

nm science enabled by waveguides

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Outline

Theoretical background

- propagation of x-rays in waveguides
- waveguide-based lensless imaging / holography

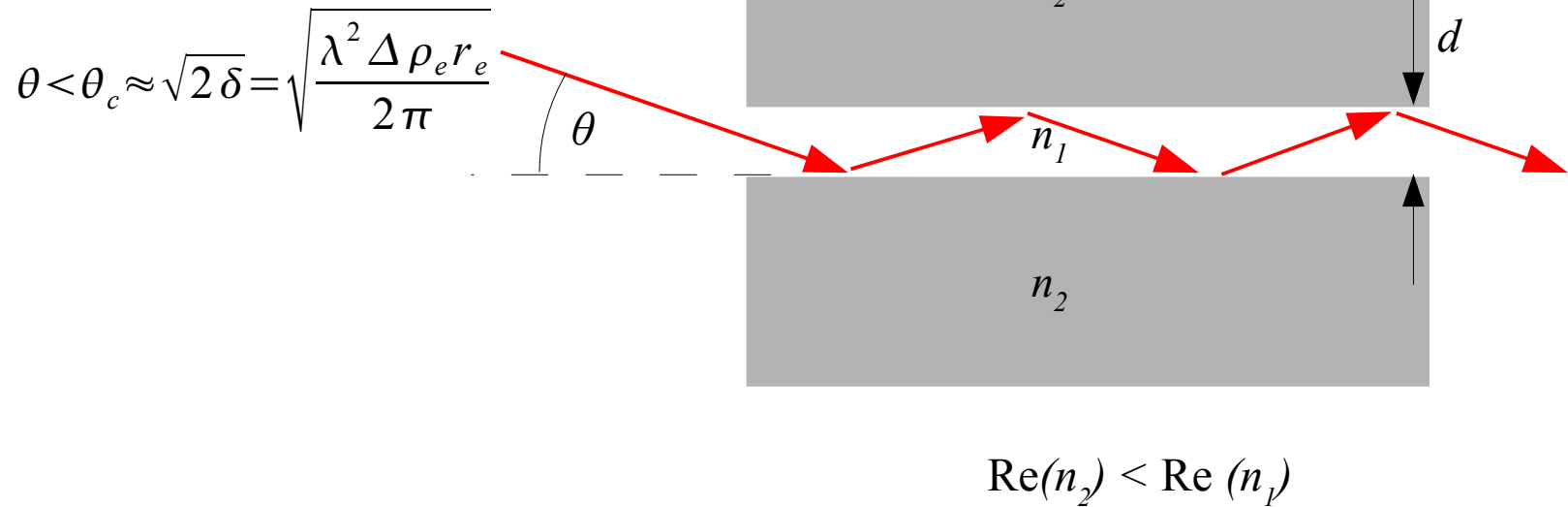
Imaging experiments

- in-line phase-contrast imaging and in-line holography
- off-axis holography

Outlook

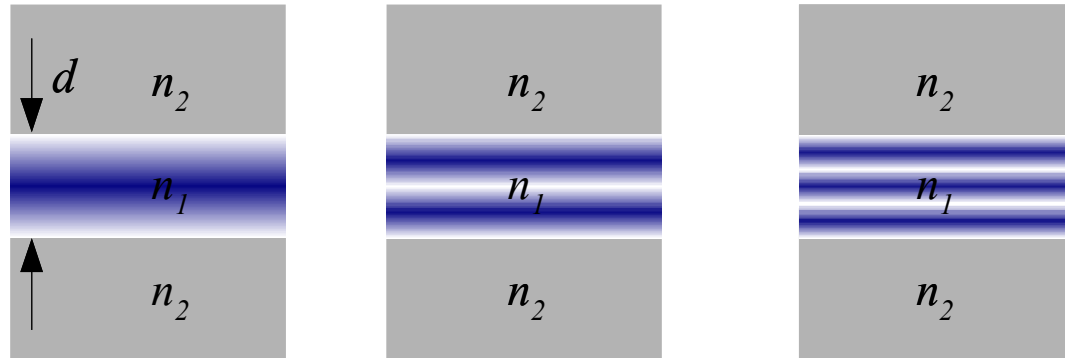
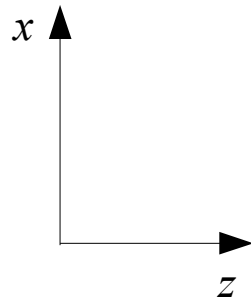
- Can we achieve 1 nm resolution by means of x-ray waveguides?

X-ray waveguides

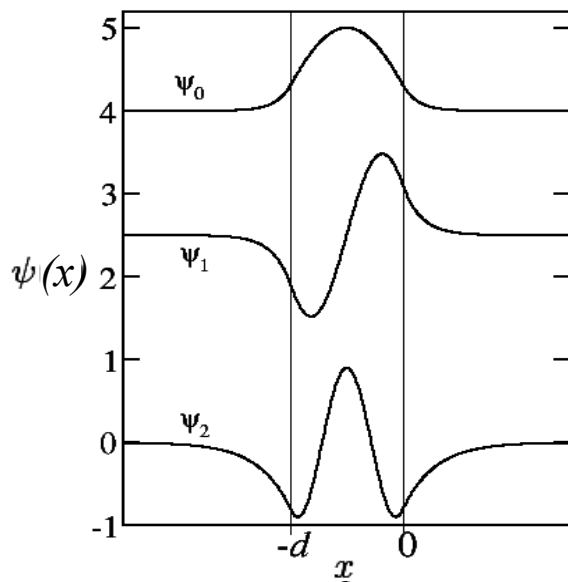


[Spiller & Segmüller, Appl. Phys.Lett. **24** (1974) 60]

Guided modes of a planar waveguide



standing waves \rightarrow guided modes

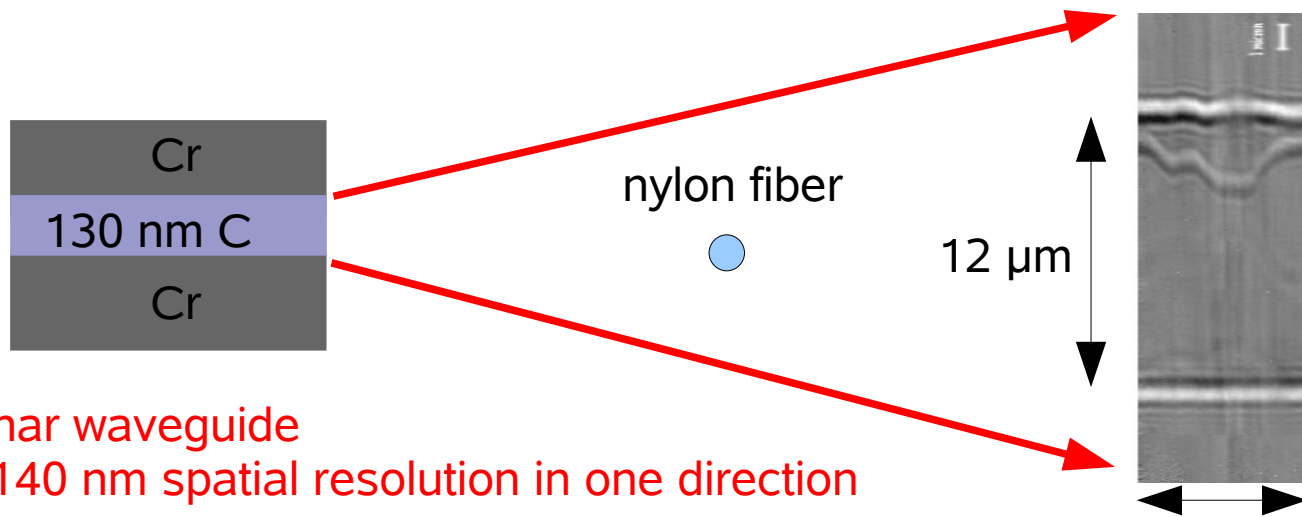
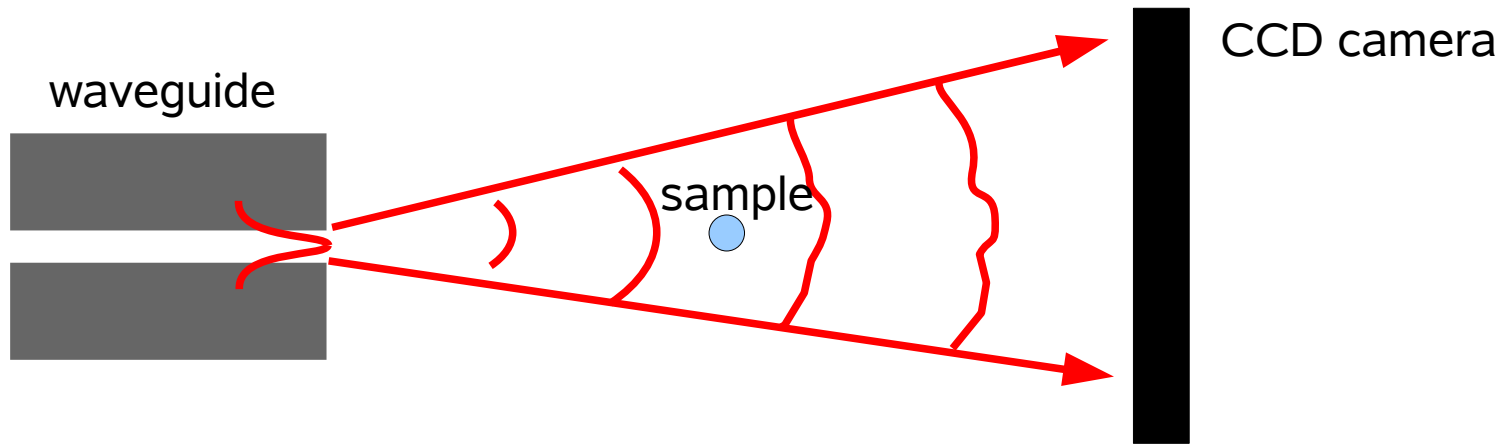


$$\psi(x) = \begin{cases} B \exp(-\gamma x), & \text{if } x \geq 0, \\ B \cos \kappa x + C \sin \kappa x, & \text{if } 0 > x > -d, \\ (B \cos \kappa d + C \sin \kappa d) \times \exp[\gamma(x + d)], & \text{if } x \leq -d, \end{cases}$$

$$\psi(x, z) = \sum_{m=0}^{N-1} c_m \psi_m(x) \exp(-i\beta_m z)$$

$$c_m = \int \psi_{\text{in}}(x) \psi_m(x) dx$$

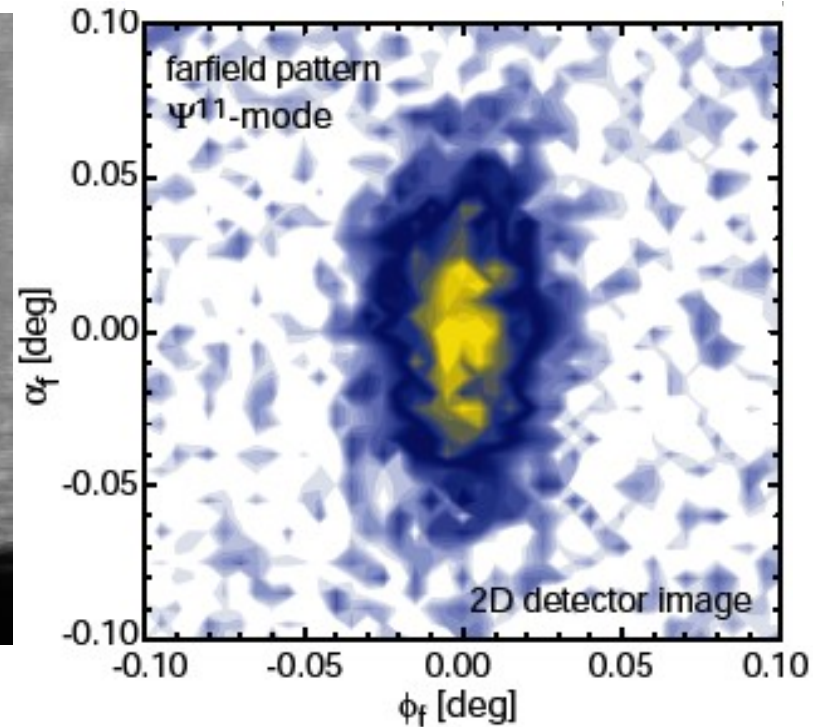
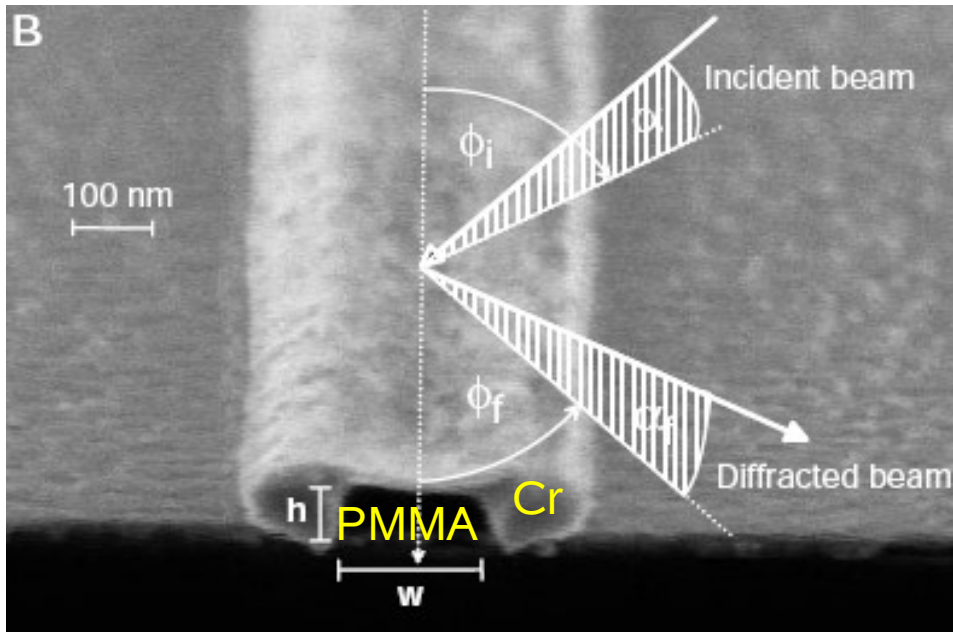
Phase-contrast imaging



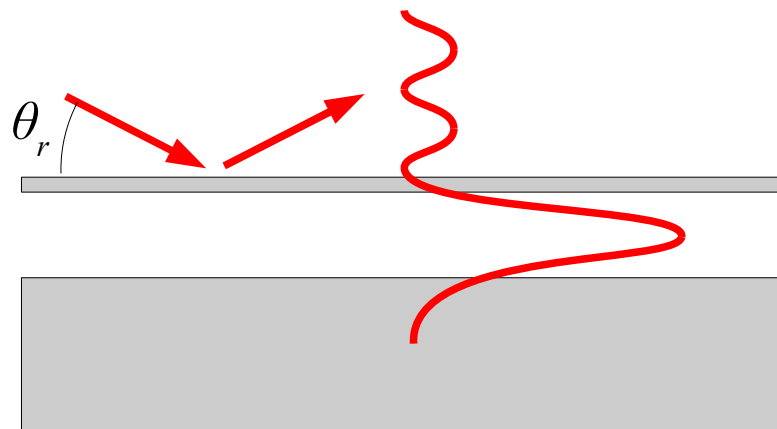
planar waveguide
→ 140 nm spatial resolution in one direction

[Lagomarsino *et al.*, Appl. Phys. Lett. **71** (1997) 2557] ≈ 500 μm

Two-dimensionally confining waveguides



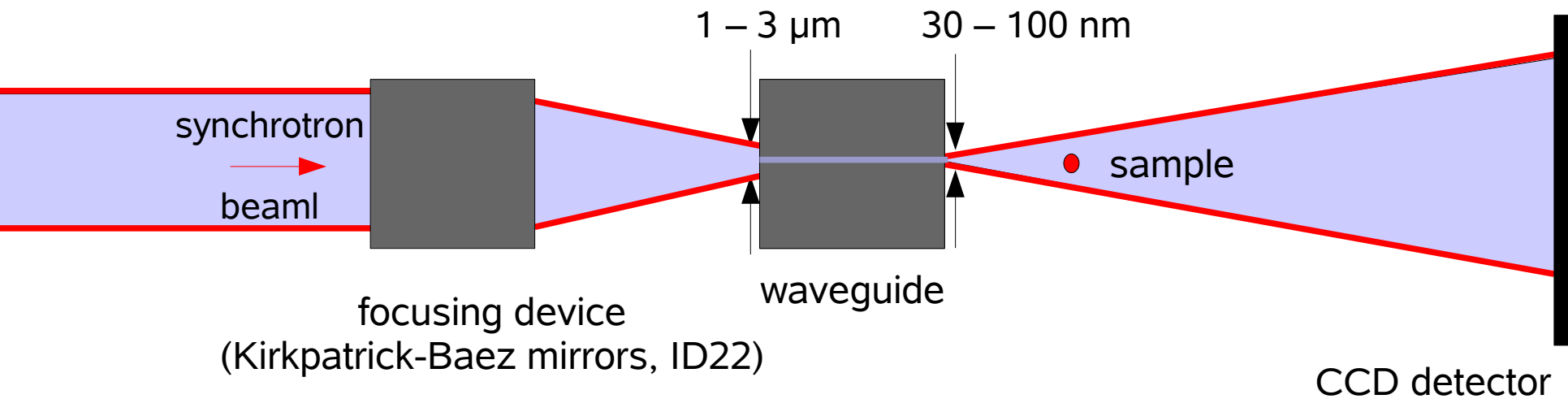
beam dimensions $69 \times 33 \text{ nm}^2$



„Resonant beam coupling“
→ flux: $2 \times 10^4 \text{ ph/s}$

[Pfeiffer *et al.*, Science **297** (2002) 231]

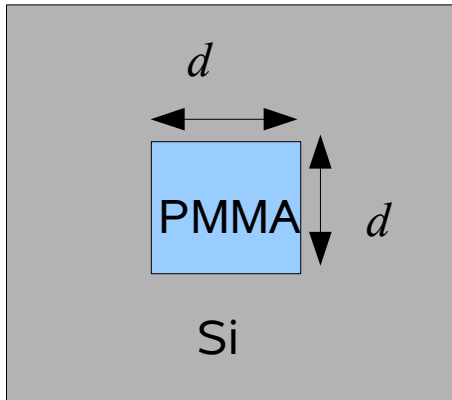
Direct coupling of a focused beam



typical flux: 10^6 - 10^7 ph/s from 2D waveguides

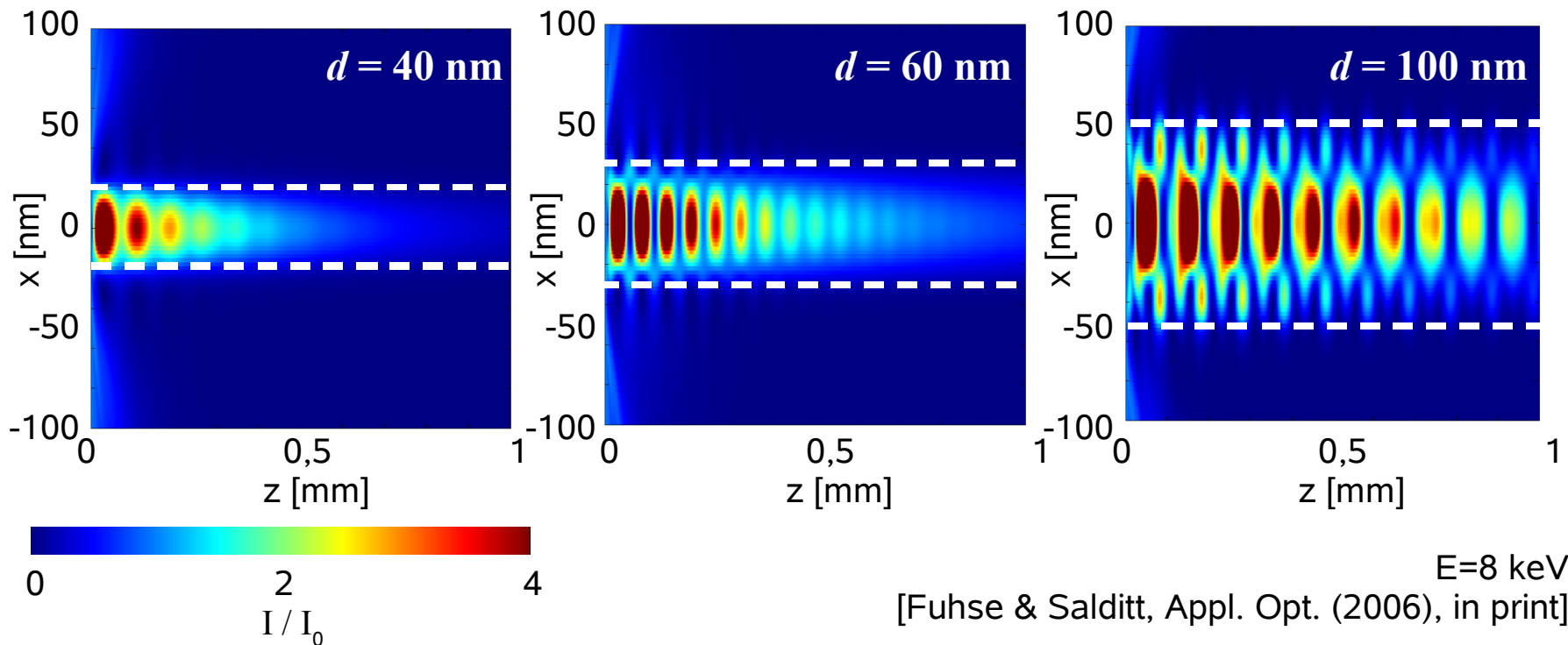
[Jarre, Fuhse, Ollinger, Seeger, Tucoulou, Salditt, Phys. Rev. Lett. **94** (2005), 074891]

Direct coupling into 2D waveguides

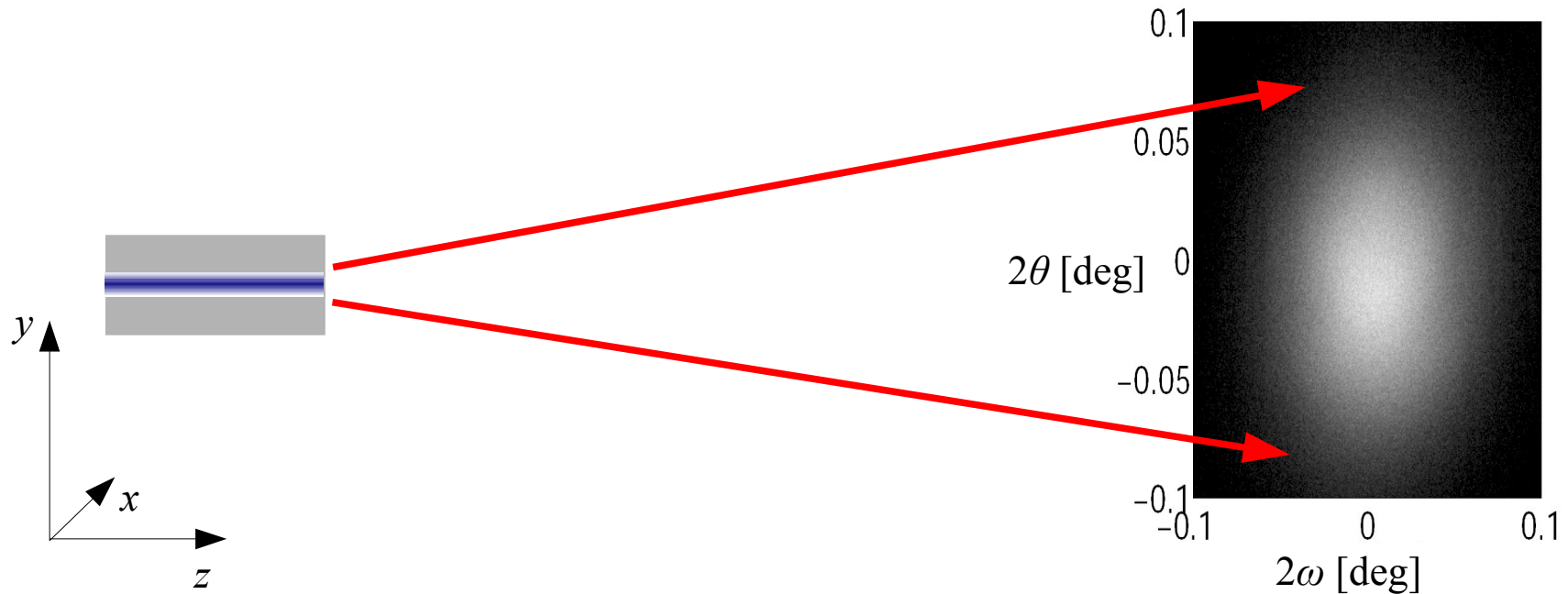


finite-difference field calculations
based on the parabolic wave equation:

- strong absorption in the cladding
- excitement of multiple guided modes
- stronger damping of higher-order modes
→ field is dominated by the fundamental mode



Far-field pattern

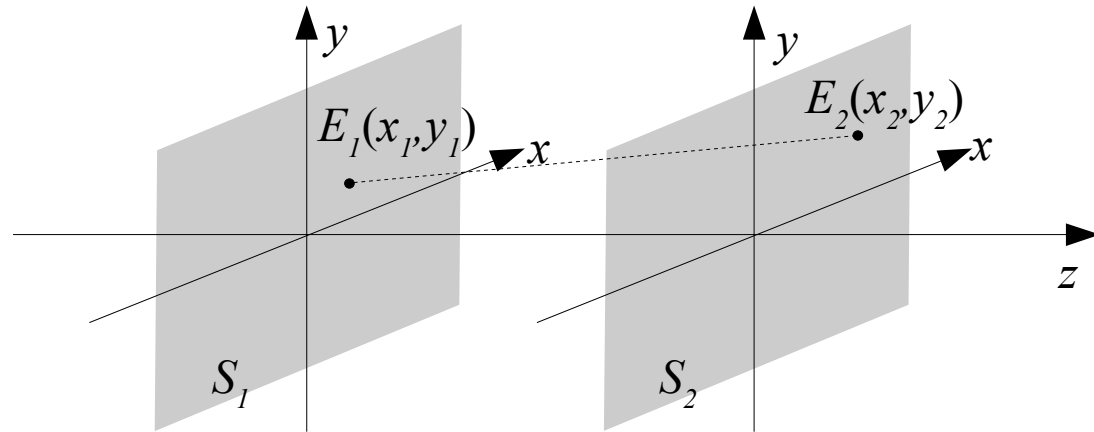


- Far-field intensity (Fraunhofer approximation):

$$I(2\theta, 2\omega) \propto \left| \int E(x,y) \exp(ik 2\theta y) \exp(-ik 2\omega x) dx dy \right|^2$$

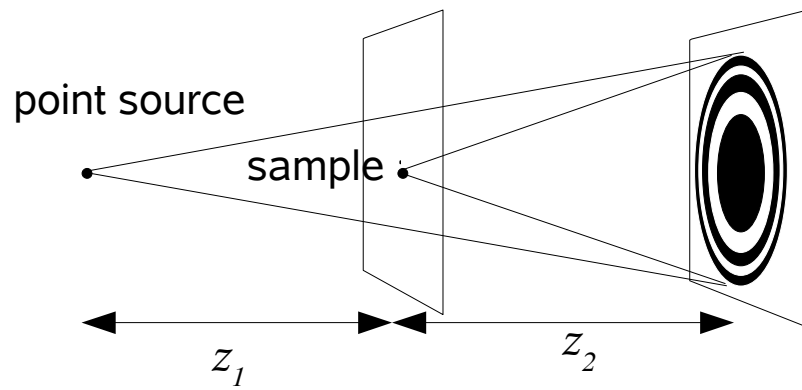
- guiding-core cross-sectional dimensions determine numerical aperture and spatial resolution → **spatial resolution \approx diameter of the guiding core**

Coherent imaging

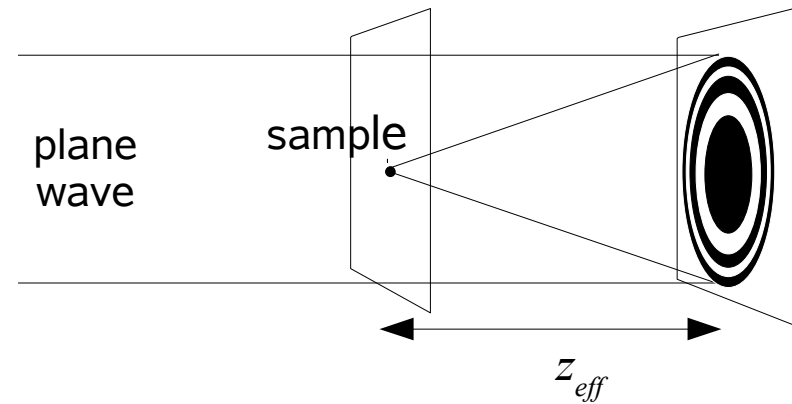


Kirchhoff's diffraction formula
(Fresnel approximation):

$$E_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \int E_1(x_1, y_1) \exp\left(ik \frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{2z}\right) dx_1 dy_1$$

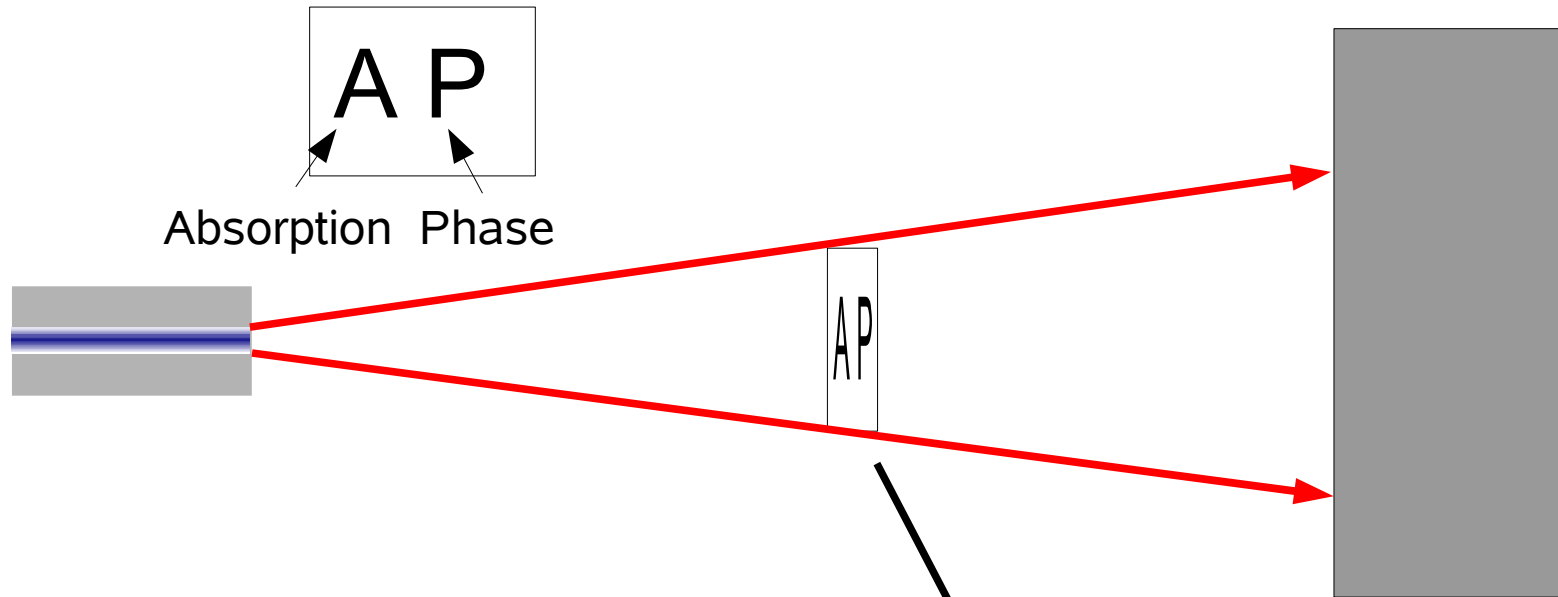


$$M = \frac{z_1 + z_2}{z_1}$$

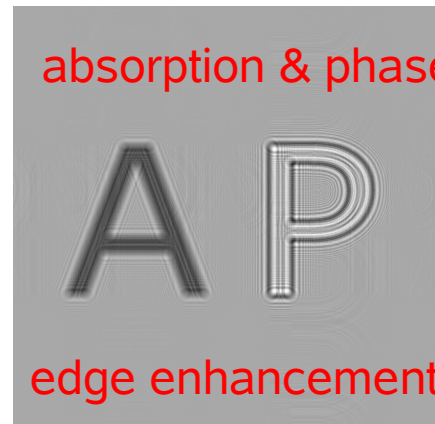


$$z_{eff} = \frac{z_1 z_2}{z_1 + z_2}$$

Imaging regimes

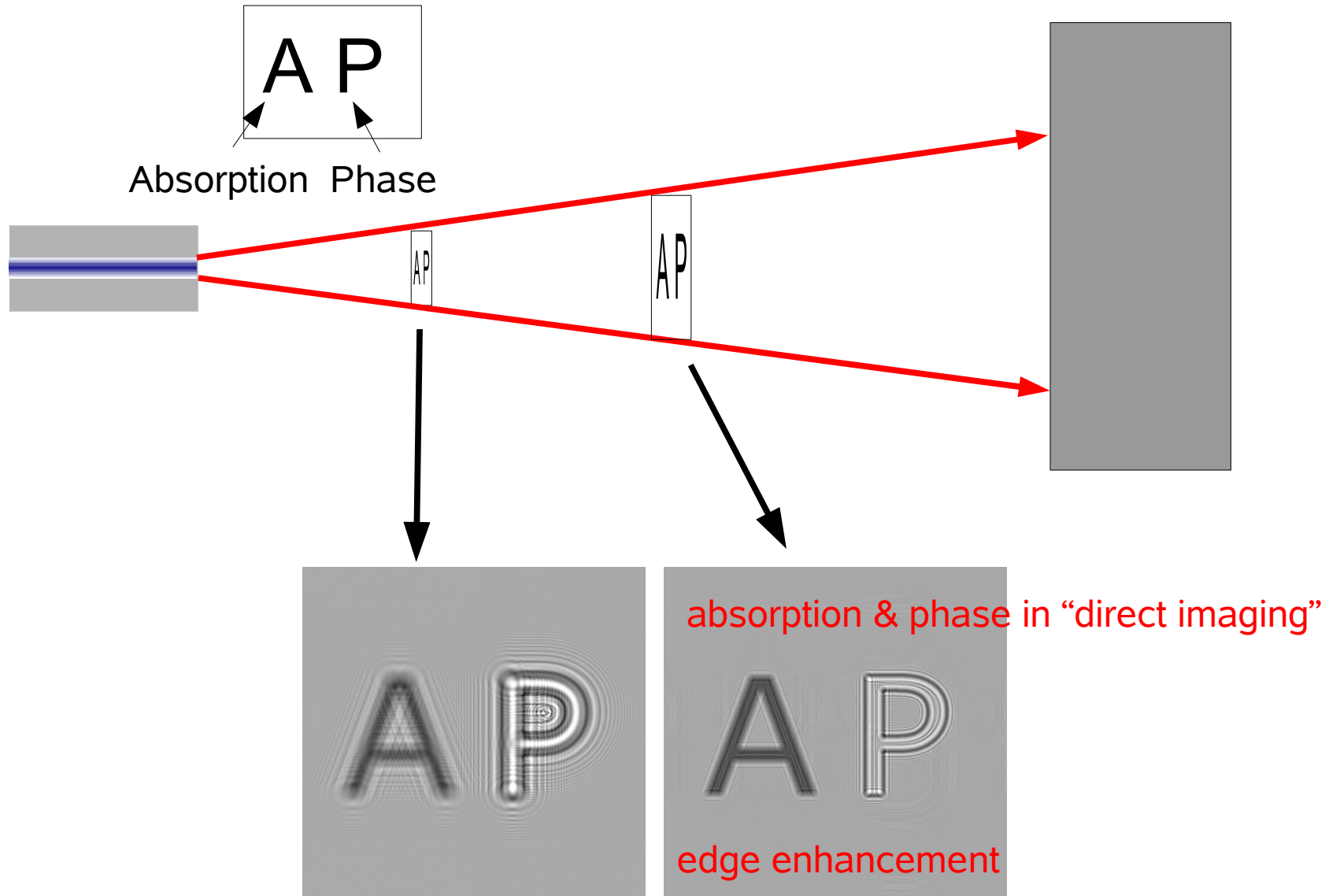


absorption & phase in "direct imaging"

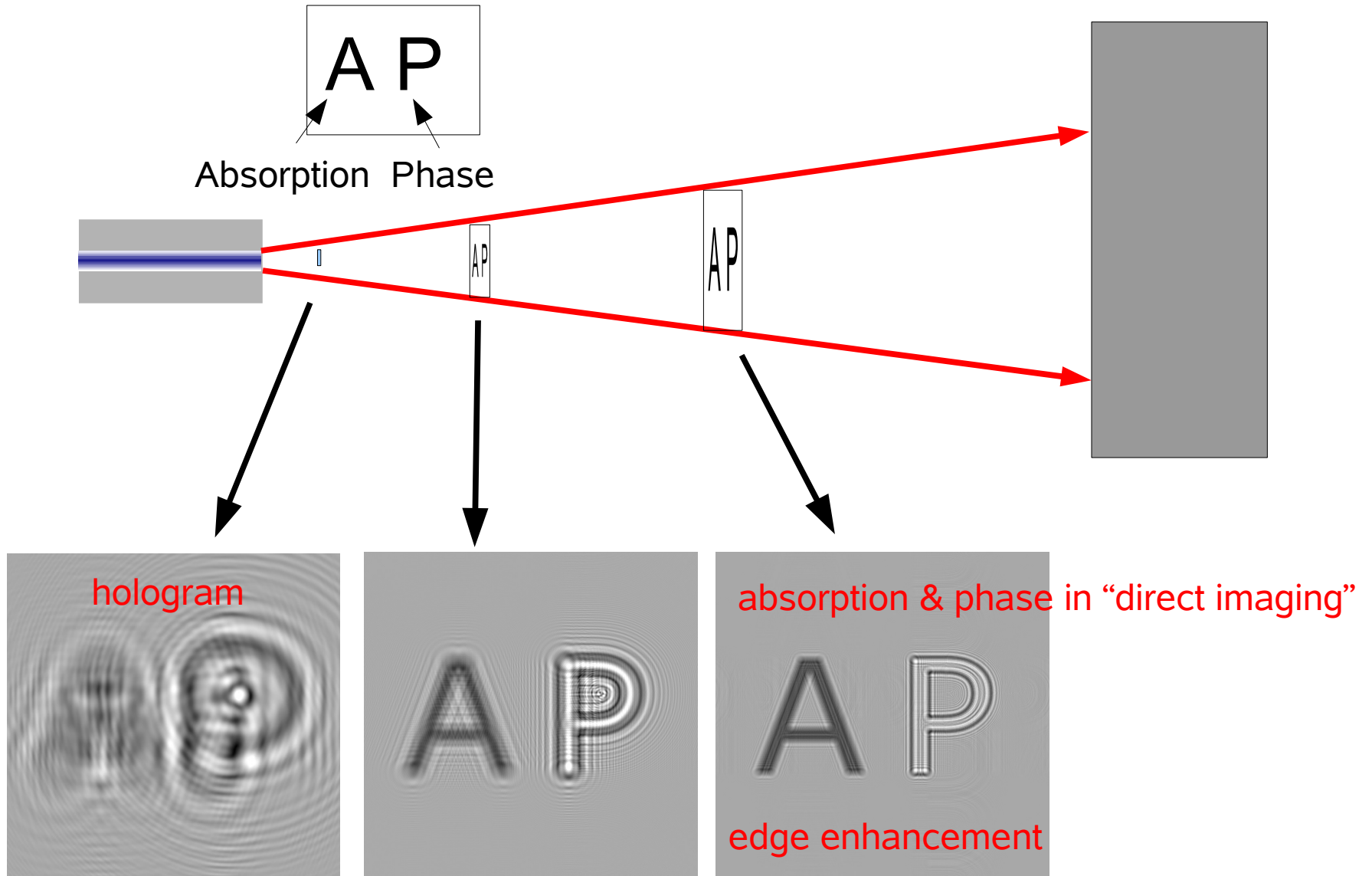


edge enhancement

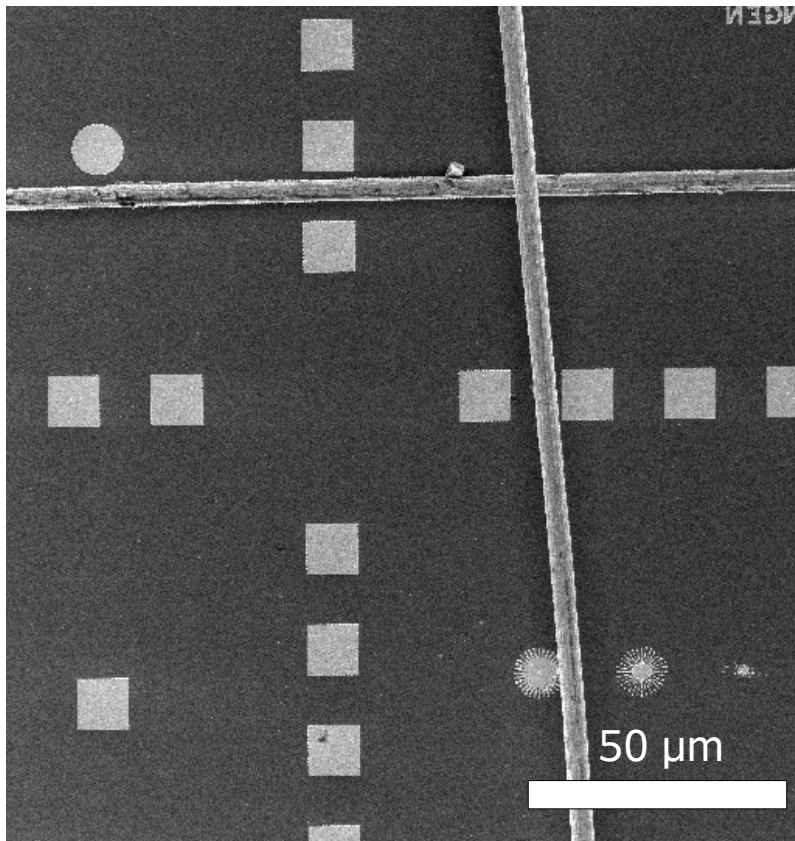
Imaging regimes



Imaging regimes



“Direct” imaging of large objects



SEM

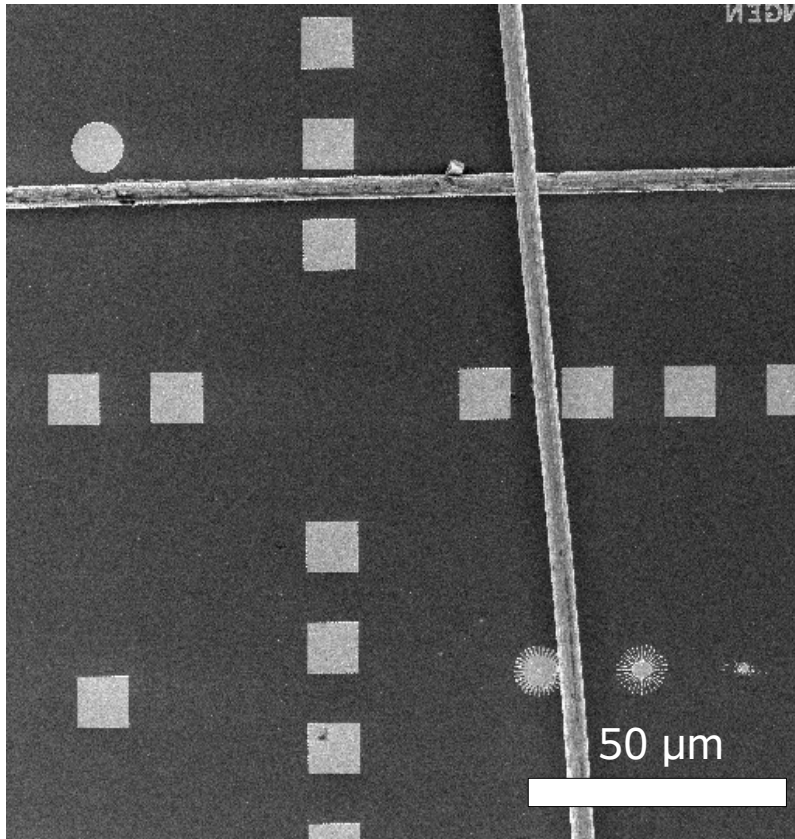
gold structures (d=150 nm)

tungsten wires (ø 4.5 μm)

→ phase-shifting sample @E=10.4 keV

→ absorbing sample @E=10.4 keV

“Direct” imaging of large objects



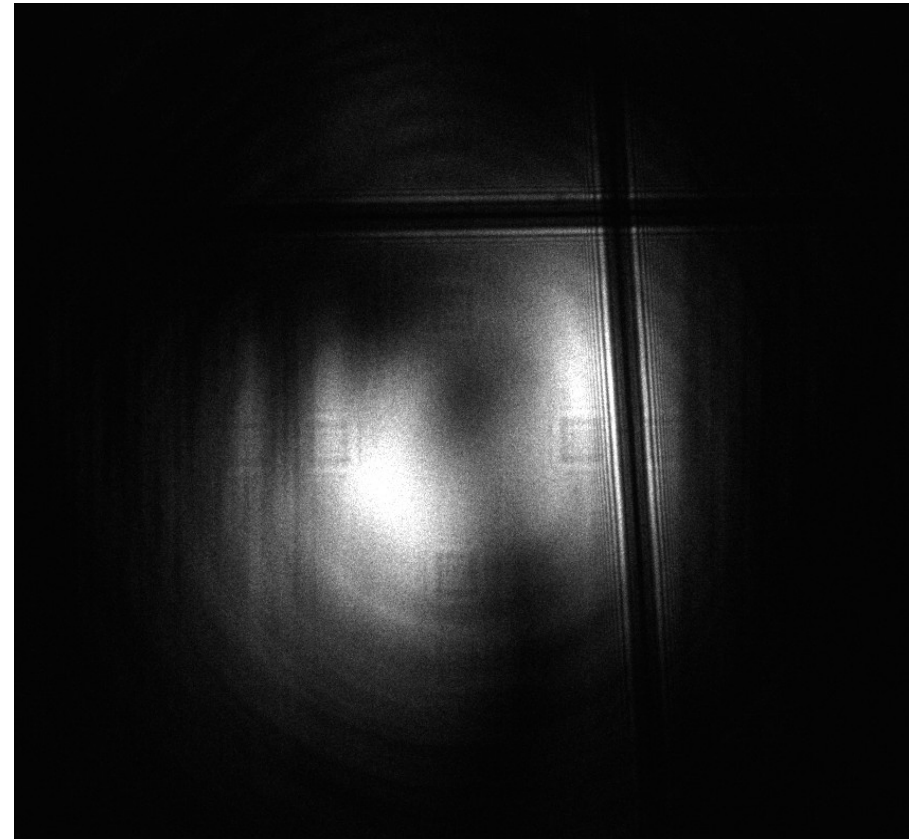
SEM

gold structures ($d=150$ nm)

tungsten wires (\varnothing 4.5 μm)

→ phase contrast (edge enhancement)

→ absorption contrast



recorded image ($E=10.4$ keV)

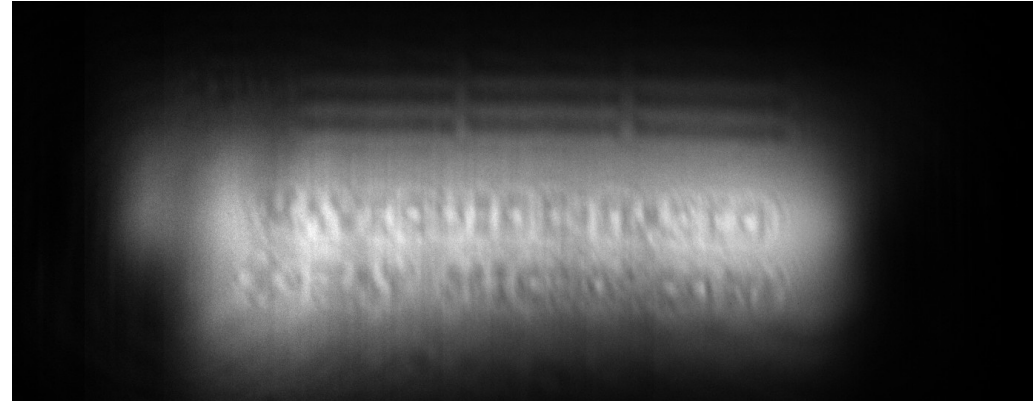
Imaging of smaller features

sample (Au, $d=150$ nm)



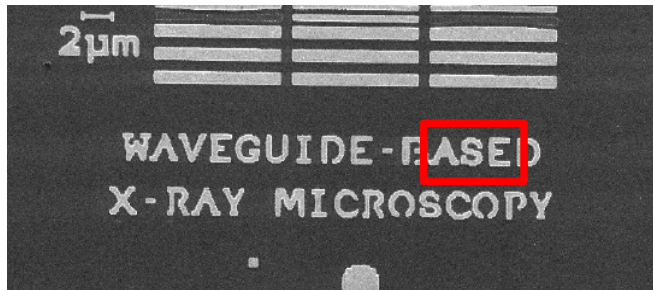
(SEM)

hologram (11 individual exposures)



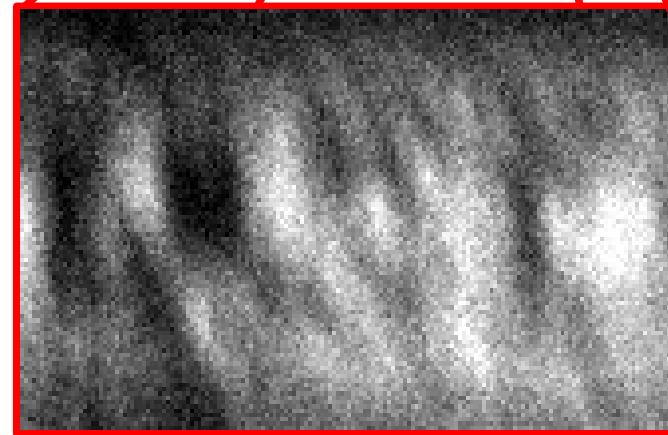
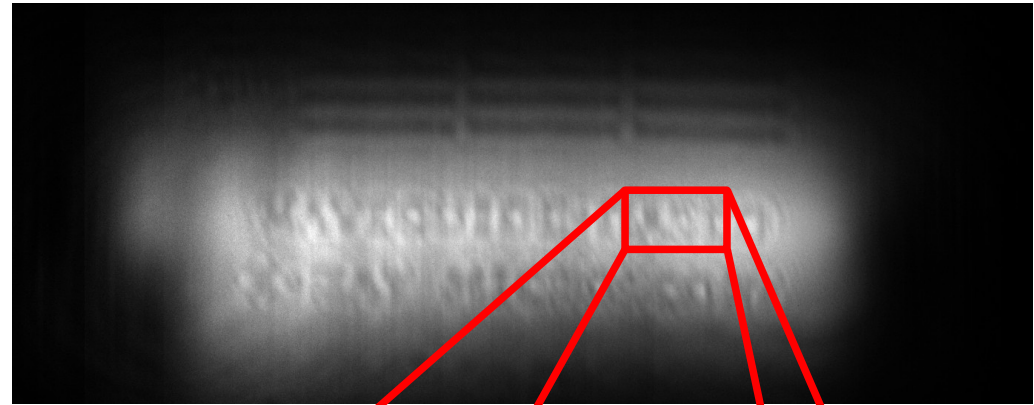
Imaging of smaller features

sample (Au, $d=150$ nm)



(SEM)

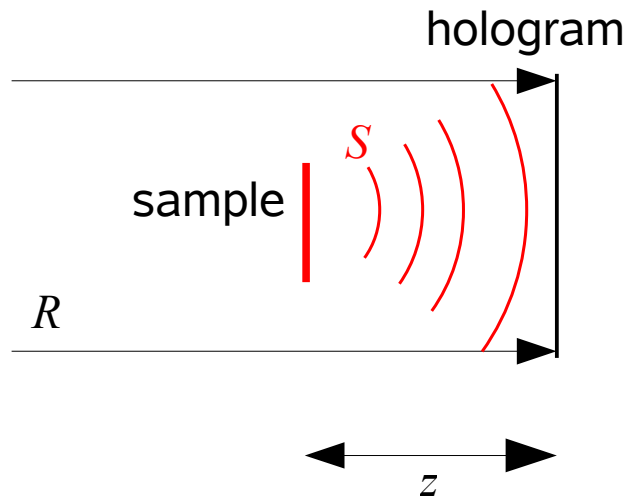
hologram (11 individual exposures)



→ holographic reconstruction required

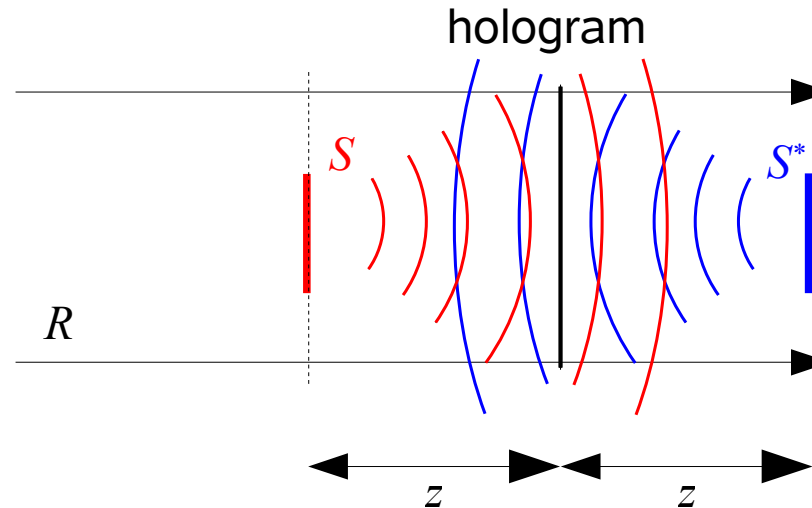
In-line holography

recording:



$$I(x, y) \sim |S(x, y) + R(x, y)|^2$$

reconstruction:



$$E_R(x, y) \sim I(x, y)R(x, y)$$

$$= (RR^* + SS^* + R^*S + S^*R)R$$

$$= |R|^2R + |S|^2R + |R|^2S + R^2S^*$$

reference beam

signal wave

phase-conjugate wave
"twin image"

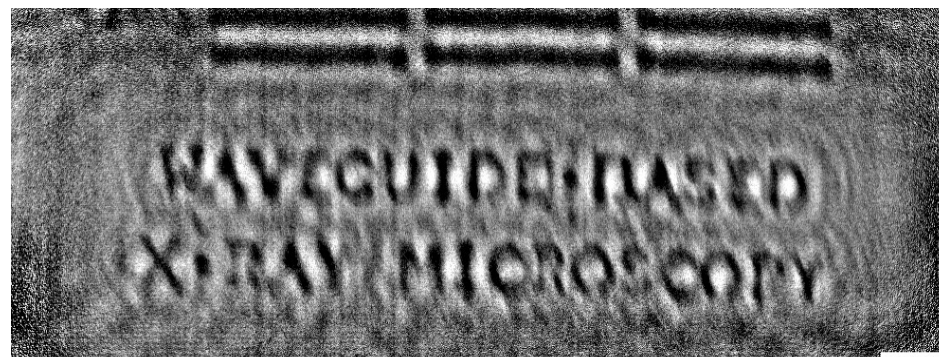
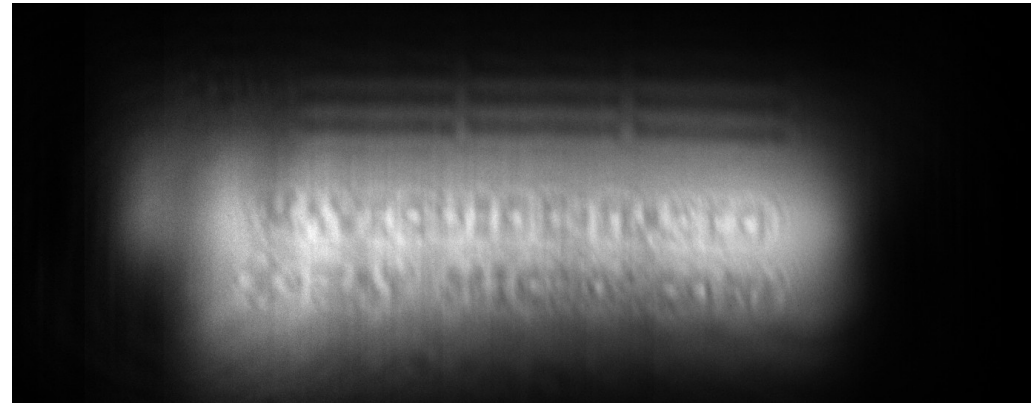
Holographic reconstruction

sample (Au, $d=150$ nm)



(SEM)

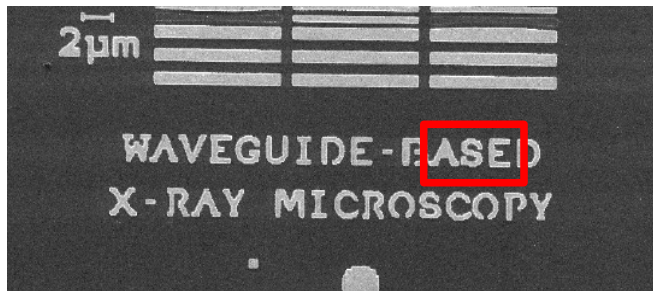
hologram (11 individual exposures)



phase of the reconstructed wave

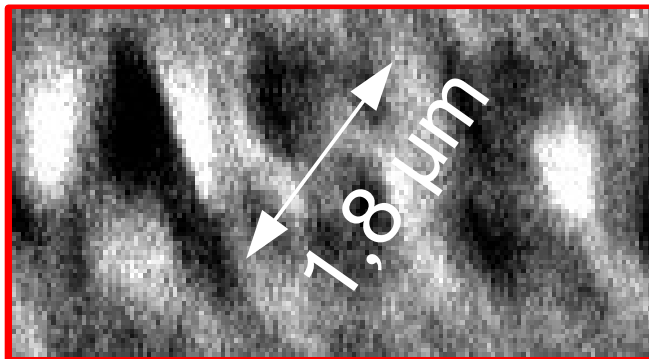
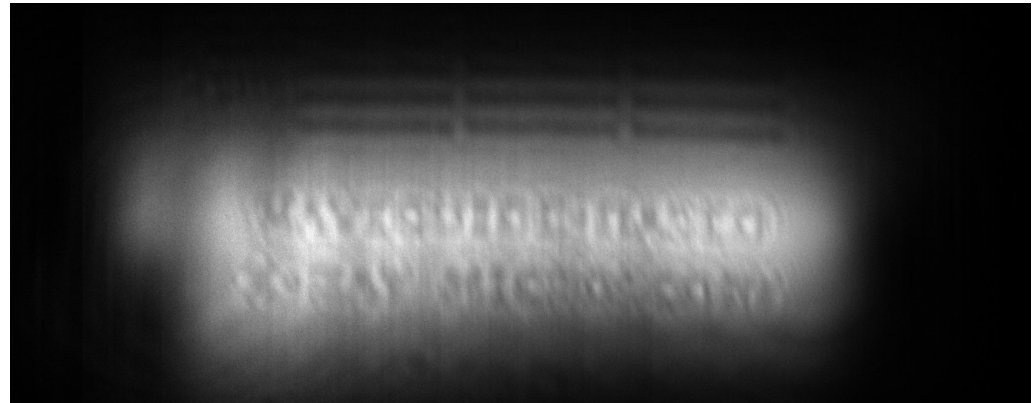
Holographic reconstruction

sample (Au, $d=150$ nm)



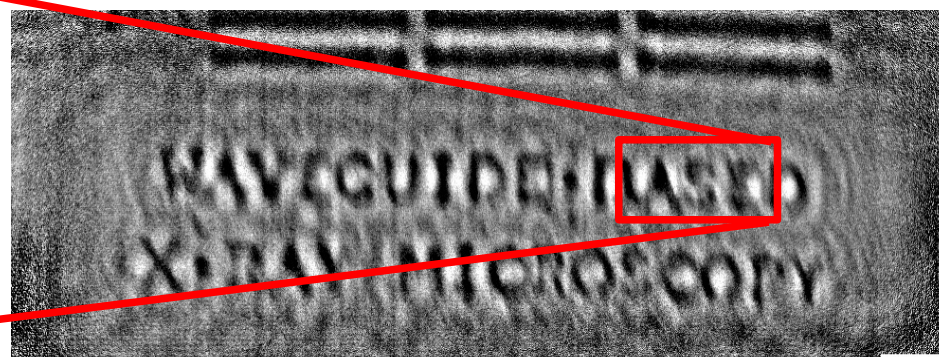
(SEM)

hologram (11 individual exposures)



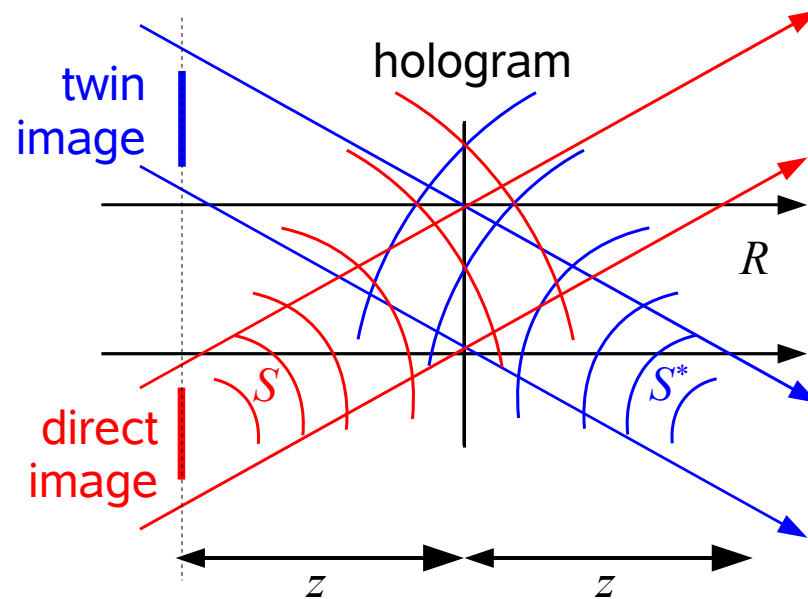
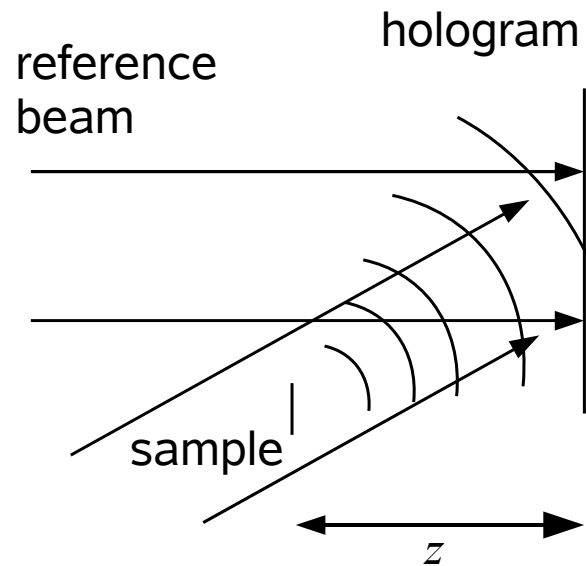
resolution ≈ 360 nm

→ but: $d_{WL} < 100$ nm



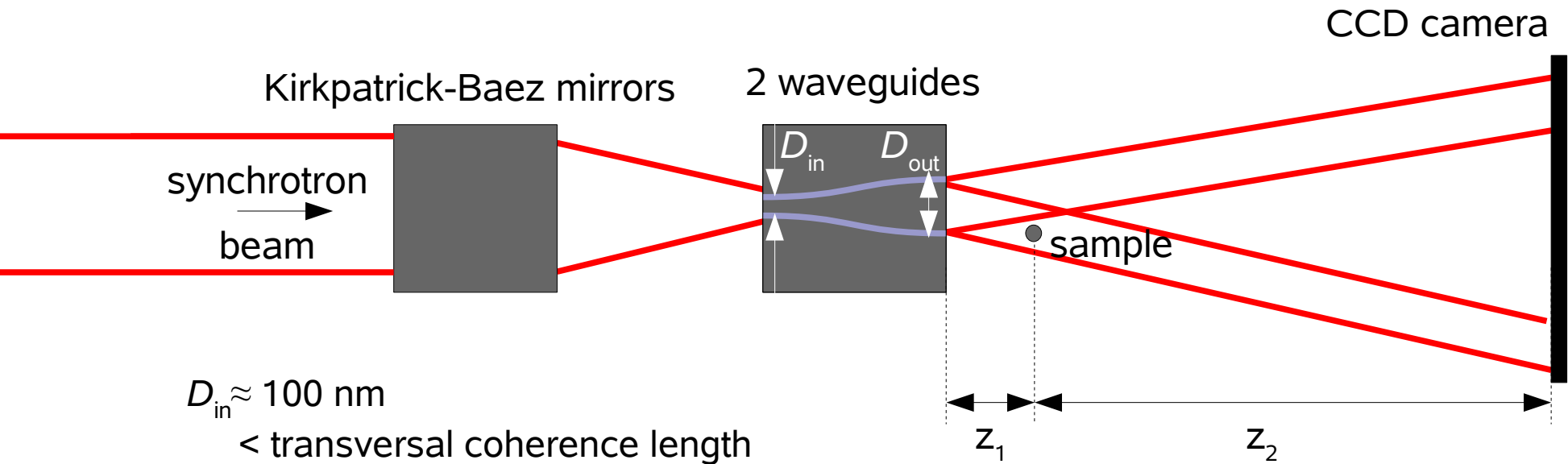
phase of the reconstructed wave

Off-axis holography



→ direct image and twin image are spatially separated!

Waveguide-based off-axis holography



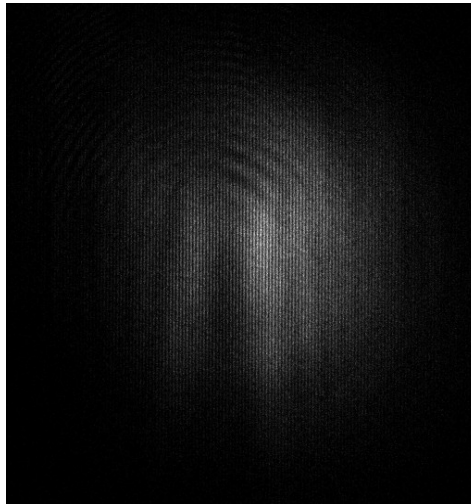
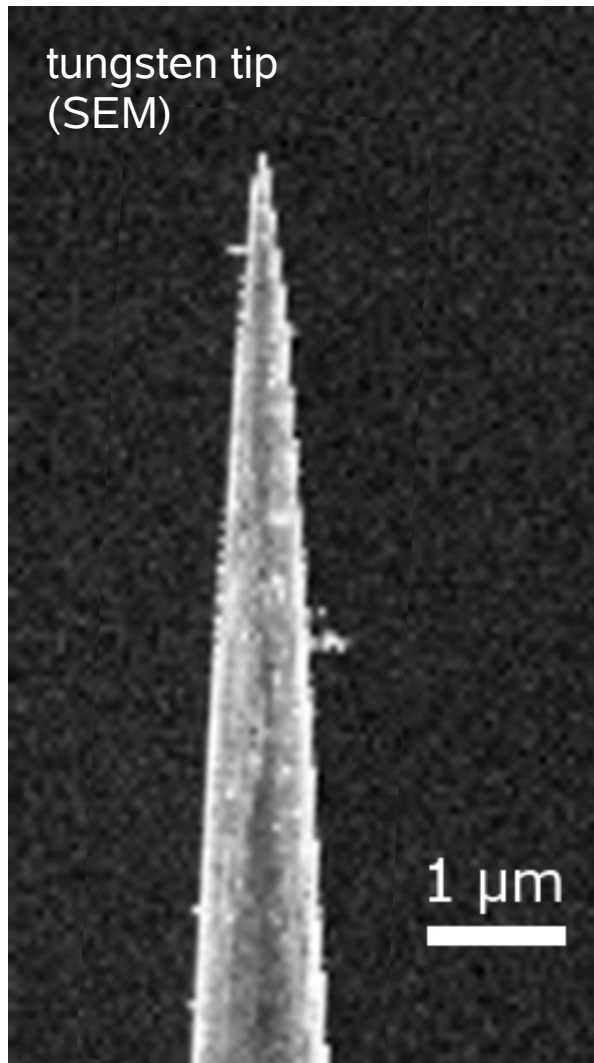
$D_{in} \approx 100 \text{ nm}$
< transversal coherence length

$D_{out} \approx 5 \text{ }\mu\text{m}$

magnified off-axis hologram

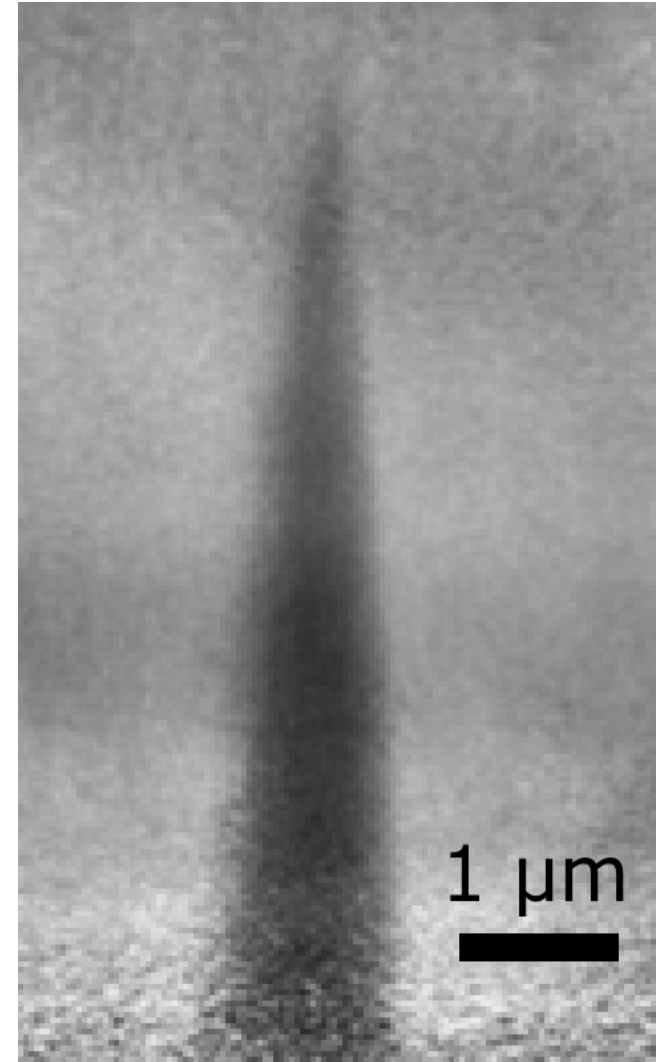
$$M = \frac{z_1 + z_2}{z_1}$$

Off-axis holography

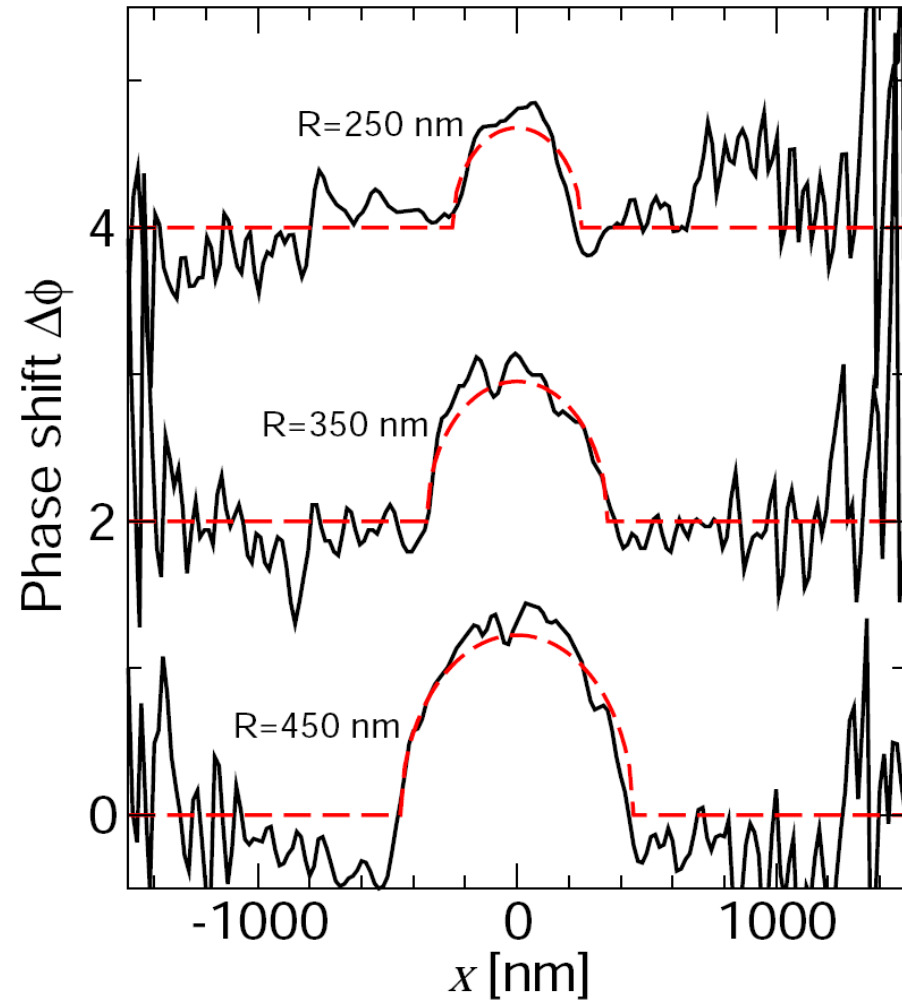
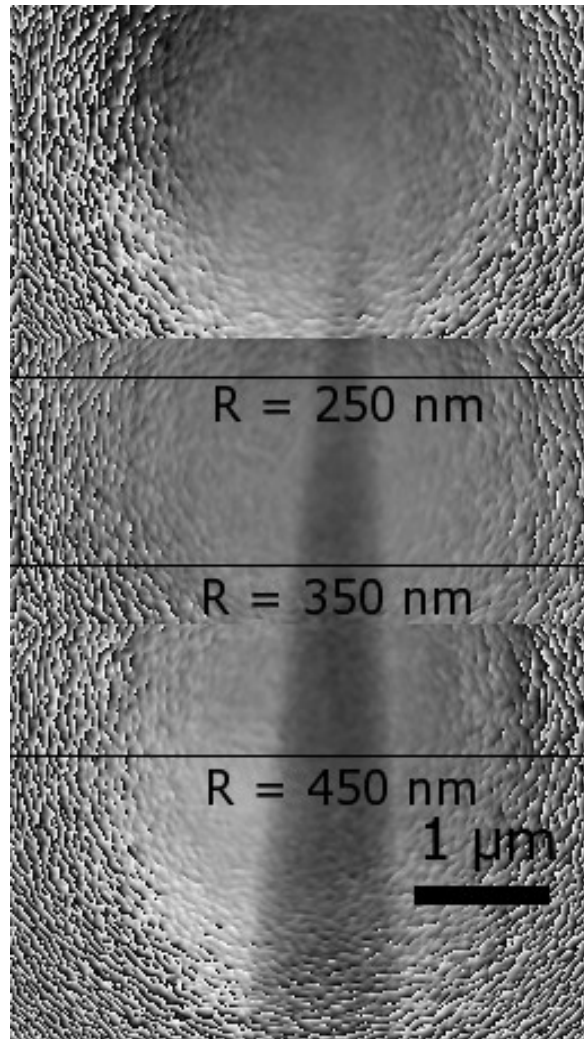


off-axis
hologram
($E=10.4$ keV)

phase of the
reconstructed wave
(from 25 holograms)

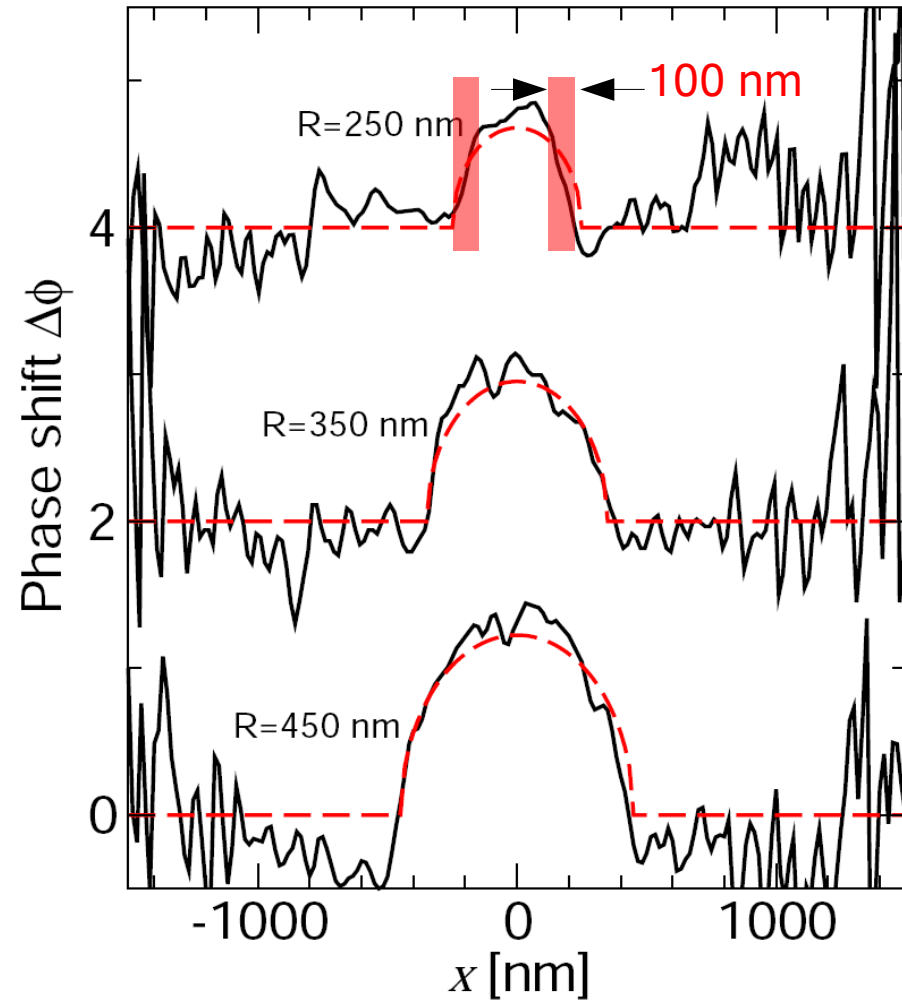
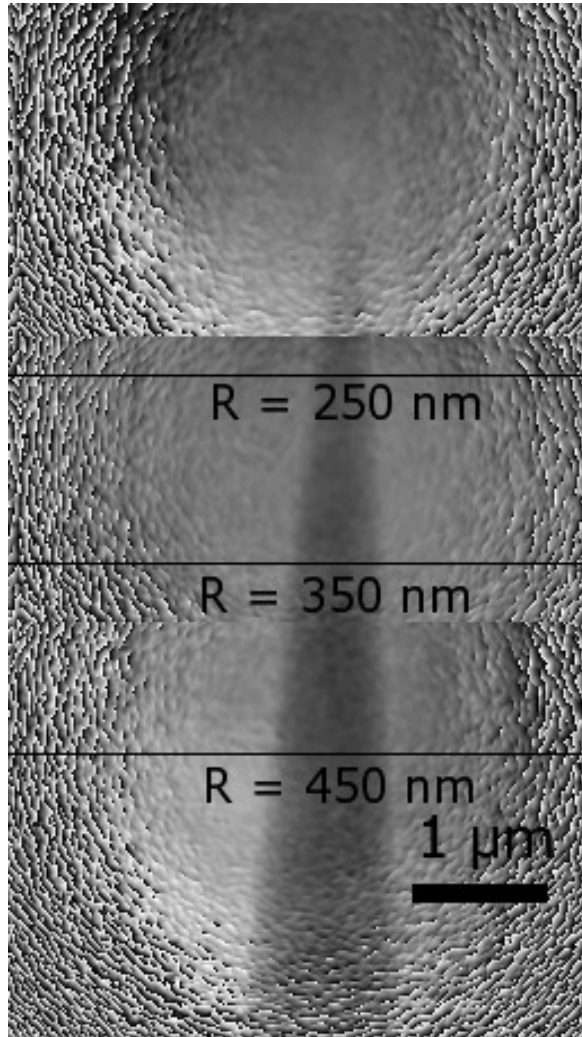


Quantitative analysis



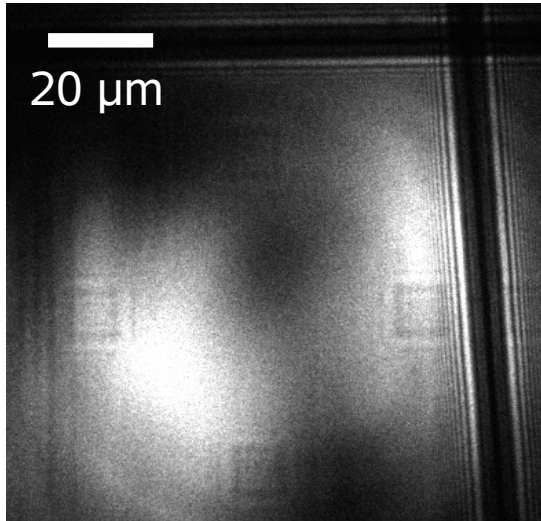
→ Phase of the reconstructed wave in quantitative agreement with the simulation!

Quantitative analysis

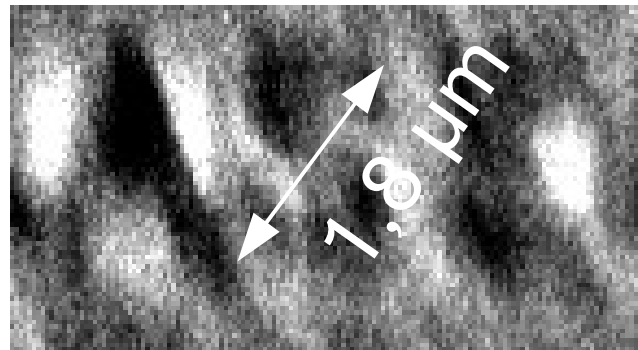


→ Phase of the reconstructed wave in quantitative agreement with the simulation!

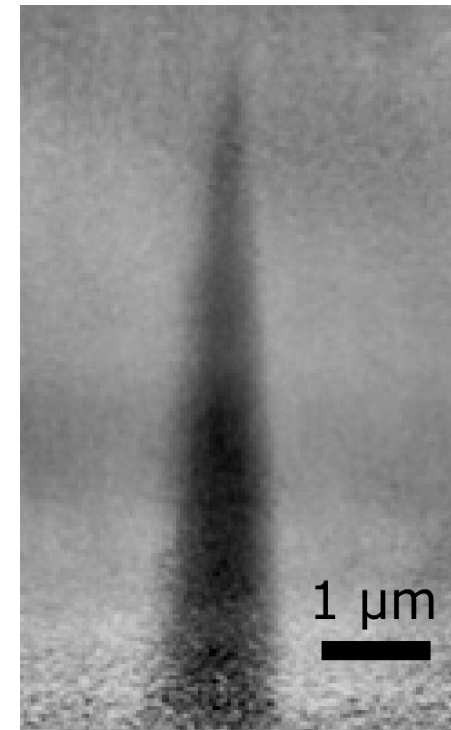
Summary: State of the art



- lensless imaging with a single waveguide:
 - “direct” imaging of large objects (phase & absorption)
 - holographic reconstruction of smaller features resolution ≈ 360 nm but: severe artifacts (“twin image”)



- off-axis holography with two waveguides
 - better image quality
 - better spatial resolution (≈ 100 nm)
 - **quantitative** determination of the phase (\rightarrow projected electron density)



Perspectives

- **3D imaging: tomography:**
small numerical aperture → large focal depth
large penetration depth (in particular in the hard-X-ray regime)

ERL seems to be a well-suited source!

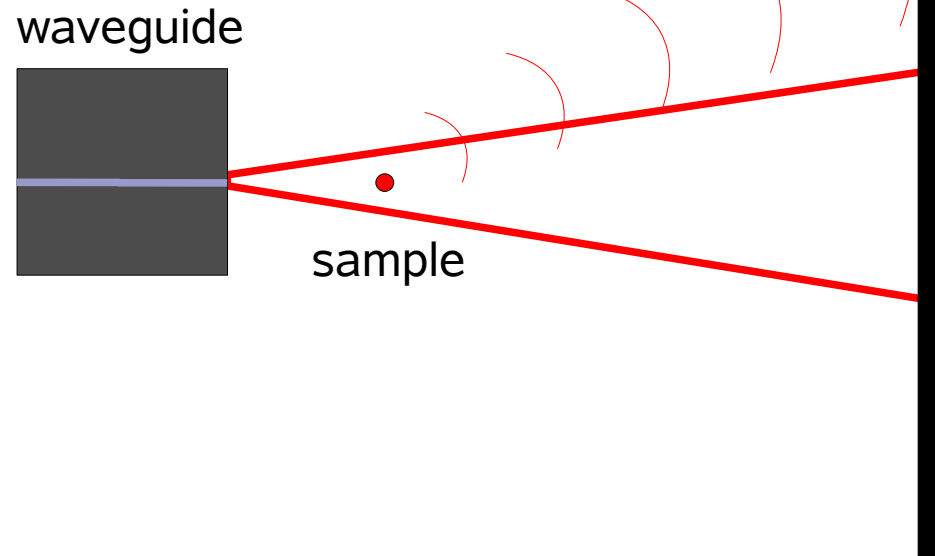
- **temporal resolution:**
single-shot experiments with highly-brilliant pulsed sources
XFEL seems well-suited
- **increase spatial resolution:**
cross-sectional dimensions of the guiding core

fundamental limit: ~ 10 nm

depending on the electron density of the utilized materials
[C. Bergemann, H. Keymeulen and J. F. van der Veen, Phys. Rev. Lett. **91**,
204801 (2003)]

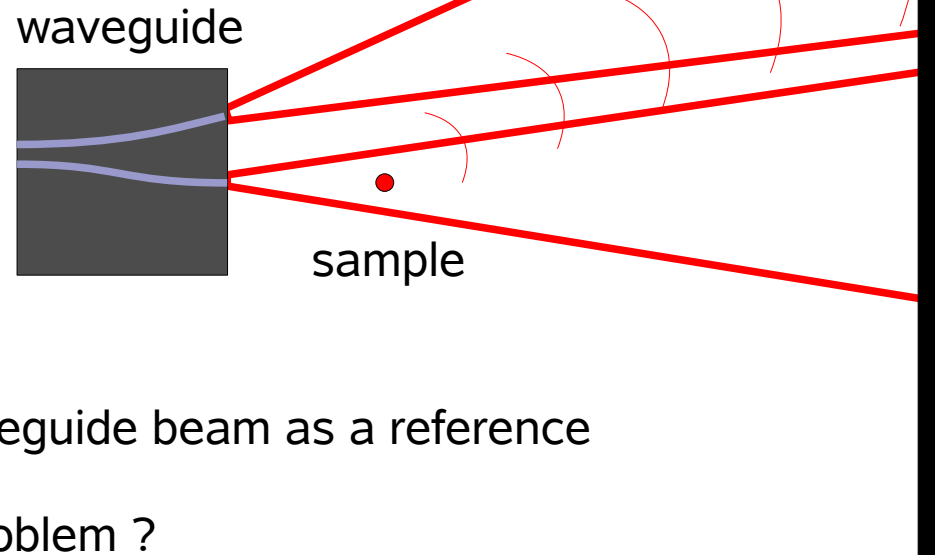
Can we achieve 1 nm resolution?

- waveguide core dimensions determine NA of the waveguide beam
spatial resolution \approx core dimensions
- higher spatial resolution requires the evaluation of radiation scattered outside the waveguide beam
- measure coherent scattering data
problem: loss of phase information



Can we achieve 1 nm resolution?

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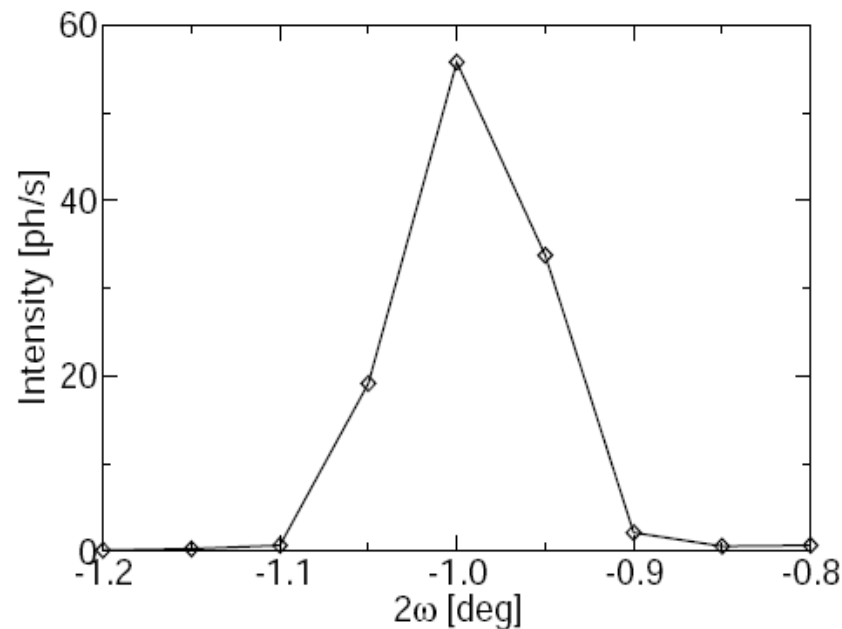
use an additional bent waveguide beam as a reference
to measure the phase!

→ solution of the phase problem ?

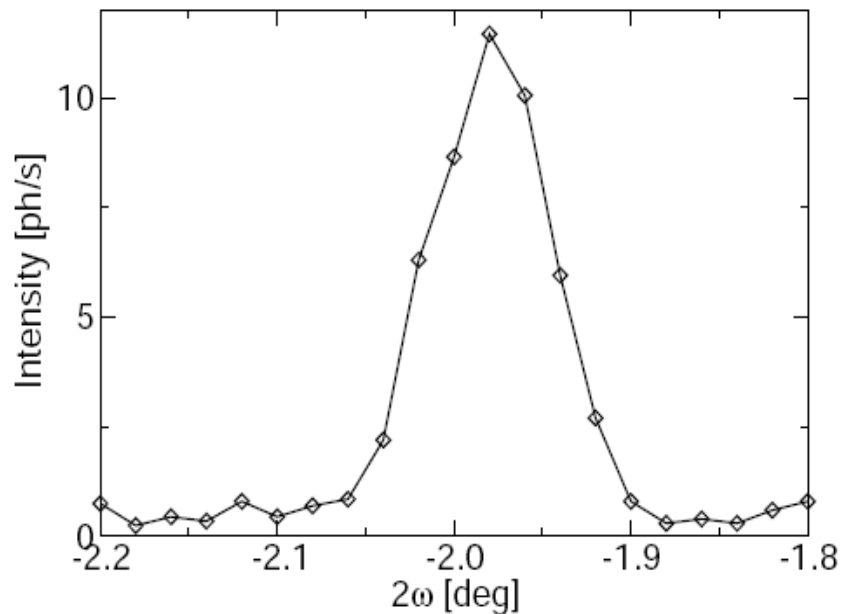
→ How far can we bend the waveguides ?

Can we achieve 1 nm resolution?

beams from bent waveguides (Si/calixarene, $d \approx 60$ nm, $E=12$ keV)
(ROBL beamline, ESRF)



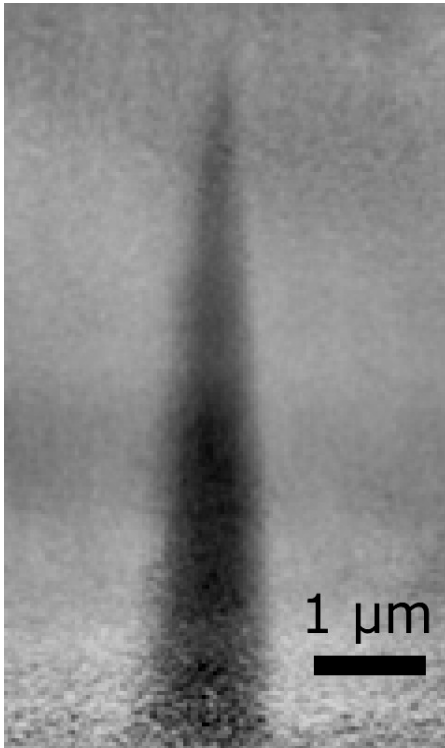
bent to 1°
losses: a factor of about 3



bent to 2°
losses: a factor of about 15

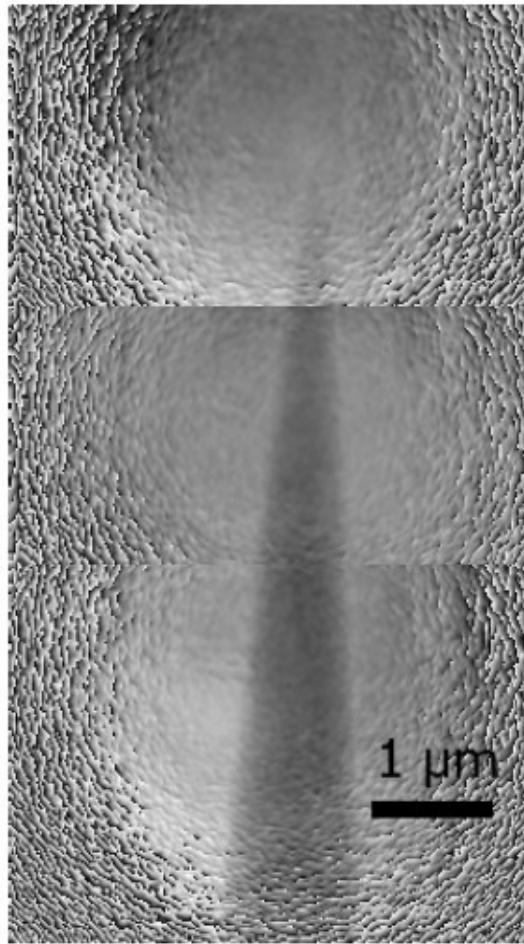
$NA = 2^\circ \rightarrow \text{spatial resolution} \approx \lambda / 2NA \approx 1.5$ nm

Acknowledgments

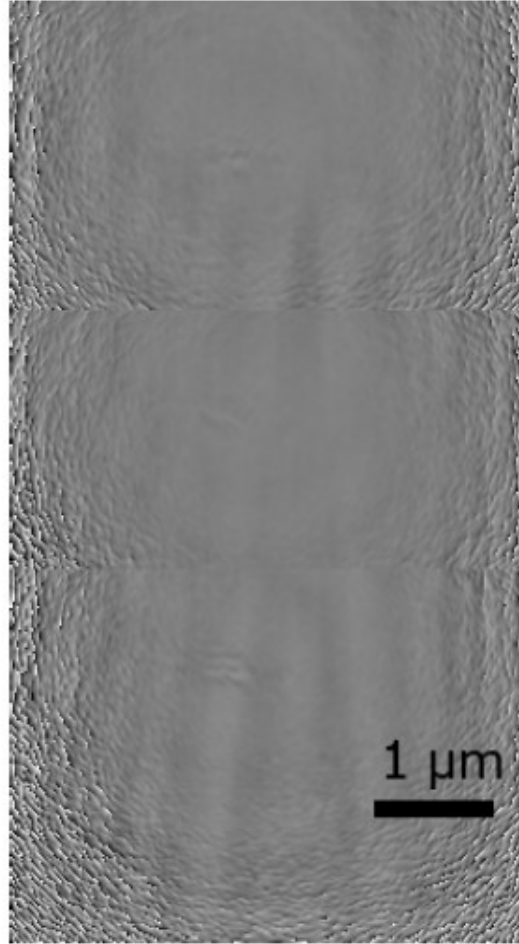


- Ansgar Jarre, Jens Seeger, Sebastian Kalbfleisch, (Institute of X-ray Physics, Göttingen)
- ESRF:
 - ID22 (Remi Tucoulou) → imaging experiments
 - ROBL (Norbert Schell) → curved waveguides
- Institute of Material Physics, Göttingen:
Talaat Al-Kassab (tungsten tips)
- German Federal Ministry of Education and Research (BMBF, grant no. 05 KS4MGA/9)

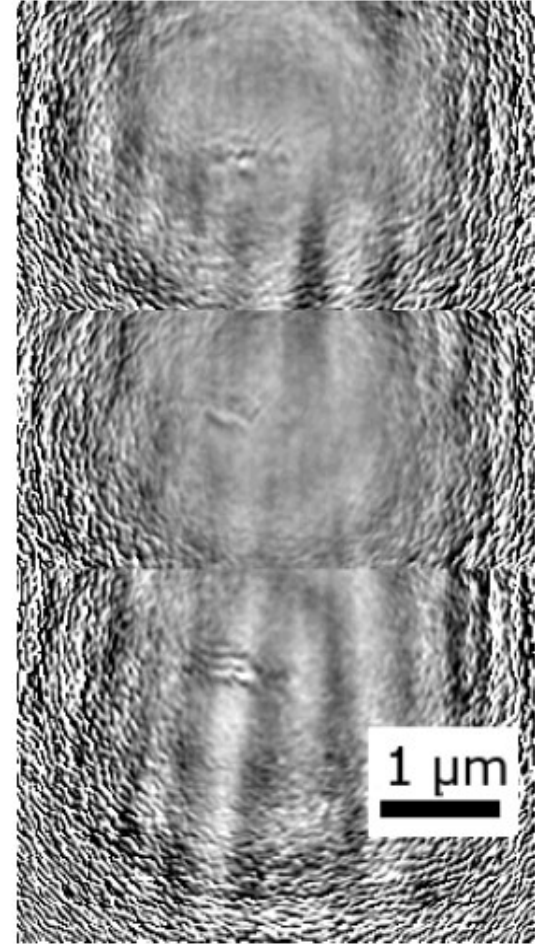
Comparison off-axis / in-line



(a) Reconstructed first-order beam (off-axis reconstruction).

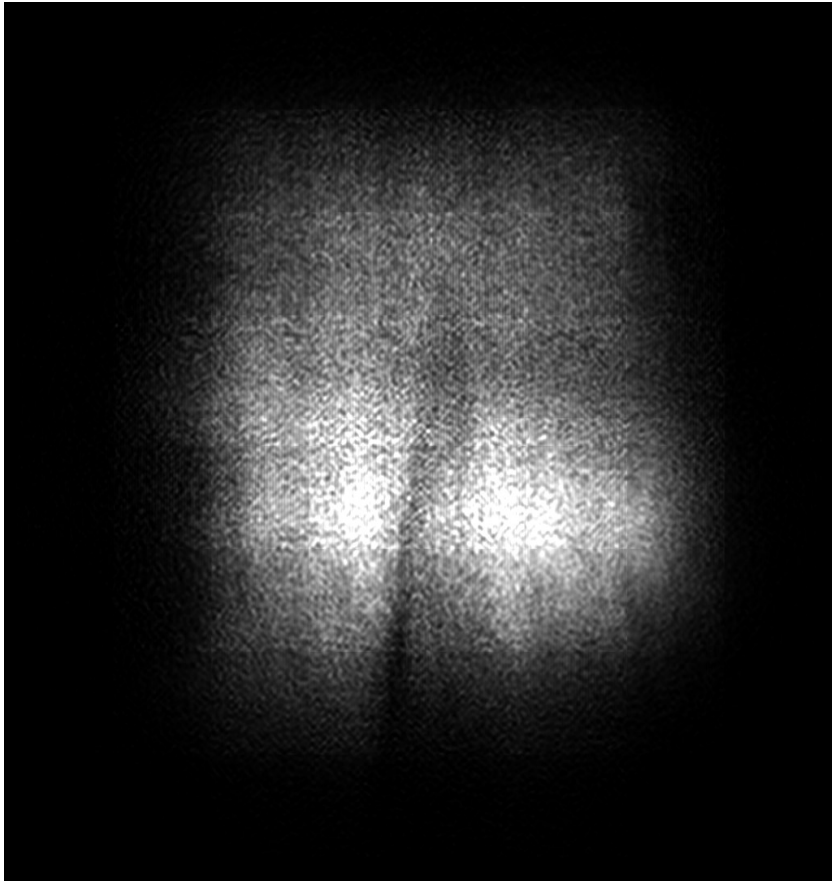


(b) Reconstructed zeroth-order beam (in-line reconstruction).

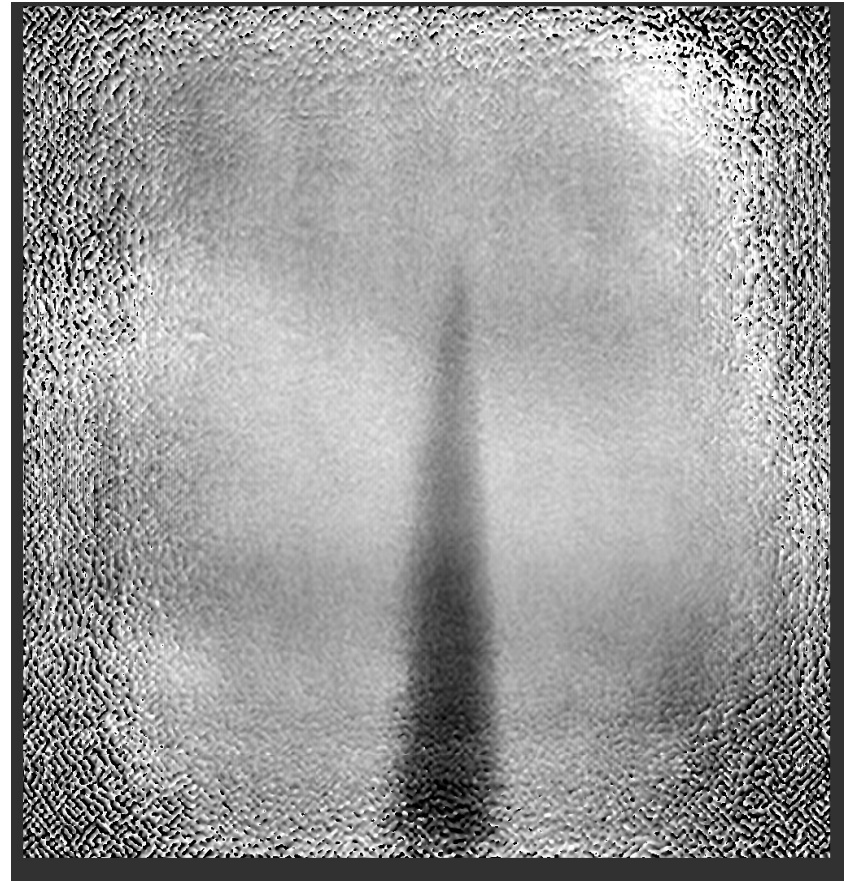


(c) Contrast-enhanced zeroth-order reconstruction.

Comparison of phase and intensity



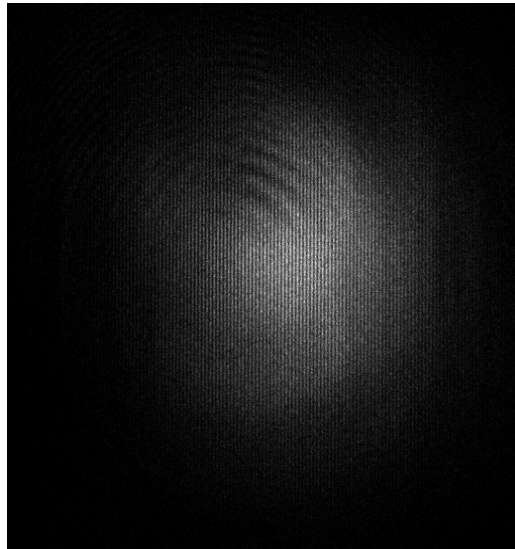
intensity



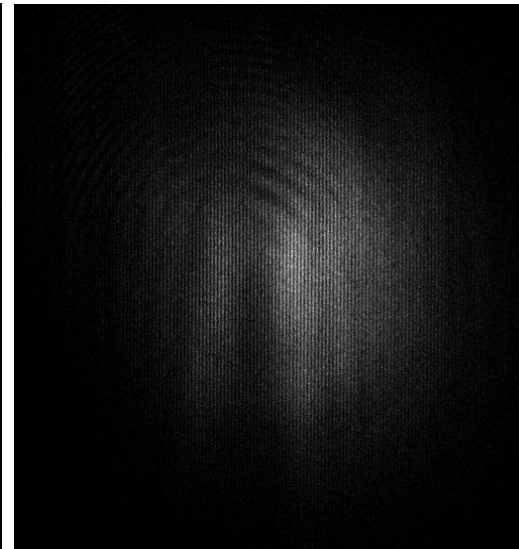
phase

Off-axis holography

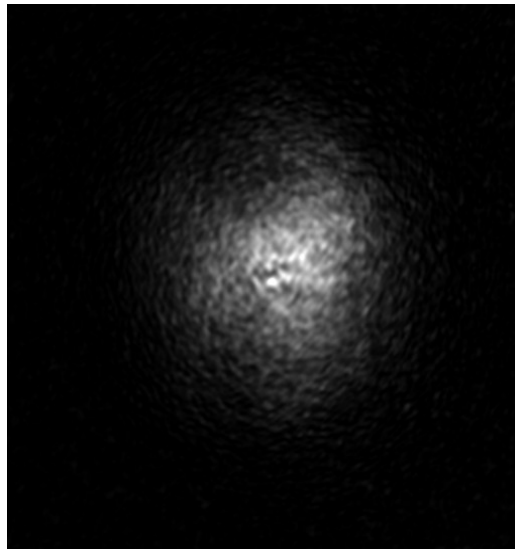
far-field
without sample



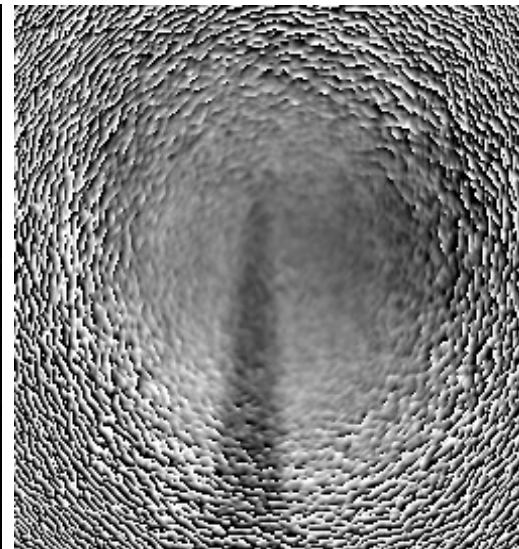
hologram of
the sample



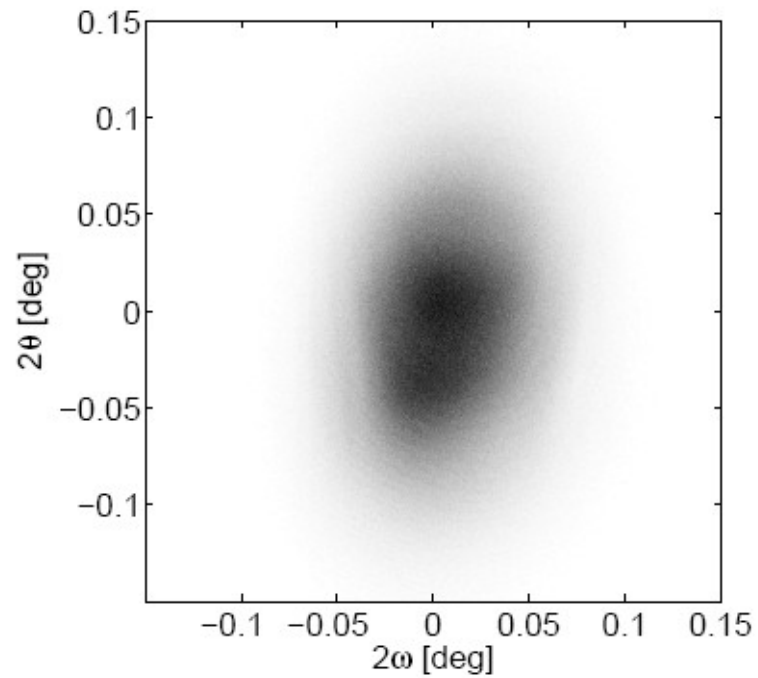
reconstruction
(intensity)



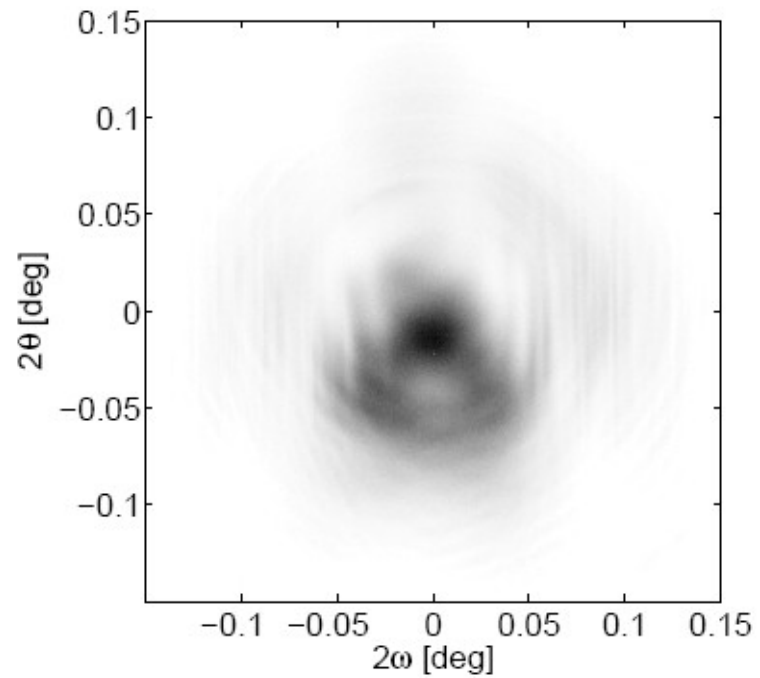
reconstruction
(phase)



Beam damage



(a) New waveguide.



(b) Used waveguide.

Fabrication of x-ray waveguides

