# Uplifting AdS/CFT to Cosmology

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hep-th/1108.5732, hep-th/1005.5403, w/Xi Dong, Shunji Matsuura, Eva Silverstein, Gonzalo Torroba, also work in progress

See also: hep-th/0407125, hep-th/0504056 (Alishahiha, Karch, Silverstein, and Tong); hep-th/0908.0756 (Polchinski and Silverstein)

# Outline

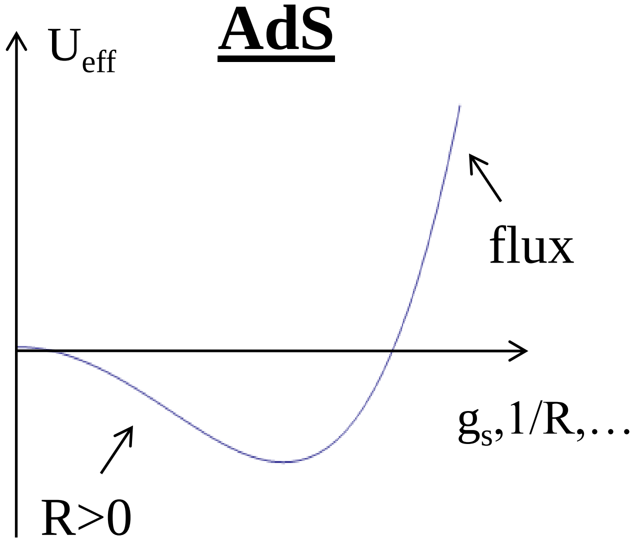
- Motivation: holographic formulation of cosmological spacetimes (dS, FRW…)
- Magnetic flavors and uplifitng
- Constructing the dS/dS correspondence
- Decay to FRW and flavor brane holography
- Prospects: field theory side

# Holographic dual of cosmological spacetime

- General considerations make dS holography hard: boundary is spacelike and disconnected, no SUSY or S-matrix, loss of causal contact, metastability and eternal inflation, gravity does not decouple, only finitely many d.o.f. are accessible.
- Thought laboratory for conceptual questions in gravity
  - Micro origin of $S_{GH}$
  - Growth of $N_{dof}$ as H decreases
  - Metastability and decay of de Sitter
  - Landscape of vacua and measure problem
- Today: brane construction of dS/dS and for FRW phase after CdL decay; parametric micro accounting of $N_{dof}$

# Uplifting AdS/CFT

- Use zeromode effective potential on Freund-Rubin solutions w/ known dual




D3 $\rightarrow$ $AdS_5$ x $S^5$

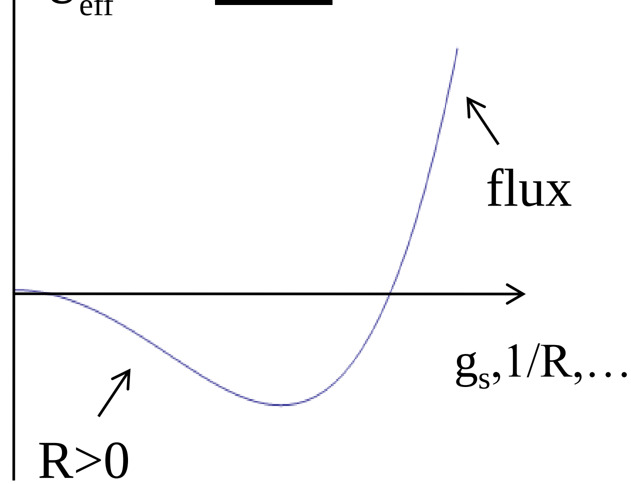
$$\frac{U}{M_5^5} \approx \left(\frac{g_s^2}{R^5}\right)^{2/3} \left(R^5\right) \left(-\frac{1}{g_s^2 R^2} + \frac{N_3^2}{R^{10}}\right)$$

$\rightarrow R^4 \approx g_s N_3$

# Uplifting AdS/CFT

- Use zeromode effective potential on Freund-Rubin solutions w/ known dual

**AdS**



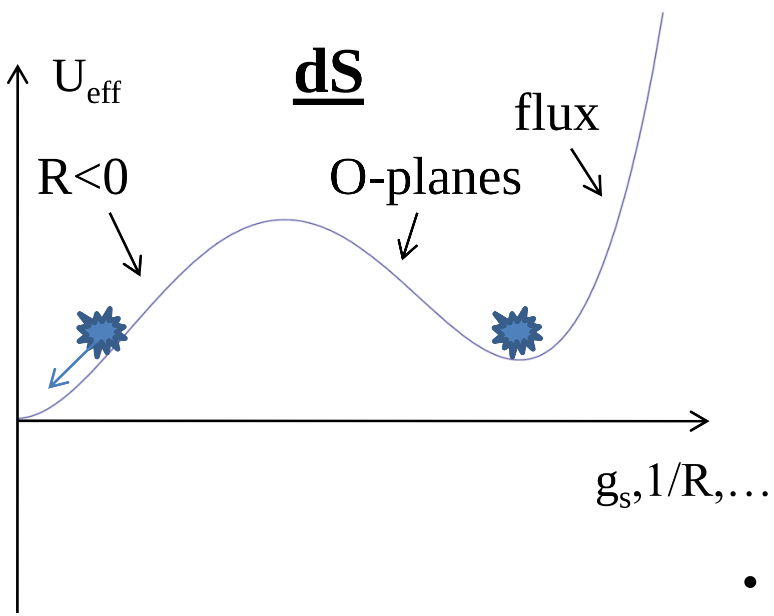

D1–D5 → $AdS_3 \times S^3 \times T^4$

$$\frac{U}{M_3^3} \approx \left(\frac{g_s^2}{R^3L^4}\right)^2 \left(R^3L^4\right)\left(-\frac{1}{g_s^2R^2}+\frac{1}{R^6}\left(\frac{N_1^2}{L^8}+N_5^2\right)\right)$$

$\rightarrow L^4 \approx N_1/N_5;\ R^2 \approx g_s N_5$

# Uplifting AdS/CFT

- "abc" structure; uplift curvature to $\Delta n = n_{branes} - n_R > 0$




- (p,q) 7-branes, stringy cosmic 5-branes (Greene et al. '90, Hellerman + McGreevy '02), KK5s can compete w/curvature.
  - F-theory techniques and GLSM control backreaction
  - $\Delta n = 0$ gives CY, e.g. 24 7-branes uplift sphere to K3
  - Time-dependent solutions with $\Delta n > 0$ (cf. Kleban + Redi '07)
  - Magnetic/dyonic flavors
- O-planes, topology provides negative term for metastable solution.
  - SO, Sp gauge groups
- FRW region: only a-term matters

# Worked example: $dS_3 \times S^3/Z_k \times T^4$

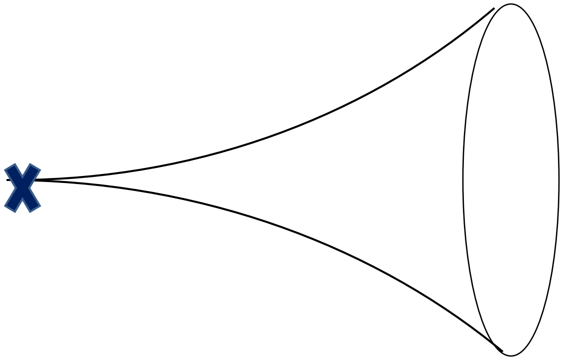
- D1-D5 system; SC5s, NS5s lift curvature to $\Delta n = \varepsilon \rightarrow$ in higher dimensions these are magnetic/dyonic flavors, in 1+1d the target space is fibered.
- Orbifold boosts strength of O-planes
- Multiple moduli ($R$, $L$, $g_s$, $R_f$) requires nested abc structure
- Weak coupling, weak curvature/large radius, and a hierarchy achieved
- Macro parameters in terms of micro inputs:

$$S_{dS} \approx M_3 R_{dS} \approx \frac{1}{\varepsilon^{3/2}} k N_1 N_5$$

1-5 strings dominate entropy; cf. quiver gauge theory w/ winding enhancement

# Brane construction

- Can use zeromode effective potential or look at Einstein equations directly for colorbranes probing transverse space $ds^2 = dw^2 + R^2(w)ds^2_\perp$
- AdS: noncompact cone over $S^5$, $S^3 \times T^4$, etc.

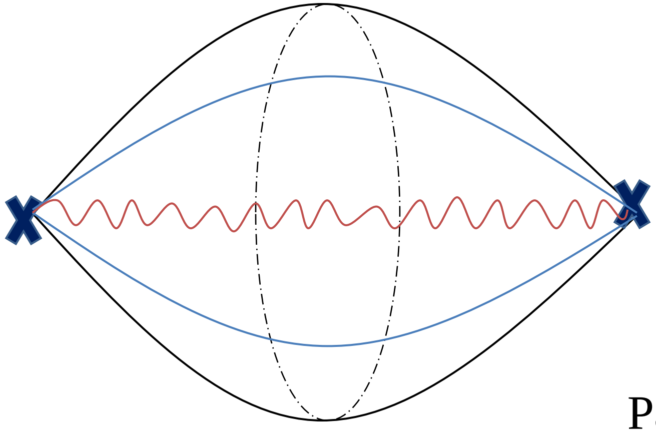


$$\frac{R(w)'^2}{R(w)^2} = \frac{1}{R(w)^2}$$ cf. Friedmann eqn.

$$\longrightarrow R(w) \sim w$$

# Brane construction

- Can use zeromode effective potential or look at Einstein equations directly for colorbranes probing transverse space $ds^2 = dw^2 + R^2(w)ds^2_\perp$
- dS: compact space; need colorbranes and antibranes!
- Unstable to Schwinger (or CdL) decay on long timescales



$$\frac{R'^2}{R^2} = -\frac{1}{R^{n_1}} + \frac{1}{R^{n_2}} \qquad n_1 < n_2$$

uplifting and negative terms

Pair of warped throats with cutoff; gravity not decoupled

# de Sitter/de Sitter

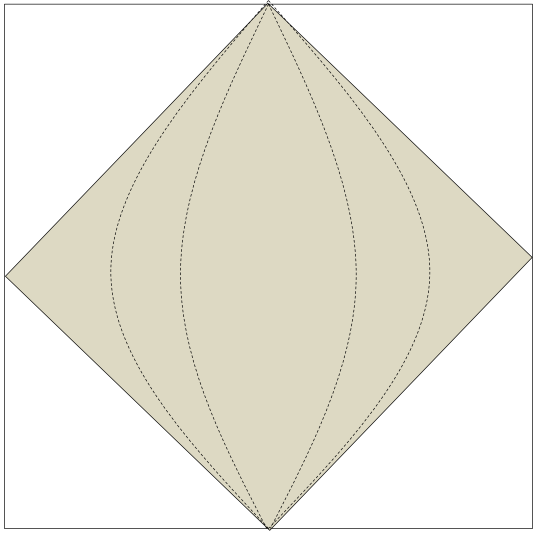
- Foliate $dS_d$ by $dS_{d-1}$

$$ds^2 = dw^2 + \sin^2\left(\frac{w}{R}\right) ds^2_{dS_{d-1}}$$

- cf. $dS_{d-1}$ slicing of AdS

$$ds^2 = dw^2 + \sinh^2\left(\frac{w}{R}\right) ds^2_{dS_{d-1}}$$

- Can slice region shown by global slices, or static patch by static patches.



- Gravity on $dS_d$ ↔ (single lattice point of) 2 QFTs cut off and coupled to each other and to gravity; thermal with T ~ $1/R_{dS}$

# de Sitter/de Sitter

- A "semiholographic" correspondence, but can in principle be iterated…
- Gravity side checks: probe branes, gravitons, heat capacity…
- QM system is similar to AdS/CFT at lengths below the AdS scale.

# Connections to other proposals?

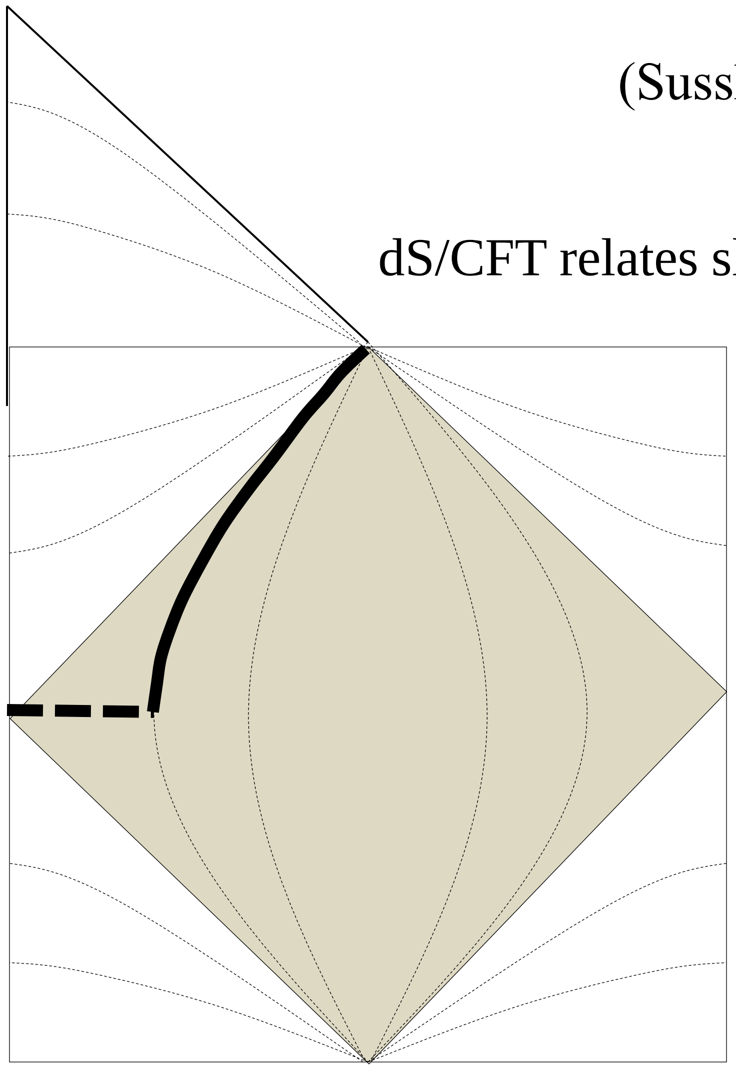
- Uplifting landed us on dS/dS. Other approaches include analytic continuation, (Strominger '01, McFadden & Skenderis '09, Anninos, Hartman and Strominger '11) embedding a dS bubble within AdS, (Freivogel, Shenker et al. '05) or focusing solely on the FRW region within a bubble. (Freivogel, Sekino, Susskind and Yeh '06)
  - Make sure not to analytically continue to imaginary flux…
- May be different aspects of a single description?
- Most descriptions are of limited precision (Harlow and Susskind '10); dS/dS can decay though, so it may have an extension to something more precise.

# An optimistic view of de Sitter

(Susskind and others…)

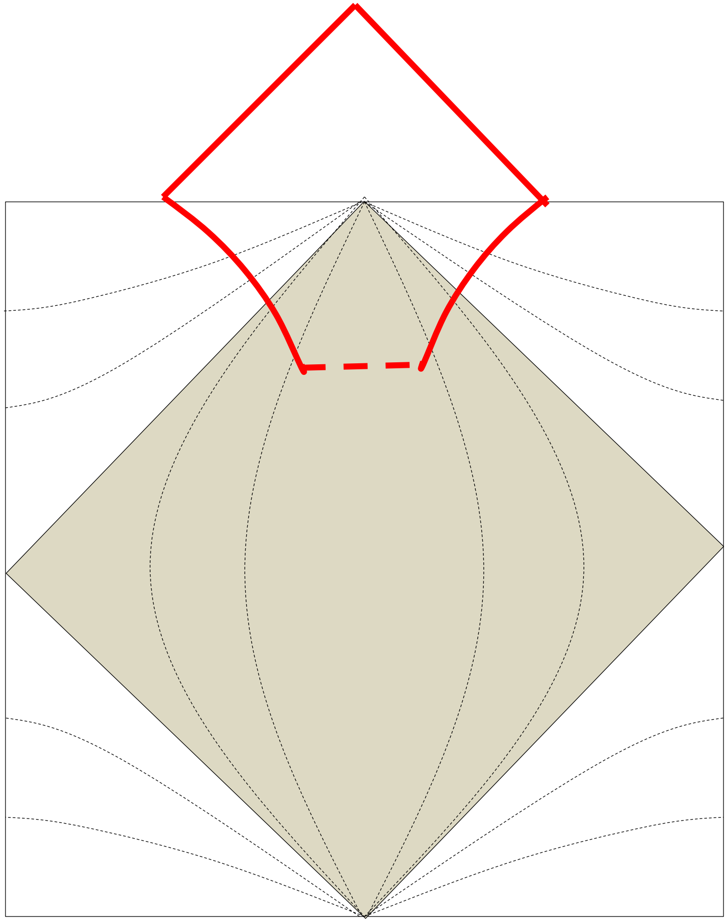
FRW/CFT relates FRW region inside bubble to boundary

dS/CFT relates slice to boundary



dS/dS relates slices to patch

# Part II: Decay to FRW and flavor brane holography



dS/dS describes a metastable region and can be continued….

Only the magnetic flavor “a” term is relevant now.

# FRW region of uplifted AdS/CFT

- Uplifting to $\Delta n > 0$ and CdL decay gives a specific metric with open slices and an uplifted sphere

$$ds^2 = -dt_s^2 + \frac{t_s^2}{c^2} dH_{d-1}^2 + \frac{t_s^2}{\hat{c}^2} dH_n^2 + (dx_f + A)^2$$

Vacuum solution if $c^2 = (d-n-2)/(d-2)$, $\hat{c}^2 = (d-n-2)/(n-1)$; RR and metric flux dilute away.

- d-dim'l Einstein frame metric

$$ds^2 = -dt_E^2 + c^2 t_E^2 dH_{d-1}^2$$

- $\Delta n = n - n_R > 0 \leftrightarrow c > 1$; singularity at $t = 0$ avoided in CdL case.

# Warped metric for FRW

- This metric has a warped product description for c > 1!

$$ds^2 = c^2 (t_{UV}^{2/c} - w^2)^{c-1} dw^2 + \left(1 - \frac{w^2}{t_{UV}^{2/c}}\right)^{c-1} (-dt_{UV}^2 + c^2 t_{UV}^2 dH_{d-2}^2)$$

Redshifting of proper energies by warp factor $f = \left(1 - \frac{w^2}{t_{UV}^{2/c}}\right)^{(c-1)/2}$

Strongly warped region should have an EFT description!

Time dependence is subdominant: $\left|\frac{\partial_{t_{UV}} f}{\partial_w f}\right| \propto t_{UV}^{-(1-1/c)} \to 0$

Alternate slicings are possible but not as clear to interpret.

# Gravity side checks: motion sickness

- Solve e.o.m for massive point particles (KK modes, closed strings, 7-7 stretched strings): these stay at constant warp factor, even when the mass is time-dependent.
  - cf. Kerr-CFT
- Solve DBI action for colorbranes → these move up the throat to regions of less warping. "Motion sickness"
  - Consistent with diluting flux and flavor sector holography
  - "Technicolor Yawn" anticipates connection to warped throats…

# Gravity side: parametrics

- Gravity decouples at late times as additional d.o.f. renormalize the Planck mass, and scalings of cutoff and $N_{dof}$ can be found using the (d-1)-dim'l quantum energy density.

$$N_{dof}\Lambda^{d-3} \equiv M_{d-1}^{d-3} \sim M_d^{d-2}\int\sqrt{-g}\,g^{ww}dw \sim M_d^{d-2}t_{UV}$$

$$H^2 \approx \frac{1}{t_{UV}^2} \sim N_{dof}\Lambda^{d-1}G_{N,d-1}$$

$$\rightarrow \Lambda \sim \frac{1}{t_{UV}},\ \ N_{dof} \sim t_{UV}^{d-2}$$

- # d.o.f. per lattice point agrees with Bousso's covariant entropy bound, which predicts $S \propto t_{UV}^{d-2}$ for the entropy within the apparent horizon, for c > 1.
- Also agrees with the Brown-York quasilocal stress-energy tensor.

# Green's functions

- Massive particles: reduces to solving geodesic equations. Using the metric $ds_d^2 = -dt^2 + (ct)^2 (d\chi^2 + \cosh^2 \chi dH_{d-2}^2)$ makes the calculation easier.
  - KK mode correlators have power law dependence on geodesic distance; cf. massive particles in AdS/CFT

$$G_{equal-t_{UV}}(\Delta x) \propto \frac{1}{(\Delta x)^{2n_{KK}}}; \;\; \Delta x = \Delta X_{geodesic}/(ct_{UV})$$

- Massless particles: requires full mode expansion in CdL geometry
  - Universal term from localized UV zero-mode; subdominant term only for c>1: (Note that $t_{UV} = t(\cosh \chi)^c$)

$$G_{equal-t}(\Delta X_{geodesic}) \sim \log \frac{\Delta X}{t} + \frac{t^{(d-2)(\sqrt{1-1/c^2}-1+1/c)}}{(\Delta X)^{(d-2)/c}} \log \frac{\Delta X}{t}$$

# Micro-accounting in FRW

- Stretched strings/junctions between flavor branes parametrized by $\{n_{str}, n_f, k_f$, group theory$\}$. E.g. in 5d:
  - core size of n strings ~ $n_{str}^{\ 1/5} < t_s \longrightarrow n_{str} \leq t_{UV}^{15/7}$
  - $O(t_s)$ windings around fiber see it is contractible $\rightarrow$

    $n_f \leq t_{UV}^{3/7}$
  - $k_f > t_s$ is a weakly bound state of stretched strings and gravitons, and it decays $\rightarrow k_f \leq t_{UV}^{3/7}$
- Assume there are no additional factors from the group theory (reasonable with certain assumptions on what constitutes an 'elementary' state)

# Micro-accounting in FRW

- Together, the number of states is $(n_{str}\, n_f\, k_f)_{max} \sim t_{UV}^3$, in agreement with the macro count of $N_{dof}$ and the covariant entropy bound!
- Not sure whether this had to work, but it works in more general dimensions…

$$n_{str} \leq t_s^{d+2m-4}, \quad n_f \leq t_s,\ k_f \leq t_s \rightarrow (n_{str}\, n_f\, k_f)_{max} \sim t_{UV}^{d-2}$$

# Summary

- Using magnetic flavor branes to uplift AdS/CFT dual pairs, we can find brane constructions realizing holography on dS and its decay to open FRW.
- For de Sitter, we have a microscopic construction of the dS/dS correspondence, and a fully stabilized example in 3D where the entropy can be parametrically expressed in terms of the microscopic parameters.
- When dS/dS decays to FRW, we have a warped region sourced by flavor branes; we have a number of gravity side checks and a suggestive match between a micro count of stretched string states and the covariant entropy bound.

# Prospects

- Want to understand field theory better
  - Connection between $\beta$ function and moduli stabilization?
  - How do magnetic flavors source time dependence in field theory?
  - What is the UV completion on the (d-1) –dimensional side of dS/dS?
- $dS_4$ worked example – possible connections to ABJM
- Connections to other versions of dS holography? Does micro construction tell us anything about the measure problem?

# Thank you!

Laboratory of Elementary Particle Physics, Cornell University
Stanford Institute for Theoretical Physics
SLAC National Accelerator Laboratory
Kavli Institute for Theoretical Physics/UC Santa Barbara
National Science Foundation
US Department of Energy Office of Science
Mr. and Mrs. William K. Bowes, Jr.
You, the audience!

# Backup slides: details of $dS_3$ worked example

# $dS_3$ worked example

- Builds upon $AdS^3 \times S^3/Z_k \times T^4$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D1$ | x | x | | | | | | | | |
| $D5$ | x | x | | | | | x | x | x | x |
| $O5$ | x | x | | x | x | | | | x | x |
| $O5'$ | x | x | x | | | x | x | x | | |
| $\rho 5$ | x | x | x | x | | | | | x | x |
| $\rho 5'$ | x | x | | | x | x | x | x | | |
| NS5 | x | x | | x | x | | x | | x | |
| NS5' | x | x | x | | | x | | x | | x |
| $D7, \overline{D7}$ | x | x | x | x | x | x | | x | x | |
| $D7', \overline{D7'}$ | x | x | x | x | x | x | x | | | x |

All branes mutually SUSY except D7-branes, anti-D7 branes

Improved constructions use D5, anti-D5s

# $dS_3$ worked example

Moduli

size of $S^3$: R

size modulus of $T^2$: $b_T + L^2$

Size of Hopf fiber: $\beta = kR_f/R$

String coupling: $\eta = g_s/(L^2R^2)$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D1$ | x | x | | | | | | | | |
| $D5$ | x | x | | | | | x | x | x | x |
| $O5$ | x | x | | x | x | | | | x | x |
| $O5'$ | x | x | x | | | x | x | x | | |
| $\rho 5$ | x | x | x | x | | | | | x | x |
| $\rho 5'$ | x | x | | | x | x | x | x | | |
| NS5 | x | x | | x | x | | x | | x | |
| NS5$'$ | x | x | x | | | x | | x | | x |
| $D7, \overline{D7}$ | x | x | x | x | x | x | | x | x | |
| $D7', \overline{D7'}$ | x | x | x | x | x | x | x | | | x |

$$U \approx 16 M_3^3 k^3 \left\{ \left( \underline{4\pi^2} - \frac{2\pi^2}{3\beta^2}\left[ \underline{24} - \underline{n_\rho - \hat{n}_\rho \left( \left( \log[\frac{L^2}{L_*^2}] \right)^2} + \frac{(b_T - b_*)^2}{L^4} \right) \right] + \frac{\pi k n_{NS5}}{\underline{L^2 \beta^3}} \right) \frac{\tilde{\eta}^4}{k} - \left( \underline{2\pi R^2} - \frac{n_{D7} R^4 \beta}{\underline{2k}} \right) \frac{\tilde{\eta}^5}{\beta^3} + 4\pi^2 \left( \underline{N_{D5}^2 L^4 + \frac{(N_{D1} + b_T^2 N_{D5})^2}{L^4} + 2 b_T^2 N_{D5}^2} \right) \frac{k \tilde{\eta}^6}{\beta^4} \right\}$$

Tuning input parameters to fix the first term at a small numerical value ε gives a hierarchy – though this is not infinitely tunable.

# dS$_3$ worked example

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D1$ | x | x | | | | | | | | |
| $D5$ | x | x | | | | | x | x | x | x |
| $O5$ | x | x | | x | x | | | | x | x |
| $O5'$ | x | x | x | | | x | x | x | | |
| $\rho 5$ | x | x | x | x | | | | | x | x |
| $\rho 5'$ | x | x | | | x | x | x | x | | |
| NS5 | x | x | | x | x | | x | | x | |
| NS5$'$ | x | x | x | | | x | | x | | x |
| $D7, \overline{D7}$ | x | x | x | x | x | x | | x | x | |
| $D7', \overline{D7'}$ | x | x | x | x | x | x | x | | | x |

Parametric solution with large curvature radius and weak string coupling!

$$R_f \sim \frac{R}{k}$$

$$R^2 \sim L^2 \sim k \sim \sqrt{\frac{N_{D1}}{N_{D5}}}$$

$$N_{D5} \sim \sqrt{\frac{1}{\varepsilon}}$$

$$g_s \sim \varepsilon$$

$$R_{dS}^2 \sim \frac{R^2}{\varepsilon}$$

$$b_T \approx b_*$$

$$U \approx 16 M_3^3 k^3 \left\{ \left( 4\pi^2 - \frac{2\pi^2}{3\beta^2}\left[ 24 - n_\rho - \hat{n}_\rho \left( \left( \log[\frac{L^2}{L_*^2}] \right)^2 + \frac{(b_T - b_*)^2}{L^4} \right) \right] + \frac{\pi k n_{NS5}}{L^2 \beta^3} \right) \frac{\tilde{\eta}^4}{k} - \left( 2\pi R^2 - \frac{n_{D7} R^4 \beta}{2k} \right) \frac{\tilde{\eta}^5}{\beta^3} + 4\pi^2 \left( N_{D5}^2 L^4 + \frac{(N_{D1} + b_T^2 N_{D5})^2}{L^4} + 2 b_T^2 N_{D5}^2 \right) \frac{k \tilde{\eta}^6}{\beta^4} \right\}$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $D1$ | x | x | | | | | | | | |
| $D5$ | x | x | | | | | x | x | x | x |
| $O5$ | x | x | | x | x | | | | x | x |
| $O5'$ | x | x | x | | | x | x | x | | |
| $\rho 5$ | x | x | x | x | | | | | x | x |
| $\rho 5'$ | x | x | | | x | x | x | x | | |
| NS5 | x | x | | x | x | | x | | x | |
| NS5′ | x | x | x | | | x | | x | | x |
| $D7, \overline{D7}$ | x | x | x | x | x | x | | x | x | |
| $D7', \overline{D7'}$ | x | x | x | x | x | x | x | | | x |

# $dS_3$ worked example

Input data

| | |
|---|---|
| $n_\rho$ | 4 |
| $\hat{n}_\rho$ | 2 |
| $\rho_*$ | $\exp(i\pi/3)$ |
| $n_{NS5}$ | 2 |
| $n_{D7}$ | 4 |
| $k$ | 44 |
| $N_{D1}$ | 156 |
| $N_{D5}$ | 5 |

Stabilized moduli

| | |
|---|---|
| $R$ | 9.2 |
| $kR_f$ | 7.5 |
| $L$ | 2.5 |
| $b_T$ | 0.48 |
| $g_s$ | 0.02 |
| $\epsilon$ | 0.002 |
| $4ac/b^2$ | 1.003 |

All moduli can be stabilized with weak string coupling, large curvature, radius and a hierarchy. $O(\alpha')$ corrections will alter the depth of the dS min but not its location.

$$U \approx 16 M_3^3 k^3 \left\{ \left( 4\pi^2 - \frac{2\pi^2}{3\beta^2}\left[ 24 - n_\rho - \hat{n}_\rho \left( \left( \log[\frac{L^2}{L_*^2}] \right)^2 + \frac{(b_T - b_*)^2}{L^4} \right) \right] + \frac{\pi k n_{NS5}}{L^2 \beta^3} \right) \frac{\tilde{\eta}^4}{k} \right. \\ \left. - \left( 2\pi R^2 - \frac{n_{D7} R^4 \beta}{2k} \right) \frac{\tilde{\eta}^5}{\beta^3} + 4\pi^2 \left( N_{D5}^2 L^4 + \frac{(N_{D1} + b_T^2 N_{D5})^2}{L^4} + 2 b_T^2 N_{D5}^2 \right) \frac{k \tilde{\eta}^6}{\beta^4} \right\}$$

# Laundry list: avoiding spills

- Additional moduli from anisotropies, axions, slippage, D7 brane-antibrane potential.
- $O(\alpha')$ corrections will alter final numerics; numbers are illustrative rather than accurate.
- D5s instead of D7s improve numerics somewhat

# Laundry list: how to remove smearing

- 10D eom vs. 3D zeromodes: include warp/conformal factors $ds^2 = e^{2A(y)} ds^2_{dS_3} + e^{-2A(y)} ds^2_{\text{int}}$ (Giddings and Maharana '05, Douglas '09, Douglas and Kallosh '10)
- Einstein equation: $\nabla^2 A - (\nabla A)^2 = -R^{(10-d)} + g_s^2 |F|^2 + g_s^2 T^{loc} + g_s^2 U$
- A << 1 for D-branes and O-planes; homogeneous approximation OK
- A ~ 1 for ρ5s; NS5s – but we have the GLSM here.