

CSR INCLUDING SHIELDING IN THE BEAM DYNAMICS CODE BMAD

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Abstract

Short bunches radiate coherently at wavelengths that are longer than their bunch length. This radiation can catch up with the bunch in bends and the electromagnetic fields can become large enough to significantly damage longitudinal and transverse bunch properties. This is relevant for many accelerators that rely on bunch compression. It is also important for Energy Recovery Linacs, where spent beams are decelerated by a large factor increasing the relative energy spread and hence increasing the impact of wake fields.

In this paper we show how the beam dynamics code Bmad computes the effect of CSR and how the shielding effect of vacuum chambers is included by the method of image charges. We compare the results to established codes: to *elegant* for cases without shielding and to a numerical solution of simplified Maxwell's equations, as well as to analytical CSR-wake formulas. Good agreement is generally found.

INTRODUCTION

The Coherent Synchrotron Radiation (CSR) wake-fields calculation that was introduced by Saldin *et al.*[3] has been generalized by Sagan[1] to include arbitrary lattice configurations of bends and drifts. As discussed in this paper, this formalism has been implemented in the particle tracking code Bmad[4, 2], and compared with two of the codes described in [5], as well as with analytic formulas.

CSR KICK

The electric field \mathbf{E} felt by a particle due to a source particle is given exactly by the Liénard-Wiechert formulas. If both particles travel on the same trajectory with velocity βc , then this electric field depends only upon the positions z and z' of the affected and source particles along the path.

The rate of energy change along the path of the affected particle is $d\mathcal{E}/ds = e \mathbf{n} \cdot \mathbf{E} \equiv K$, where \mathbf{n} is the unit tangent vector. Unfortunately, this expression has a singularity as path length separation distance $\zeta = z - z'$ goes to zero. To alleviate this problem, we use the regularization procedure of [3] to split \mathbf{E} into a space charge (SC) part and a CSR part, as in

$$\mathbf{E}_{\text{SC}} \equiv \frac{e}{4\pi\epsilon_0} \frac{\mathbf{n}}{\gamma^2 \zeta^2}, \quad \mathbf{E}_{\text{CSR}} \equiv \mathbf{E} - \mathbf{E}_{\text{SC}}, \quad (1)$$

so that

$$K \equiv K_{\text{CSR}} + K_{\text{SC}} = e \mathbf{n} \cdot \mathbf{E}_{\text{CSR}} + e \mathbf{n} \cdot \mathbf{E}_{\text{SC}}. \quad (2)$$

For charges distributed smoothly over a line with density $\lambda(z)$, the CSR kick is therefore

$$K_{\text{CSR}}^{\text{tot}} = \int_{-\infty}^{\infty} dz' \lambda(z') K_{\text{CSR}}(z, z'). \quad (3)$$

Within a bend of radius R , K_{CSR} is highly peaked in amplitude near $\zeta = 0$, especially for simulations at ultra-relativistic energies. One way of resolving this peak is to integrate by parts, giving

$$K_{\text{CSR}}^{\text{tot}} = \int_{-\infty}^{\infty} dz' \frac{d\lambda(z')}{dz'} I_{\text{CSR}}(z, z'), \quad (4)$$

$$I_{\text{CSR}}(z, z') = - \int_{-\infty}^{z'} dz'' K_{\text{CSR}}(z, z''). \quad (5)$$

In an accelerator, the particle trajectories are typically straight lines and arcs of circles, as in drifts and bends, and source and kicked particles are not necessarily in the same element. Assuming small angles and relativistic velocities, K_{CSR} has been worked out for such a geometry in [2], yielding

$$K_{\text{CSR}}(z, z') = 4r_c m c^2 \gamma^4 \tau^2 \left\{ \frac{(\tau^2 - \alpha^2)(\alpha - \tau \kappa)}{R(\tau^2 + \alpha^2)^3} + \frac{\tau^2 - \alpha^2 + 2\tau \alpha \kappa}{(\tau^2 + \alpha^2)^3} \right\} - \frac{r_c m c^2}{\gamma^2 \zeta^2} \quad (6)$$

Here α , τ , κ and ζ are second order polynomials in angles and $1/\gamma$. Equation (6) can be integrated to yield

$$I_{\text{CSR}}(z, z') = -r_c m c^2 \left(\frac{2\gamma(\tau + \alpha \kappa)}{\tau^2 + \alpha^2} - \frac{1}{\gamma^2 \zeta} \right). \quad (7)$$

This is the main result of [2], and is the basis for CSR calculations in Bmad.

CSR IN BMAD

The above algorithm for simulating CSR has been implemented as part of the Bmad [4] subroutine library for

relativistic charged-particle simulations. Bmad simulates a beam as a set of particles. The beam is tracked through a lattice element by dividing the element into a number of slices. Tracking through a slice involves first propagating the particles independently from each other and then applying the CSR kicks. To calculate the energy kick, the beam is divided longitudinally into N_b bins. For computing the charge in each bin, each beam particle is considered to have a triangular charge distribution, and the overlap of the triangular charge distribution with a bin determines that particle's contribution to the total charge in that bin. Increasing the particle width smooths the distribution at the cost of resolution.

The charge density λ_i at the center of the i^{th} bin is taken to be $\lambda_i = \rho_i / \Delta z_b$ where ρ_i is the total charge within the bin and Δz_b is the bin width. The charge density is assumed to vary linearly in between the bin centers. The CSR energy kick for a particle at the center of the j^{th} bin after traveling a distance ds_{slice} according to Eq. (4) is then

$$d\mathcal{E}_j = ds_{\text{slice}} \sum_{i=1}^{N_b} (\lambda_i - \lambda_{i-1}) \frac{I_{\text{CSR},j-i} + I_{\text{CSR},j-i+1}}{2}, \quad (8)$$

where $I_{\text{CSR},j} \equiv I_{\text{CSR}}(\zeta = j \Delta z_b)$.

CHAMBER WALLS

The simulation incorporates the shielding of the top and bottom chamber walls by using image currents. Here, because the image current is well separated from the actual beam, there are no singularities to deal with, K_{sc} does not have to be subtracted, and a straight forward integration is done using Eq. (3),

$$d\mathcal{E}_j \quad (\text{image}) = 2 * ds_{\text{slice}} \quad (9)$$

$$\times \sum_{k=1}^{N_i} (-1)^k \sum_{i=1}^{N_b} q_i * K(z = (j - i)\Delta z_b, y = kh),$$

where $q_i = \lambda_i \Delta z_b$ is the charge in a bin, h is the chamber height, and k indexes the image currents at vertical displacement $y = \pm kh$. The number of image layers N_i needs to be chosen large enough so that the neglected image currents do not have a significant effect on the simulation results. Because the relevant angles are not small, the image charge kick K must be calculated without the small angle approximation, as in Eq. (6). The full Liénard-Wiechert fields are therefore used, where the retarded positions are found numerically.

COMPARISON BETWEEN BMAD, THE AGOH AND YOKOYA CODE, AND ELEGANT

In order to validate our method, we compare simulations from Bmad with a code developed by Agoh and

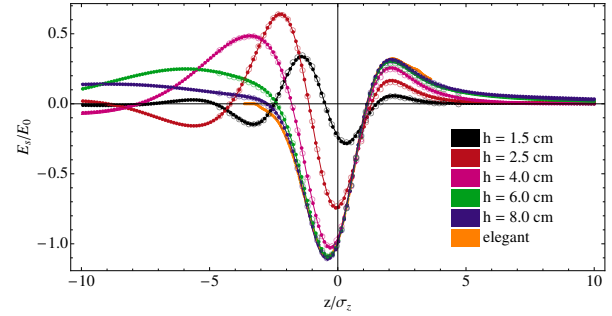


Figure 1: The steady state ($R = 10\text{m}$, $\sigma_z = 0.3\text{mm}$) for varying chamber heights h . The A&Y code (dots), Bmad (circles), and the analytic shielded CSR-wake (lines) agree well, and elegant agrees with the data at large chamber heights.

Yokoya (A&Y), which calculates CSR wake fields by directly integrating Maxwell's equations on a mesh representing a rectangular beam chamber [6]. Comparisons are also made with the particle tracking code elegant [7], as well as with analytic expressions for the CSR-wake given in [3], [8], and [10]. Electric fields are normalized by $E_0 = 2Nrcmc^2 / (\sqrt{2\pi}(3R^2\sigma_z^4)^{1/3})$, where σ_z is the bunch length for a Gaussian distribution.

Figure 1 shows the steady-state CSR kick in a bend as a function of z for various values of the chamber height. The parameters used are the same as in Fig. 1 of [6]. Excellent agreement is seen between Bmad, the analytic shielded CSR-wake, and the A&Y code. For the largest chamber size all codes agree with elegant.

The principal detrimental effects of the CSR-wake in an accelerator are energy loss and increase in energy spread of a bunch. The transverse bunch distribution can also be damaged and is mostly influenced by increases of the energy spread which, through dispersive orbits, couples to

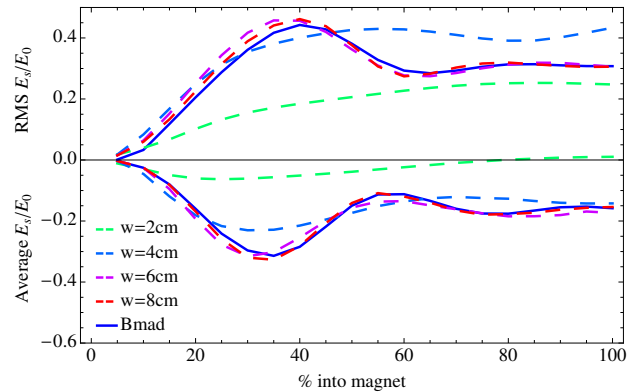


Figure 2: Average and RMS longitudinal electric fields in the Cornell ERL's CE magnet for various chamber widths using the A&Y code, compared to Bmad, which has infinite chamber width. Here $R = 87.9\text{m}$, $\sigma_z = 0.3\text{m}$, and the magnet length is 6.6m .

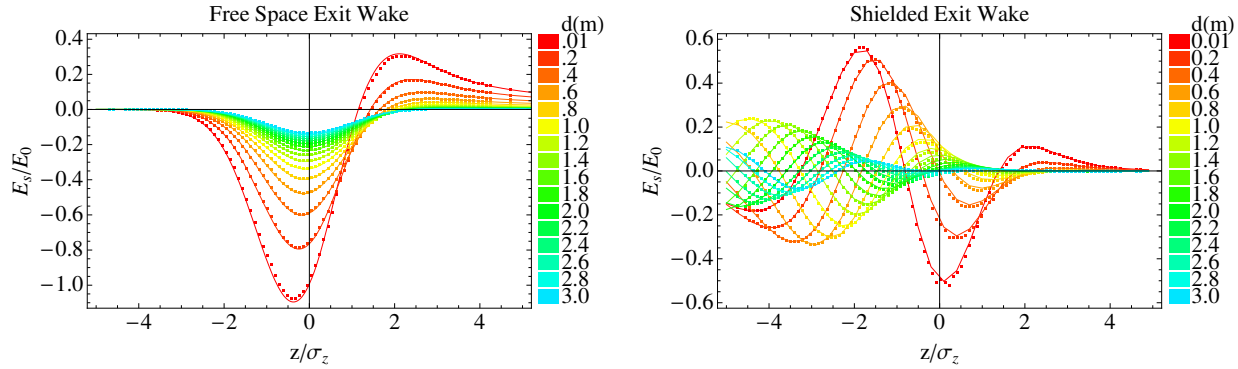


Figure 3: Exit wake-field as a function of the length d from the end of the magnet for free space (left) and with shielding (right). Bmad (dots) shows excellent agreement with the analytic formula (top lines) in the free space case, and with numerical integration over image bunches in the shielded case (bottom lines). Here $R = 10\text{m}$, $\sigma_z = 0.3\text{mm}$.

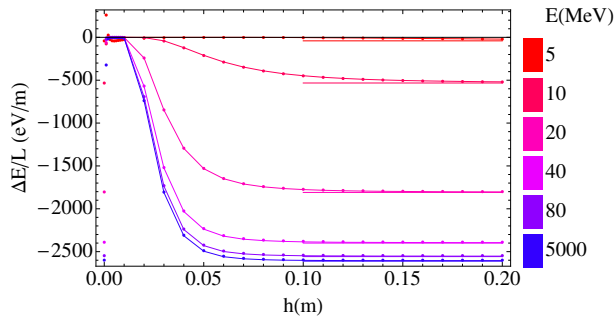


Figure 4: Average energy losses versus shielding height for various energies. Bmad (dots) agrees well with numerical integration of the shielded power spectrum of [9] down to 2mm. Analytic steady state losses are indicated as horizontal lines. Here $R = 10\text{m}$, $\sigma_z = 0.3\text{mm}$.

transverse motion. Figure 2 shows the average and RMS electric field across the bunch distribution as a function of distance into the magnet. One sees that, in this shielded case, the chamber width w in the A&Y code begins to change the wake field when it is comparable or less than 4cm, the height of the chamber. It shows that ignoring the chamber width, as in Bmad, is a reasonable approximation when the chamber is wider than the height.

The method in this paper can correctly account for the CSR-wake in a drift section following a magnet. Using the same parameters as in Fig. 1, Fig. 3 shows this wake as a function of the distance d from the end of the magnet in free space and with shielding. The free space case, for which an analytic formula exists [10], shows excellent agreement between Bmad and analytic formulas. The shielded case shows good agreement with numerical integration over image bunches.

Our final test of Bmad compares the total coherent energy lost for various particle energies and chamber heights to the integration of the shielded power spectrum given in [9]. Figure 4 shows excellent agreement for energies down

to 5MeV and chamber heights down to 2mm. At smaller heights, the number of image layers used ($N_i = 64$) is insufficient to correctly model the CSR-wake.

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