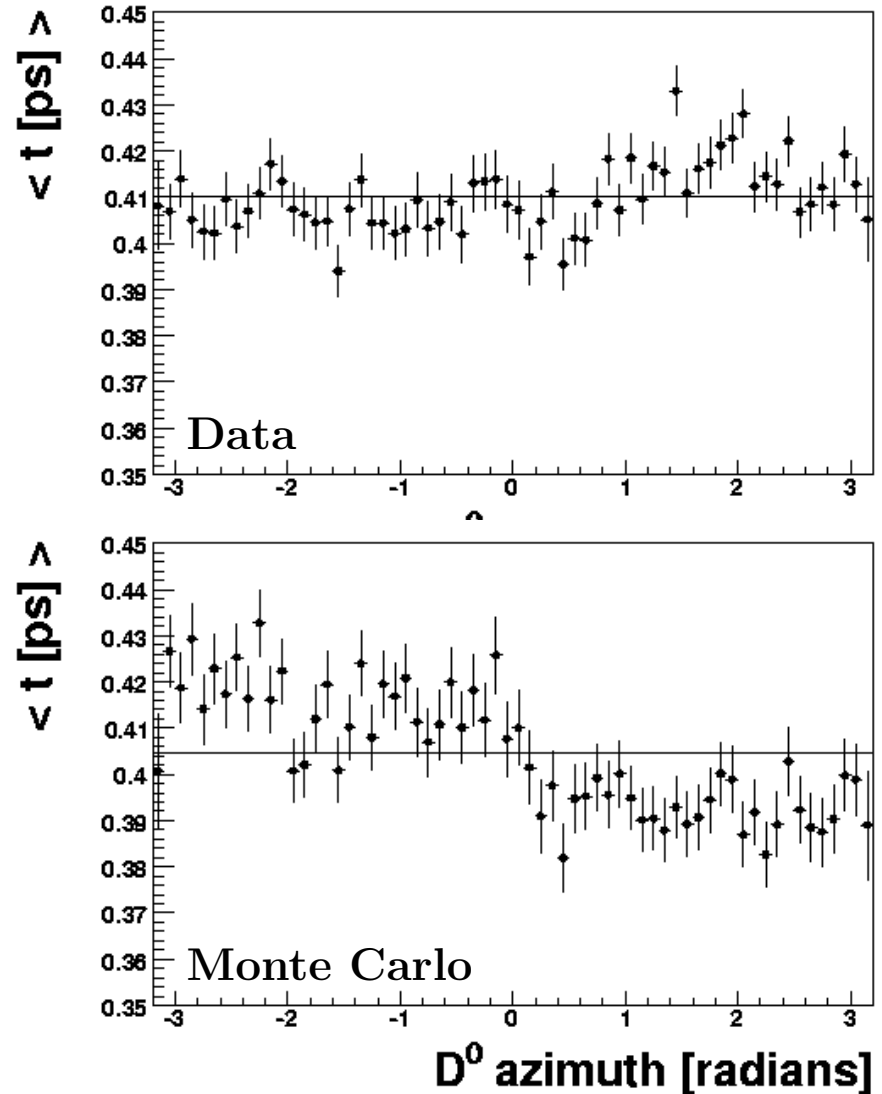
Experimental systematic uncertainties

- Blinding still in effect for systematic studies.
- Summary:

Source	x [%]	y [%]
SVT misalignment	0.0279	0.0826
Fit bias	0.0745	0.0662
Charge-flavor correlation (mistagging)	0.0487	0.0398
Event selection	0.0395	0.0508
Efficiency map	0.0367	0.0175
Background Dalitz-plot distribution	0.0331	0.0142
D^0 mass window	0.0250	0.0250
Proper lifetime PDF	0.0134	0.0128
Signal and background yields	0.0109	0.0069
Mixing in background	0.0103	0.0082
Dalitz-plot normalization	0.0106	0.0053
Proper lifetime error PDF	0.0058	0.0087
Experimental systematics	0.1177	0.1302

Detector misalignment

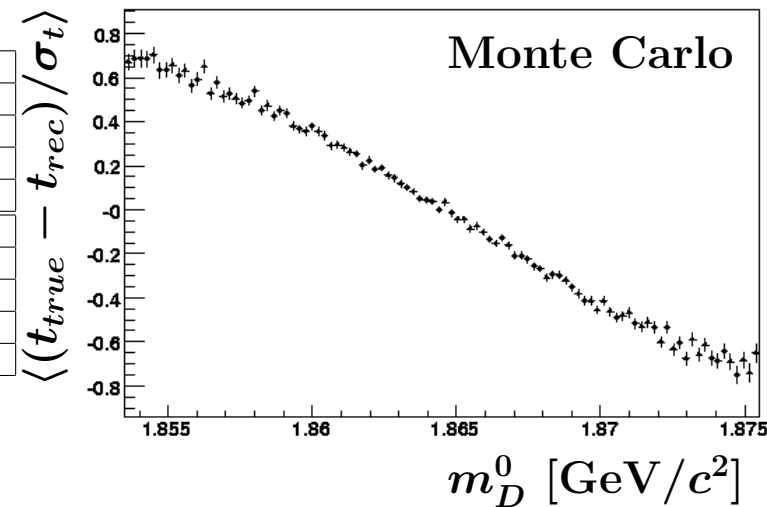
- **SVT subdetector positions** are not perfectly known, and ‘creep’ over time.
- Study this effect by making simulated events with SVT **deliberately misaligned** by taking difference in measured positions between different times - probably an overestimate.
- Run the D^0 mixing fit on these misaligned events, and compare with results for the same events without misalignment.



Determining other systematics

- D^0 mass window: Shift the **central value of the m_D box**.
- Fit bias: Fit Monte Carlo event samples with known mixing parameters, look for consistent bias.
- Mistagging: Vary the amount of category-2 background assumed to be incorrectly tagged.
- Event selection: Vary the allowed ranges of t , σ_t , and the size of the $m_D - \Delta m$ box.
- Efficiency map: Use the histogram directly instead of parametrising it.
- Background Dalitz-plot distribution: Take backgrounds from Monte Carlo or from data sidebands.

True (x, y) [%]	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
(1.00, 1.00)	0.001	0.007	-0.037	-0.074	-0.027
(1.00, -1.00)	-0.009	-0.103	-0.084	-0.047	0.046
(-1.00, 1.00)	0.038	0.006	-0.058	0.027	-0.025
(-1.00, -1.00)	-0.048	-0.063	-0.064	-0.014	0.007
True (x, y) [%]	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
(1.00, 1.00)	-0.004	0.007	-0.036	0.043	-0.063
(1.00, -1.00)	0.001	-0.155	-0.082	-0.040	-0.003
(-1.00, 1.00)	0.000	-0.006	-0.035	-0.020	0.004
(-1.00, -1.00)	0.019	-0.024	-0.001	-0.046	0.138



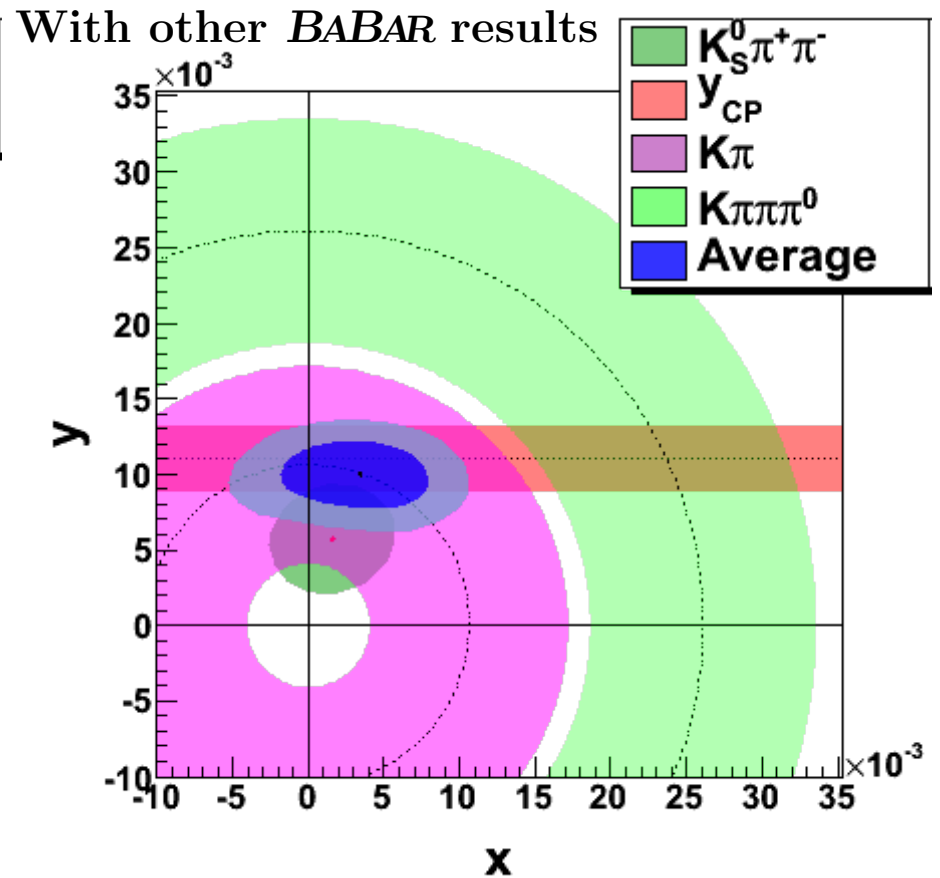
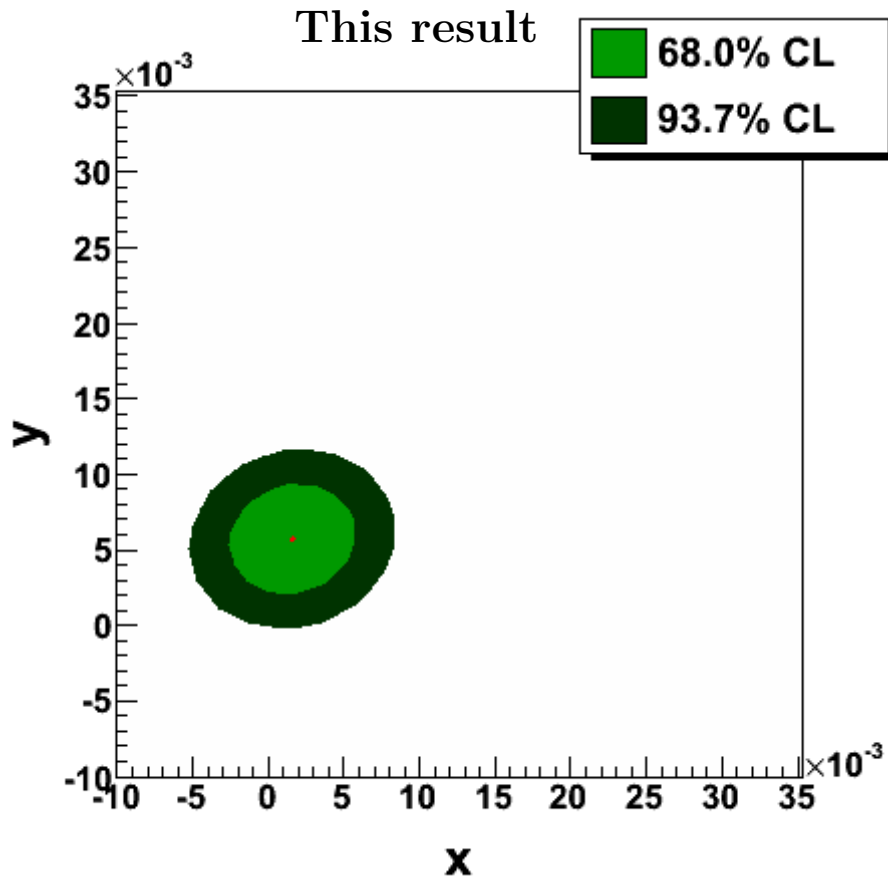
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- Proper lifetime PDF: Vary resolution-model parameters (determined in initialisation step) within errors.
 - Signal and background yields: Vary each yield (from mass fit) within its errors.
 - Mixing in background: Allow category-2 PDF to have separate mixing parameters from signal.
 - Dalitz-plot normalization: Use a finer grid in the normalisation integral.
 - Proper lifetime error PDF: For signal, instead of using a σ_t distribution that varies across the Dalitz plot, use a global one; also replace the data distribution with a signal-Monte-Carlo one. For background, replace the Monte Carlo distribution with a sideband-data one.

Resonance model systematics

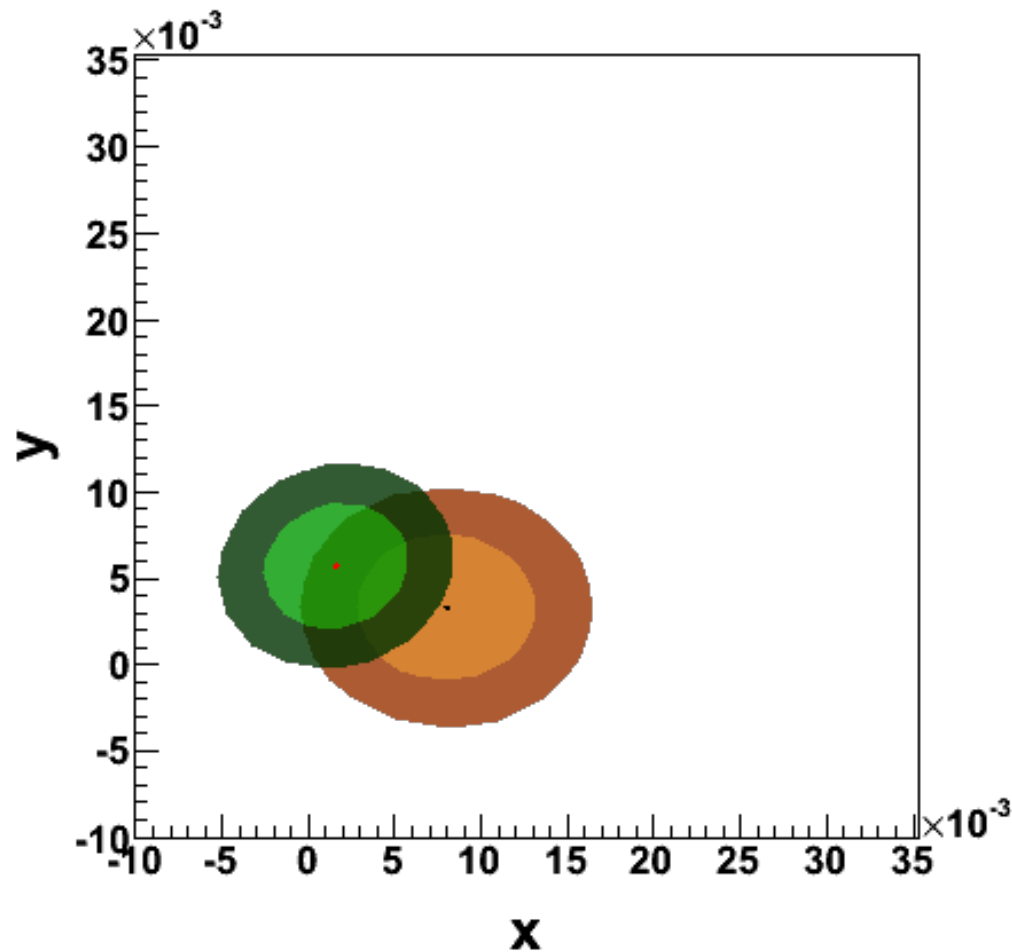
- The decay model is **not the decay physics**. If you put in a different model, you get a different fit result!
- We measure model-dependent systematics using 10 toy Monte Carlo samples generated with the nominal model. Each toy sample is fitted with the nominal model, and with each of the alternative models, and the mean change in fit result is taken as the uncertainty due to an alternative model.

Systematic source	$x(\%)$	$y(\%)$
$\pi\pi$ S -wave: K -matrix solution-I	0.0121 ± 0.0116	-0.0077 ± 0.0077
$\pi\pi$ S -wave: K -matrix solution-IIa	-0.0033 ± 0.0020	0.0020 ± 0.0012
$\pi\pi$ S -wave: Alternative NR term production vector	-0.0040 ± 0.0032	-0.0174 ± 0.0052
$\pi\pi$ P -wave: $\rho(770)$ and $\omega(782)$ float mass and width	0.0279 ± 0.0284	-0.0080 ± 0.0227
$\pi\pi$ P -wave: $\rho(770)$ BW line shape	-0.0010 ± 0.0063	0.0052 ± 0.0052
$K\pi$ P -wave: $K^*(1680)$ mass variation	-0.0125 ± 0.0023	0.0020 ± 0.0031
$K\pi$ P -wave: $K^*(1680)$ width variation	-0.0033 ± 0.0017	0.0025 ± 0.0015
$K\pi$ P -wave: $K^*(1680)$ mass and width from PDG	-0.0172 ± 0.0042	0.0037 ± 0.0046
$K\pi$ D -wave: $K_2^*(1430)$ mass variation	0.0013 ± 0.0014	-0.0007 ± 0.0014
$K\pi$ D -wave: $K_2^*(1430)$ width variation	-0.0005 ± 0.0013	0.0012 ± 0.0009
More $K_s^0\pi^+\pi^-$ resonances: $K^*(1410)$ and $\rho(1450)$	-0.0001 ± 0.0036	-0.0010 ± 0.0025
K-matrix, LASS, $K^*(892)$, $\phi(1020)$, and $g_{K\bar{K}}$ parameters	0.0678	0.0532
KK S -wave: $a_0(980)$ mass variation	0.0001 ± 0.0004	0.0010 ± 0.0002
KK S -wave: $g_{\eta\pi}$ variation	0.0003 ± 0.0009	0.0032 ± 0.0006
KK S -wave: $f_0(1370)$ mass variation	-0.0003 ± 0.0004	-0.0012 ± 0.0006
KK S -wave: $f_0(1370)$ width variation	-0.0001 ± 0.0002	-0.0005 ± 0.0004
KK S -wave: $f_0(1370)$ from E791	-0.0004 ± 0.0004	-0.0009 ± 0.0007
KK S -wave: $a_0(1450)$ mass variation	-0.0002 ± 0.0004	0.0007 ± 0.0003
KK S -wave: $a_0(1450)$ width variation	0.0001 ± 0.0003	0.0003 ± 0.0002
More $K_s^0K^+K^-$ resonances: $a_0(1450)$ DCS and $f_0(980)$	-0.0007 ± 0.0013	-0.0003 ± 0.0026
Fewer $K_s^0K^+K^-$ resonances: $f_0(1370)$ and $f_2(1270)$	-0.0165 ± 0.0109	0.0226 ± 0.0091
$\pi\pi - KK$ D -waves: $f_2(1270)$ mass variation	-0.0007 ± 0.0009	-0.0008 ± 0.0008
$\pi\pi - KK$ D -waves: $f_2(1270)$ width variation	0.0006 ± 0.0012	0.0006 ± 0.0010
$\pi\pi - KK$ $P-, D$ -waves: Blatt-Weisskopf factors	0.0025 ± 0.0058	0.0026 ± 0.0077
$\pi\pi - KK$ $P-, D$ -waves: resonance frame	-0.0244 ± 0.0248	-0.0233 ± 0.0173
$\pi\pi - KK$ $P-, D$ -waves: Helicity formalism	0.0005 ± 0.0245	-0.0172 ± 0.0170
Total Dalitz model systematics	0.0830	0.0685

Comparison of results



Comparison with Belle result



$$x = (0.16 \pm 0.27)\%$$
$$y = (0.57 \pm 0.25)\%$$

$$x = (0.80 \pm 0.34)\%$$
$$y = (0.33 \pm 0.28)\%$$

Other mixing results

- **BABAR**

$$\begin{aligned}
 x &= (0.16 \pm 0.27)\% \\
 y &= (0.57 \pm 0.25)\% \\
 x'^2 &= (-0.024 \pm 0.036)\% \\
 y' &= (0.97 \pm 0.54)\%
 \end{aligned}$$

- **Belle:**

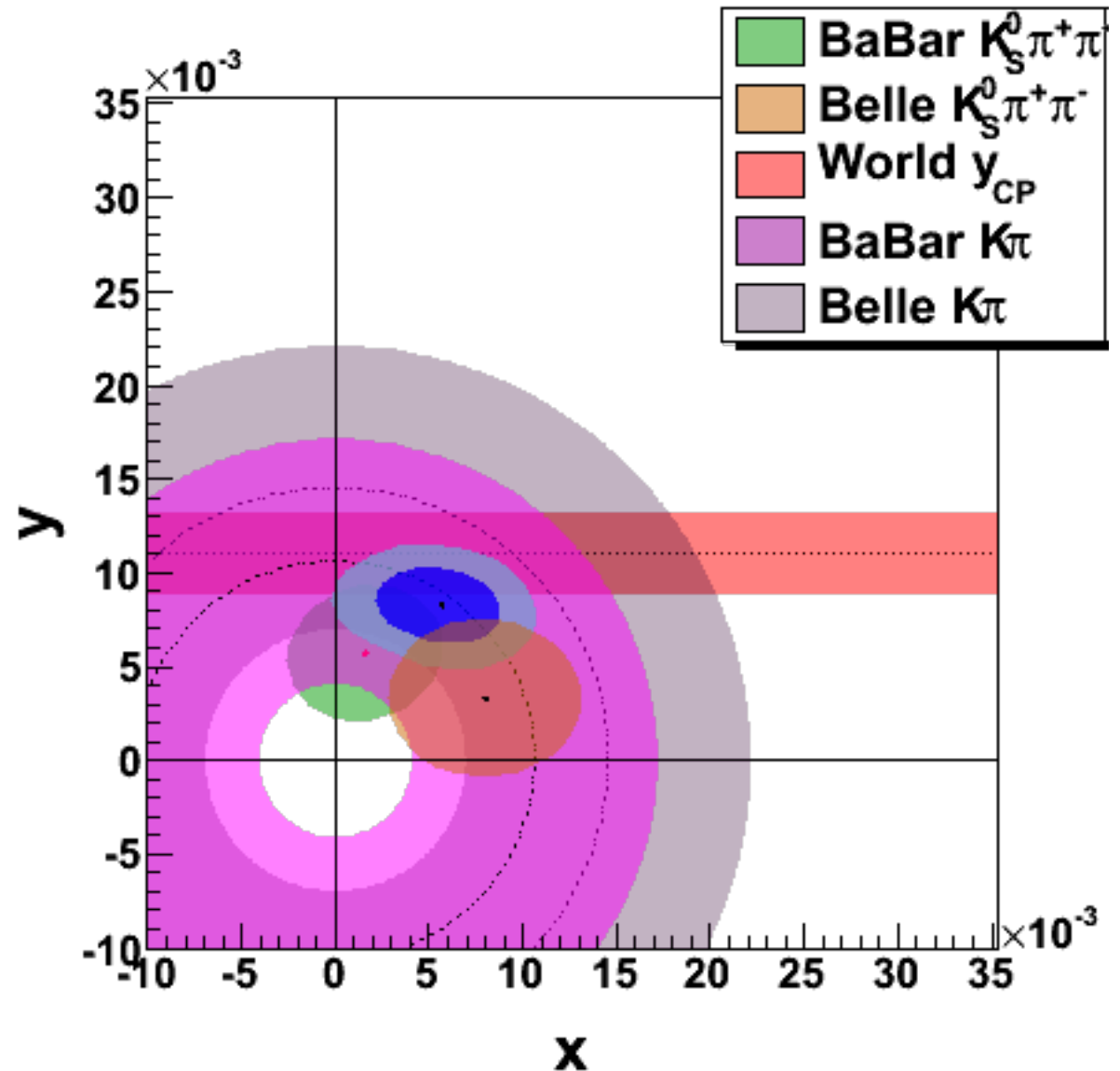
$$\begin{aligned}
 x &= (0.80 \pm 0.34)\% \\
 y &= (0.33 \pm 0.28)\% \\
 x'^2 &= 0.018_{-0.023}^{+0.021}\% \\
 y' &= 0.06_{-0.39}^{+0.40}\%
 \end{aligned}$$

- **CDF:**

$$\begin{aligned}
 x'^2 &= (-0.012 \pm 0.025)\% \\
 y' &= (0.85 \pm 0.76)\%
 \end{aligned}$$

- **HFAG:**

$$y_{CP} = (1.107 \pm 0.217)\%.$$



Summary and outlook

- Final numbers:

$$x = (0.16 \pm 0.23 \pm 0.12 \pm 0.08)\%$$

$$y = (0.57 \pm 0.20 \pm 0.13 \pm 0.07)\%$$

$$x_{D^0} = (0.00 \pm 0.33)\%$$

$$y_{D^0} = (0.55 \pm 0.28)\%$$

$$x_{\bar{D}^0} = (0.33 \pm 0.33)\%$$

$$y_{\bar{D}^0} = (0.59 \pm 0.28)\%.$$

- No evidence for CP violation.

- Future analyses in the same vein:

- Add other decay channels (eg $\pi^+\pi^-\pi^0, K_S^0 K^\mp \pi^\pm$).

- Measure q/p :

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$

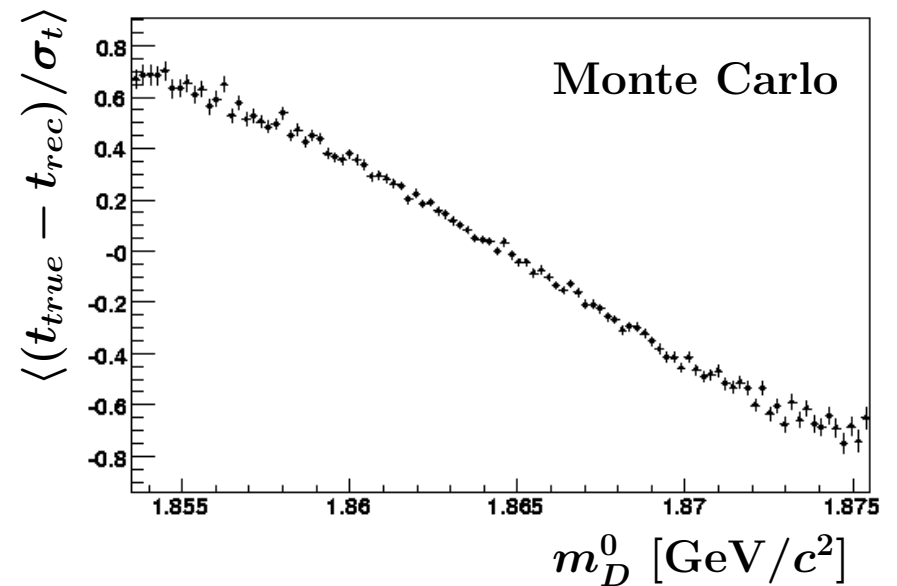
$$|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$

- A high-luminosity flavour factory could improve the statistical error by a factor 10; we would then be much more sensitive to CP violation.

Backup slides

D^0 mass uncertainty

- D^0 mass is not perfectly known.
- Reconstructed decay time is correlated with reconstructed D^0 mass.
- The effect on t is large, order tens of femtoseconds, but cancels out on average; it has a small effect on x , y and τ .
- If you are wrong about the D^0 mass, you might move your entire decay-time distribution up or down!
- Study this effect in data and in simulated events by moving the allowed mass window up and down.
- Assign 0.025% uncertainty in both x and y from this effect.



Mistagging

- With correct D^0 reconstruction and random slow pion, you will still have the correct pion charge about 50% of the time.
- From simulation, we get the right charge ('lucky event') in 54% of category-2 events.
- When we are not lucky, the Dalitz plot is reversed!
- Naively we expect this to be a large effect if we do not know the unlucky fraction quite exactly.
- Test in two ways:
 - Create toy Monte Carlo with a known unlucky fraction, and fit it assuming no unlucky events. This assumes that the entire sample is category 2! In data it is only a few percent.
 - Fit the data using different lucky fractions.
- Both methods agree: Effect is quite small. There just aren't very many category-2 events.
- Assign 0.01% uncertainty in both x and y .

Unlucky fraction of Category-2 events	Fit x [%]	Δx [%]	Fit y [%]	Δy [%]
0.0%	0.836	0.000	0.971	0.000
0.5%	0.714	-0.122	0.849	-0.122
1.0%	0.729	-0.107	0.820	-0.151
1.5%	0.653	-0.183	0.745	-0.226
2.5%	0.531	-0.305	0.656	-0.315
5.0%	0.291	-0.545	0.600	-0.371