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# Adiabatic Refractive Lenses for Making nm Sized Hard X-Ray Beams

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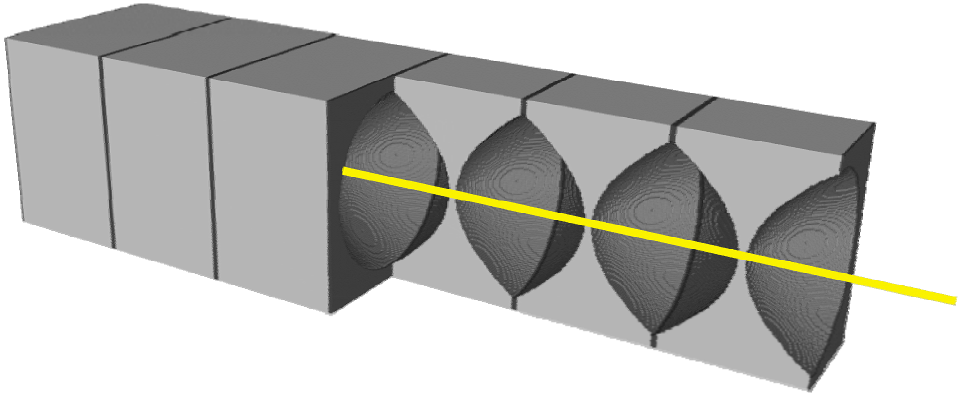
bmb+f
Großgeräte
der physikalischen
Grundlagenforschung

# Refractive X-Ray Optics

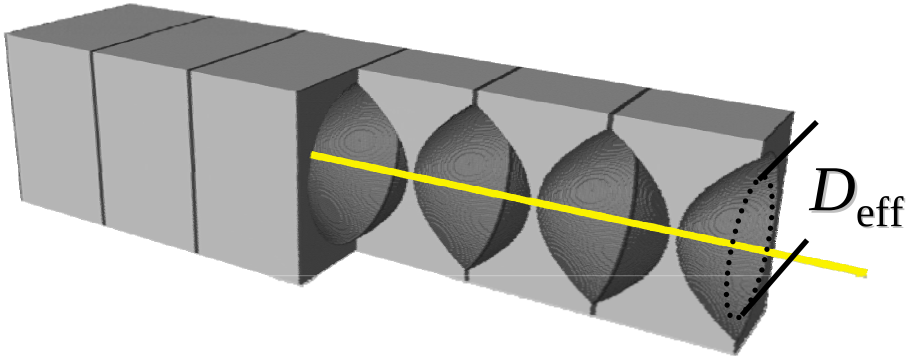
- first realized in 1996 (Snigirev et al.)
- a variety of refractive lenses have been developed since
- applied in full field imaging and scanning microscopy
- most important to achieve optimal performance:

aspherical lens shape

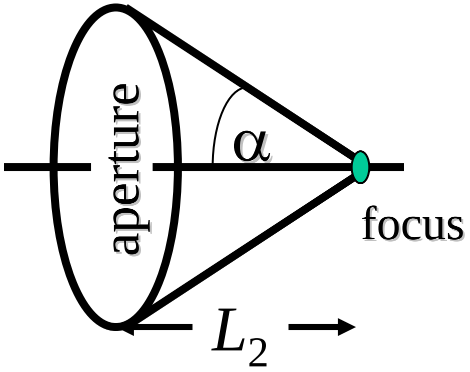


parabolic

# Effective Aperture and Diffraction Limit



Numerical aperture:



$D_{\mathrm{eff}}$ limited by:

- geometric aperture $2R_0$
- attenuation inside lens material (includes Compton scattering)

⟶ low *Z* lens material

$$NA = \sin\alpha = \frac{D_{\mathrm{eff}}}{2L_2}$$

Diffraction limit:

$$d_t = 0.75 \cdot \frac{\lambda}{2NA}$$

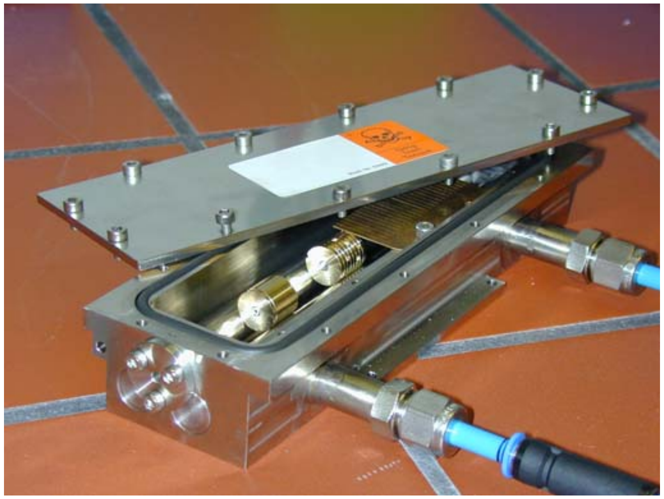
# Numerical Aperture

large *f*: aperture dominated by attenuation

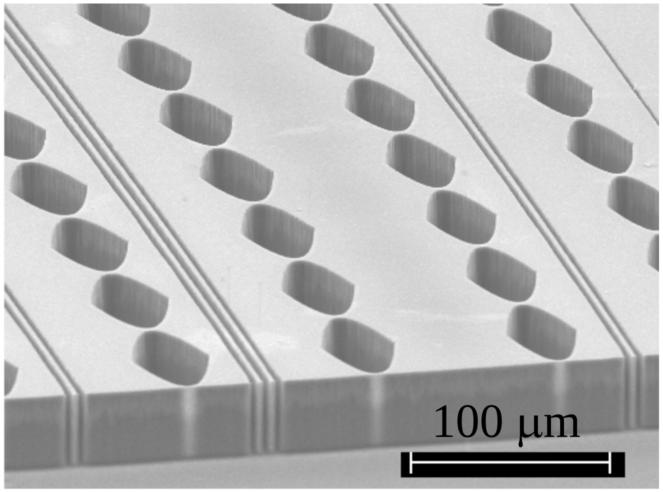
$$D_{\text{eff}} = 4\sqrt{\frac{f\delta}{\mu}} \propto \sqrt{f}$$

→ reduce μ/$\delta$ (low *Z* lens material)

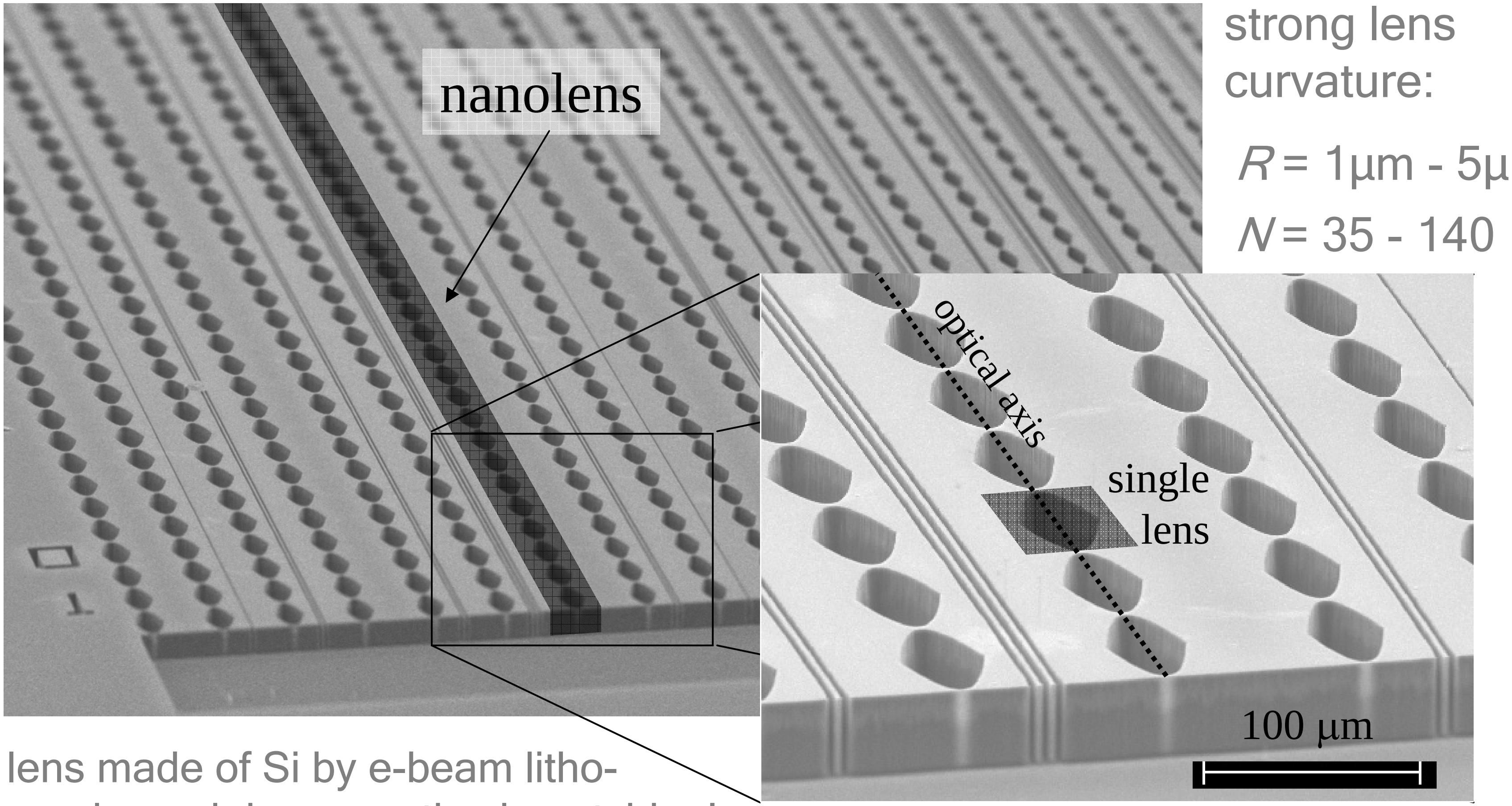
→ $NA = D_{\text{eff}} / 2f \propto 1/\sqrt{f}$ : reduce focal size to minimum



→



# Nanofocusing Lenses (NFL)



strong lens curvature:

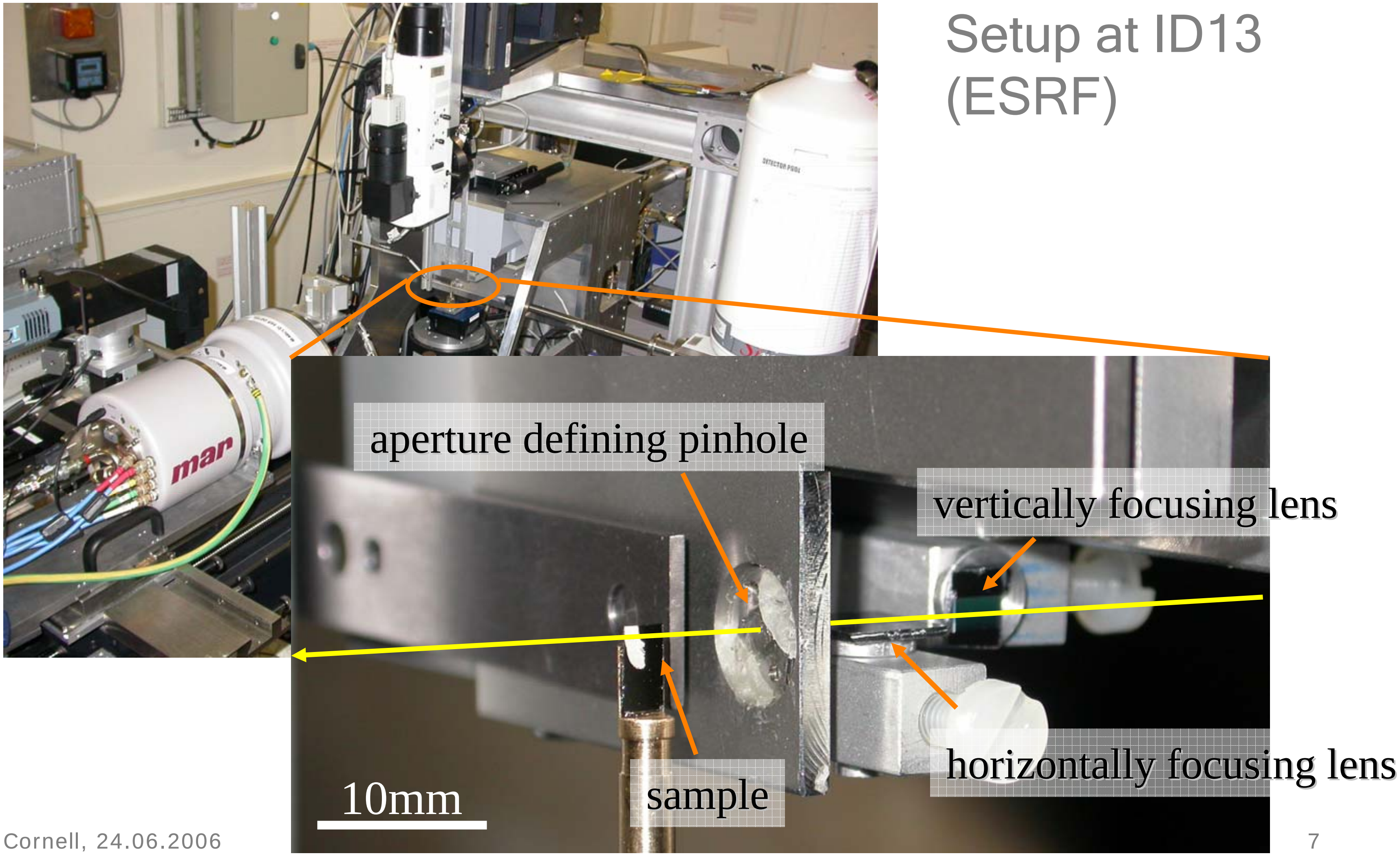
$R$ = 1µm - 5µm

$N$ = 35 - 140

lens made of Si by e-beam lithography and deep reactive ion etching!

APL 82, 1485 (2003)

# Crossed Nanofocusing Lenses



Setup at ID13
(ESRF)

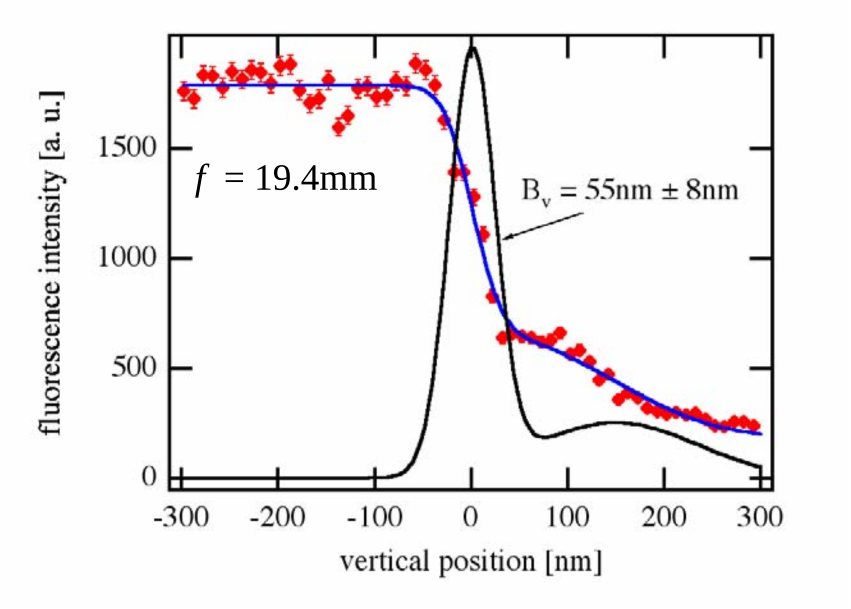
# Focusing with NFLs

Si lens: $E = 21\text{keV}$, $L_1 = 47\text{m}$

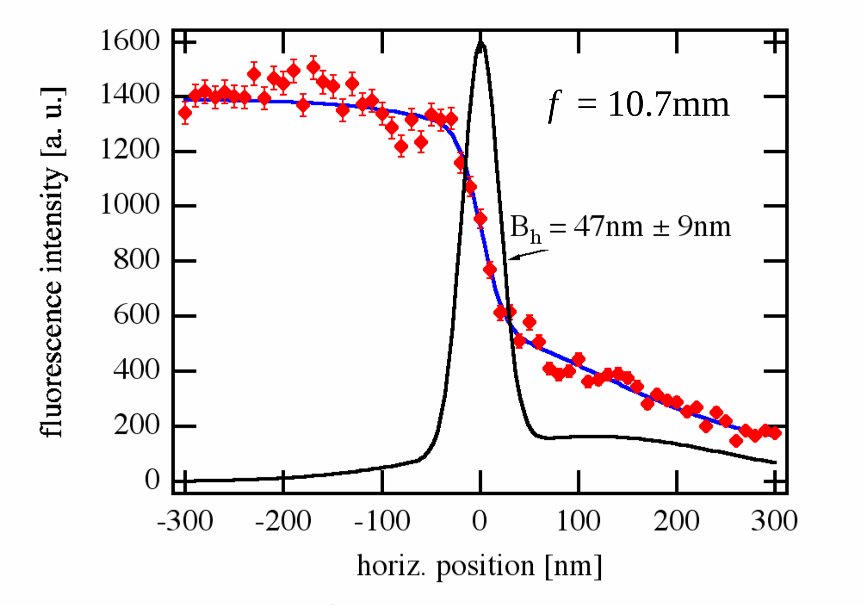
source:
ID13 low-β invac. undulator

source size: 150 x $60\mu\text{m}^2$

vertical focus: 55nm



horizontal focus: 47nm



demagnification:
~ 2400 x 4400

flux: $1.7 \cdot 10^8$ph/s

DOF = 42 x 86 $\mu\text{m}^2$

APL **87**, 124103 (2005)

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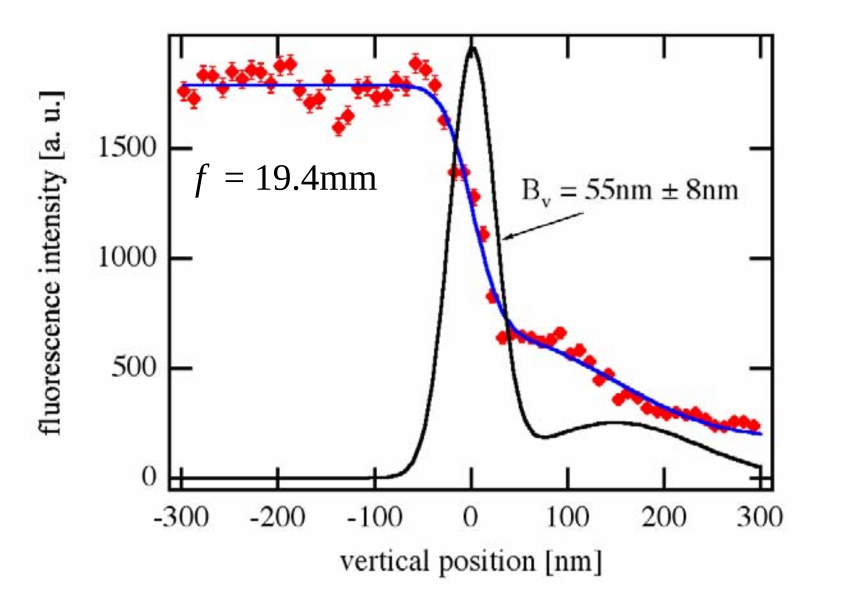
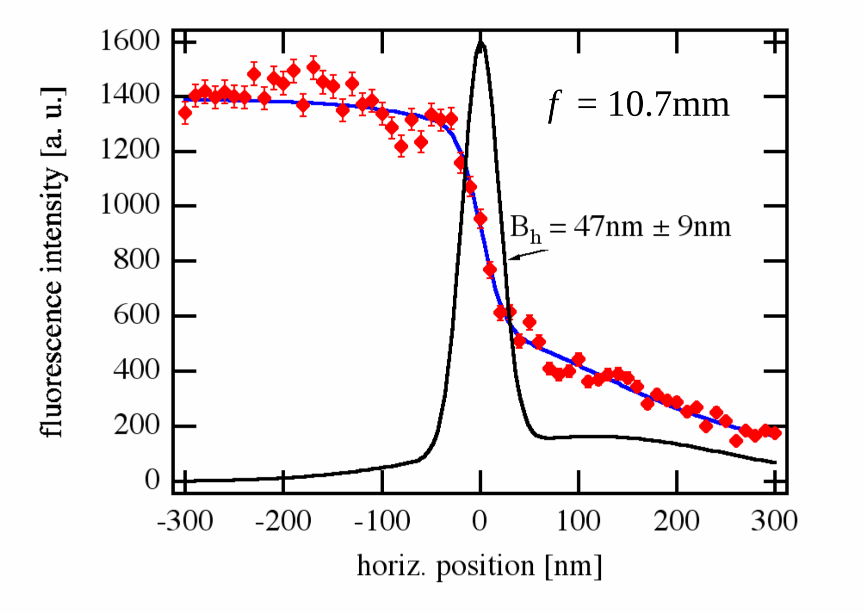


horizontal focus: 47nm

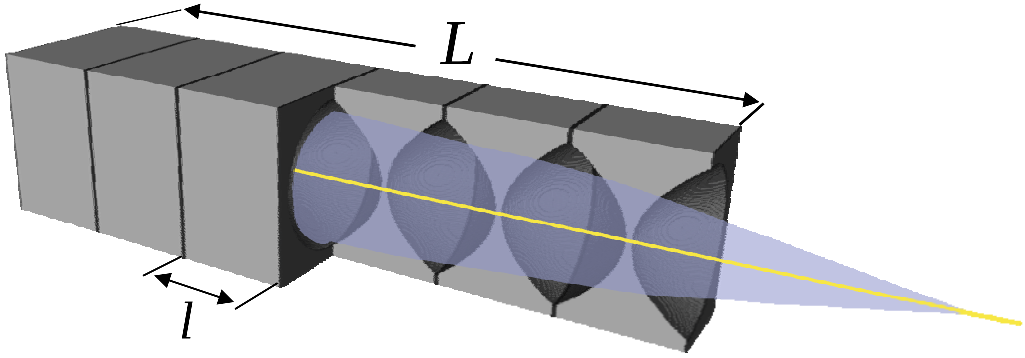


roughness: ~ 10nm rms

APL **87**, 124103 (2005)

# Effective Aperture and Diffraction Limit

Nanofocusing lens:



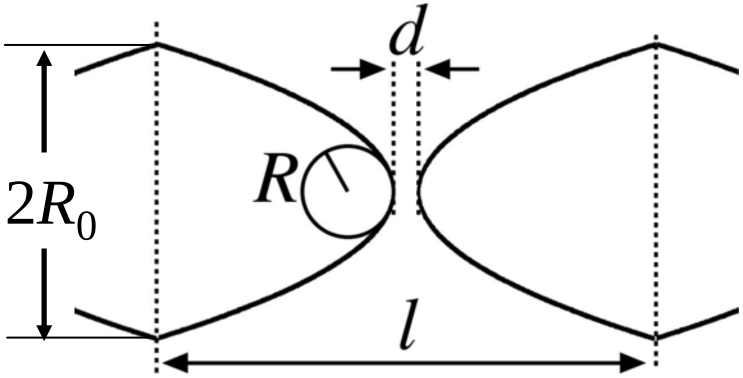
$$f_{\min} = \sqrt{f_0 L} = \sqrt{\frac{Rl}{2\delta}}$$

with $f_0 = \dfrac{R}{2N\delta}$

lens short (attenuation negligible):

$$D_{\text{eff}} < 2R_0 \approx 2\sqrt{Rl}$$

$$NA = \frac{D_{\text{eff}}}{2f_{\min}} \le \frac{2\sqrt{Rl}}{2\sqrt{\frac{Rl}{2\delta}}} = \sqrt{2\delta}$$



# Numerical Aperture of NFLs

$$NA = \frac{D_{\mathrm{eff}}}{2f_{\mathrm{min}}} \leq \sqrt{2\delta}$$

Always smaller than critical angle of total reflection

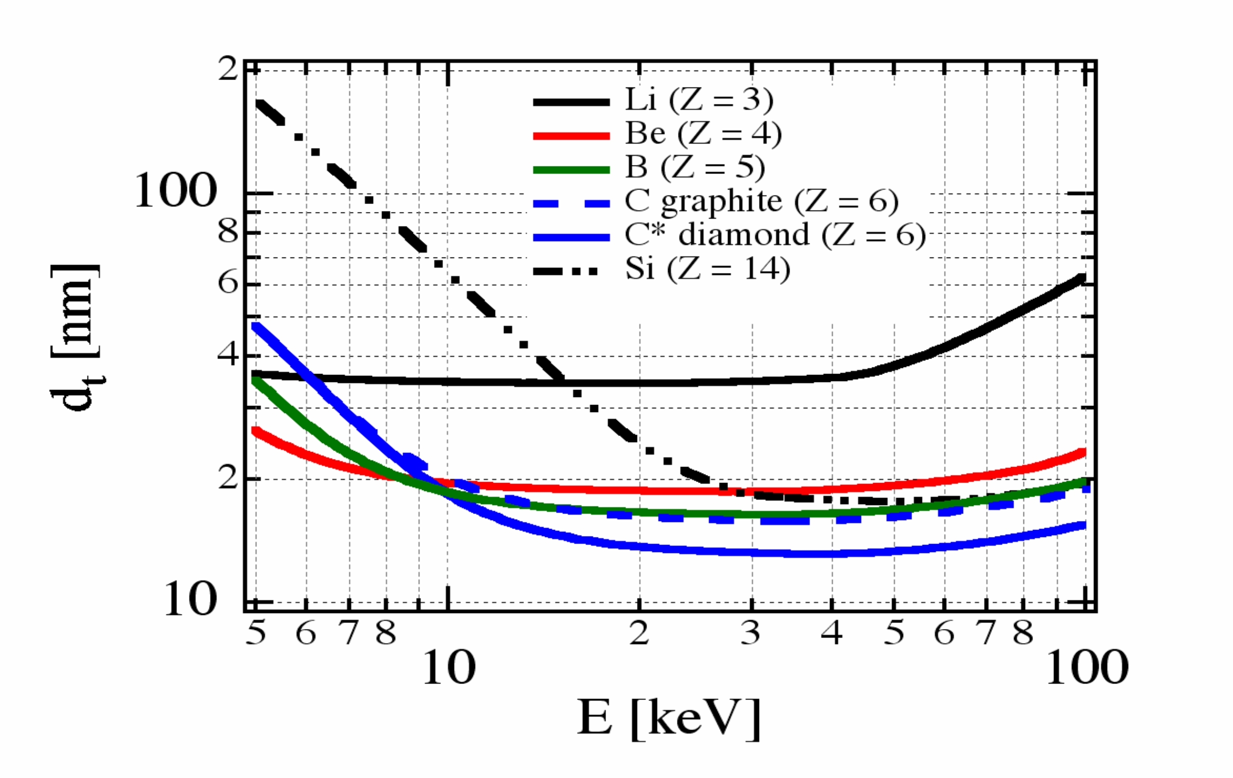
Limits diffraction limit of NFLs to

$$d_t \geq 0.75 \cdot \frac{\lambda}{2\sqrt{2\delta}} > 10\mathrm{nm}$$

for useful lens materials

# Effective Aperture and Diffraction Limit

Diffraction limit:



$N = 100$

$l \geq 0.084$

$R = 0.5$ - 50µm

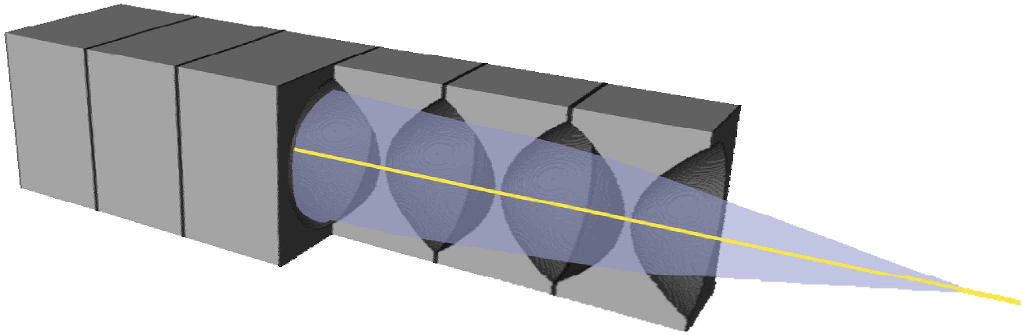
bounded by

$$0.75\frac{\lambda}{2\sqrt{2\delta}} \propto \text{const.}$$

Best materials: high density and low Z

APL 82, 1485 (2003)

# Refractive Power per Unit Length

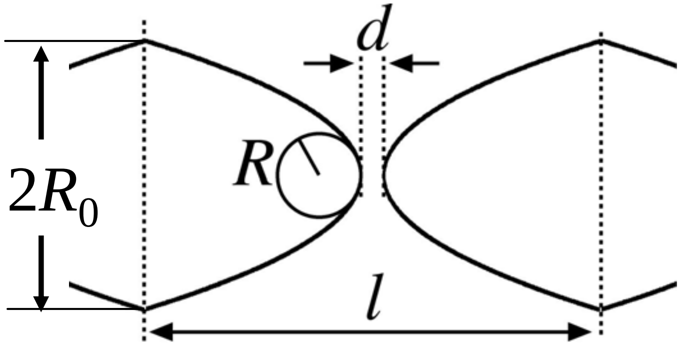


$$\omega^2 = \frac{1}{lf_s} = \frac{2\delta}{lR} \approx \frac{2\delta}{R_0^2}$$

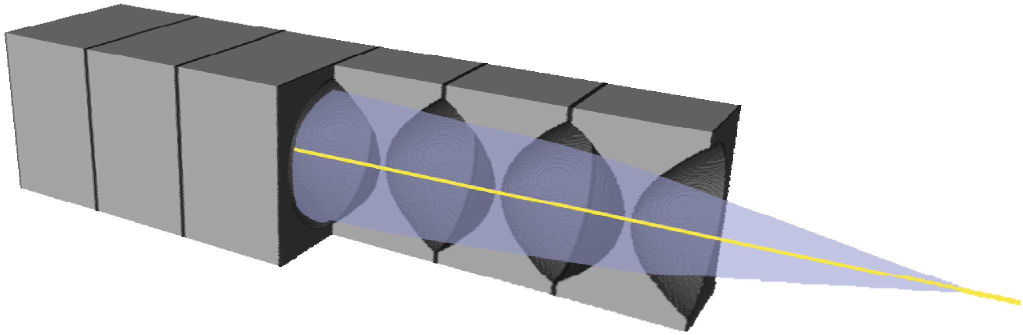
For large number of lenses:

$$r'' = \frac{d^2 r}{dz^2} = -\omega^2 r$$

Beam oscillates inside of lens (analogy to harm. oscillator)



# Refractive Power per Unit Length

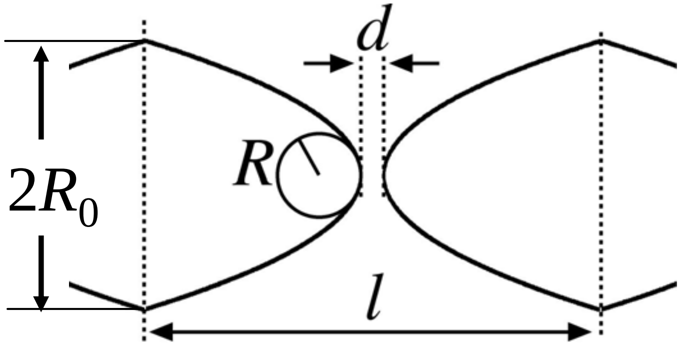


$$\omega^2 = \frac{1}{lf_s} = \frac{2\delta}{lR} \approx \frac{2\delta}{R_0^2}$$

- increases with decreasing $R_0$
- beam converges to focus inside of lens

  aperture can be decreased without loss toward exit of lens

  increase $\omega^2$ toward exit of lens



# Adiabatically Focusing Lens

adjust $R_0$ to fit the converging beam as it is focused:

$$\omega^2 = \frac{2\delta}{l_j R_j} \approx \frac{2\delta}{R_{0j}^2}$$

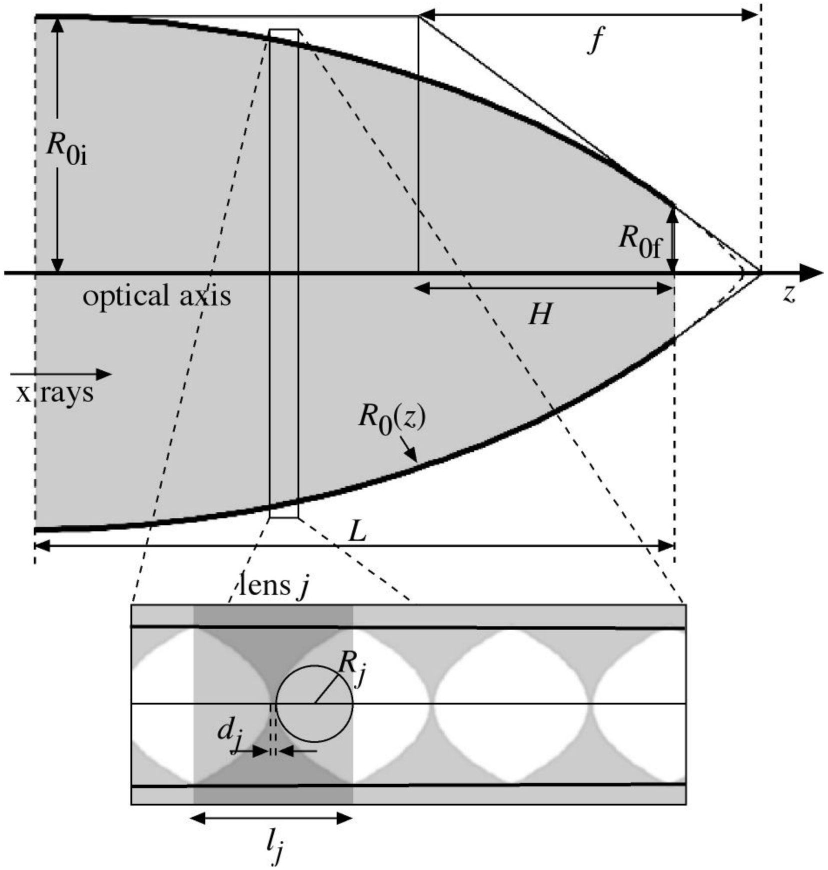
Solve

$$r'' = -\omega(z) r$$

for peripheral ray $R_0(z)$

PRL **94**, 054802 (2005)

adiabatically focusing lens (AFL)



# Adiabatically Focusing Lens

$$R_0'' = -\frac{2\delta}{R_0}$$

First integral:

$$\frac{1}{2}(R_0')^2 + 2\delta \log(R_0) = E$$

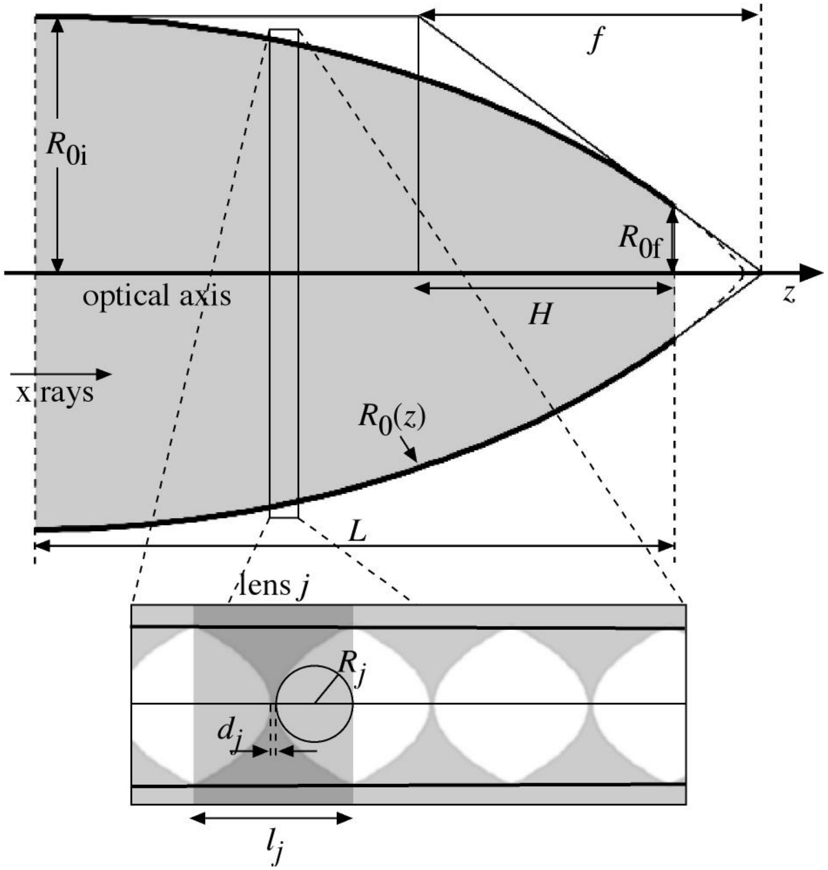
*E* defined by initial conditions

For example:

$$R_0' = 0,\ R_0 = R_{0i}$$

PRL **94**, 054802 (2005)

adiabatically focusing lens (AFL)



# Adiabatically Focusing Lens

First order differential eq.:

$$R_0' = \sqrt{4\delta \log \frac{R_{0i}}{R_0}}$$

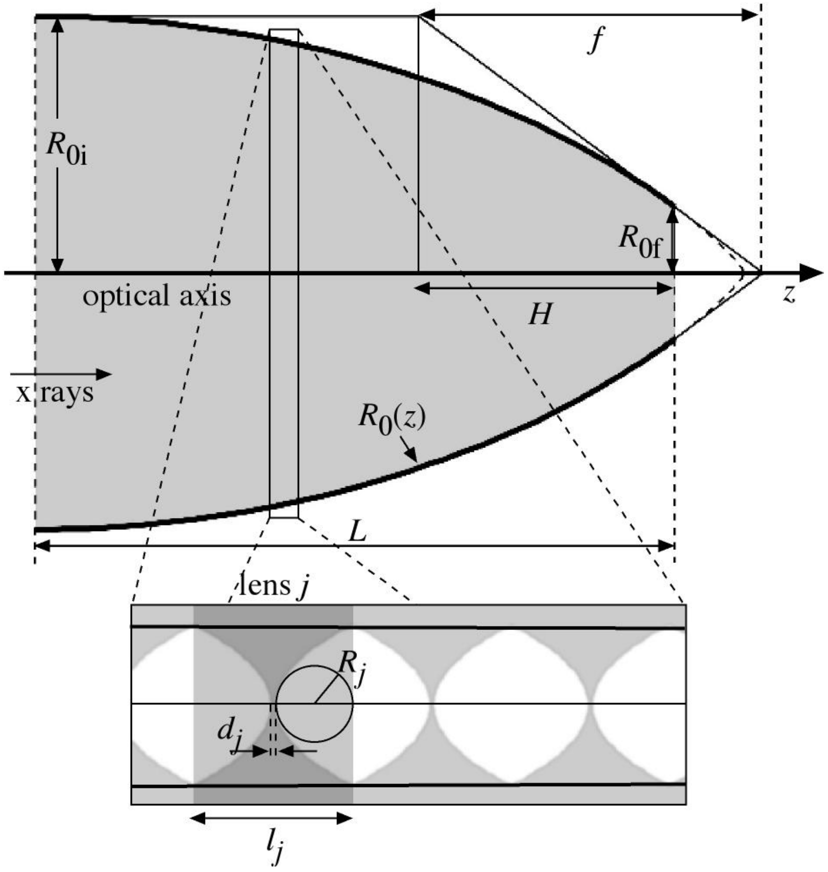
Solution shown to the right

$$f = \frac{R_{0i}}{\sqrt{4\delta \log \frac{R_{0i}}{R_{0f}}}}$$

$$D_{\mathrm{eff}} = 2R_{0i}\sqrt{\frac{2}{\mu L}\left[1-\exp\left(-\frac{\mu L}{2}\right)\right]}$$

PRL **94**, 054802 (2005)

adiabatically focusing lens (AFL)



# Adiabatically Focusing Lens

Numerical aperture: (char. aperture)

$$NA = \sqrt{\delta}\sqrt{4\frac{a}{R_{0i}}\left[1-\exp\left(-\frac{R_{0i}}{a}\right)\right]\log\frac{R_{0i}}{R_{0f}}}\text{ , with }\qquad a = \frac{2\sqrt{\delta}}{\sqrt{\pi\mu}}$$

$\delta$ large: high density $\rho$
*a* large: low absorption (low Z)

$\Big\}$ material parameters

→ optimal material: diamond (high density, low Z)

$R_{0i}$ set to maximize *NA* (0.6 - 1·*a*)
$R_{0f}$ set to minimal value

$\Big\}$ fabrication parameters

PRL **94**, 054802 (2005)

# Example AFL

Diamond lens:

low atomic number *Z* and high density $\rho$

*N* = 1166 individual lenses
entrance aperture: 18.9µm
exit aperture: 100nm
*f* = 2.3mm

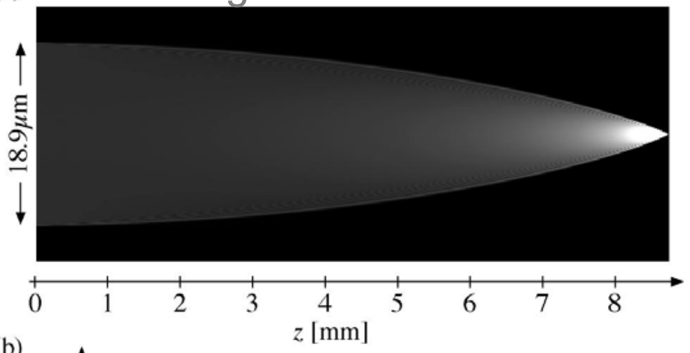
diffraction limit: 4.7nm

compare to NFL:

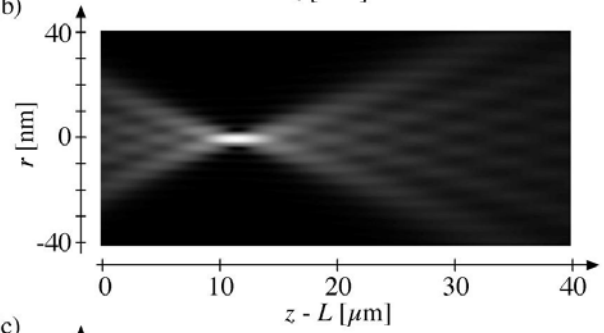
same aperture

diffraction limit: 14.2nm

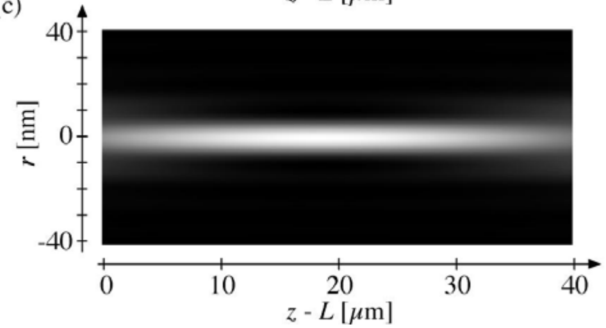
(a) contracting wave field inside lens



(b)



(c)



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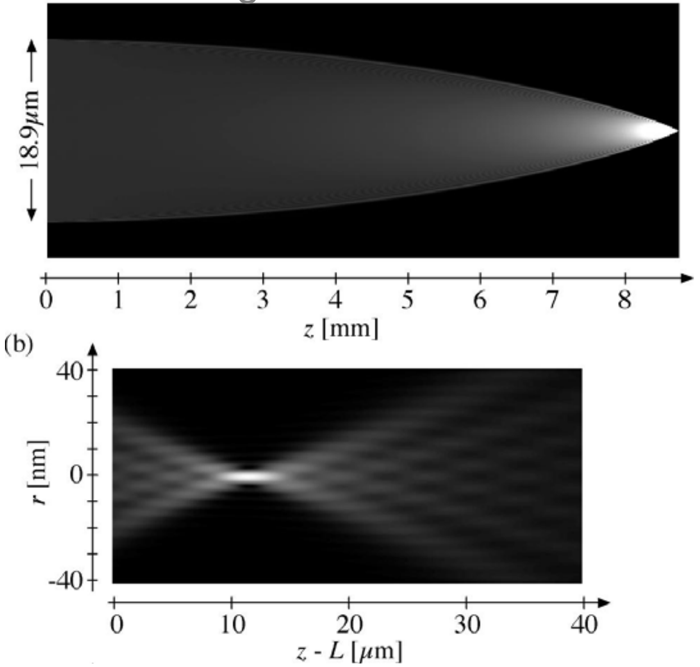


Flux in focus (@20 keV, same focus size

ERL hi-coh (15pm, 10mA): $\sim 10^{11}$ ph/s

DOF = 1.1µm

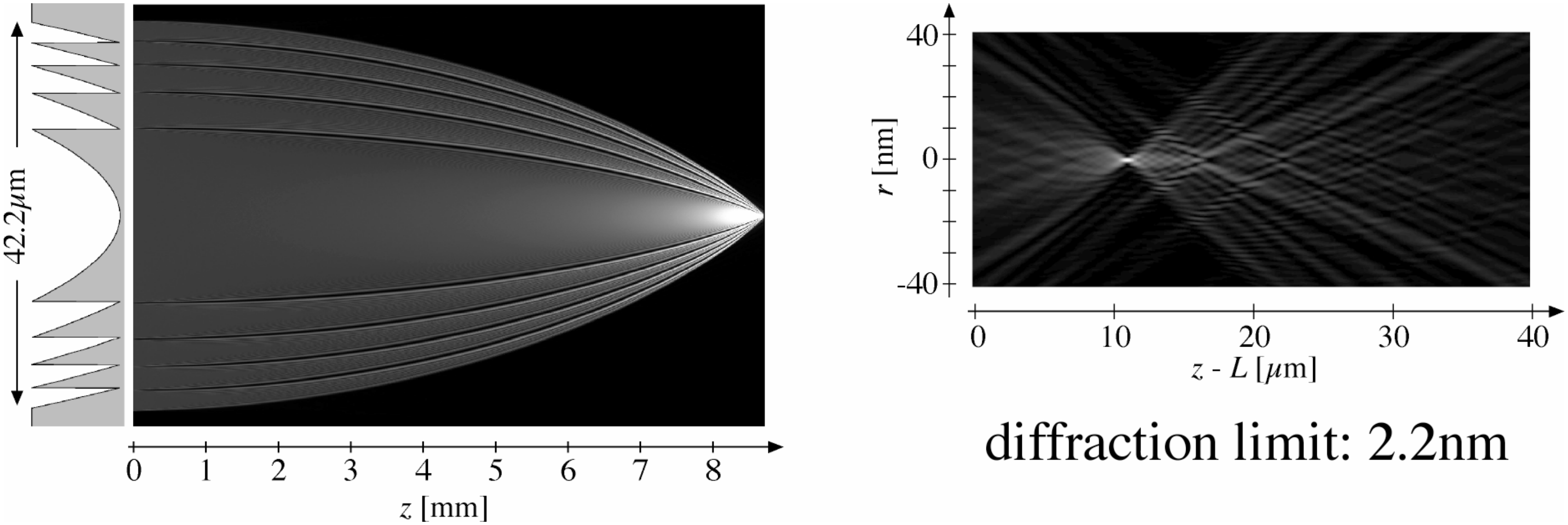
ERL hi-coh (8pm, 25mA): $\sim 10^{12}$ ph/s

ESRF, Invac. undulator: $\sim 10^{9}$ ph/s

$\longrightarrow$ $10^{7}$ - $10^{8}$ ph/Å$^{2}$/s!!

# Example AFL

kinoform lens: segment size follows converging beam



diffraction limit: 2.2nm

→ No sharp fundamental limit! Practical implementation difficult!

but

→ No atomic resolution in direct imaging with refractive lenses!

PRL **94**, 054802 (2005)

# Refractive Lenses: Summary

Numerical aperture limited:

- limited density of low $Z$ materials limits $\delta$
- characteristic aperture $a$ limits initial aperture $R_{0i}$ (as result of attenuation)
- fabrication and atomic structure limits exit aperture $R_{0f}$

# Wave Propagation Through FZP

parabolic wave equation:

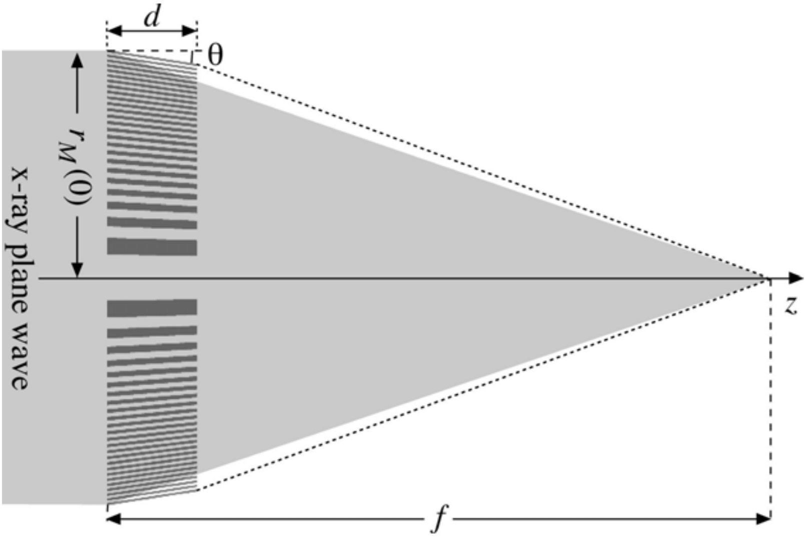
$$2ik\frac{\partial u}{\partial z}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2\left(n^2(x,y,z)-1\right)u=0$$

$n(x,y,z)=1-\delta(x,y,z)+i\beta(x,y,z)$ complex potential!

Ni/vac. zone plate

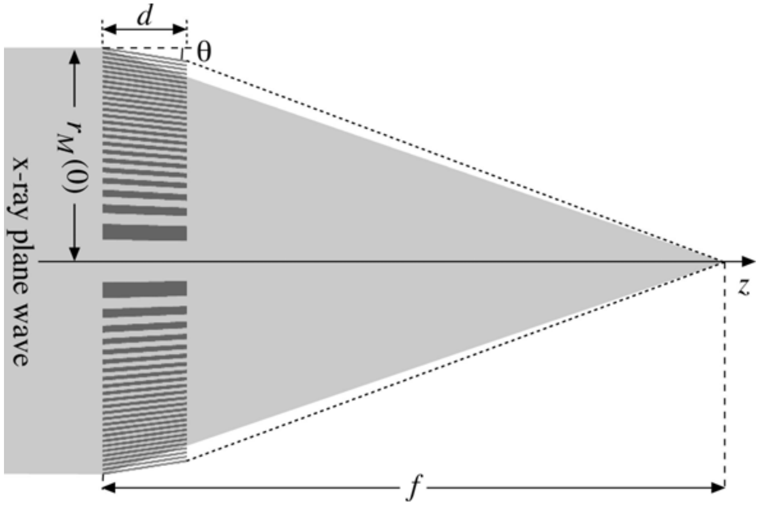
$E$ = 20 keV, $r_M(0)$ = 0.8µm

$\Delta r_M$ = 1nm



(inspired by poster by
F. Pfeiffer at XRM2005)

# Wave Field Inside FZP

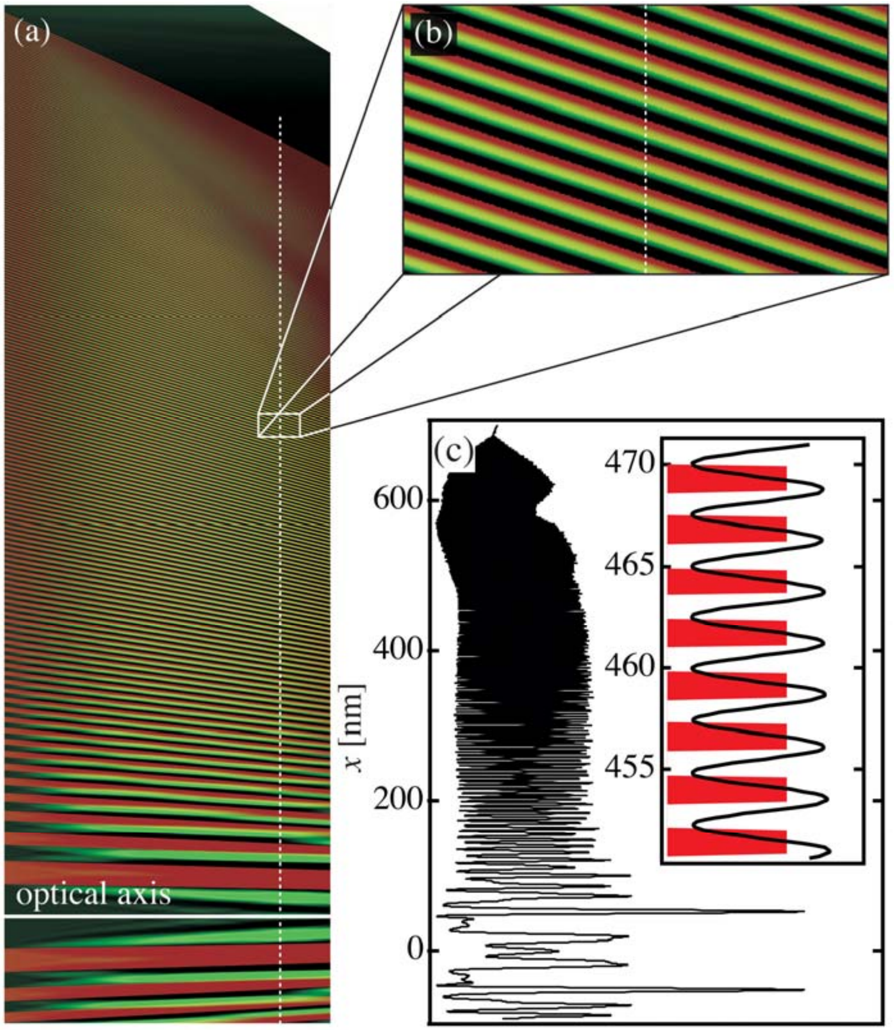


ideal tilted FZP

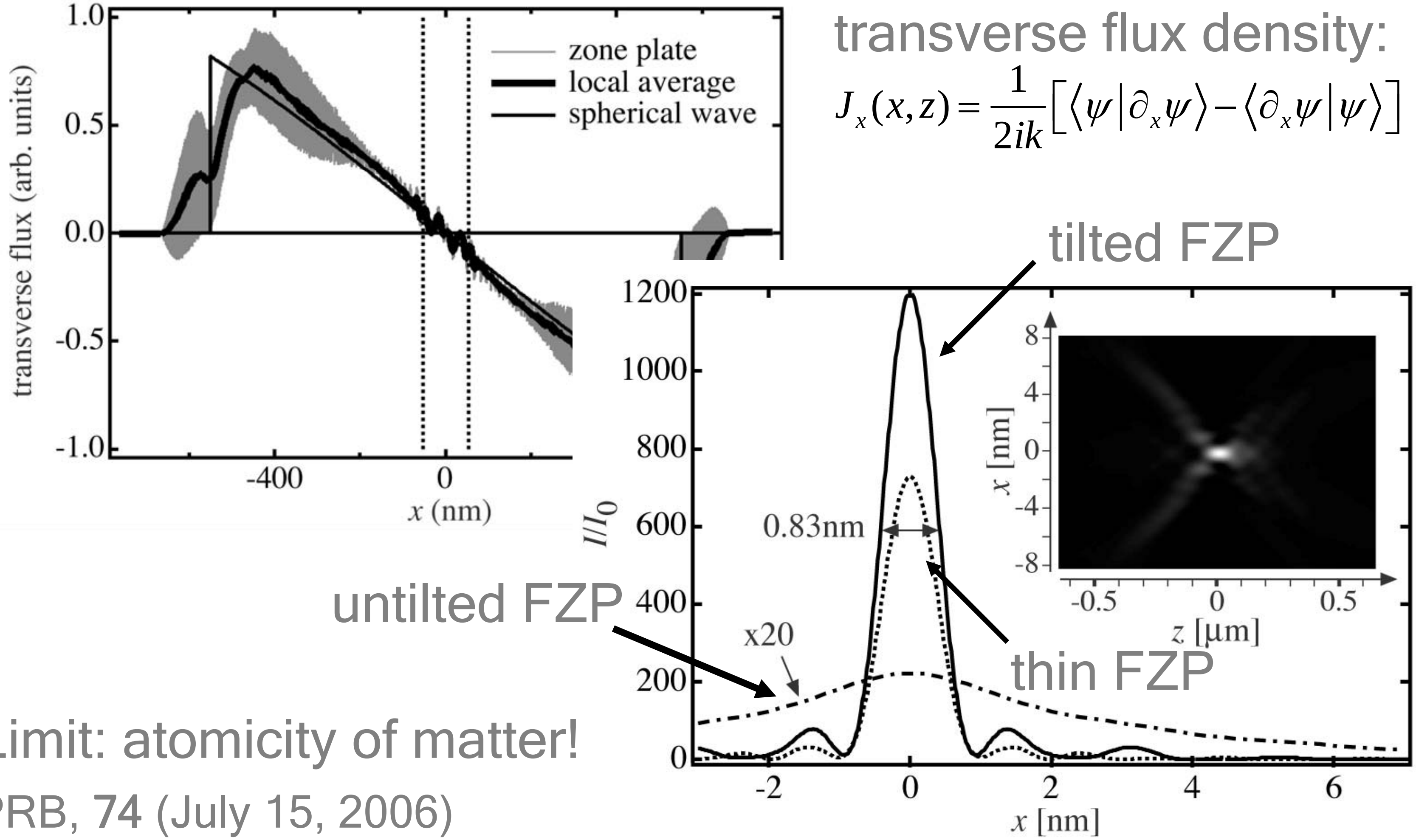
[Kang, et al., PRL 96 127401 (2006)]

incoming plane wave

propagate exit wave field to focus



# FZP Focus



transverse flux density:

$$J_x(x,z) = \frac{1}{2ik}\left[\langle\psi|\partial_x\psi\rangle - \langle\partial_x\psi|\psi\rangle\right]$$

tilted FZP

untilted FZP

thin FZP

Limit: atomicity of matter!

PRB, **74** (July 15, 2006)

# FZP: Summary

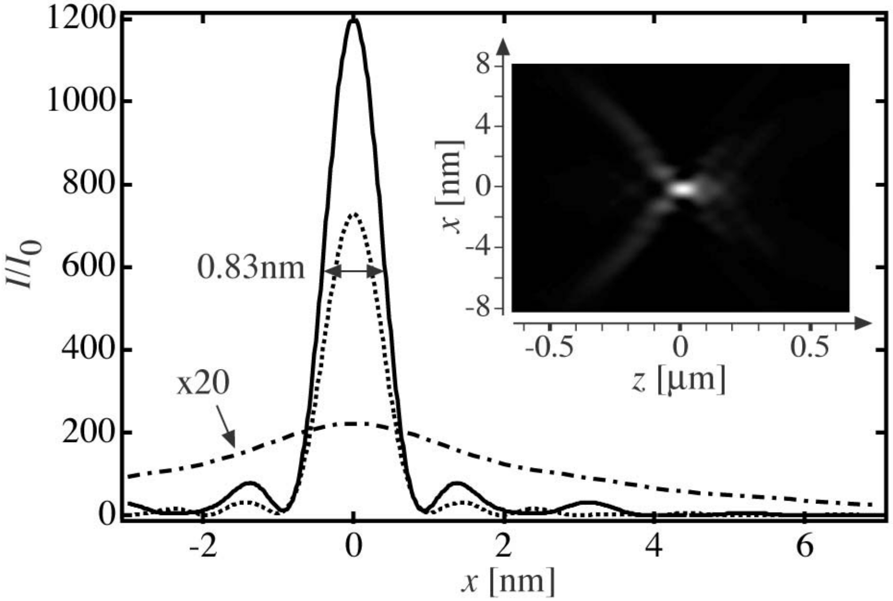
no limit as long as matter is homogeneous

multilayers have been shown to behave homogeneously down to below 2 nm *d*-spacing (1 nm layers)

high efficiency, since only one diffraction order is excited!

atomicity will limit zone placement!

other optics may be calculated similarly!



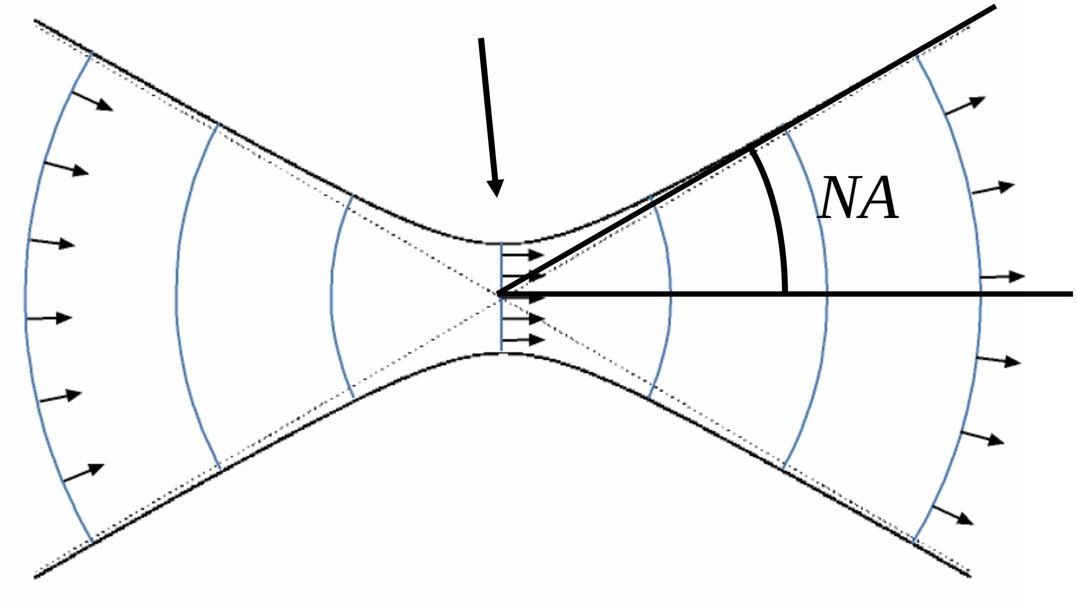
PRB, **74** (July 15, 2006)

# Wave Front in Diffraction Limited Focus

divergence angle:
numerical aperture

$$d_t = \frac{2\sqrt{2\ln 2}}{\pi}\frac{\lambda}{2NA} \approx 0.75\frac{\lambda}{2NA}$$

Gaussian limited plane wave

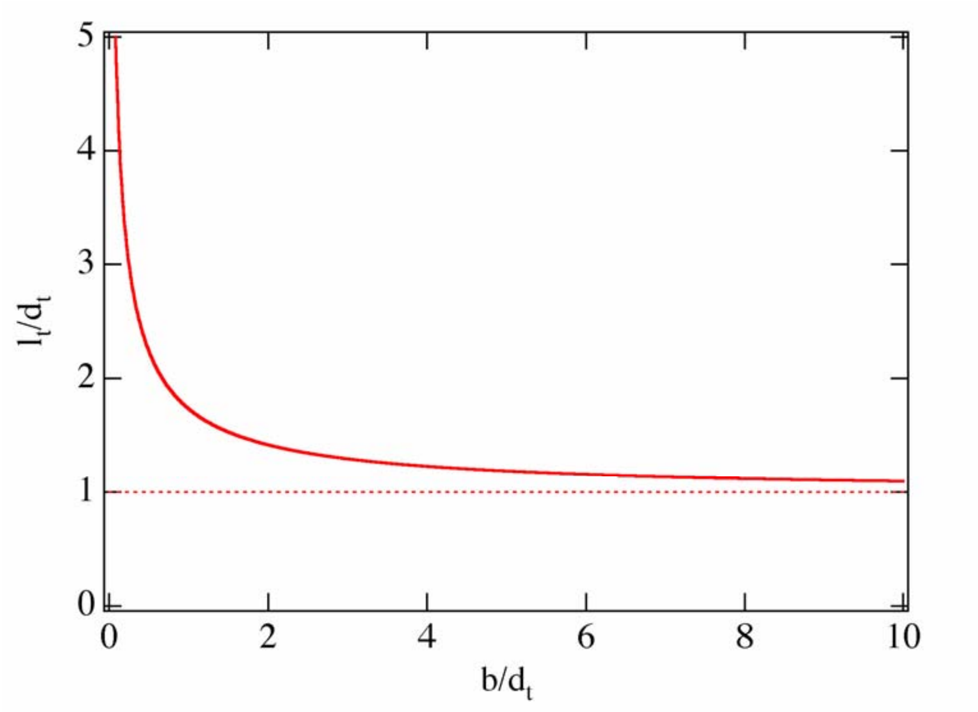


- coherent diffraction
- XPCS, XFCS

XFCS, J. Wang, et al.,
PRL 80, 1110 (1998)

# Wave Front in Diffraction Limited Focus

Lateral coherence length in nanofocused beam:



$$l_t = d_t\sqrt{1+\frac{2d_t^2}{b^2}}$$

$d_t$: diffraction limit
$b$: geometric image of source

Coherent diffraction at nanoparticles possible,
as long as particles are smaller than diffraction limit.

# Coherence in the Focus

Si NFL @ ID13, $E = 15$keV nominal parameters:

FWHM focus size:

66 x 75 $\text{nm}^2$

diff. limit:

54 x 71 $\text{nm}^2$

↓

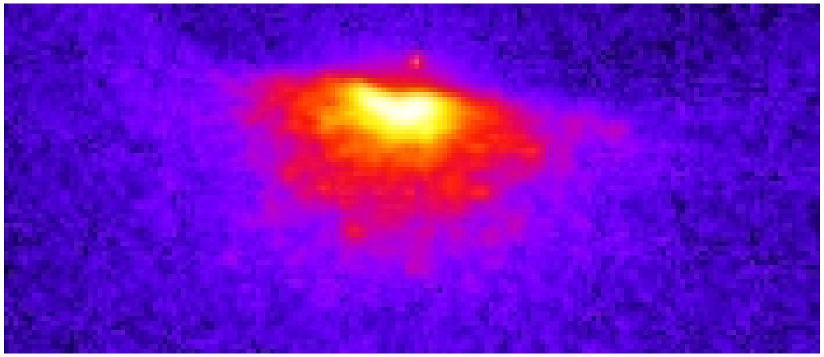
lateral coherence length:

120 x 300 $\text{nm}^2$

divergence (*NA*):

0.58 x 0.43 $\text{mrad}^2$

Preliminary experiment:
Diffraction from Fe-particles
(~ 40nm diam., on $Si_3N_4$ membrane, from R. Röhlsberger)



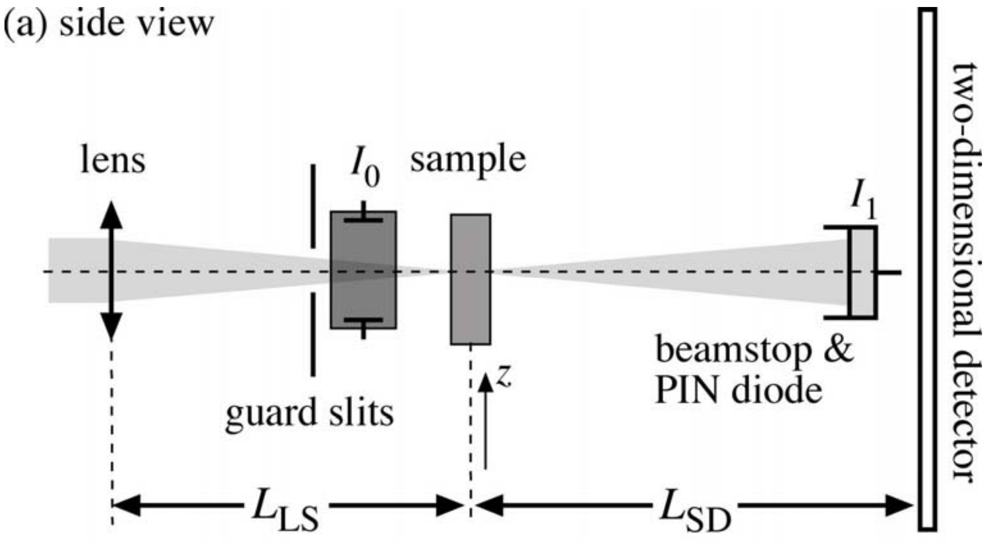
(exposure: 10 s)

Visibility reduced:
mechanical instability!

horiz. beam size: 120nm

# Scanning Coherent Diffraction Microscopy

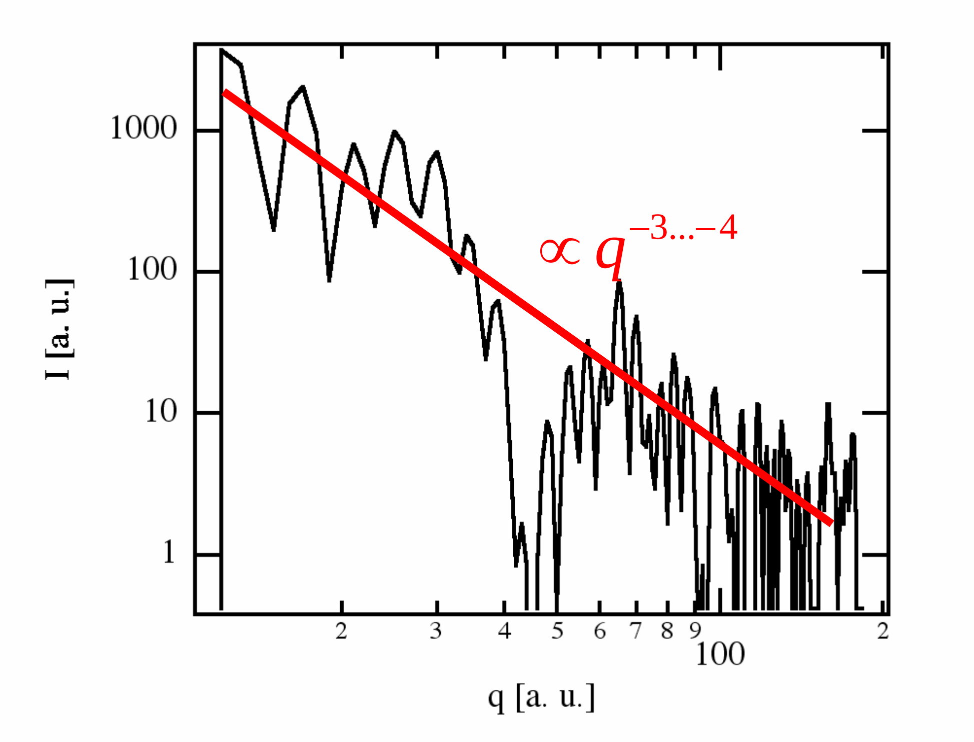
bridge the „small“ gap to atomic resolution by using coherent diffraction imaging contrast



„arbitrary“ samples: support defined by illumination

# Scanning Coherent Diffraction Microscopy

bridge the „small“ gap to atomic resolution by using coherent diffraction imaging contrast



smaller *q*-range (WAXS)

reduced requirements on dynamic range of detector

larger detector pixels

short local exposure (up to $10^7$ph/Å$^2$)

# Conclusion

Refractive optics:

- hard x-ray beams of 5nm seem feasible
  (limiting factor is attenuation and atomicity of matter)
- kinoform lenses would reduce focus size (feasibility?)

Fresnel zone plates (tilted):

- focus below 1 nm should be feasible
  (limiting factor is atomicity of matter)

Challenging experiment:

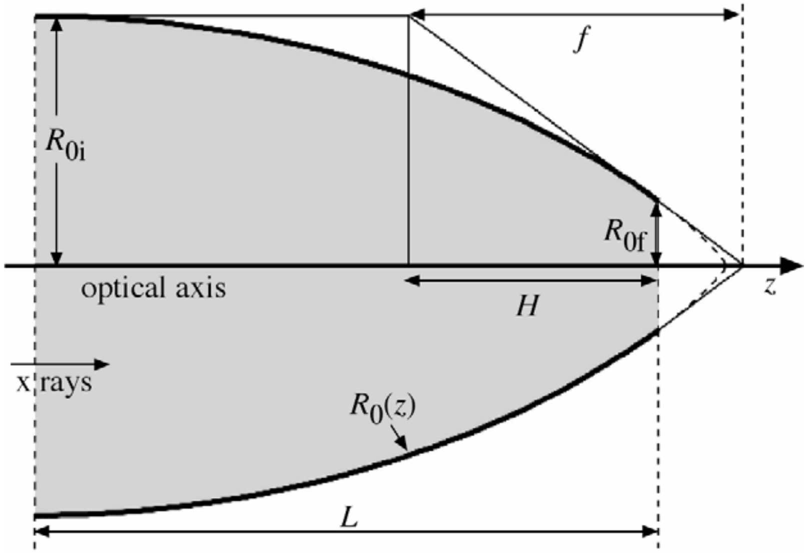
- scanning coherent diffraction microscopy

# AFLs Made of Silicon

entrance aperture: $2R_{0i}$ = 20µm
exit aperture: $2R_{0f}$ = 1µm
energy: 10 - 20keV in 500eV steps

properties:

$f$ = 2.7mm
$d_t$ = 12.6nm



as horizontal lens in x-ray nanoprobe (e. g. ID13 ESRF):

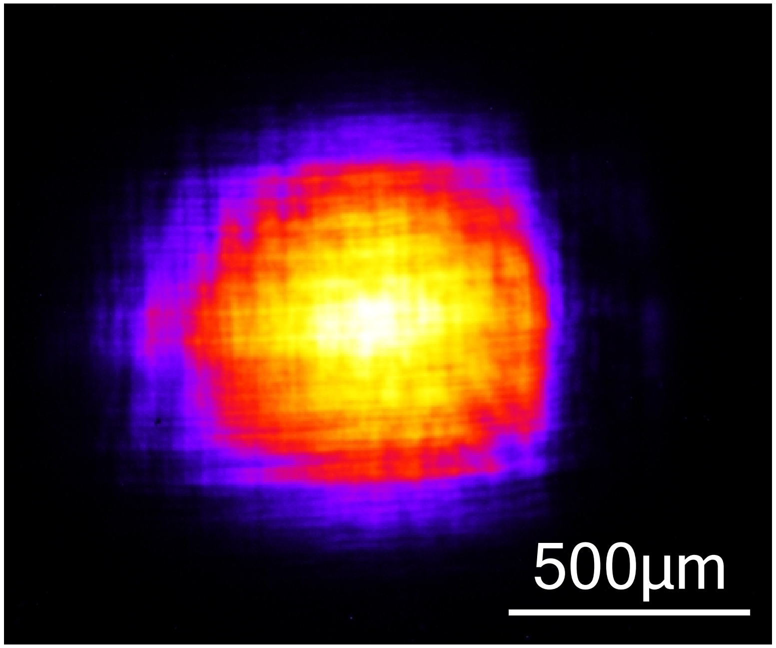
$L_1$ = 47m, source size: 150µm

horizontal focus: 15.3nm
(17400 x reduction)

# Far Field of Focus: Aberrations

Si NFL @ ID13, $E$ = 15keV

Far field image of focus:



Structure:

irregularities in lens shape

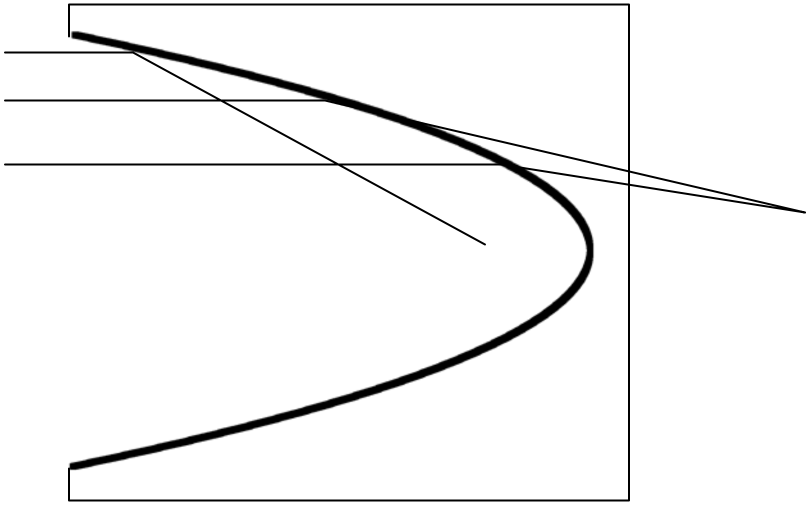
reconstruction of lens shape?

[Quiney, et al., Nat. Phys. **2**, 101 (2006)]

detector dist.: 800mm
log (I)

# Optimal Numerical Aperture of Single Lens

First scenario:



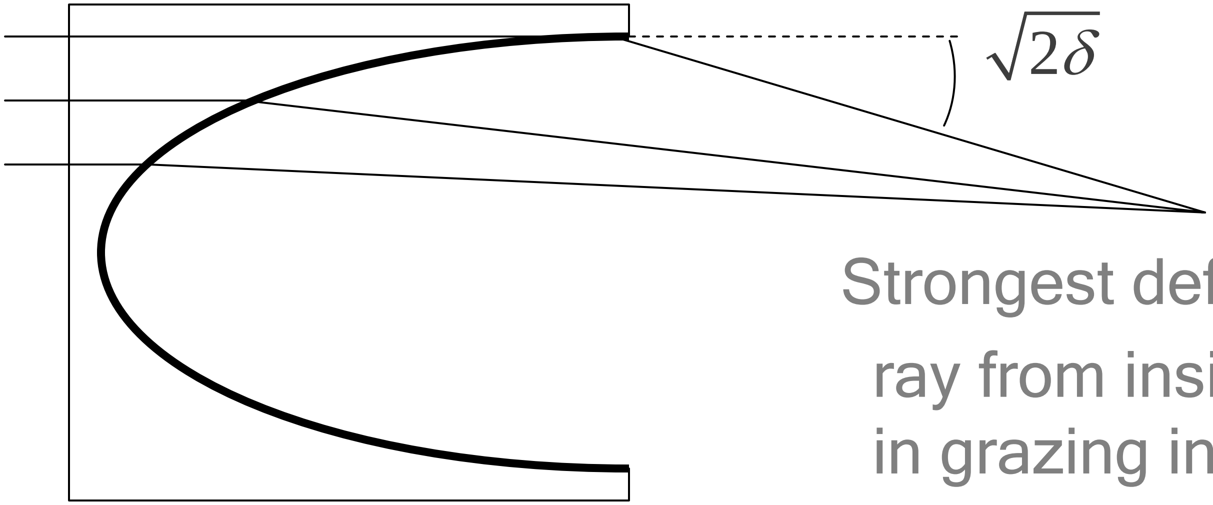
Works as long as ray is not totally reflected

Deflection angle < $\sqrt{2\delta}$

NA limited by $\sqrt{2\delta}$ even for non-absorbing material

# Optimal Numerical Aperture of Single Lens

## Second scenario:



Strongest deflection:

ray from inside the material
in grazing incidence

deflection angle < $\sqrt{2\delta}$

NA limited by $\sqrt{2\delta}$

→ make more than one refraction to increase *NA*