

Electron and muon $g-2$ experiments: present and future

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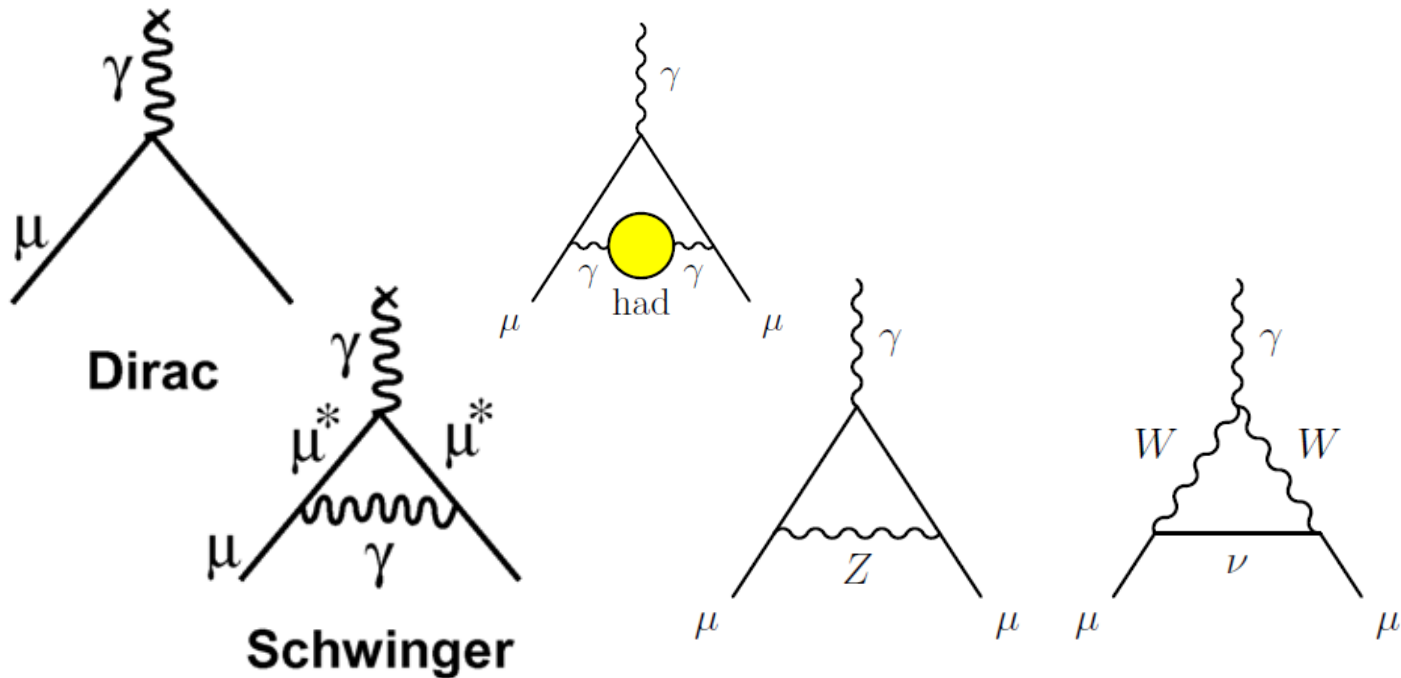
Outline

- Polarised beams in storage ring accelerators
 - Physics motivation
 - Quantities measured
- Resonant depolarisation experiments with electrons
 - Apparatus and detector choices
 - Physics results
- Fermilab muon $g-2$ experiment (E989)
 - Proposed calorimeter for GeV decay electrons
 - Coherent betatron oscillations
- Future electron experiments
 - Circular Unruh effect

POLARISED BEAMS IN STORAGE RINGS

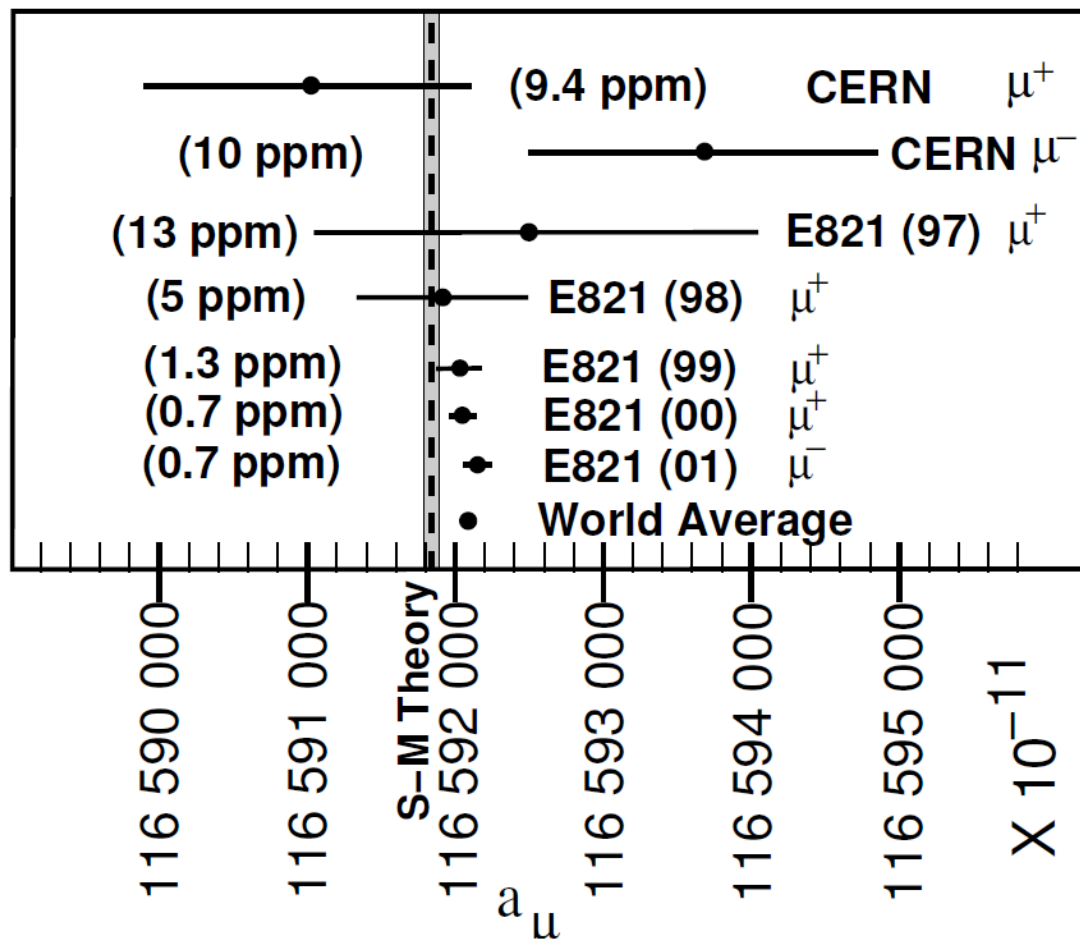
Magnetic moment

$$g_{\mu} = 2.002\,331\,841\,78(108)(66)$$



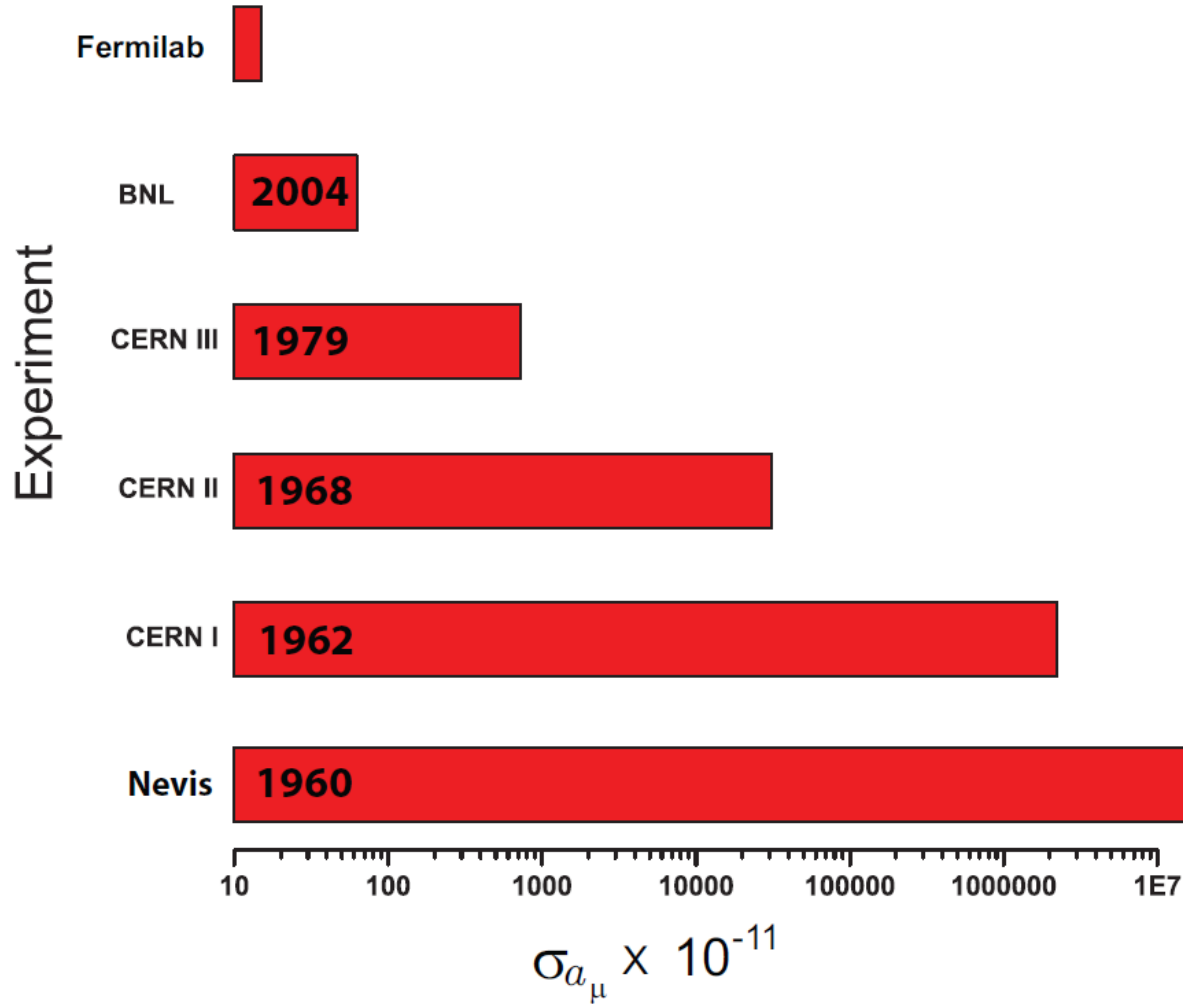
Miller, et al. (2007) Rep. Prog. Physics 70, 795.
Beringer, et al. (2012) Phys. Rev. D 86, 010001.

Physics motivation



Fermilab E989 Conceptual Design Report (2013)

Fermilab muon g-2 experiment (E989)



What is measured in g-2 experiments?

- Thomas – BMT equation

$$\vec{\Omega}_{BMT} = -\frac{q_e}{\gamma m_e c} \left[(1 + a\gamma)\vec{B}_\perp + (1 + a)\vec{B}_\parallel - \left(a\gamma + \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right]$$

- Describes rate of spin precession
- For a storage ring without transverse electric fields

$$\vec{\Omega}_{BMT} \times \vec{S} \equiv f_{spin} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} [(1 + a\gamma)]$$

- Compare with the cyclotron frequency

$$f_{cyc} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c}$$

- Measurement of frequencies

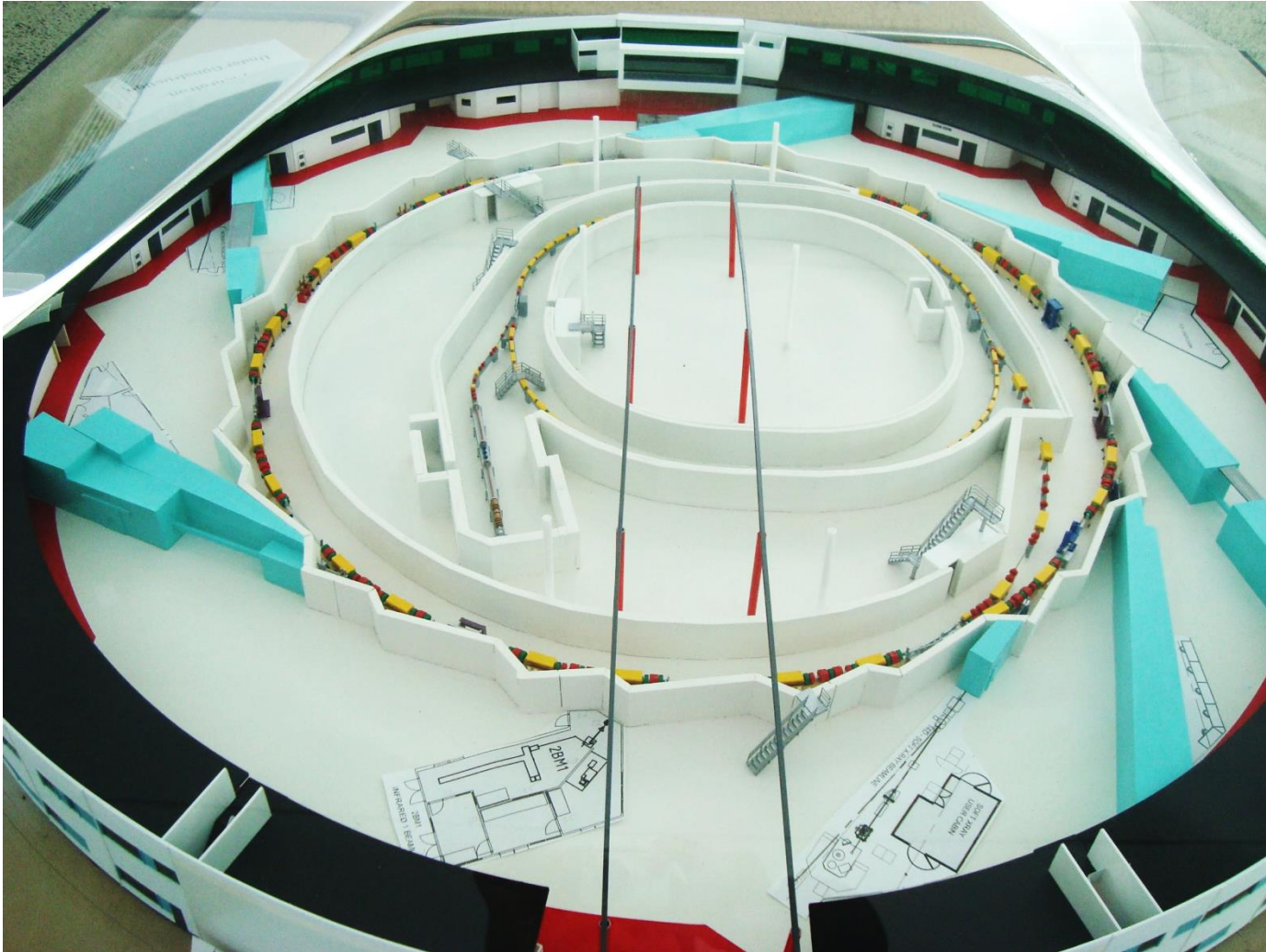
Arnaudon 1995 Z. Phys. C. 66, 45

RESONANT DEPOLARISATION EXPERIMENTS WITH ELECTRONS

Australian Synchrotron



Australian Synchrotron



Storage ring

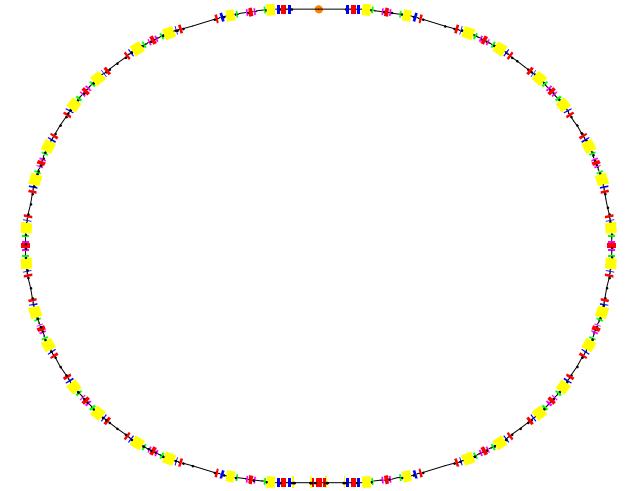
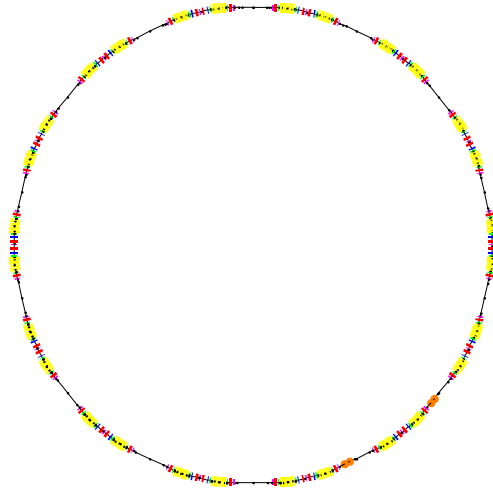


- 3 GeV electron ring
- Periodicity 14, Double Bend Achromat
- 216.0 m circumference
- RF 500 MHz, 3.0 MV
 - 4 × CW klystron



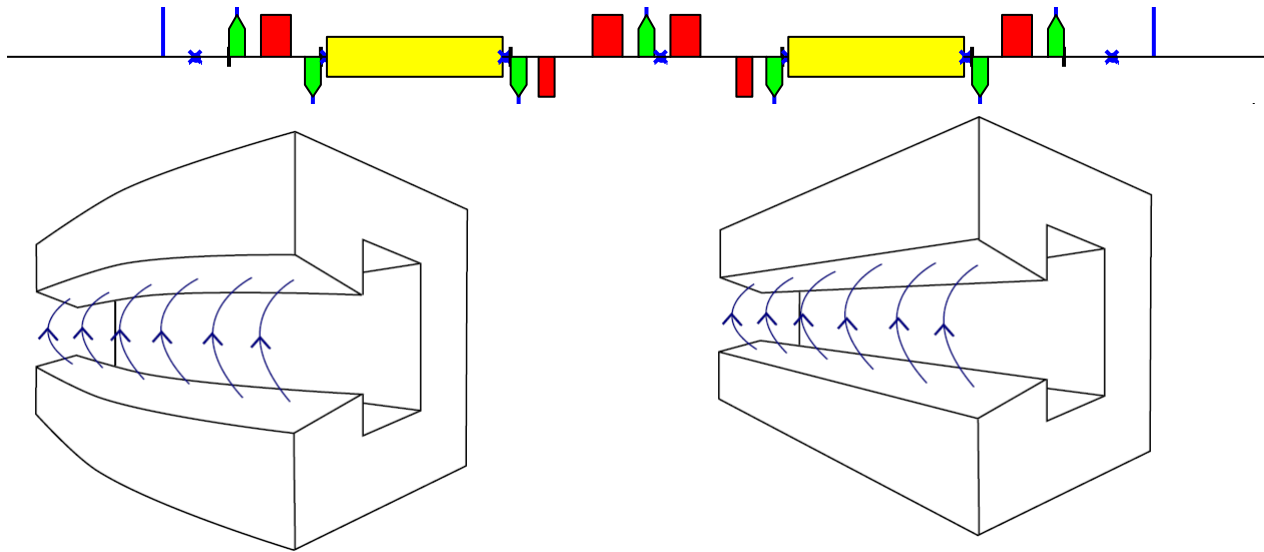
Storage ring light sources

- 3 GeV
- DBA cell
 - Gradient dipoles
- Like many other low horizontal emittance rings



Motivation – electron rings

- Lower horizontal emittance by incorporating defocussing gradient into bending magnet
 - DBA, TBA, MBA, TME lattices heading in this direction
 - Can eliminate quadrupoles from lattice

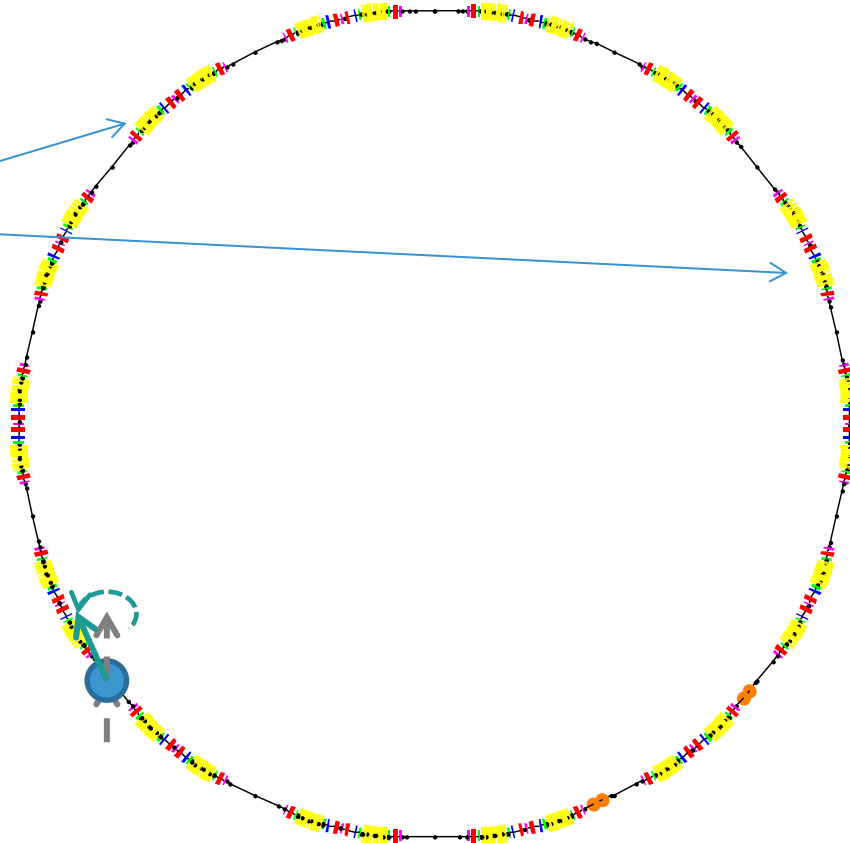


- Build a straight, rectangular magnet with defocussing gradient
 - What equation describes the particle motion?

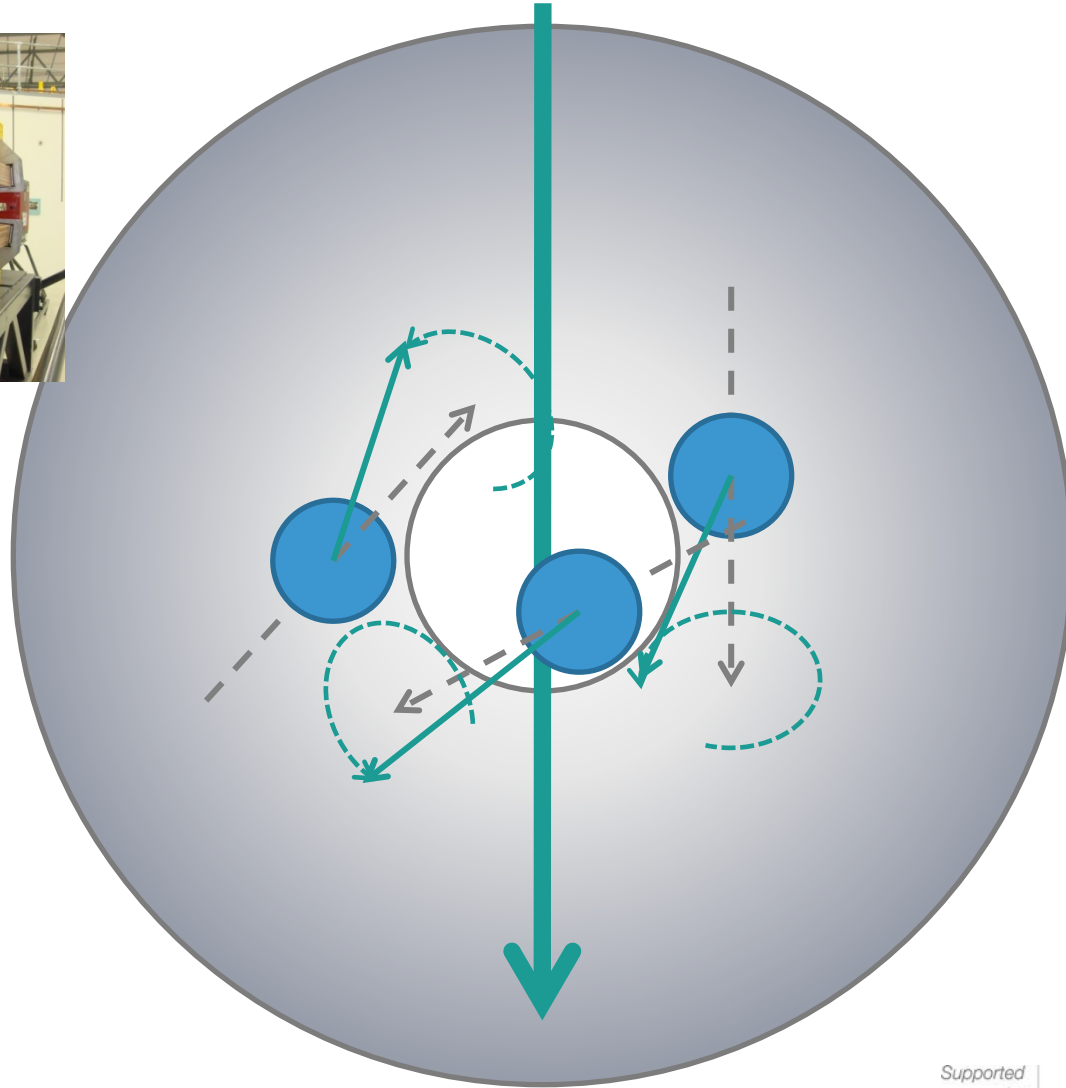
Electron spin resonant depolarisation

- Precision measurement of beam energy and momentum compaction
 - Tells us about value of dispersion function where there is bending
- Require:
 - Spin-polarised electron beam
 - Spin precession
 - Method of depolarising the beam
 - Method of monitoring beam polarisation
- These measurements will be compared to simulations of the trajectory through the bending magnet

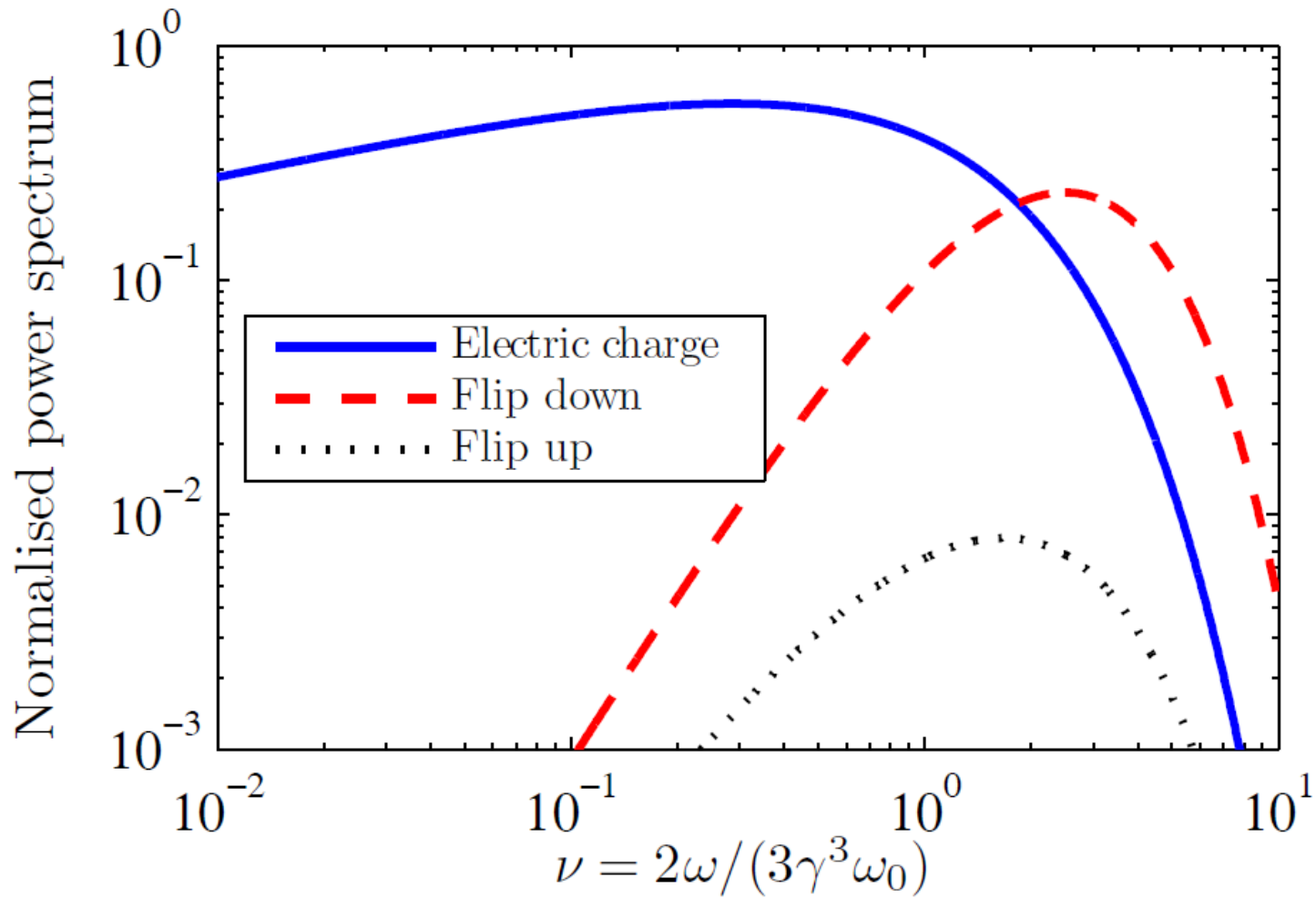
Resonant spin depolarisation



Sokolov-Ternov Effect



Sokolov-Ternov Effect



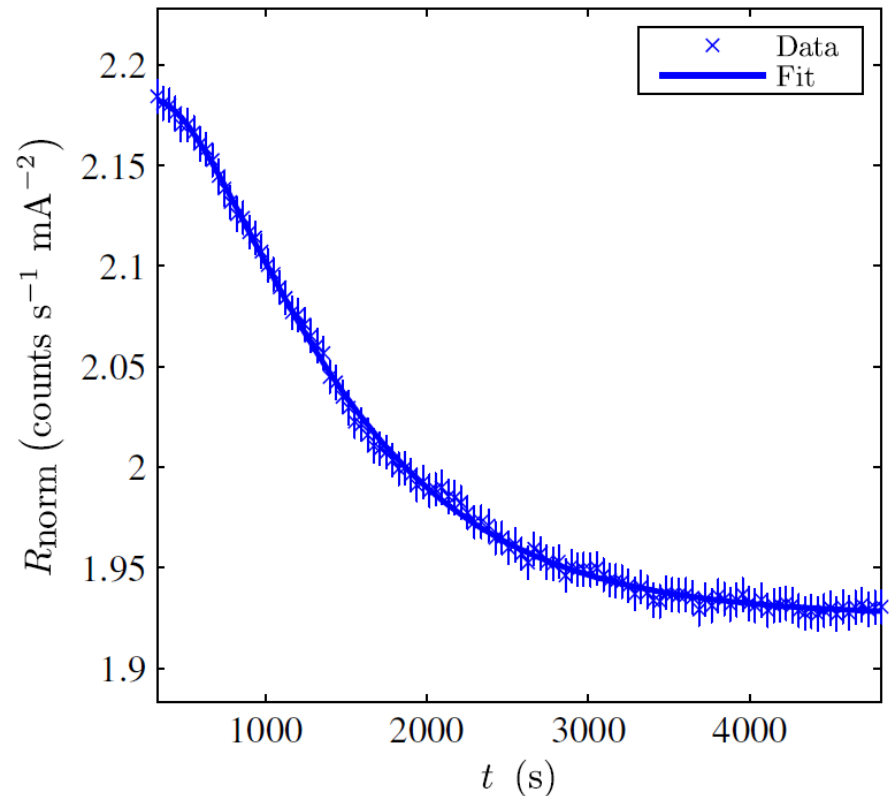
Polarised beam

We get it for free!
Sokolov-Ternov effect

$$\tau_{ST} = \frac{8}{5\sqrt{3}} \frac{m_e \rho^2 R}{\hbar \gamma^5 r_e}$$

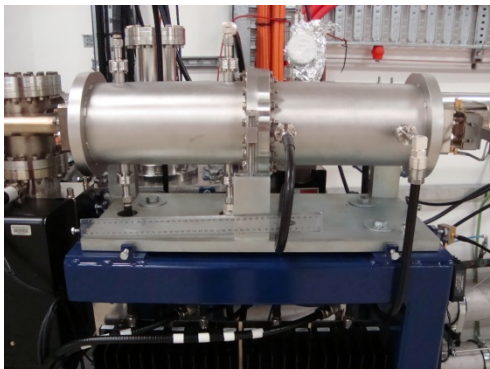
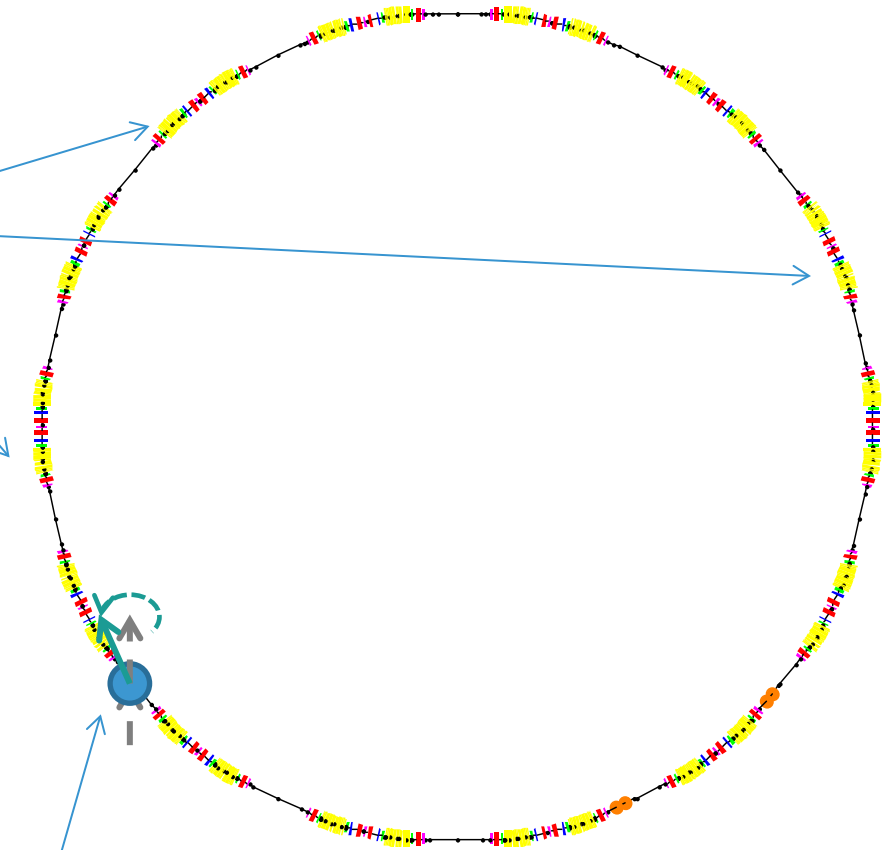
Sokolov & Ternov 1964 Sov. Phys.
Dokl. 8, 1203-5.

Ring	Measured	Model
AS	τ 806(21)	807
SPEAR3	τ 840(12)	1003

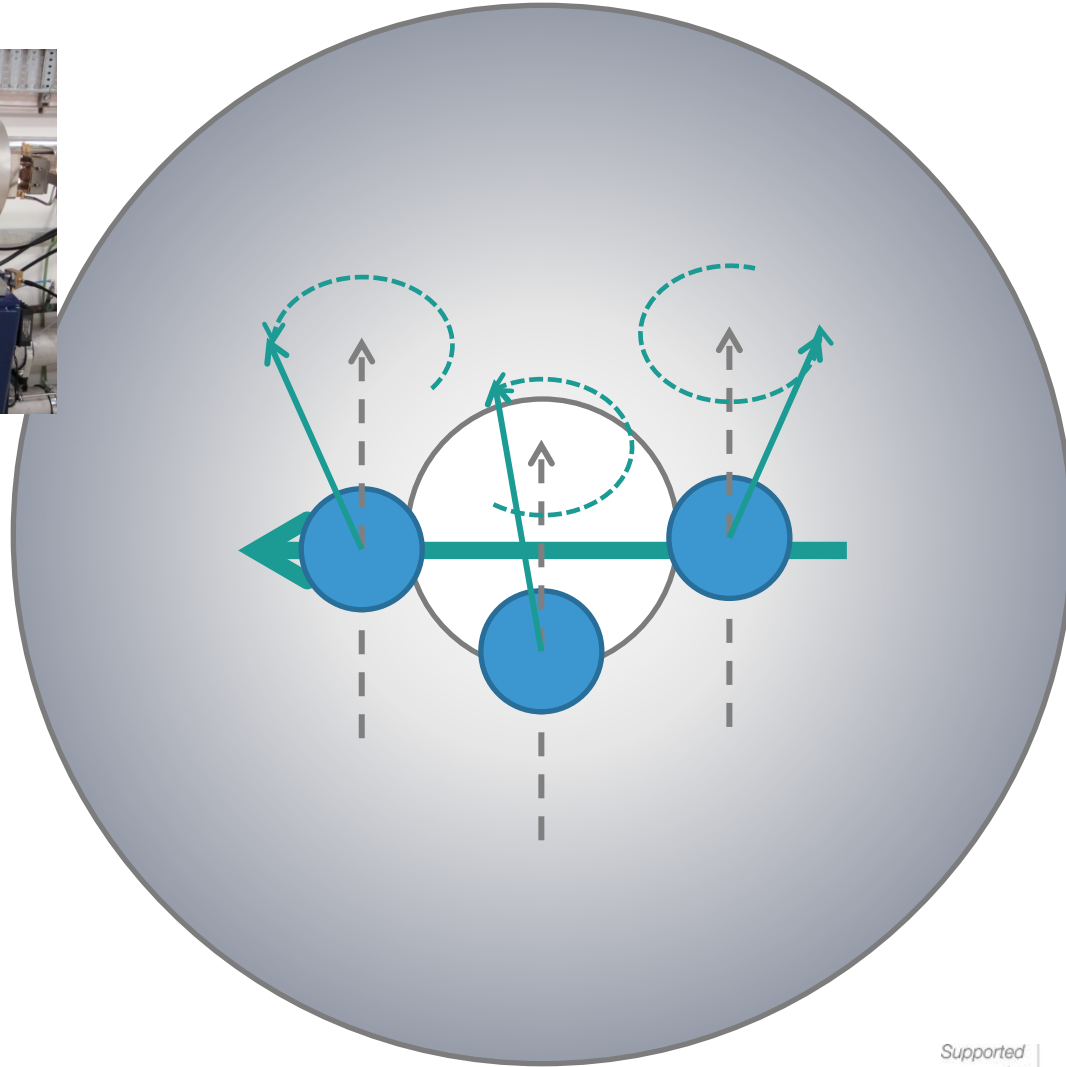
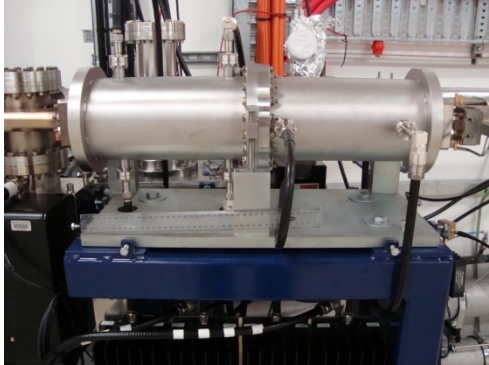


Wootton et al. (2013) Phys. Rev. ST –
Accel. Beams. 16, 074001

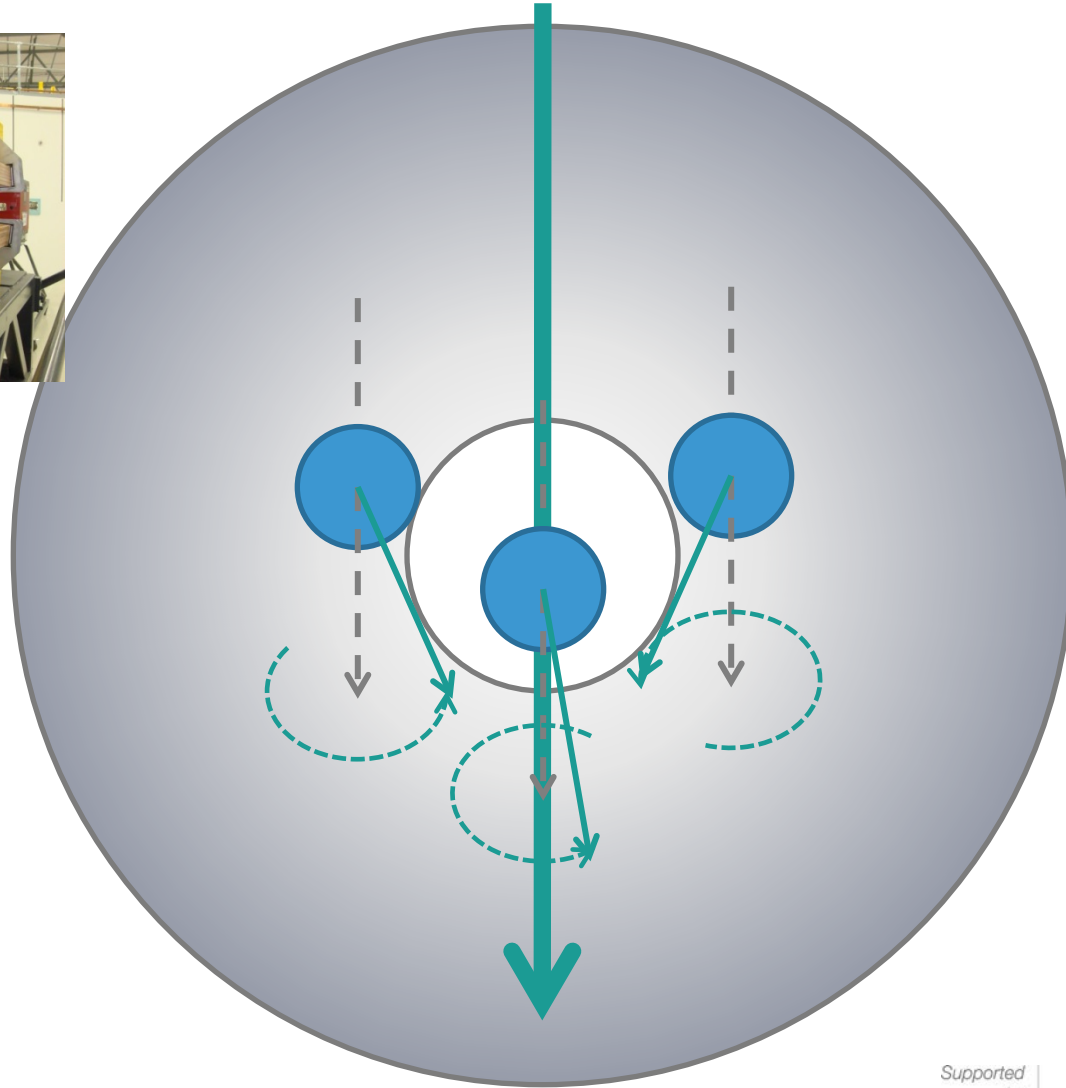
Resonant spin depolarisation



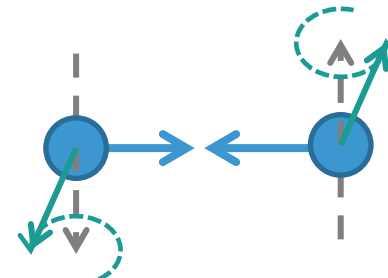
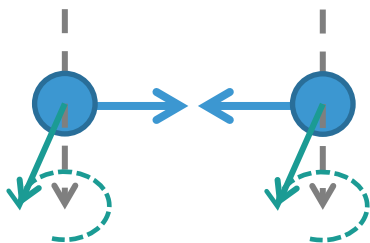
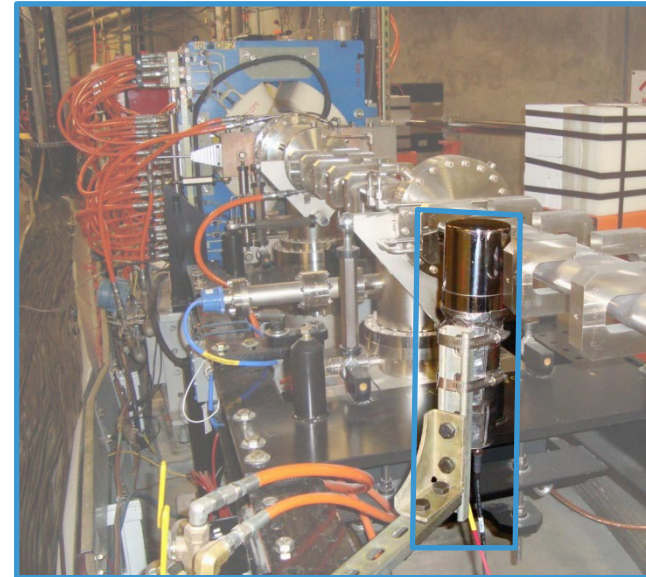
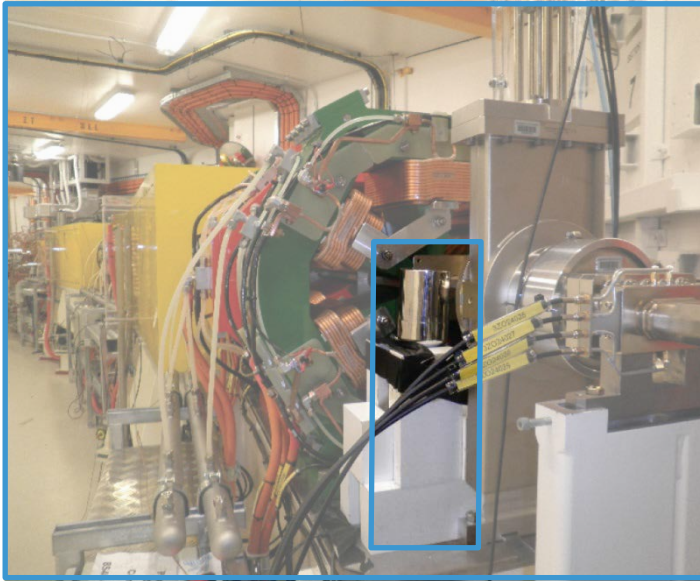
Resonant spin depolarisation



Sokolov-Ternov Effect



Polarisation monitor

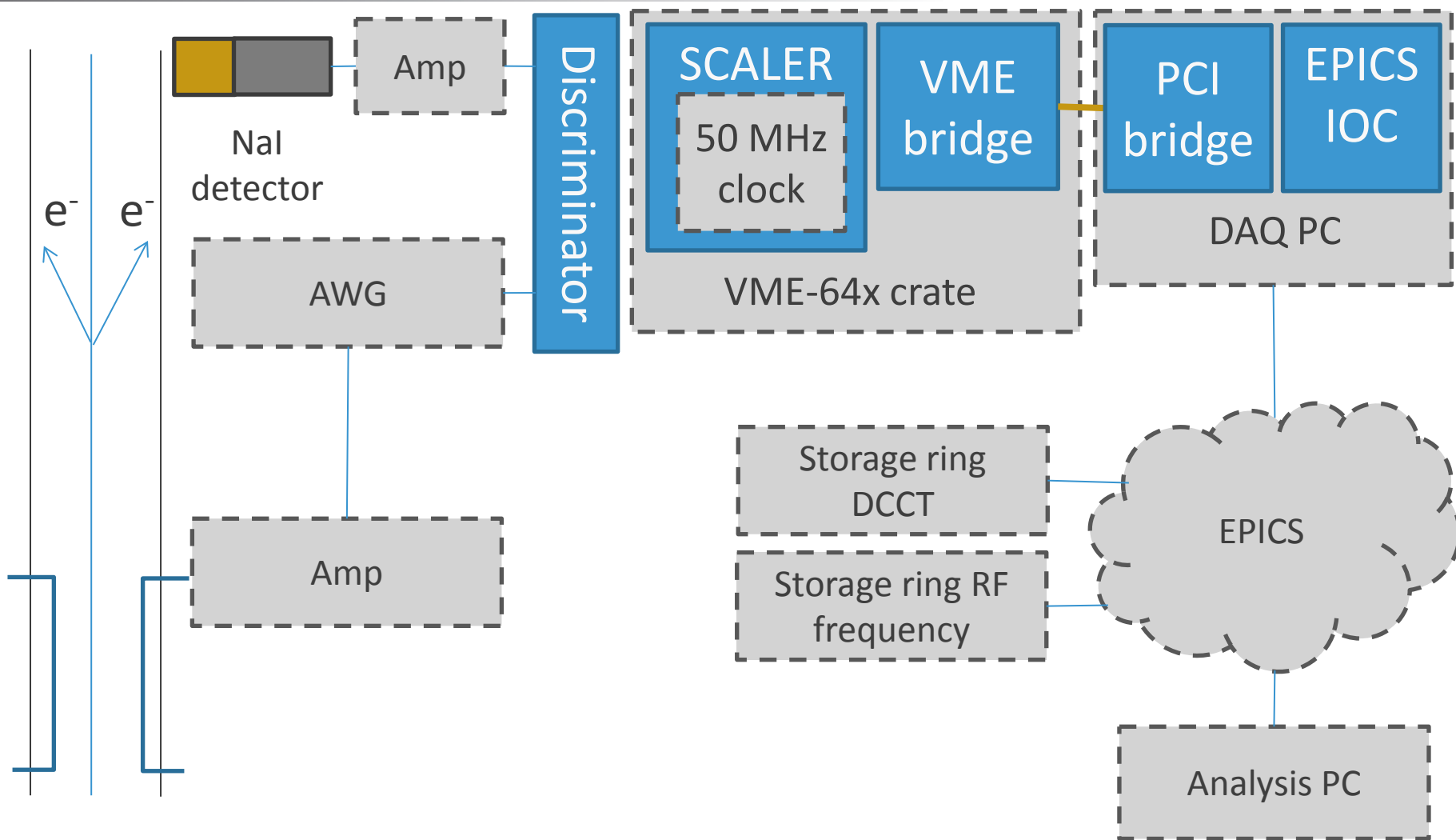


- Loss rate from Touschek scattering strongly dependent upon polarisation

Bernardini et al., 1963 Phys. Rev. Lett. 10, 407.

Serednyakov 1976, Sov Phys JETP 71, 2025. Supported by

Apparatus



Thomas – BMT equation

- Describes adiabatic spin evolution

$$\vec{\Omega}_{BMT} = -\frac{q_e}{\gamma m_e c} \left[(1 + a\gamma)\vec{B}_\perp + (1 + a)\vec{B}_\parallel - \left(a\gamma + \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right]$$

- For a storage ring light source, simplify to

$$\vec{\Omega}_{BMT} \times \vec{S} \equiv f_{spin} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c} [(1 + a\gamma)]$$

- By comparison with the cyclotron frequency

$$f_{cyc} = -\frac{q_e |\vec{B}_\perp|}{\gamma m_e c}$$

$$\therefore \nu_{spin} = a\gamma$$

$$a = 0.001\ 159\ 652\ 180\ 76(27)$$

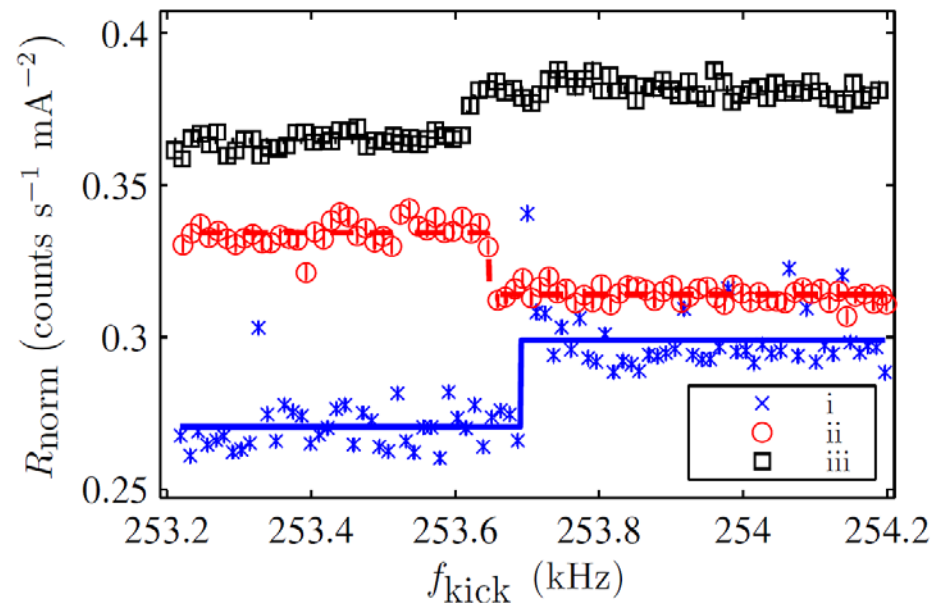
ν_{spin} is a frequency

Arnaudon 1995 Z. Phys. C. 66, 45

Beam energy measurement

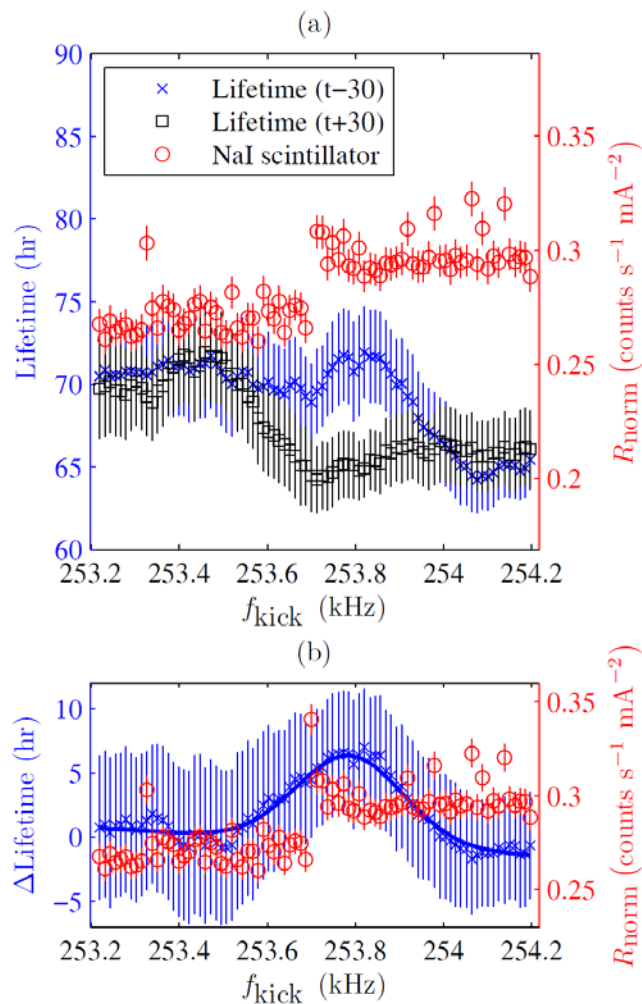
- 0.25362(2) MHz
- 2.997251(7) GeV

Ring		Measured
AS	E	3.013408(8)
SPEAR3	E	2.997251(7)

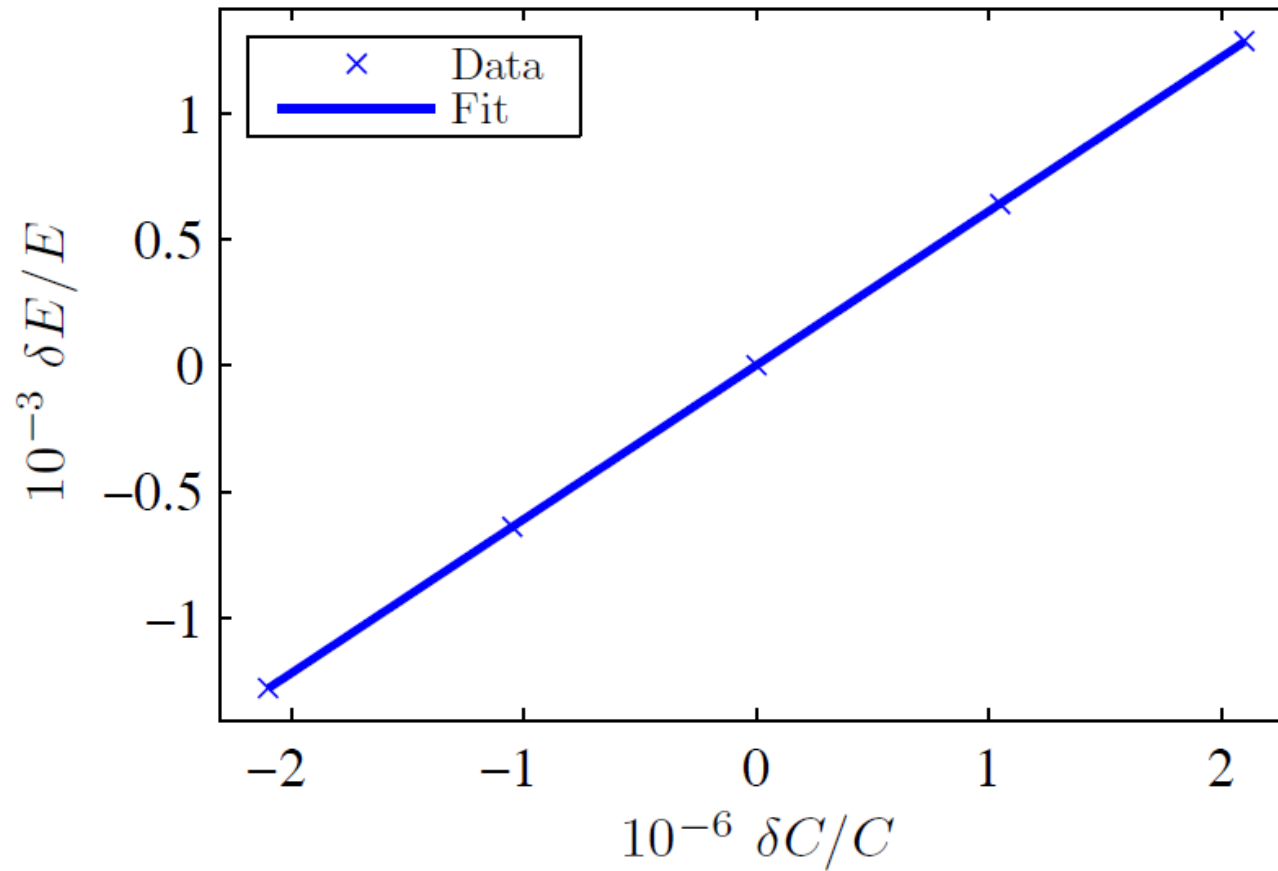


Wootton et al. (2013) Phys. Rev. ST –
Accel. Beams. 16, 074001

Beam Energy Measurement – Detector Choice



Momentum compaction factor



Wootton et al. (2013) Phys. Rev. ST – Accel. Beams. 16, 074001

Measured MCF

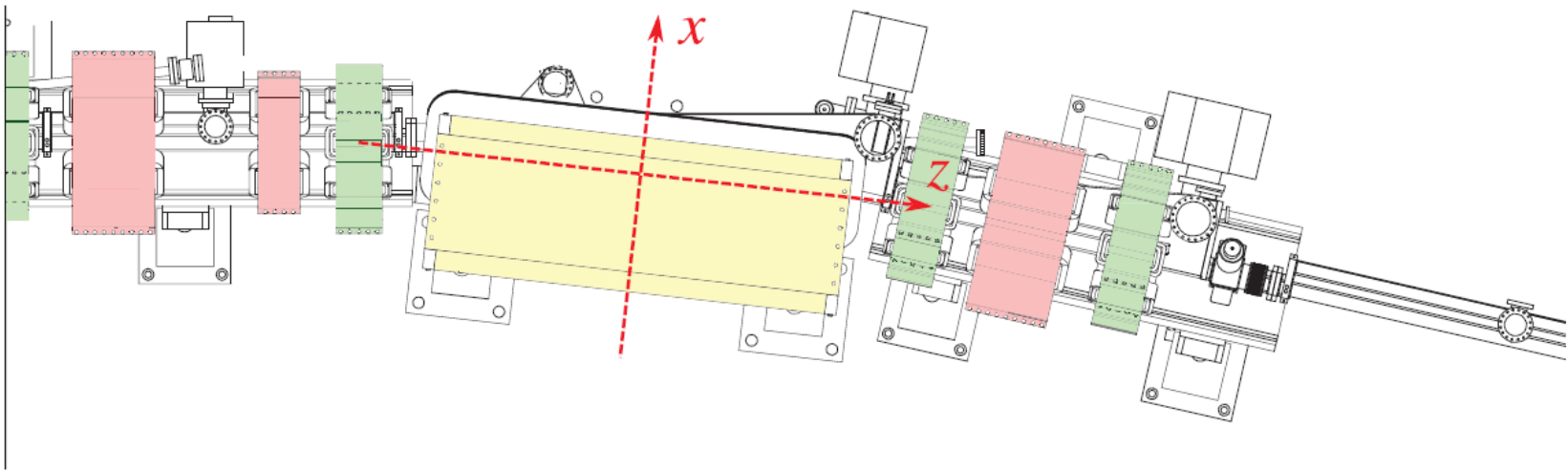
- Measured the momentum compaction factor
 - What does that tell us?

$$\alpha_c = \frac{1}{C} \oint_0^C \frac{\eta_x(s)}{\rho(s)} ds$$

- Tells us about value of dispersion function where there is bending
- Bending radius
 - How do we model the bending magnet?

Coordinate system for following equations

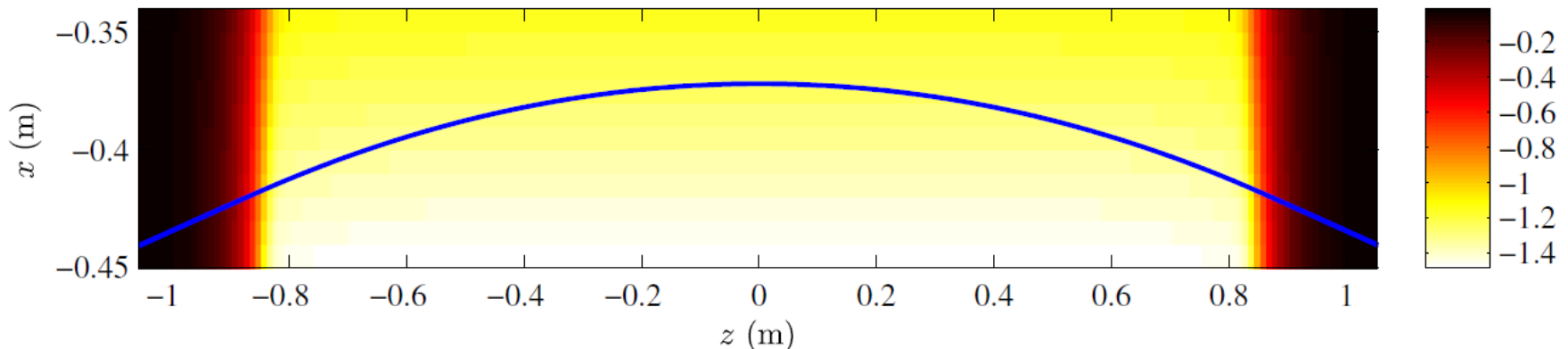
- We are trying to evaluate the trajectory, so we cannot use the normal curvilinear coordinates
- Define rectangular coordinate system along main bending magnet axis



Wootton et al. (2013) Phys. Rev. ST – Accel. Beams. 16, 074001

Numerical evaluation of trajectory

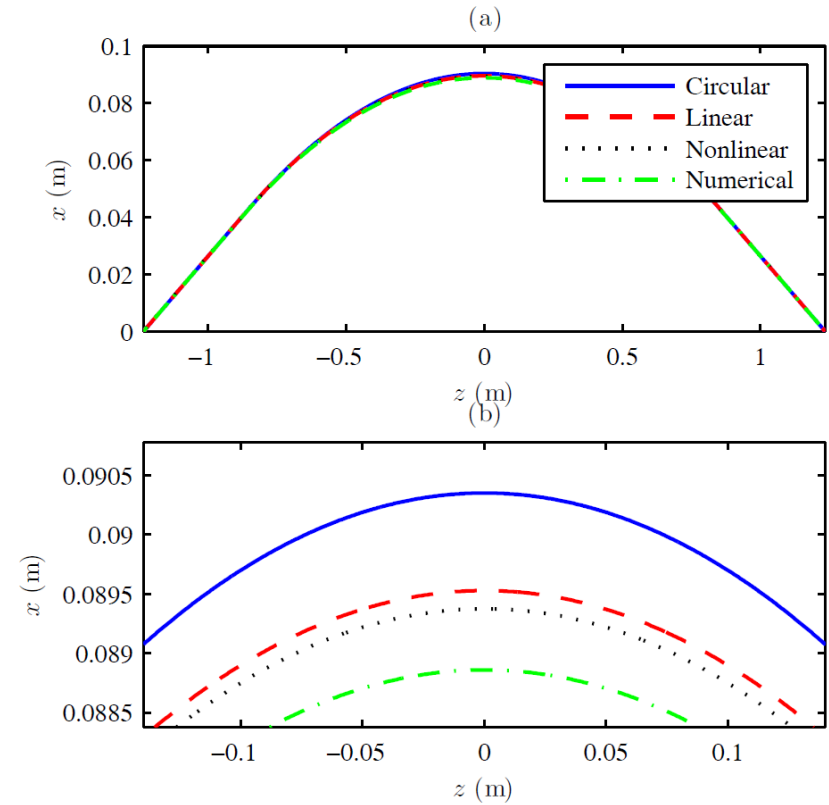
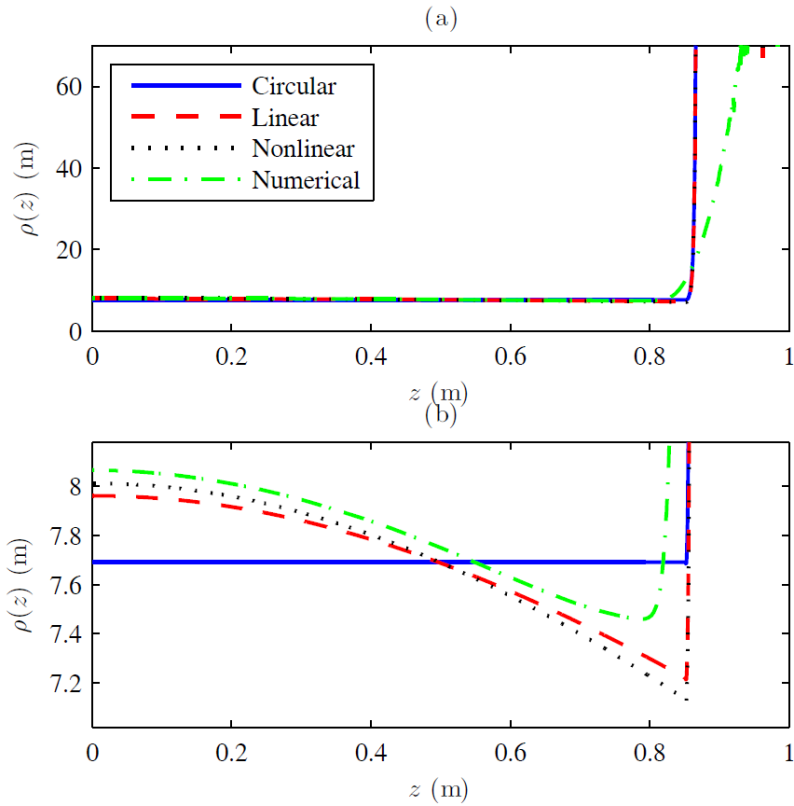
- Analytical models of trajectory
 - Circular
 - Linear hyperbolic cosine
 - Nonlinear hyperbolic cosine



- Magnetic field measured on horizontal mid-plane with Hall probe (2D – map)
- Fourth-order Runge-Kutta integration of trajectory
 - Constraints on total deflection angle, equal position at entrance and exit

Yoon 2004 NIMA, 9, 523

Comparison of models

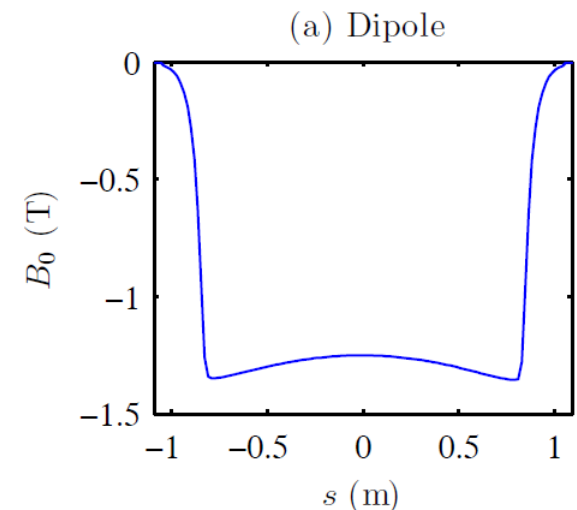


Wootton et al. (2013) Phys. Rev. ST – Accel. Beams. 16, 074001

Momentum compaction factor

Approach	AS	SPEAR3
Linear hyperbolic cosine	0.00205	0.00162
Numerical	0.00211	0.00165
Measured	0.00211(5)	0.00164(1)

- Within uncertainty, measured agrees with numerical integration and disagrees with linear hyperbolic cosine.
- A better model for the trajectory than usual hyperbolic cosine
- Accuracy in model comes from correct distribution of the dipole field



Summary

- Rectangular magnet with defocussing gradient
 - Circular, linear hyp. cosine, nonlinear hyp. cosine, numerical trajectories
- Simulation of momentum compaction factor
- Within uncertainty, measured momentum compaction factor agrees with numerical integration and disagrees with linear hyperbolic cosine trajectory
- Model accuracy comes from correct distribution of the dipole field

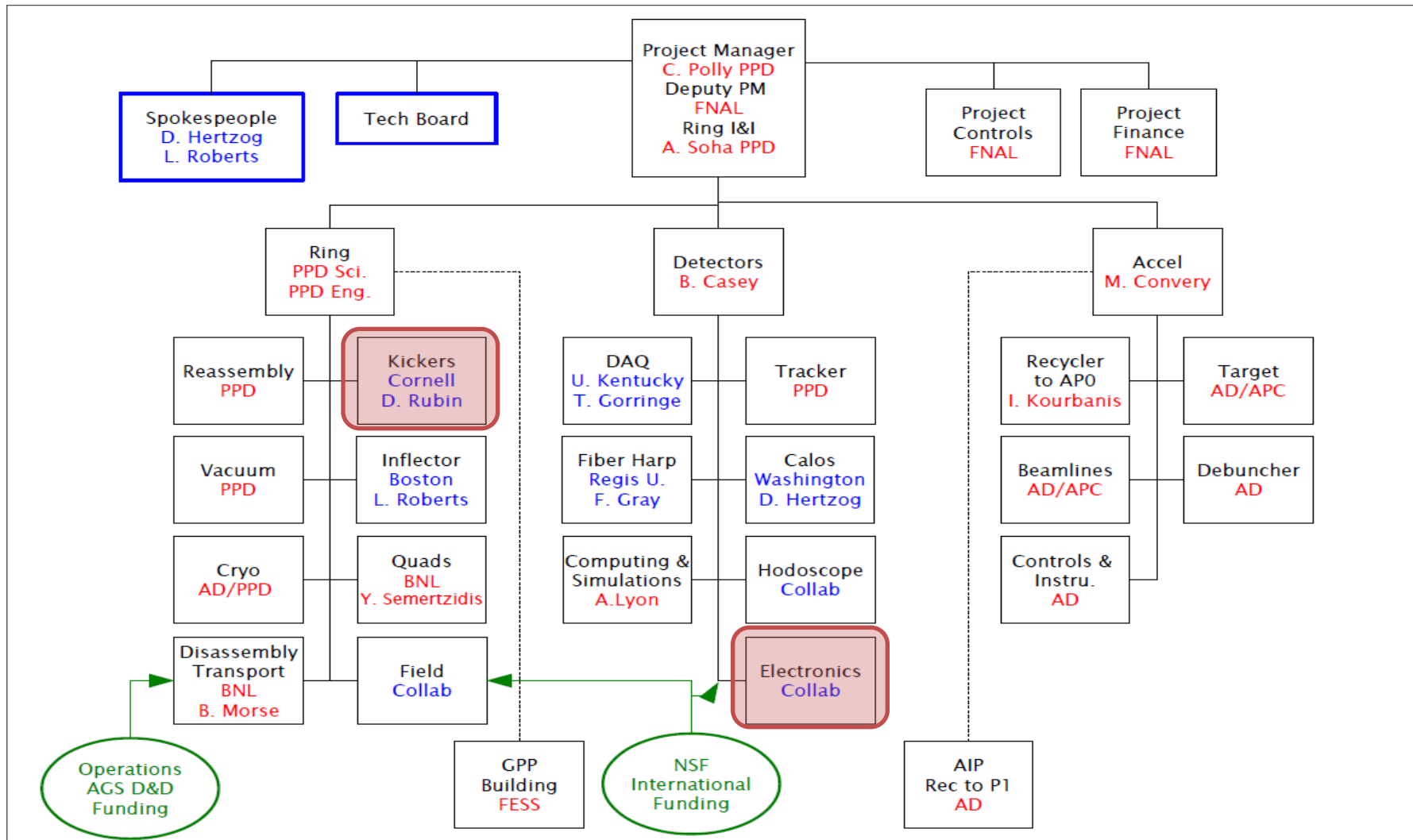
PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **16**, 074001 (2013)

Storage ring lattice calibration using resonant spin depolarization

K. P. Wootton,^{1,*} M. J. Boland,^{1,2} W. J. Corbett,³ X. Huang,³ G. S. LeBlanc,² M. Lundin,⁴ H. P. Panopoulos,^{1,†}
J. A. Safranek,³ Y.-R. E. Tan,² G. N. Taylor,¹ K. Tian,³ and R. P. Rassool¹

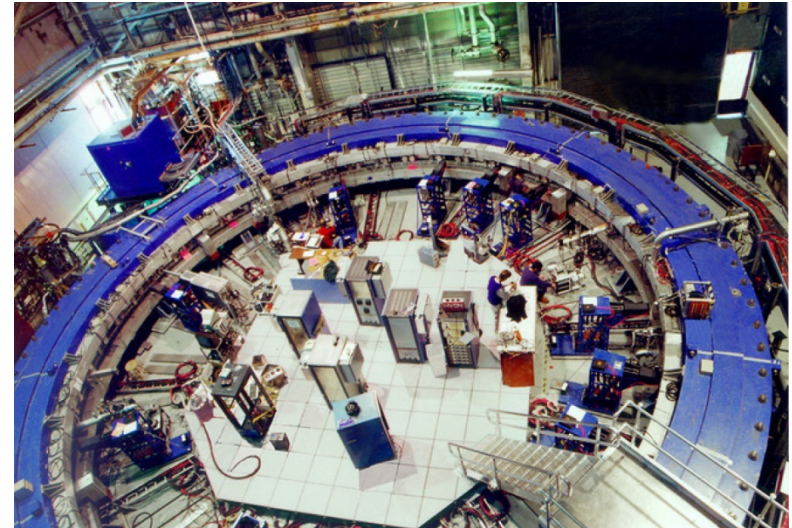
FERMILAB MUON G-2 EXPERIMENT (E989)

Fermilab muon g-2 collaboration



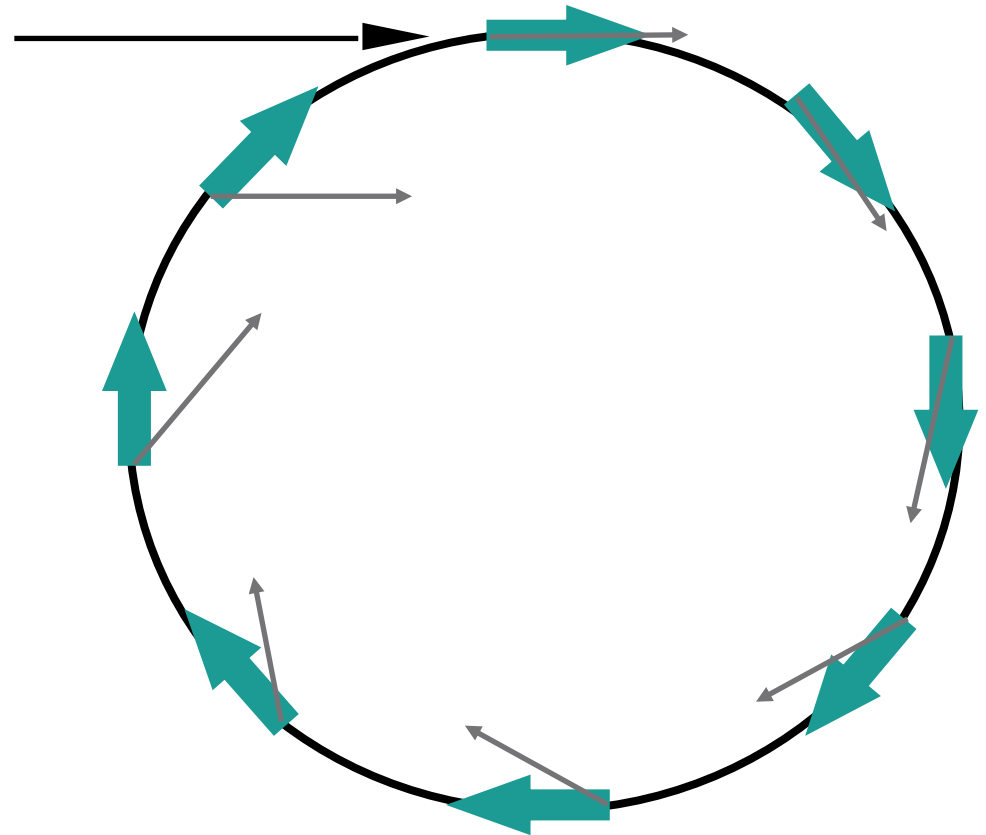
Muon g-2 accelerators

- Tertiary muon beam
- Proton beam
 - $p = 8.9 \text{ GeV}/c$
 - 2×10^{20} protons on target per year
- Pion beam
 - $p = 3.11 \text{ GeV}/c$
 - $\gamma\tau = 570 \text{ ns}$
- Muon beam
 - $p = 3.094 \text{ GeV}/c$
 - $\gamma\tau = 64 \mu\text{s}$



Muon storage ring

- Injected muon beam longitudinal polarisation
- Orbital angular momentum precesses at the revolution (cyclotron) frequency
- Spin precession advances ahead of orbital angular momentum
 - Thomas precession



Thomas – BMT equation

- Describes adiabatic spin evolution

$$\vec{\Omega}_{BMT} = -\frac{q_\mu}{\gamma m_\mu c} \left[(1 + a\gamma)\vec{B}_\perp + (1 + a)\vec{B}_\parallel - \left(a\gamma + \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right]$$

- For a storage ring **with** transverse electric fields

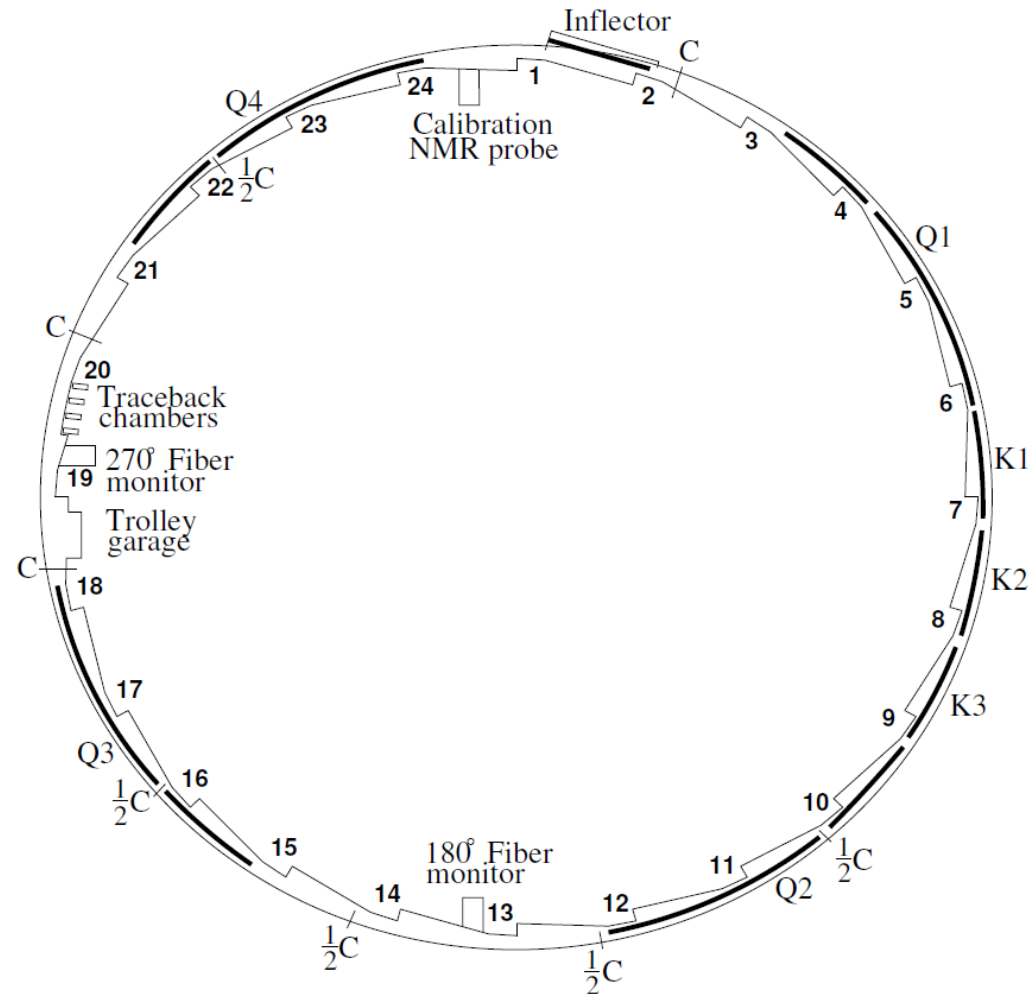
$$\vec{\Omega}_{BMT} \times \vec{S} \equiv f_{spin} = -\frac{q_\mu |\vec{B}_\perp|}{\gamma m_\mu c} \left[(1 + a\gamma) - \left(a\gamma + \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right]$$

- ‘Magic momentum’ Lorentz factor $\gamma = 29.3$, $\left(a\gamma + \frac{\gamma}{\gamma+1} \right) = 0$

Arnaudon 1995 Z. Phys. C. 66, 45

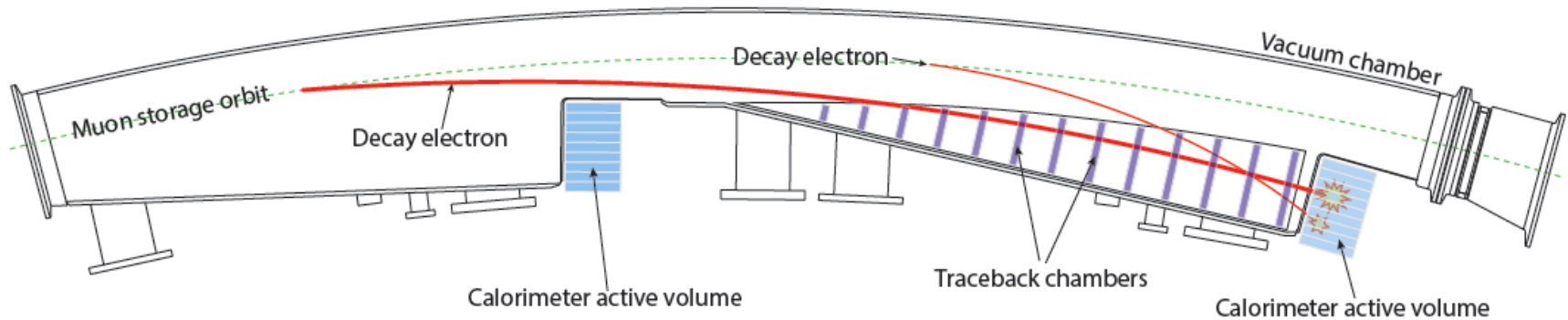
Muon storage ring

- Uniform bending field
 - Radius $\rho = 7.1$ m
 - Field $B = 1.45$ T
- Electrostatic quadrupoles
 - Vertical focussing
- Weak-focussing ring
 - Cyclotron $T_{\text{cyc}} = 149$ ns
 - Betatron tune $Q_x = 0.930$
 - $Q_y = 0.370$
 - Spin tune $Q_{\text{spin}} = 0.034$
- Decay electron detectors
 - Trackers
 - Calorimeters



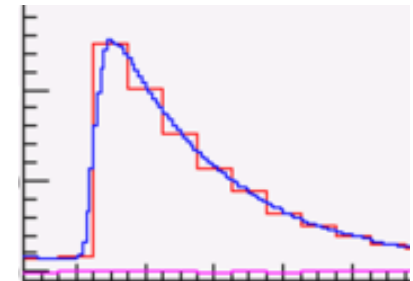
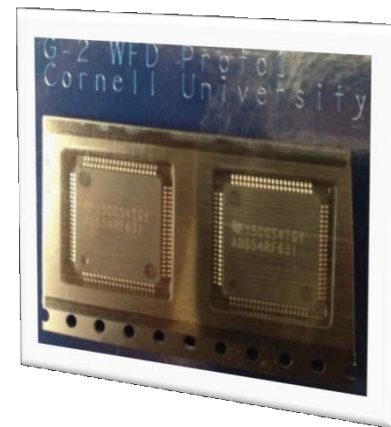
(The Muon (g-2) Collaboration), Phys. Rev. D, 73, 072003 (2006)

Decay electron detectors



Hertzog (The Muon (g-2) Collaboration) Fermilab PAC, 22 Jan 2014

- Calorimeter
 - PbF_2 scintillator
 - Decay time ≈ 18 ns
- ADC
 - Texas Instruments ADS54RF63
 - 12-bit ADC
 - 550 M samples per second



<http://www.ti.com/product/ads54rf63>

Decay electron detectors



MRI Submitted

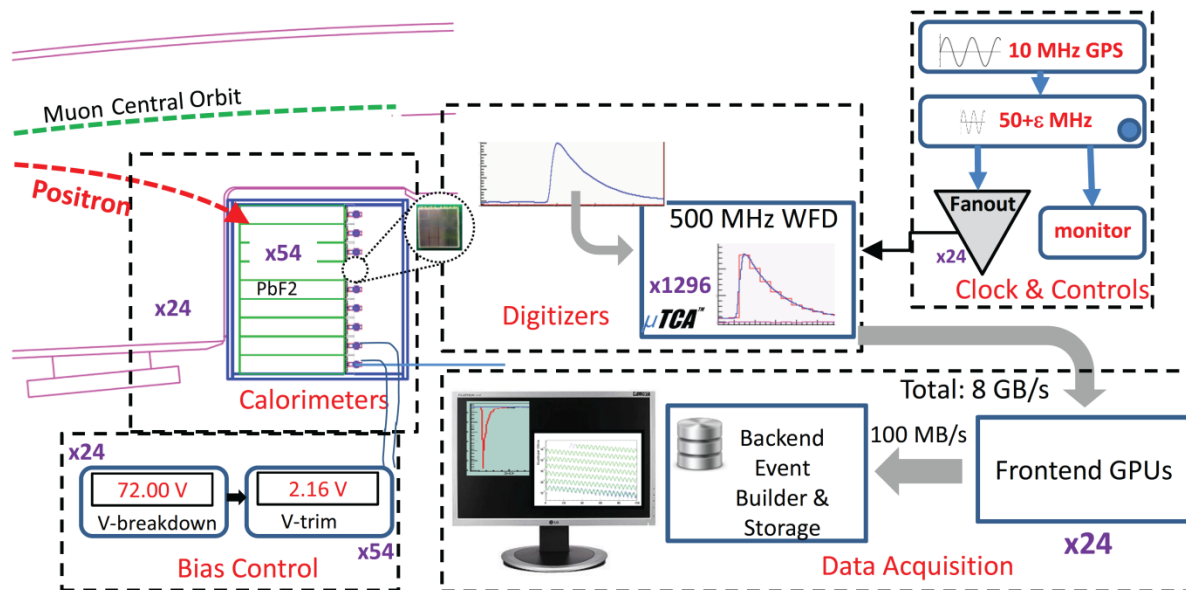
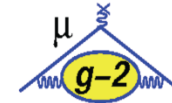


Figure 2: Schematic of the ω_a instrumentation organized by dedicated systems. The participating institutes have well-defined responsibilities: Calorimeter (Washington, Shanghai), Bias Control (Virginia, JMU), Digitizer (Cornell), Clock & Controls (Illinois), Data Acquisition (Kentucky)

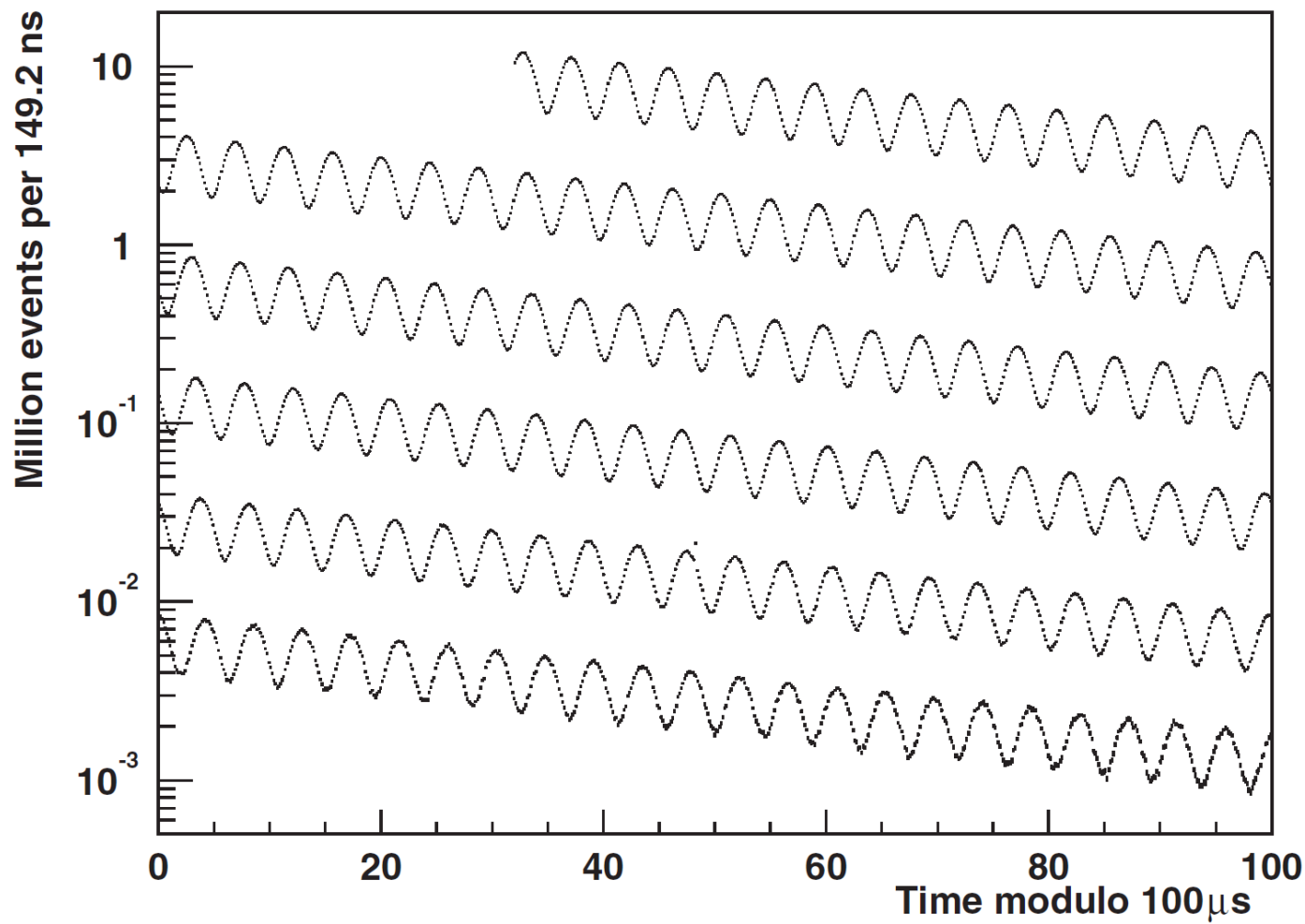
2/19/2013

Detector Update, Muon g-2 PGM

10

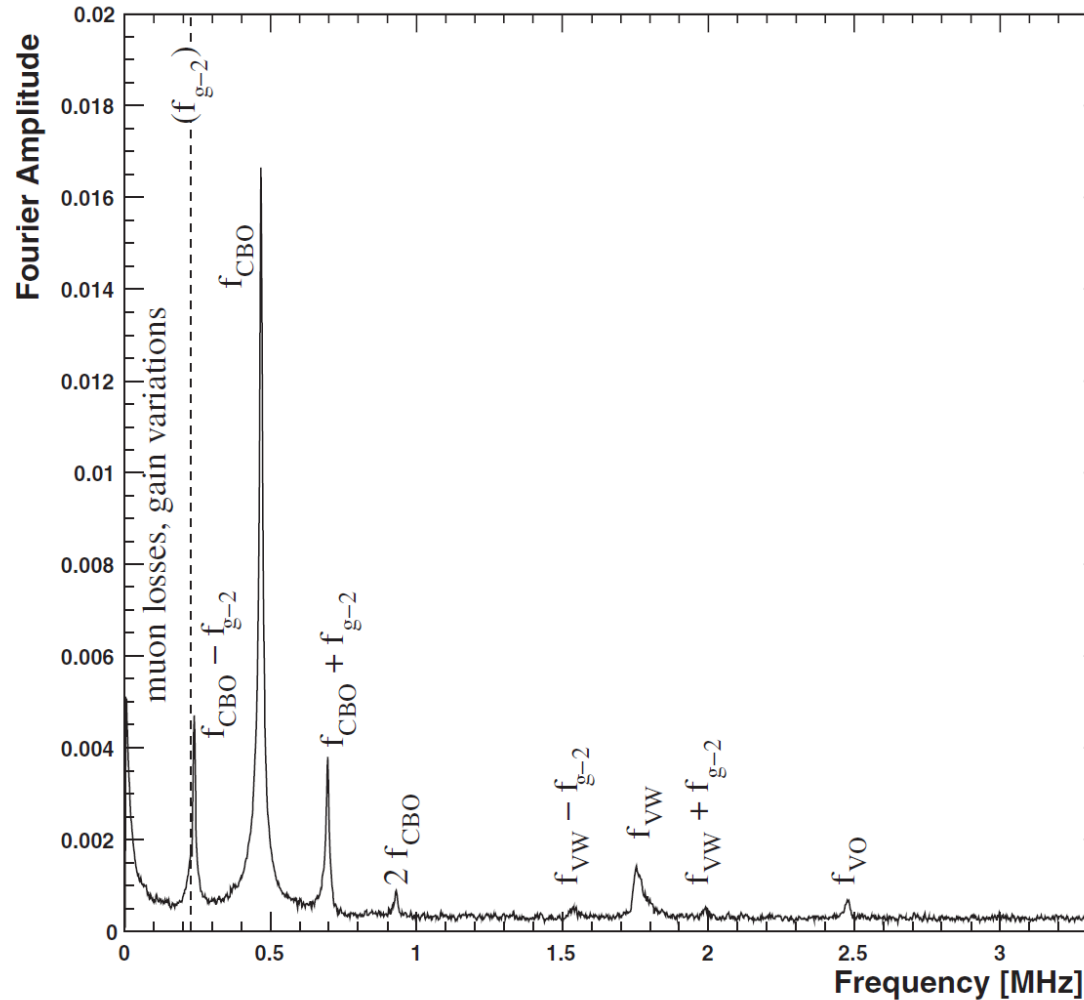
Hertzog (The Muon (g-2) Collaboration) Fermilab PAC, 22 Jan 2014

Decay electron detectors



(The Muon (g-2) Collaboration), Phys. Rev. D, 73, 072003 (2006)

Fourier transform – tune space



(The Muon ($g-2$) Collaboration), Phys. Rev. D, 73, 072003 (2006)

Summary

- Aim to measure muon anomalous magnetic moment to new precision
- Exploit magic momentum of 3.094 GeV
- Measure spin tune, cyclotron tune
 - Important to minimise systematic and statistical uncertainties
 - Avoid horizontal betatron tune

FUTURE ELECTRON BEAM G-2 EXPERIMENTS

Circular Unruh Effect

- Electromagnetic analogue of Hawking radiation
- Need accelerations possible approaching black holes
 - Of the order $a = 10^{20} \text{ m s}^{-2}$ ($E = 1400 \text{ MV m}^{-1}$)
 - Linacs $E \approx 10 - 100 \text{ MV m}^{-1}$

- Circular accelerations

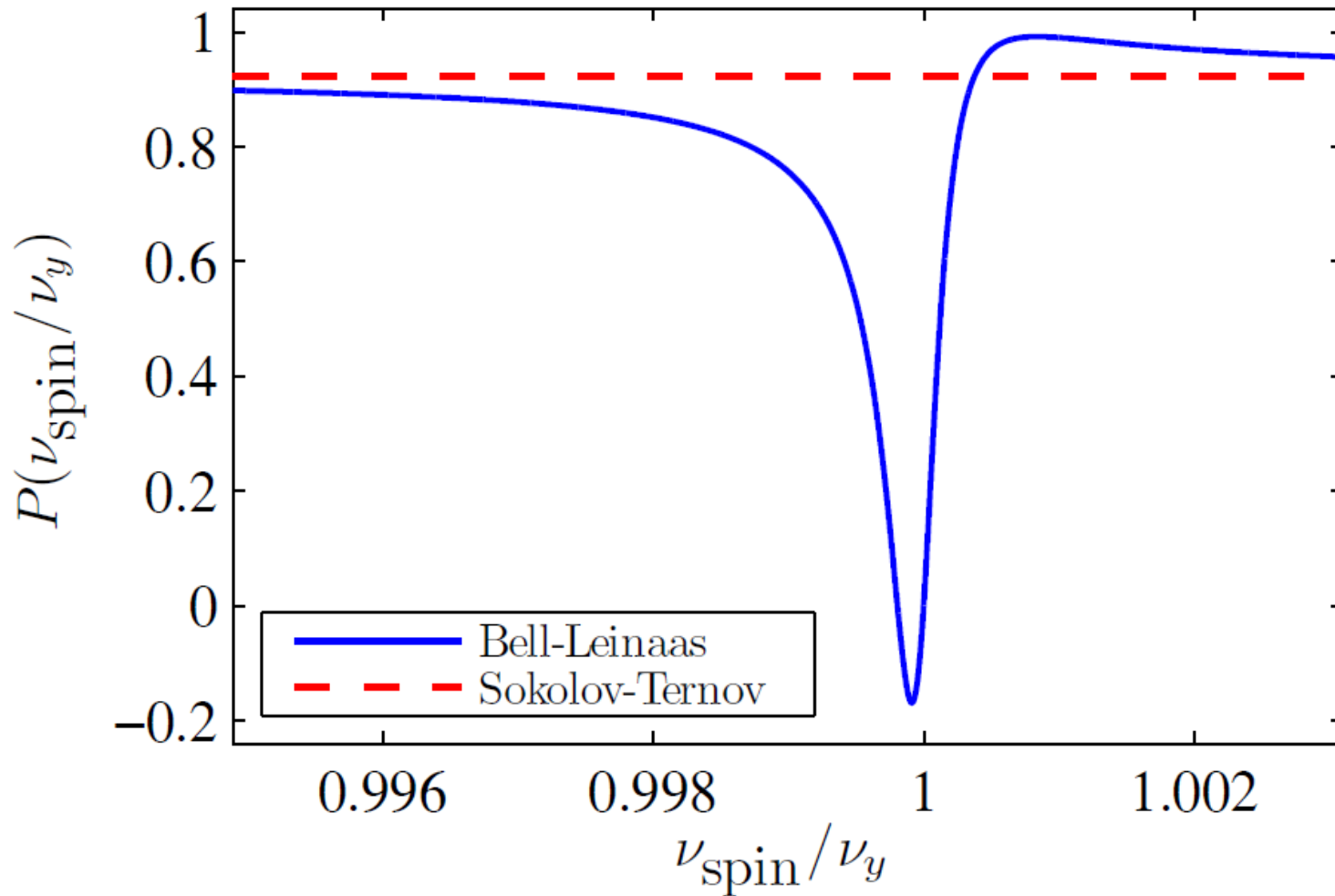
$$a = \frac{\gamma^2 c^2}{\rho}$$

- For $\gamma = 6000$, $\rho = 8 \text{ m}$

$$a = 10^{23} \text{ m s}^{-2}$$

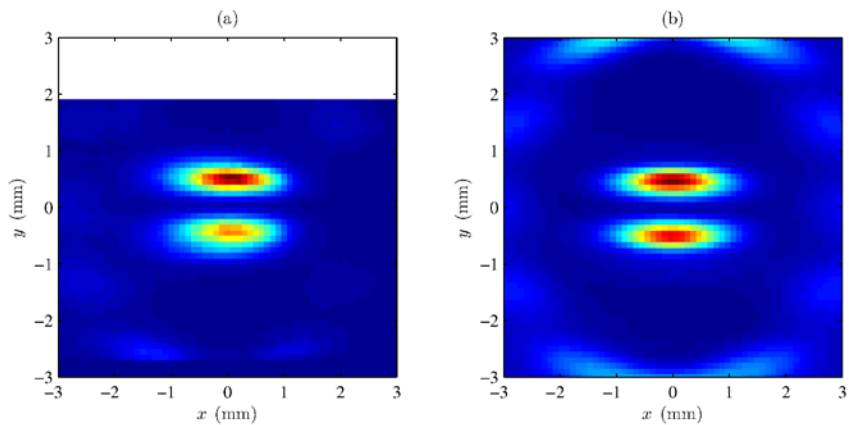
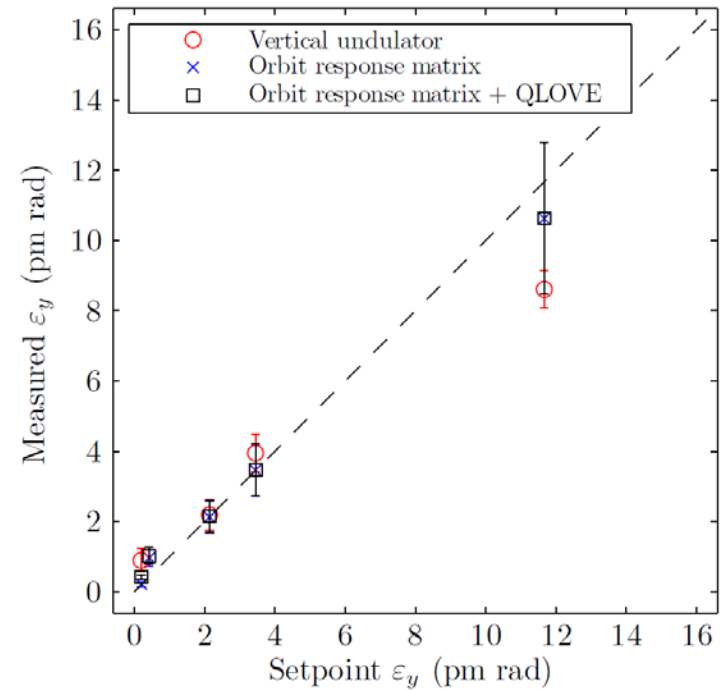
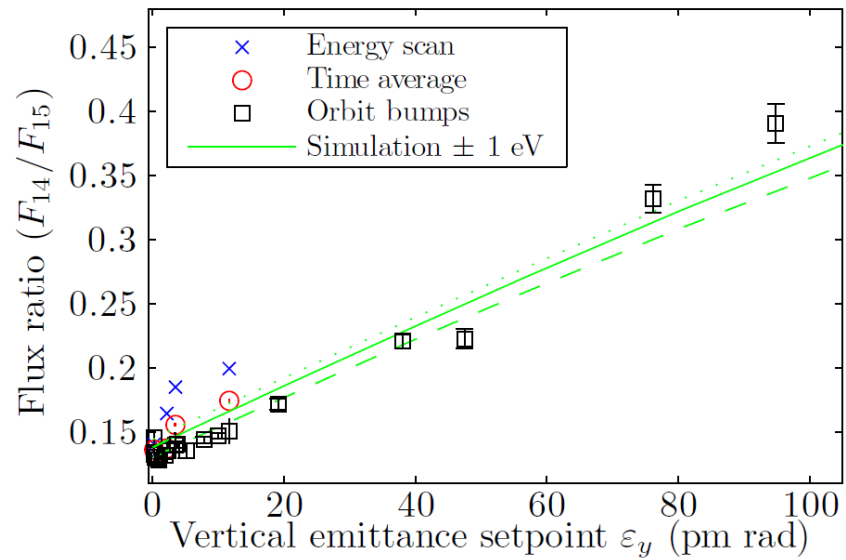
Bell and Leinaas (1983) Nucl. Phys. B 212, 131

Bell-Leinaas Effect



Bell and Leinaas (1983) Nucl. Phys. B 212, 131

Future electron beam g-2 experiments



K.P. Wootton, et al. (2012) Phys. Rev. Lett. 109, 194801.

Conclusions

- Spin is not a property often considered in accelerators
- Measure frequencies
- Determine either:
 - Properties of the accelerator (assume a known)
 - Properties of the beam (assume accelerator known)
- Electron experiments demonstrated calibration of unconventional bending magnet
- Fermilab muon $g-2$ experiment precision measurement
- Proposed experiment of circular Unruh effect using ultralow vertical emittance electron rings

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Turning bright ideas into brilliant outcomes



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