

# Coherent Diffraction Imaging with Nano- and Microbeams

Why does lensless imaging need lenses?

Mark A Pfeifer

Cornell High Energy Synchrotron Source  
Cornell University  
Ithaca, NY 14850  
map322@cornell.edu

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# Outline

Lensless  
imaging with  
lenses

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Coherent  
diffraction –  
theory

Coherent  
diffraction –  
theory

Coherent  
diffraction –  
practice

Structured  
illumination

Comparison of  
imaging  
methods

A few (crazy)  
ideas

- Coherent diffraction imaging and why it will never work for x-rays.
- Coherent diffraction imaging - how it works.
- Can you really use a lens for that?
- Why should I use phasing algorithms when I have a lens?
- Putting it all together.

# Some thoughts by Sayre

*Acta Cryst.* (1952). 5, 843

**Some implications of a theorem due to Shannon.** By D. SAYRE, *Johnson Foundation for Medical Physics, University of Pennsylvania, Philadelphia 4, Pennsylvania, U. S. A.*

(Received 3 July 1952)

Shannon (1949), in the field of communication theory, has given the following theorem: If a function  $d(x)$  is known to vanish outside the points  $x = \pm a/2$ , then its Fourier transform  $F(X)$  is completely specified by the values which it assumes at the points  $X = 0, \pm 1/a, \pm 2/a, \dots$ . In fact, the continuous  $F(X)$  may be filled in merely by laying down the function  $\sin \pi a X / \pi a X$  at each of the above points, with weight equal to the value of  $F(X)$  at that point, and adding.

Now the electron-density function  $d(x)$  describing a single unit cell of a crystal vanishes outside the points  $x = \pm a/2$ , where  $a$  is the length of the cell. The reciprocal-lattice points are at  $X = 0, \pm 1/a, \pm 2/a, \dots$ , and hence the experimentally observable values of  $F(X)$  would suffice, by the theorem, to determine  $F(X)$  everywhere, if the phases were known. (In principle, the necessary points extend indefinitely in reciprocal space, but by using, say, Gaussian atoms both  $d(x)$  and  $F(X)$  can be effectively confined to the unit cell and the observable region, respectively.)

For centrosymmetrical structures, to be able to fill in the  $|F|^2$  function would suffice to yield the structure, for sign changes could occur only at the points where  $|F|^2$  vanishes. The structure corresponding to the  $|F|^2$  function is the Patterson of a single unit cell. This has

twice the width of the unit cell, and hence to fill in the  $|F|^2$  function would require knowledge of  $|F|^2$  at the half-integral, as well as the integral  $h$ 's. This is equivalent to a statement made by Gay (1951).

I think the conclusions which may be stated at this point are:

1. Direct structure determination, for centrosymmetric structures, could be accomplished as well by finding the sizes of the  $|F|^2$  at half-integral  $h$  as by the usual procedure of finding the signs of the  $F$ 's at integral  $h$ .

2. In work like that of Boyes-Watson, Davidson & Perutz (1947) on haemoglobin, where  $|F|^2$  was observed at non-integral  $h$ , it would suffice to have only the values at half-integral  $h$ .

The extension to three dimensions is obvious.

## References

- BOYES-WATSON, J., DAVIDSON, E. & PERUTZ, M. F. (1947). *Proc. Roy. Soc. A*, **191**, 83.  
GAY, R. (1951). Paper presented at the Second International Congress of Crystallography, Stockholm.  
SHANNON, C. E. (1949). *Proc. Inst. Radio Engrs.*, N.Y. **37**, 10.

## Some implications of a theorem due to Shannon (Nyquist frequency)

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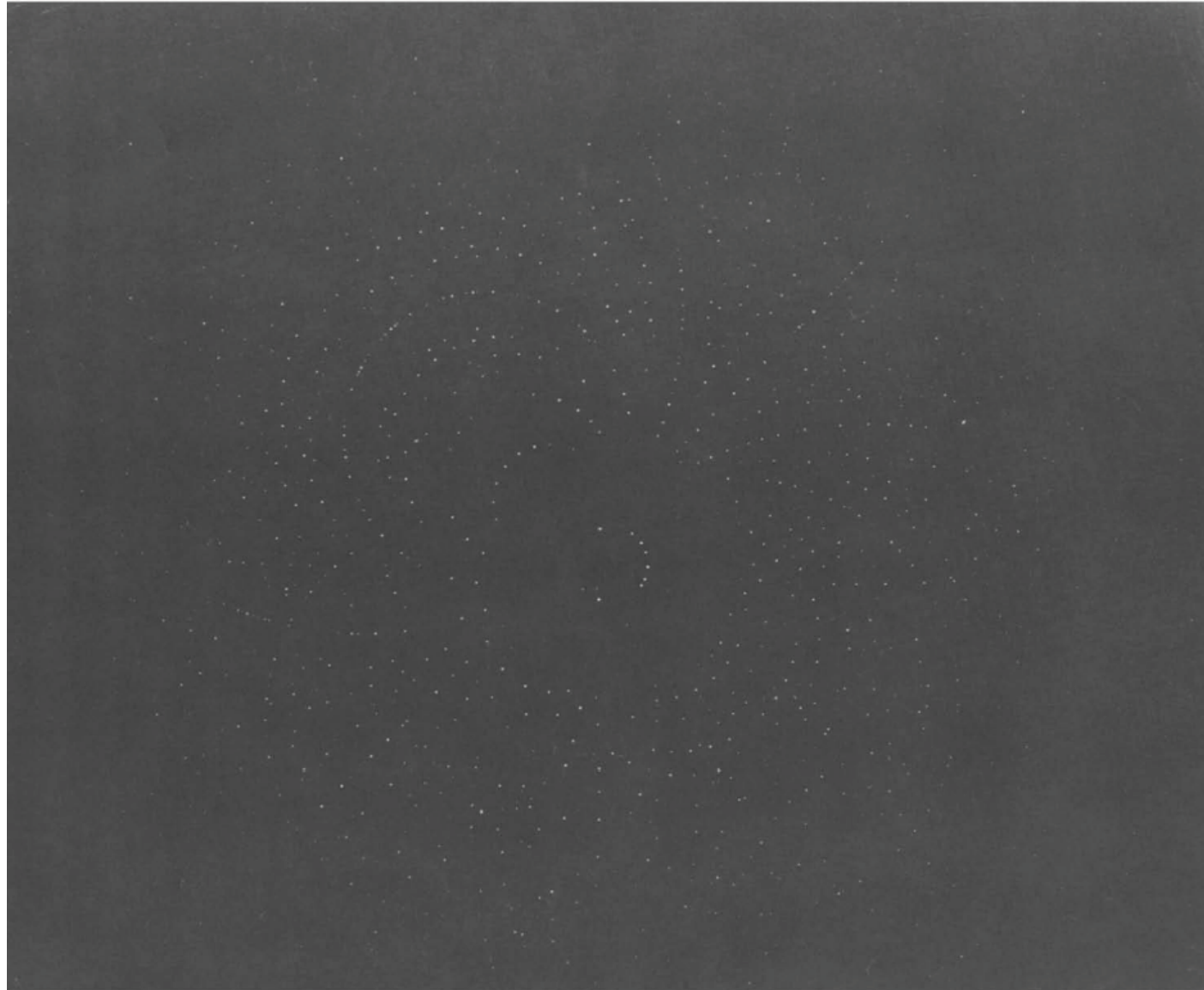
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# Fourier intensity measurements and the autocorrelation

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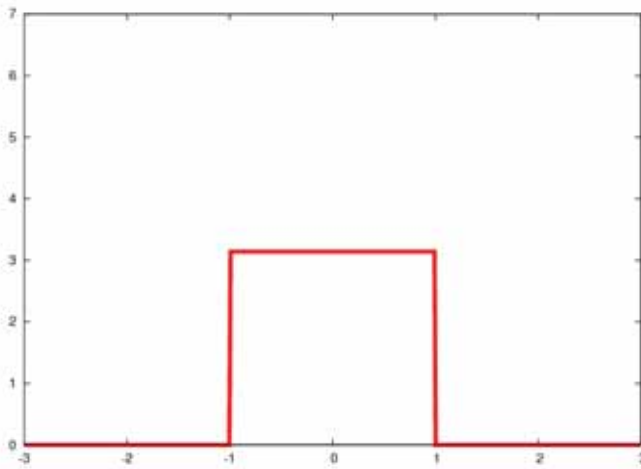
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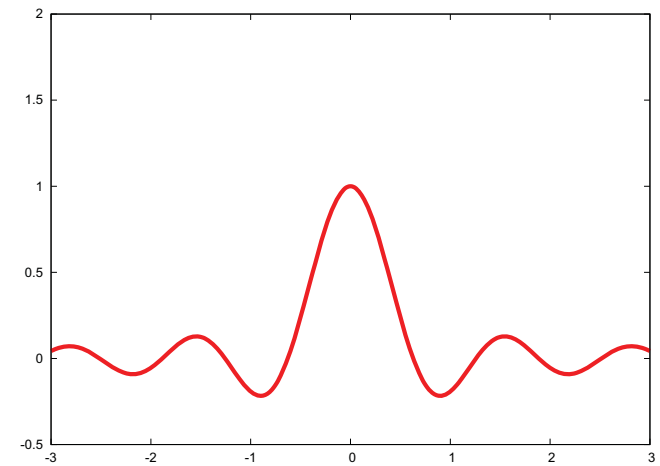
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density



amplitude



# Fourier intensity measurements and the autocorrelation

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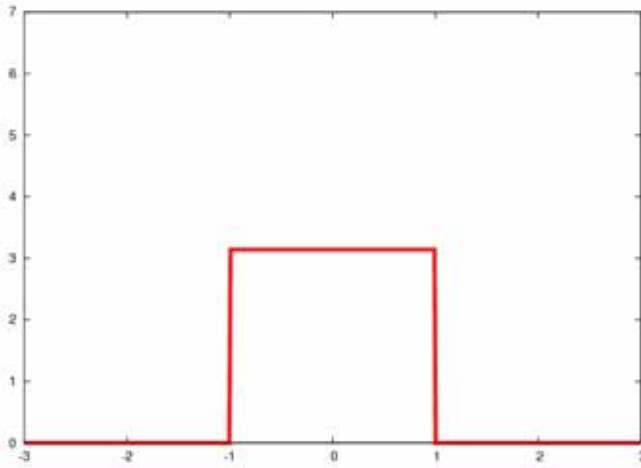
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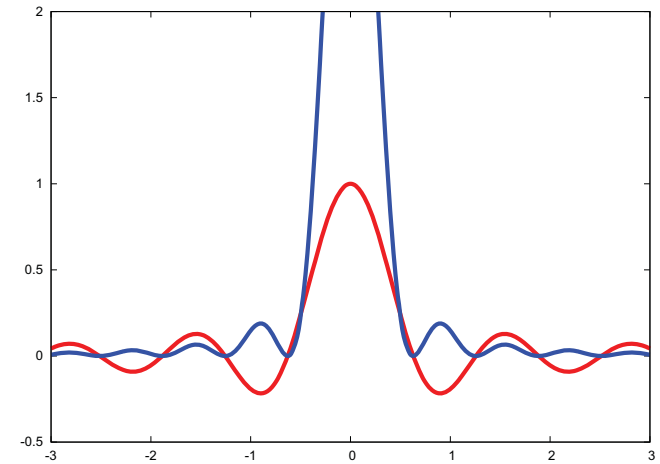
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intensity



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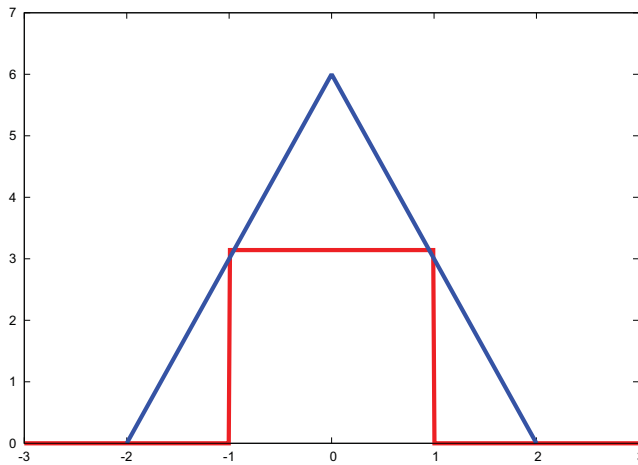
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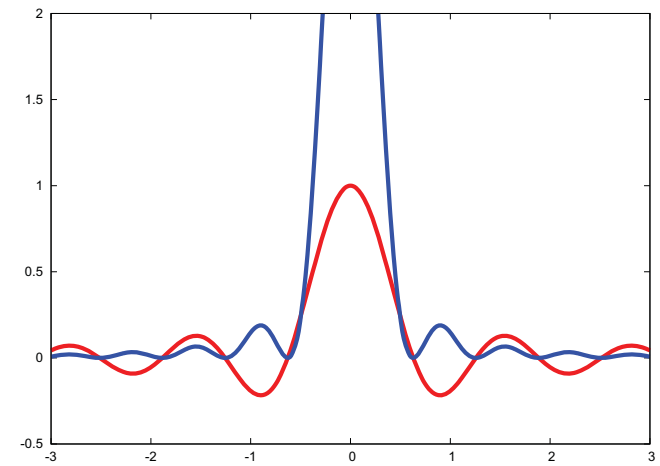
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## autocorrelation



## intensity



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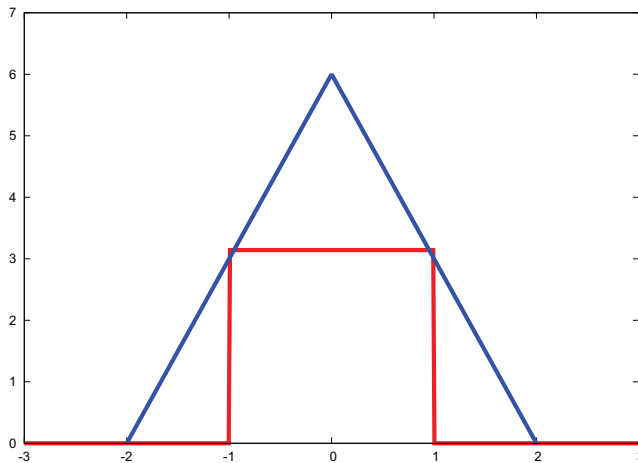
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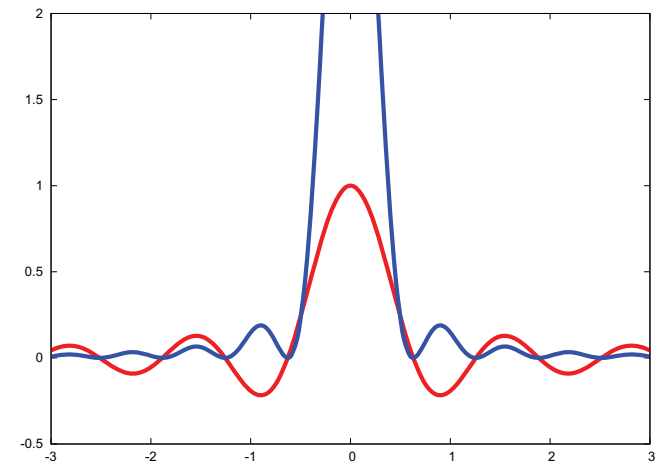
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intensity



- If only intensity measured, sampling requirement doubled.
- In greater than one dimension, autocorrelation *uniquely* determines original function.



# Wavefront reconstruction

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- $e^-$ : Gerchberg and Saxton - *Optik* 1971, 1972
  - intensity measurements in exit plane and far field
  - alternately enforced known intensities, kept phase estimates, propagated with FFT
  - algorithm equivalent to steepest descents
- visible light: Fienup
  - finite source size, positivity, and measured far-field
  - correct for atmospheric disturbances
  - diagnose aberration in Hubble's main mirror
  - explored range of algorithms with degrees of feedback

# Born and Wolf

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Born and Wolf, *Principles of Optics*:

“In the method of wave-front reconstruction, a background is artificially added. There are no conditions to be satisfied regarding symmetry or even periodicity. On the other hand this method is not suitable for X-ray analysis owing to the practical impossibility of producing a strong coherent background.”

# Born and Wolf

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## Rotating anode

- $L_T \approx \frac{\lambda d}{2s} = \frac{0.14nm * 2m}{2 * 1mm} = 70nm$
- fractional coherence  $\sim 2^{-14}$

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## APS undulator A

- fractional coherence  $\sim 8^{-4}$
- $2.5^{11}$  ph/s/0.1%

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- fractional coherence  $\sim 8^{-4}$
- $2.5^{11}$  ph/s/0.1%

## ERL helical mode delta

- fractional coherence  $\sim 2^{-1}$
- $4.8^{14}$  ph/s/0.1%

# First X-ray demonstration

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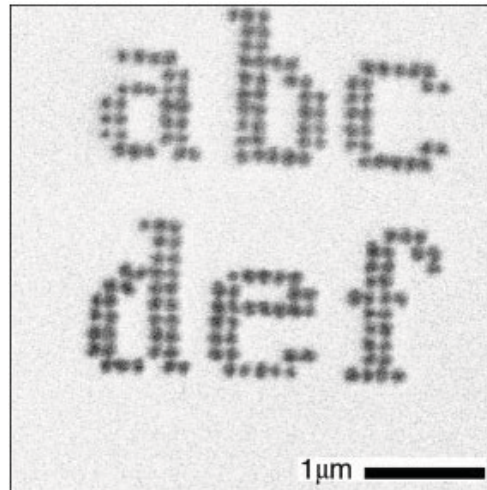
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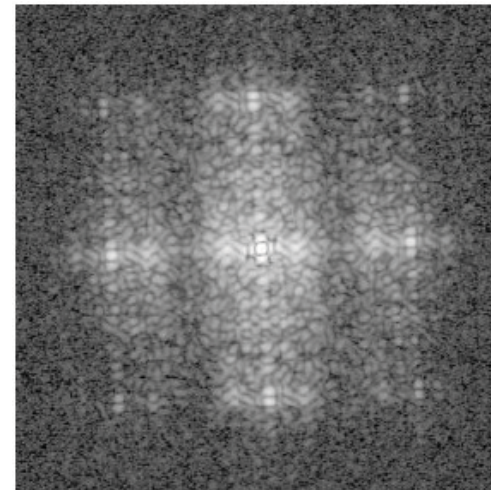
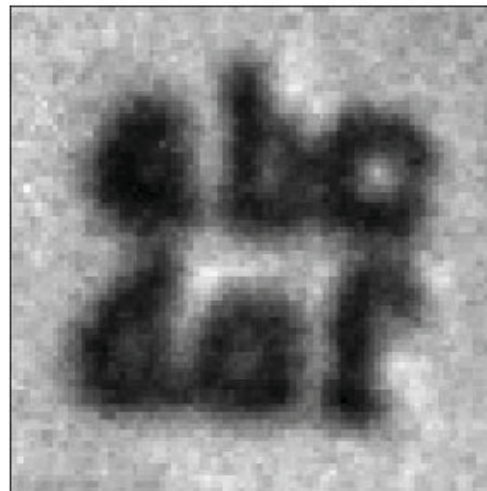
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Miao *JOSA* 1998.

SEM



VLM



Diffraction



CDI

# CDI moves into 3D and biology

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Au decorated Si<sub>3</sub>N<sub>4</sub> pyramids  
Chapman, *JOSA* 2006.

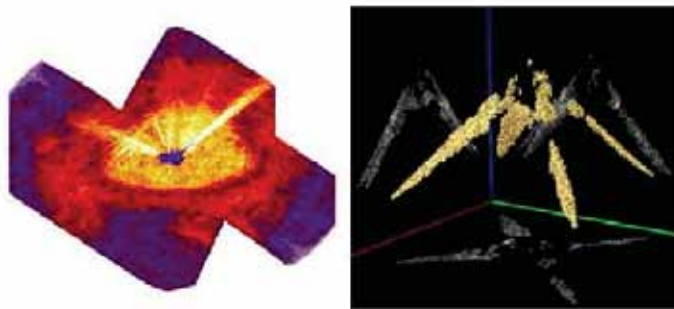
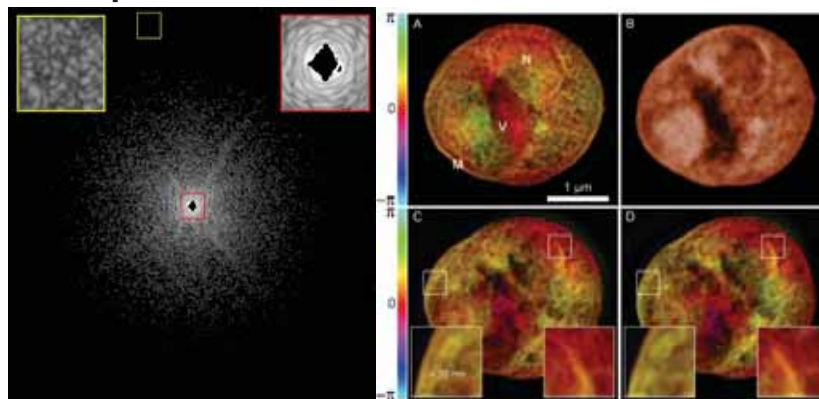
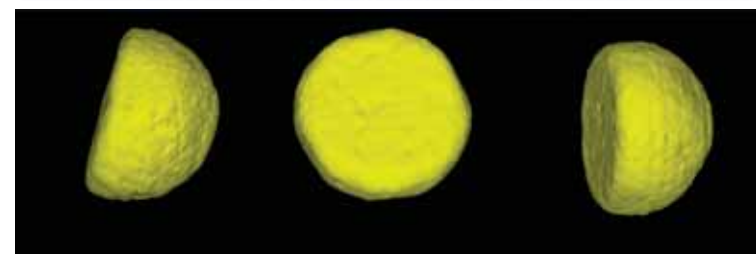
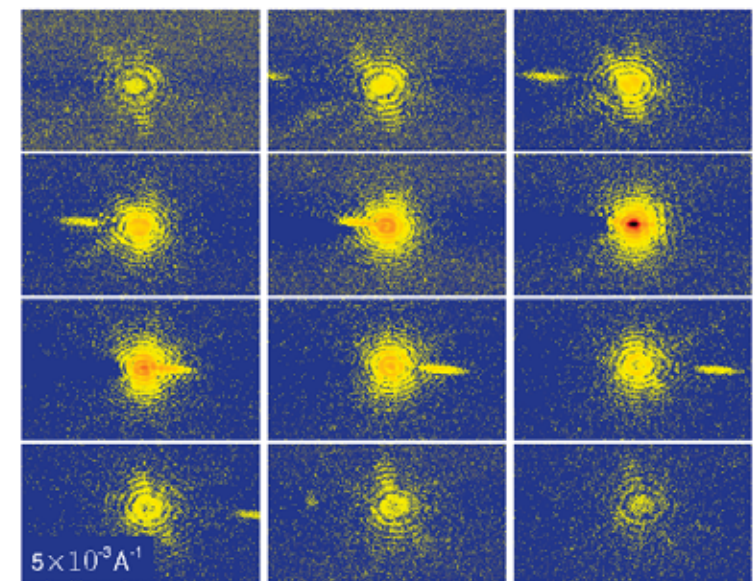


Fig. 4. Three dimensional diffraction pattern (left) (with a quadrant removed for visualization) and reconstructed 3D images<sup>19)</sup> (right) showing the isosurface as well as the projection images of the sample.

Yeast cells  
Shapiro *PNAS* 2005.



Pb crystals - strain mapping  
Pfeifer, *Nature* 2006.

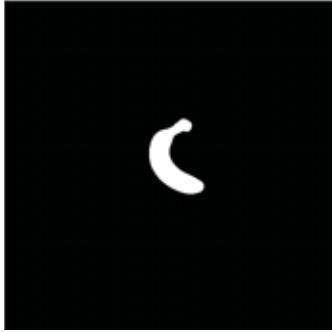




# A nice flow chart showing how CDI works

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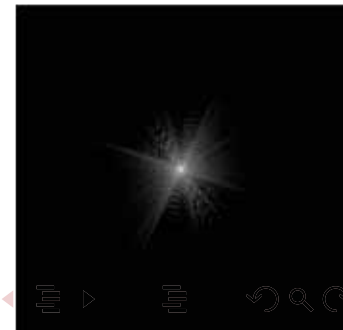
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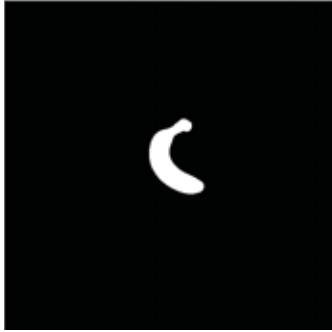
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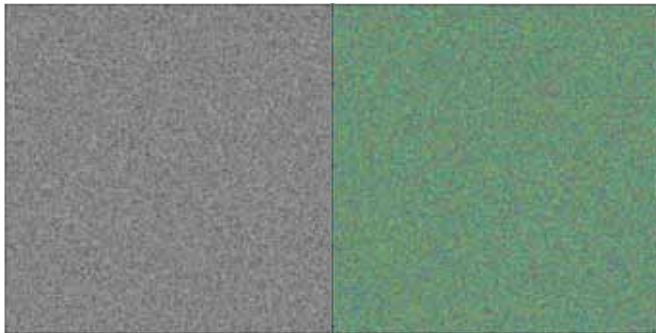
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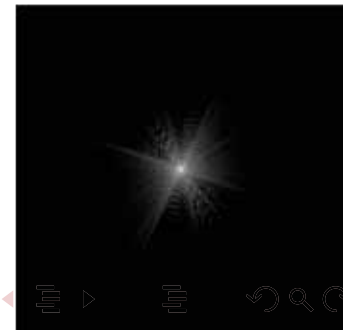
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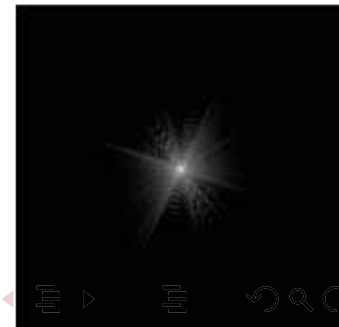
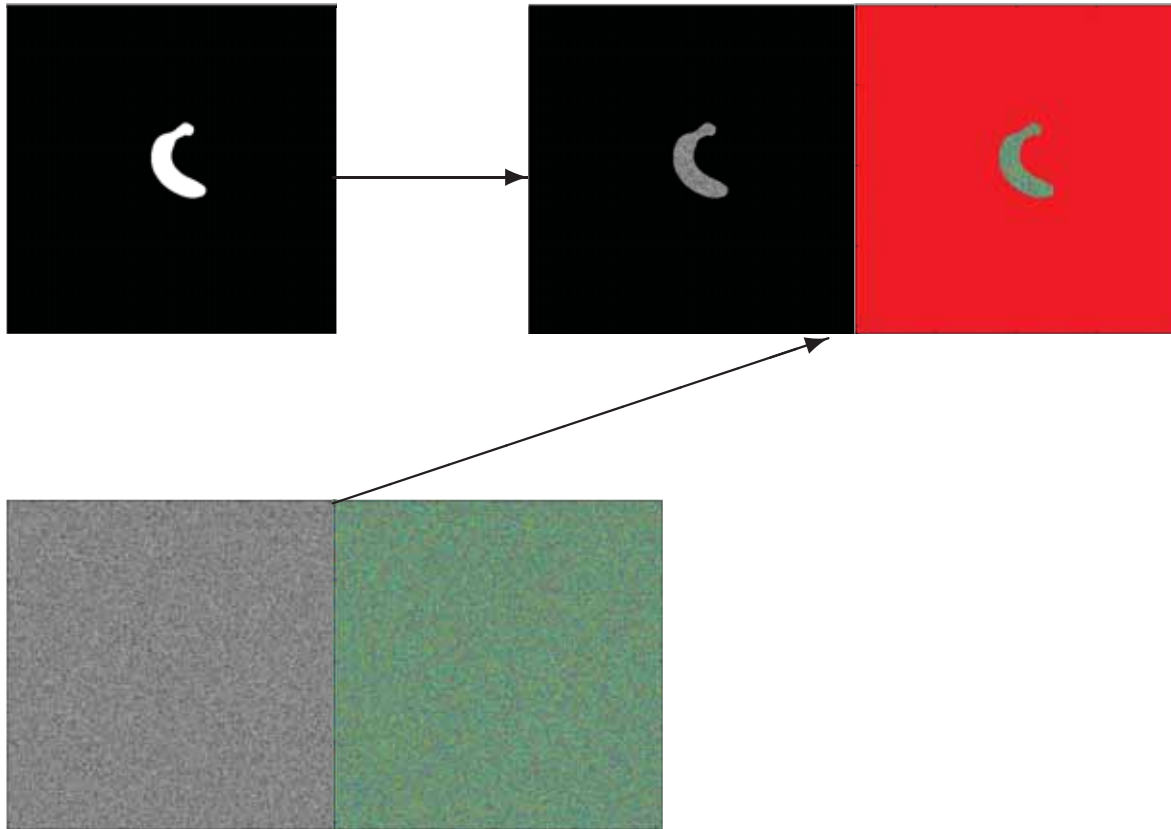
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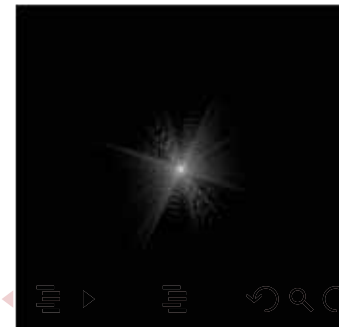
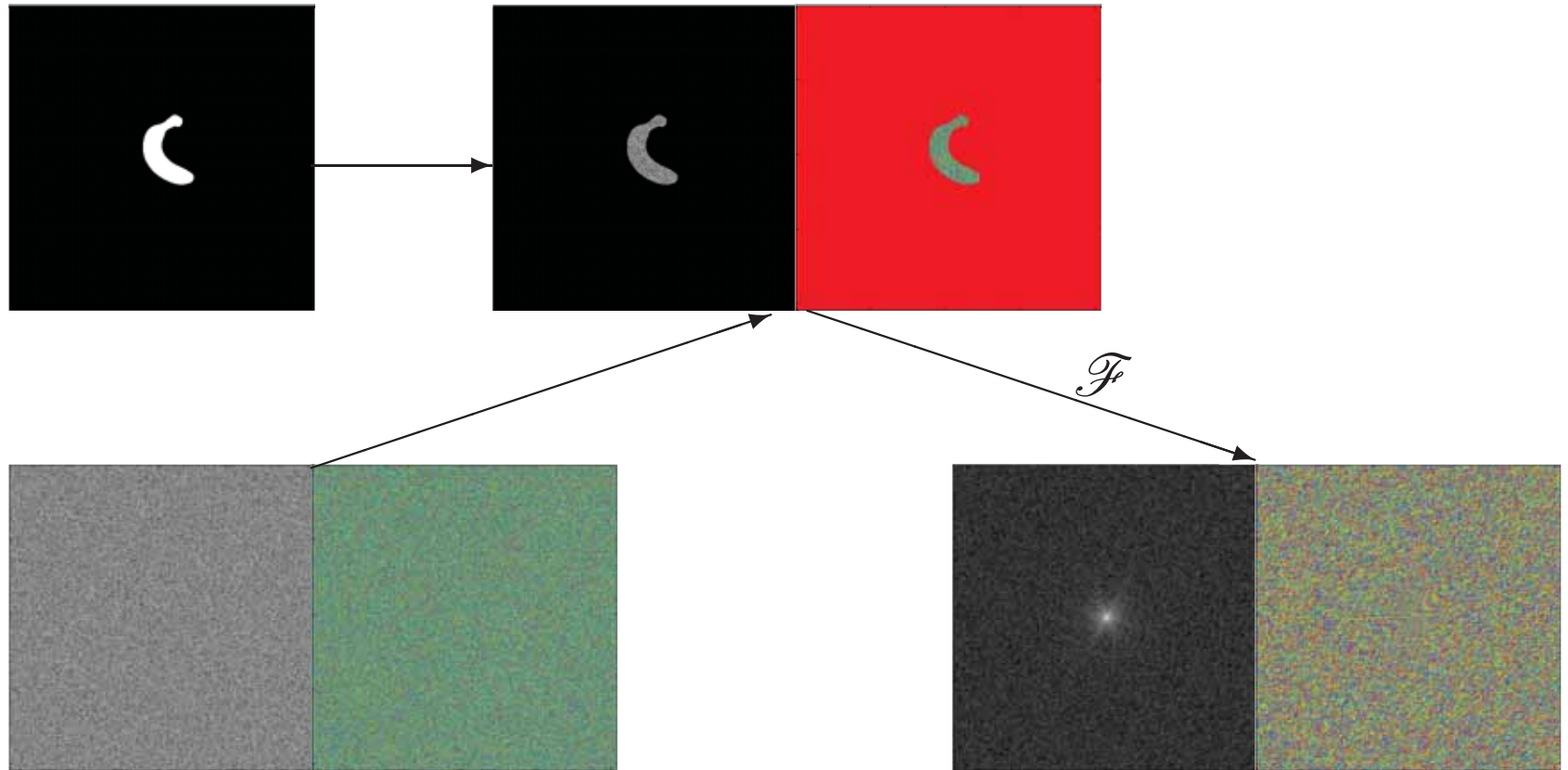
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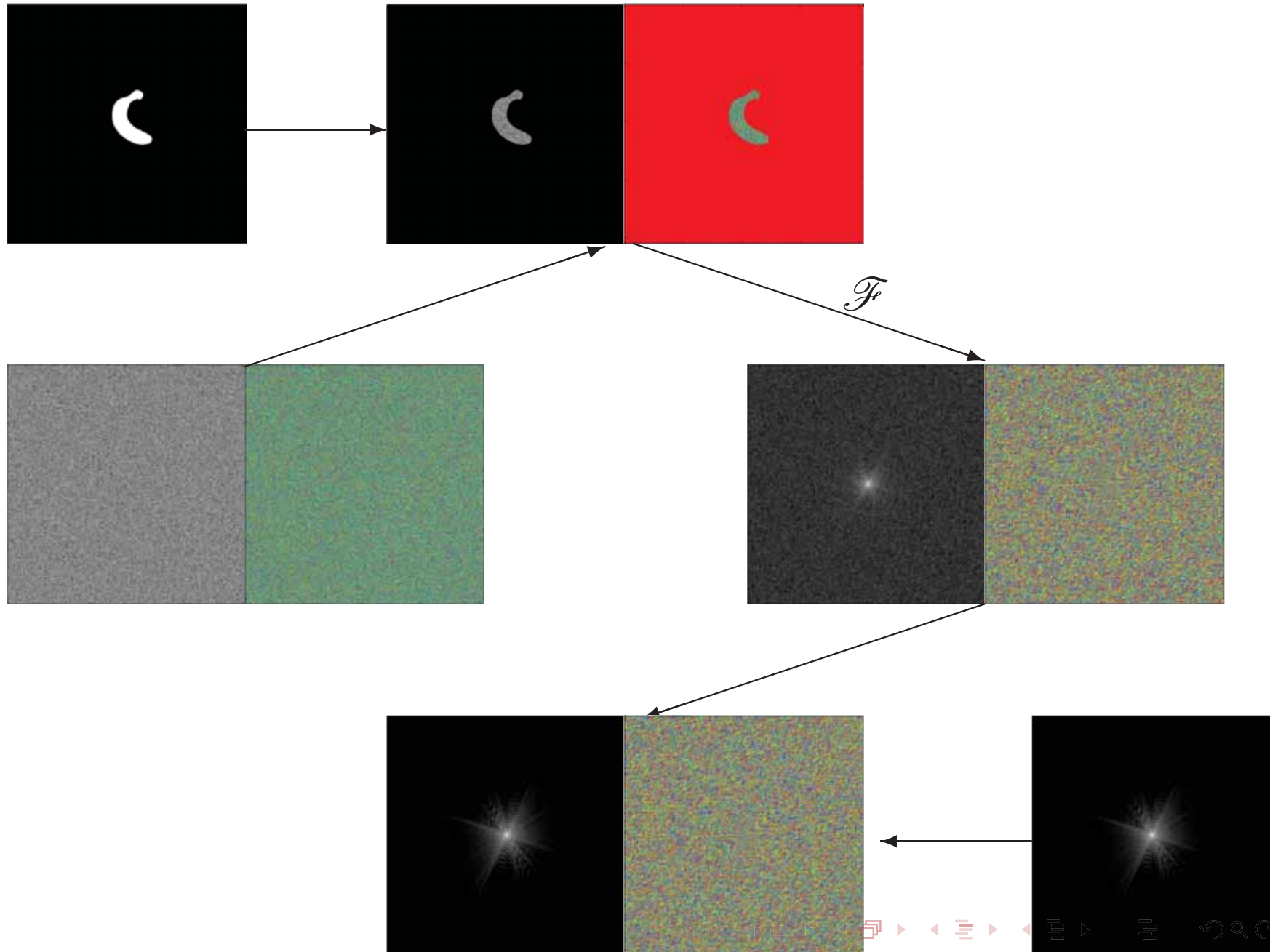
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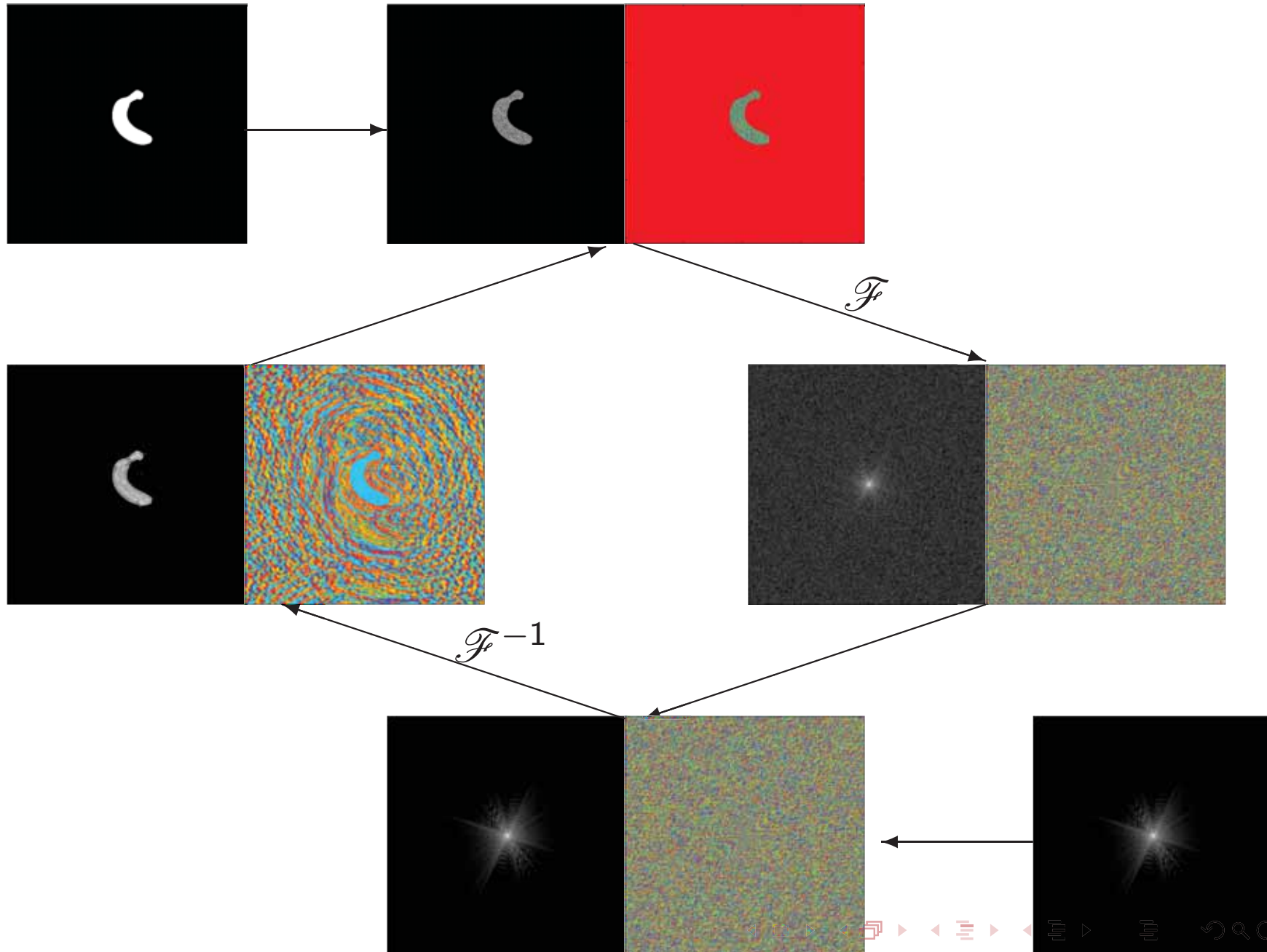
Coherent diffraction – practice

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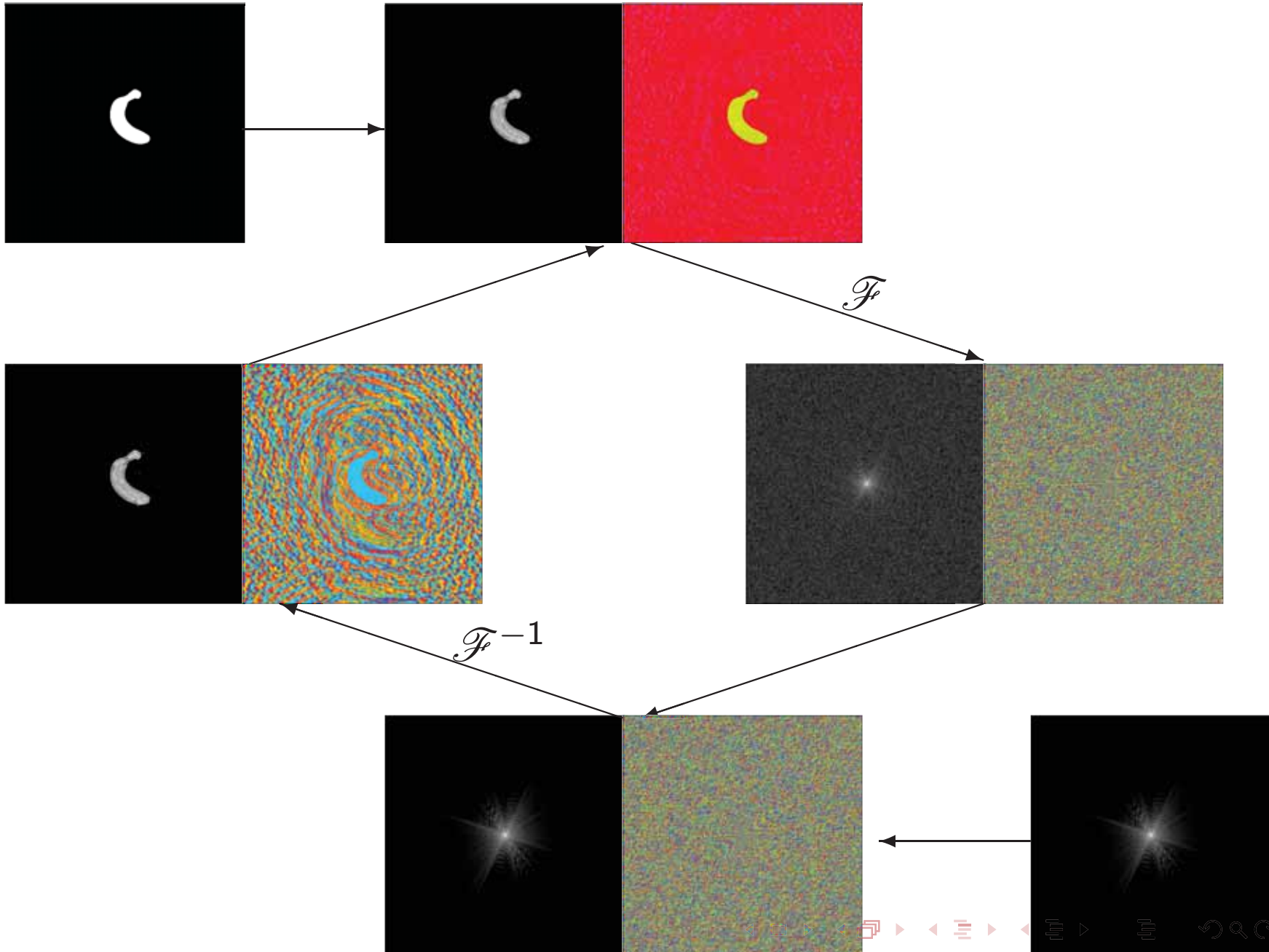
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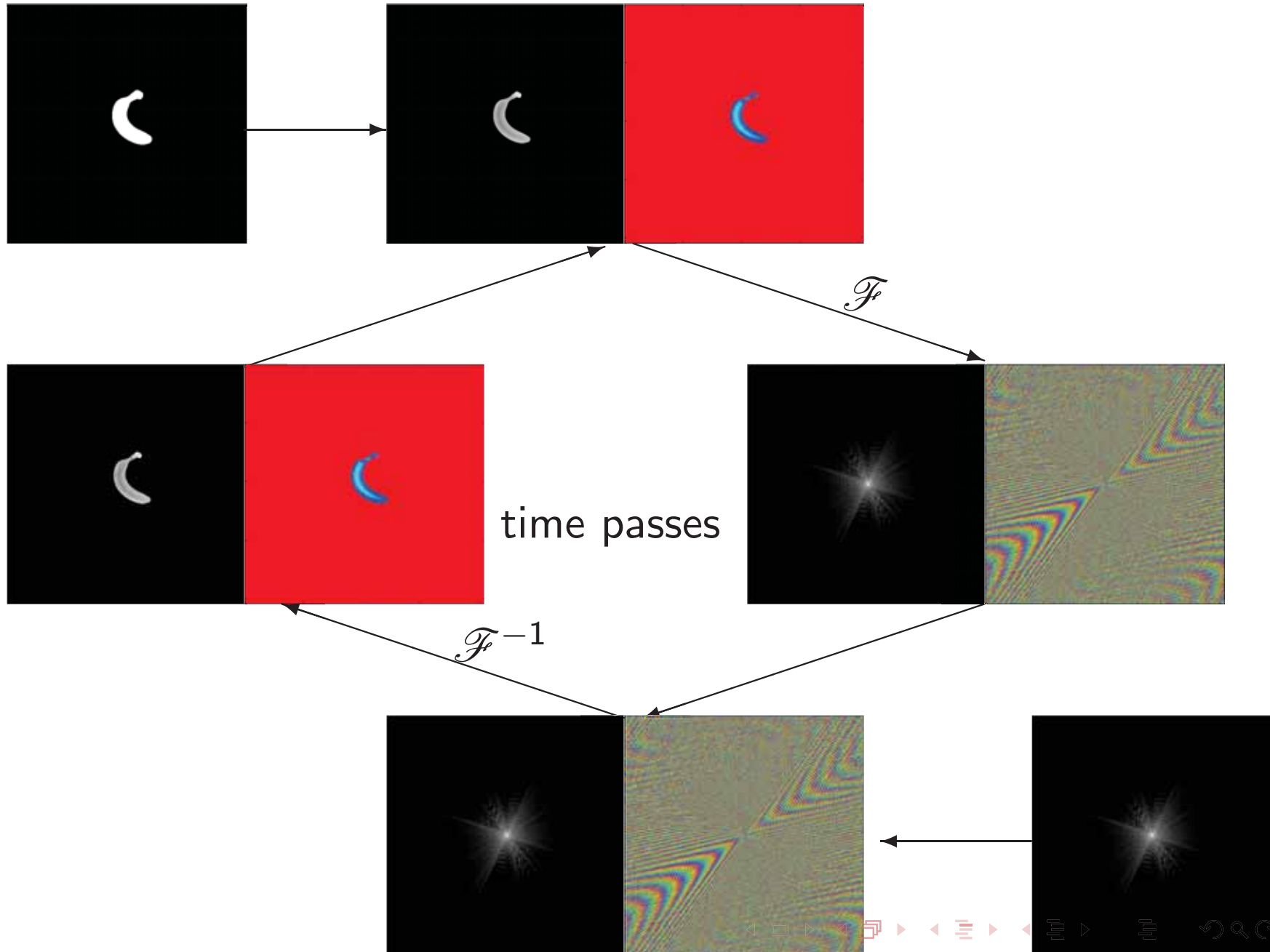
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# Missing information

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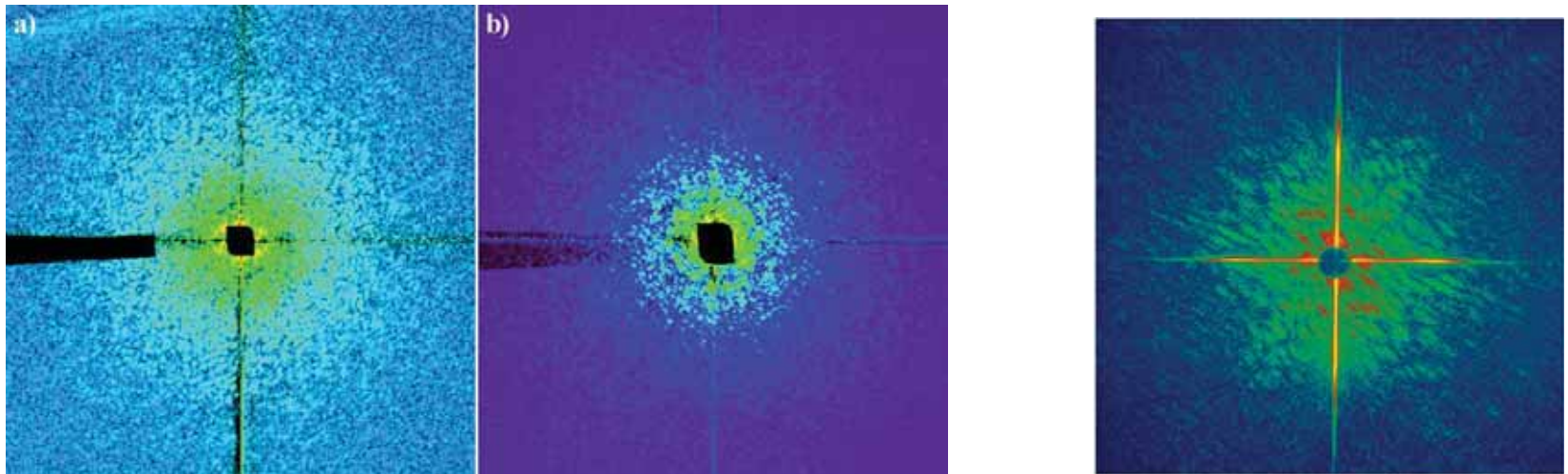
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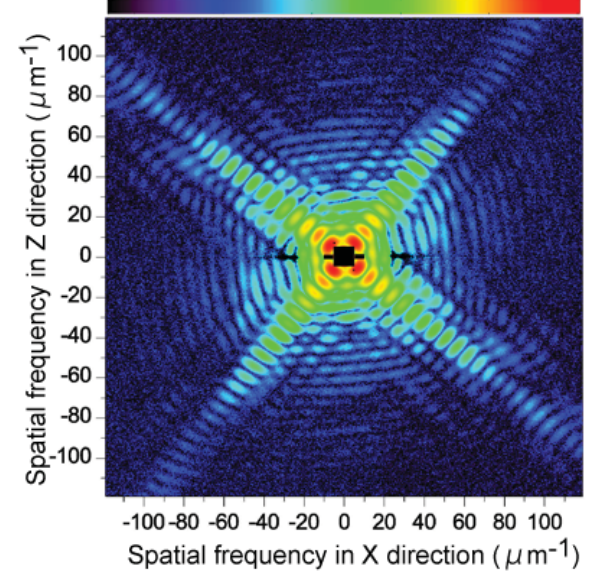
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Coherent X-ray diffraction intensity (arb. unit)

$10^0$   $10^1$   $10^2$   $10^3$   $10^4$   $10^5$



# Fresnel CDI – Geometry

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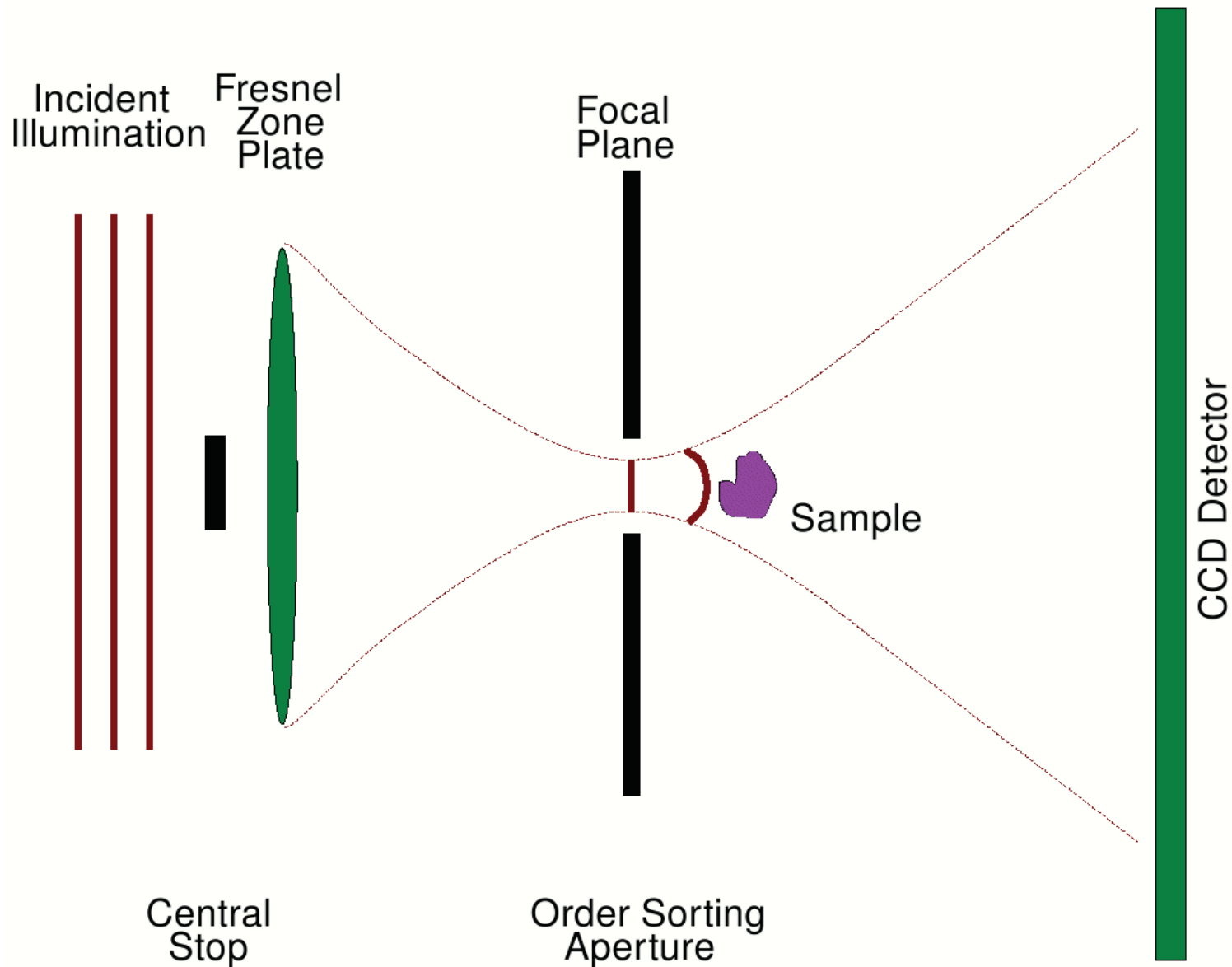
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# Fresnel CDI – Illumination

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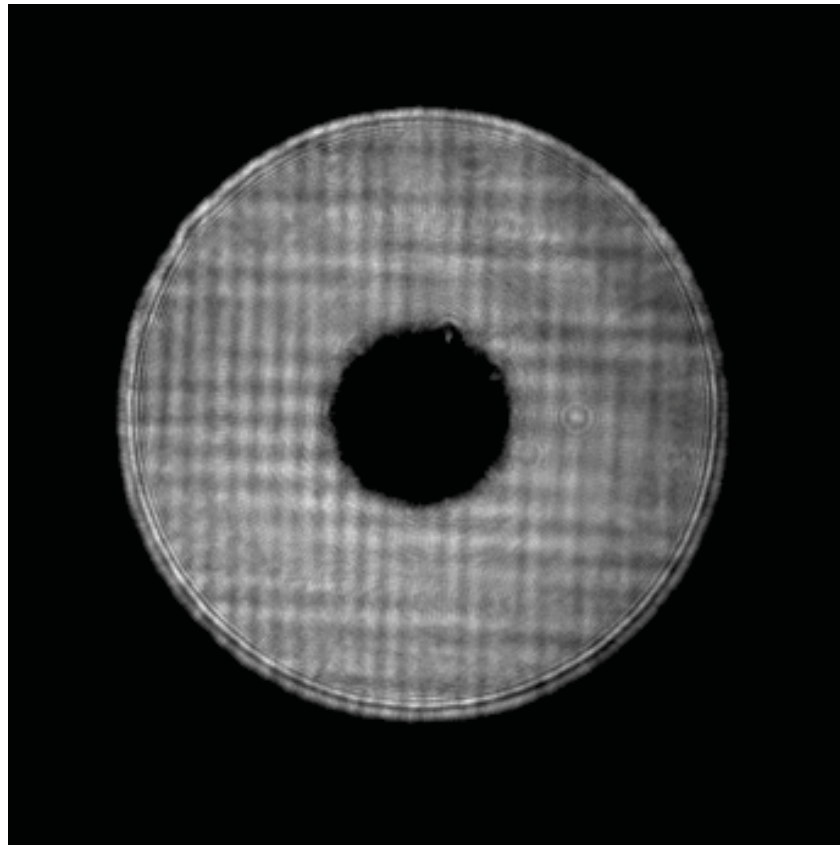
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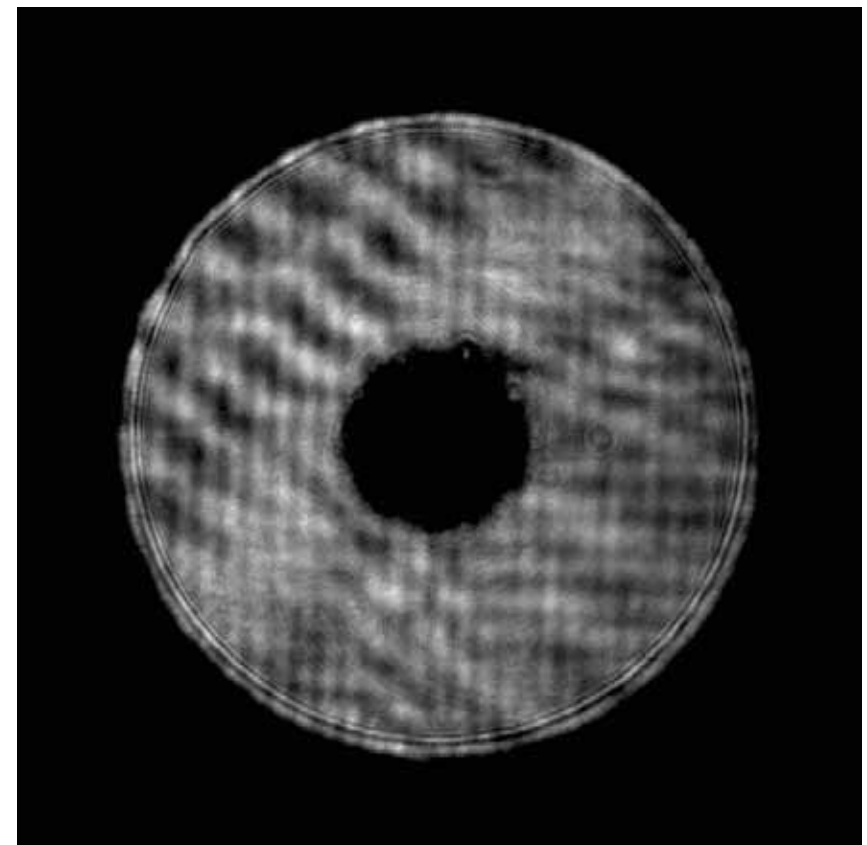
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far field from Fresnel zone  
plate



diffraction from Au test  
pattern

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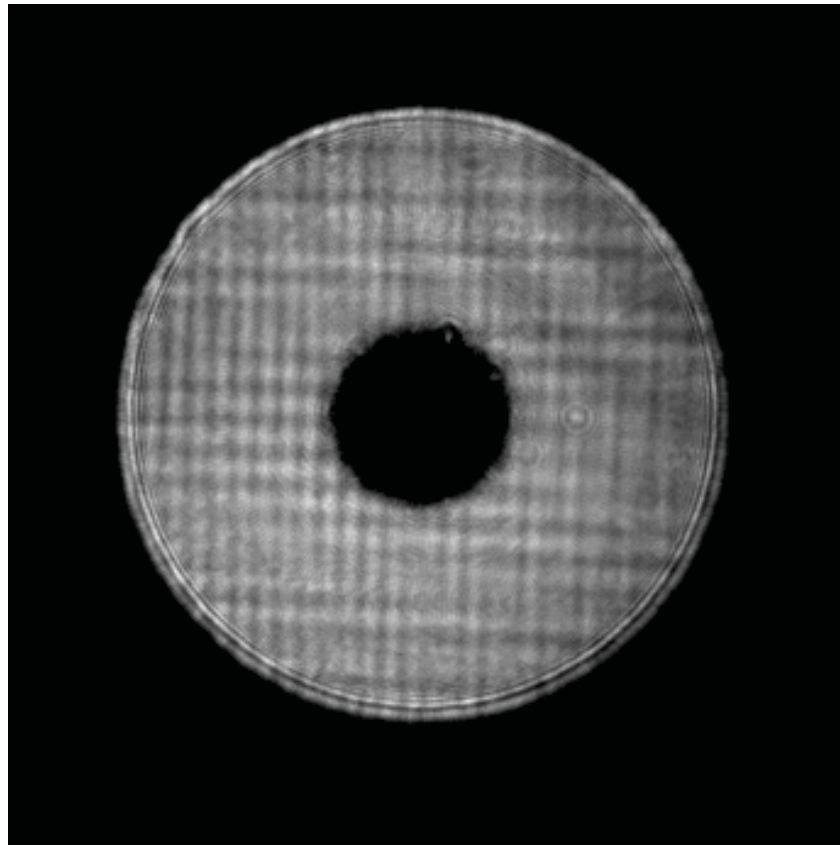
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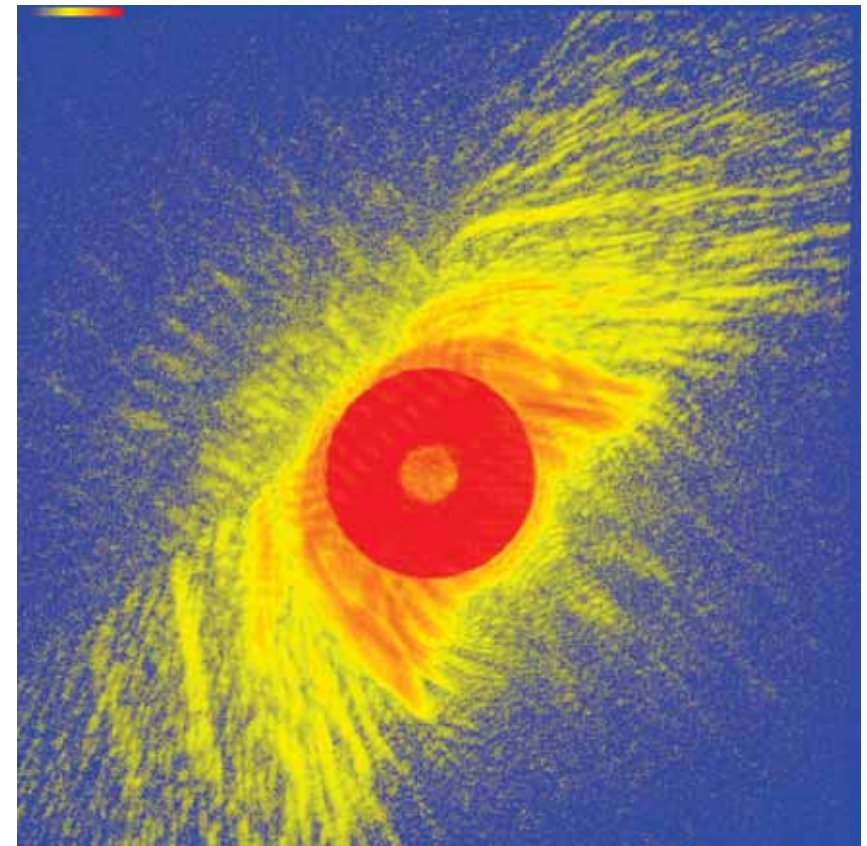
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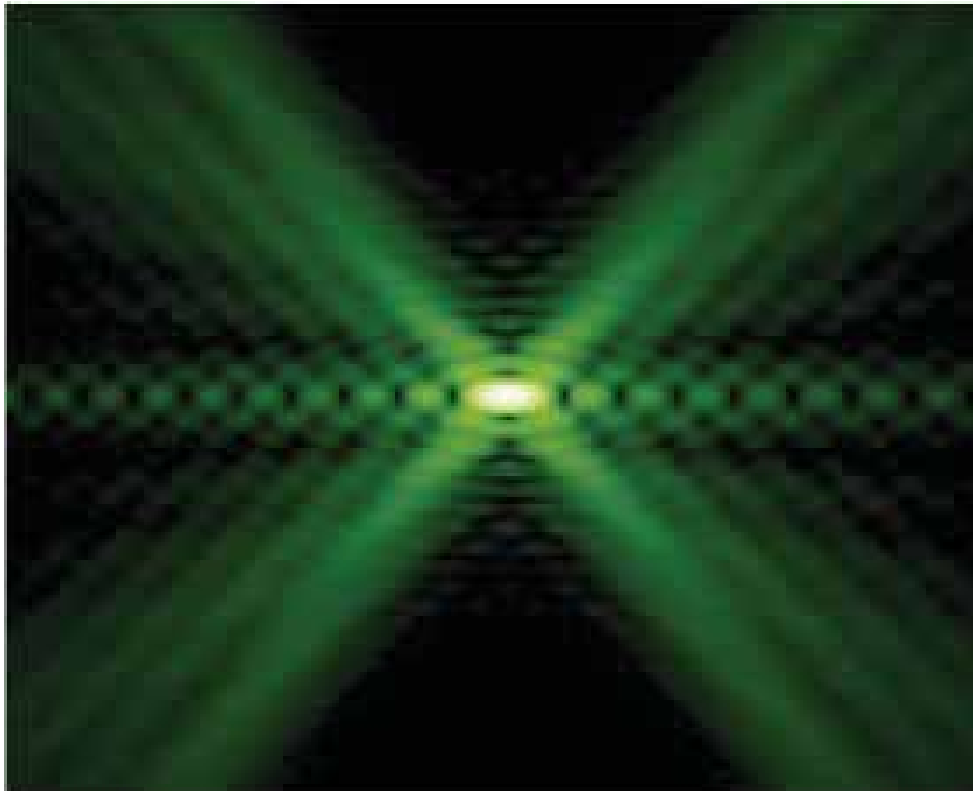
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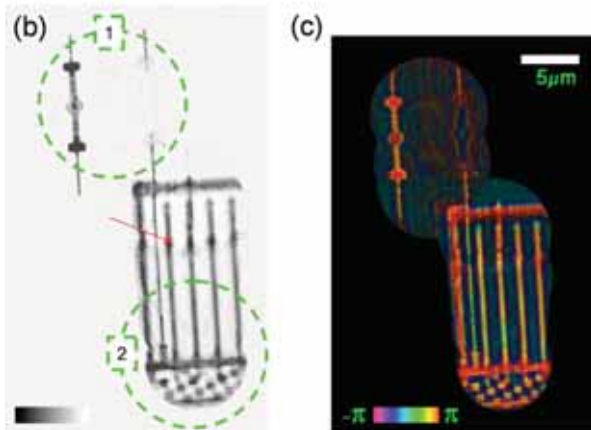
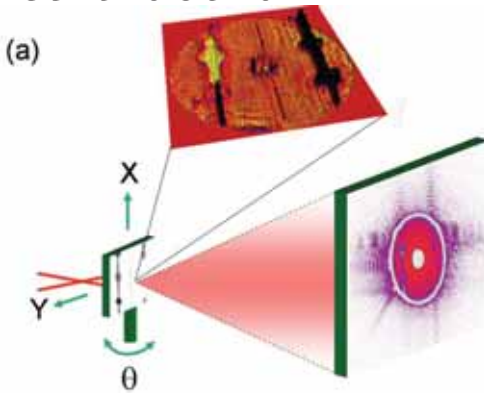
Harry Quiney, *Nature Physics* 2006.



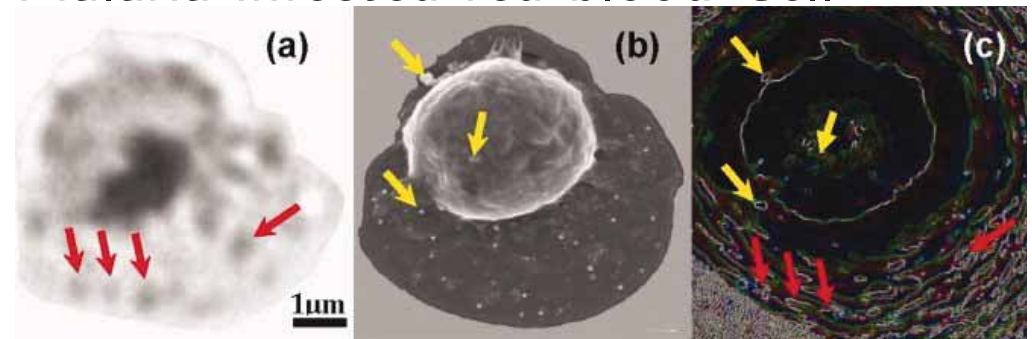
Complex wave field reconstructed in region of focus.  
Only need far-field intensity pattern and knowledge of focal  
length.

# Fresnel CDI – Sample Results

Brian Abbey *APL*  
2008.  
Interconnect from  
circuit board



Garth Williams *Cytometry* 2008.  
Malaria infected red blood Cell



comparison of light microscopy, SEM  
and FCDI

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theory

Coherent  
diffraction –  
practice

Structured  
illumination

Comparison of  
imaging  
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A few (crazy)  
ideas

# Ptychography

Lensless  
imaging with  
lenses

M. Pfeifer

Coherent  
diffraction –  
theory

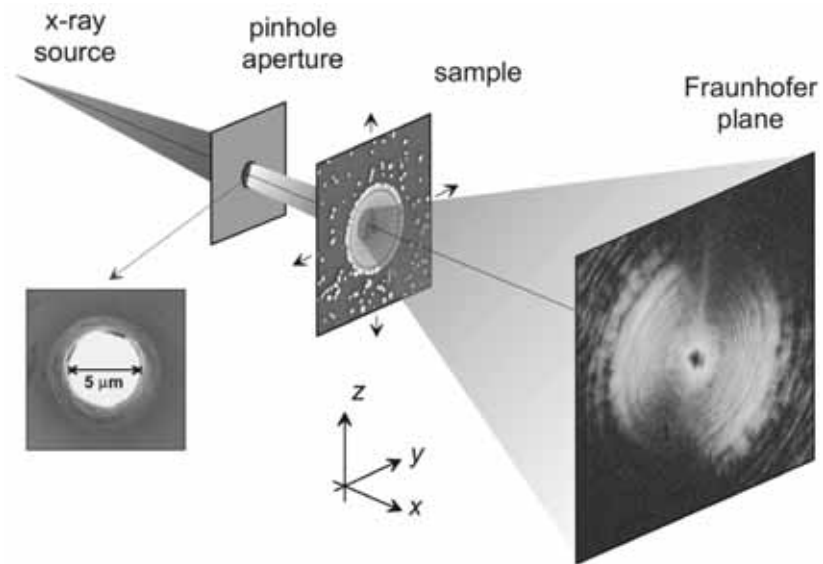
Coherent  
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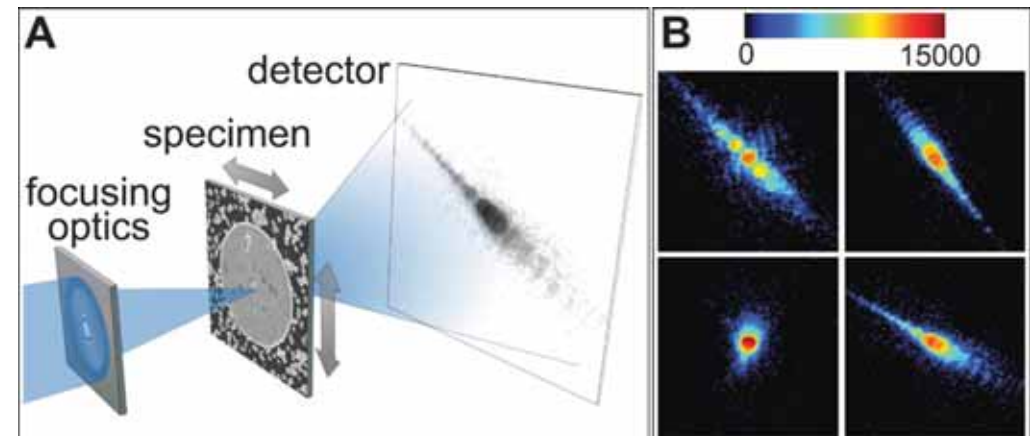
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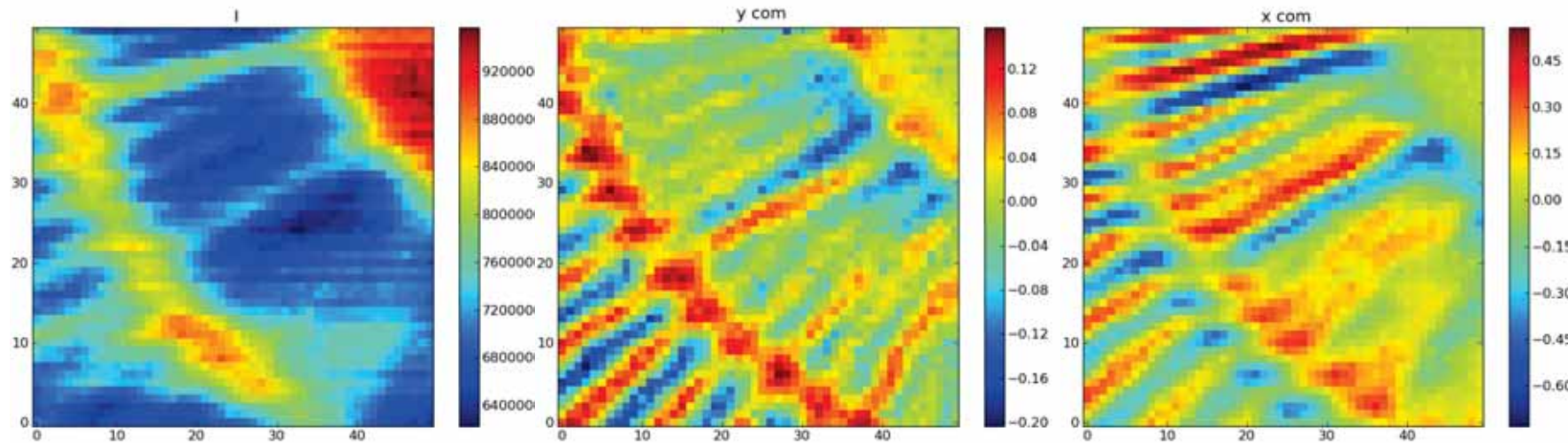
Rodenburg *PRL* 2007.  
Collect diffraction patterns  
from overlapping regions

Thibault *Science* 2008.  
Focused illumination  
Beam recovered



# Ptychography

## Au star test pattern measured at APS 2-ID-B with MMPAD



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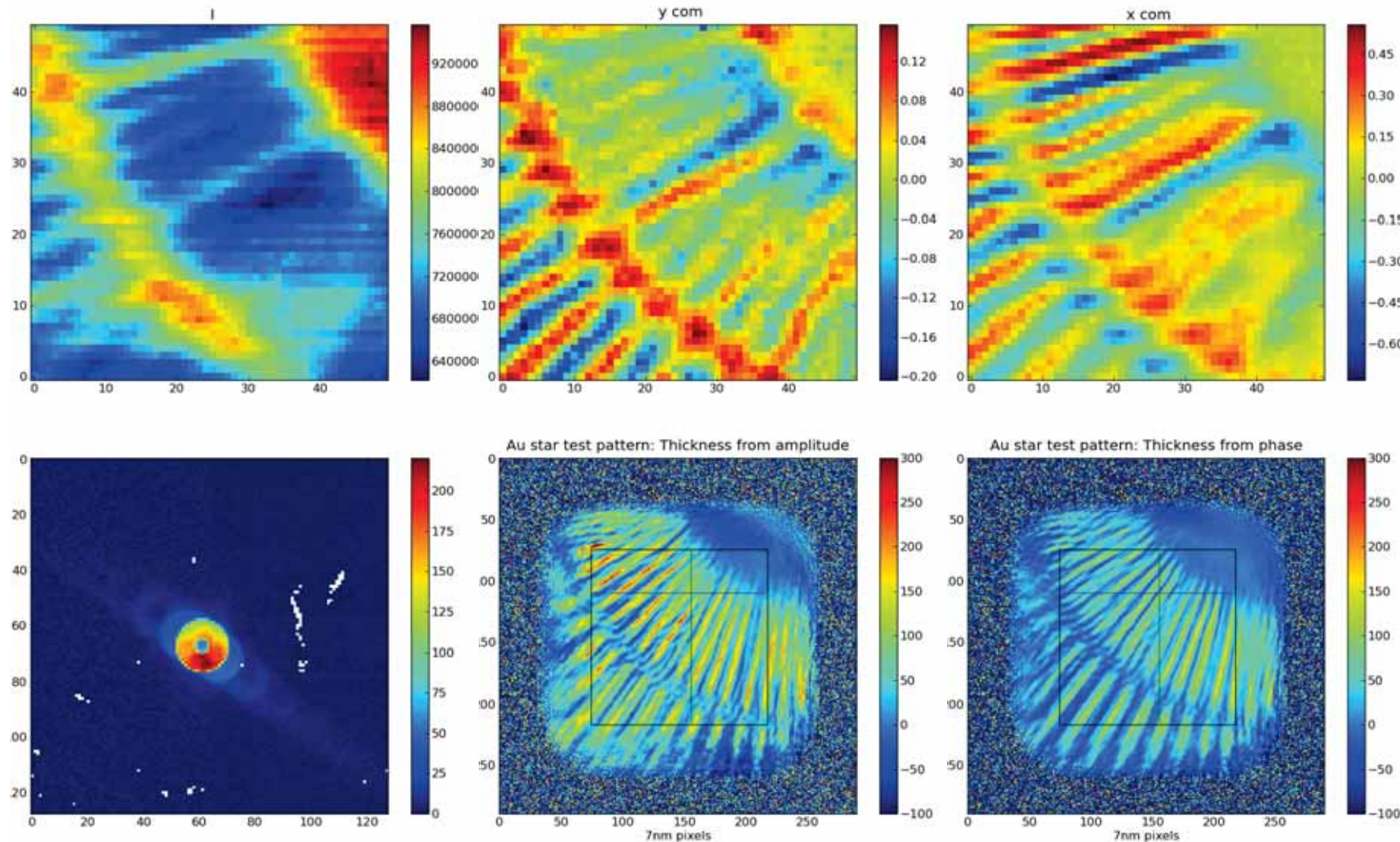
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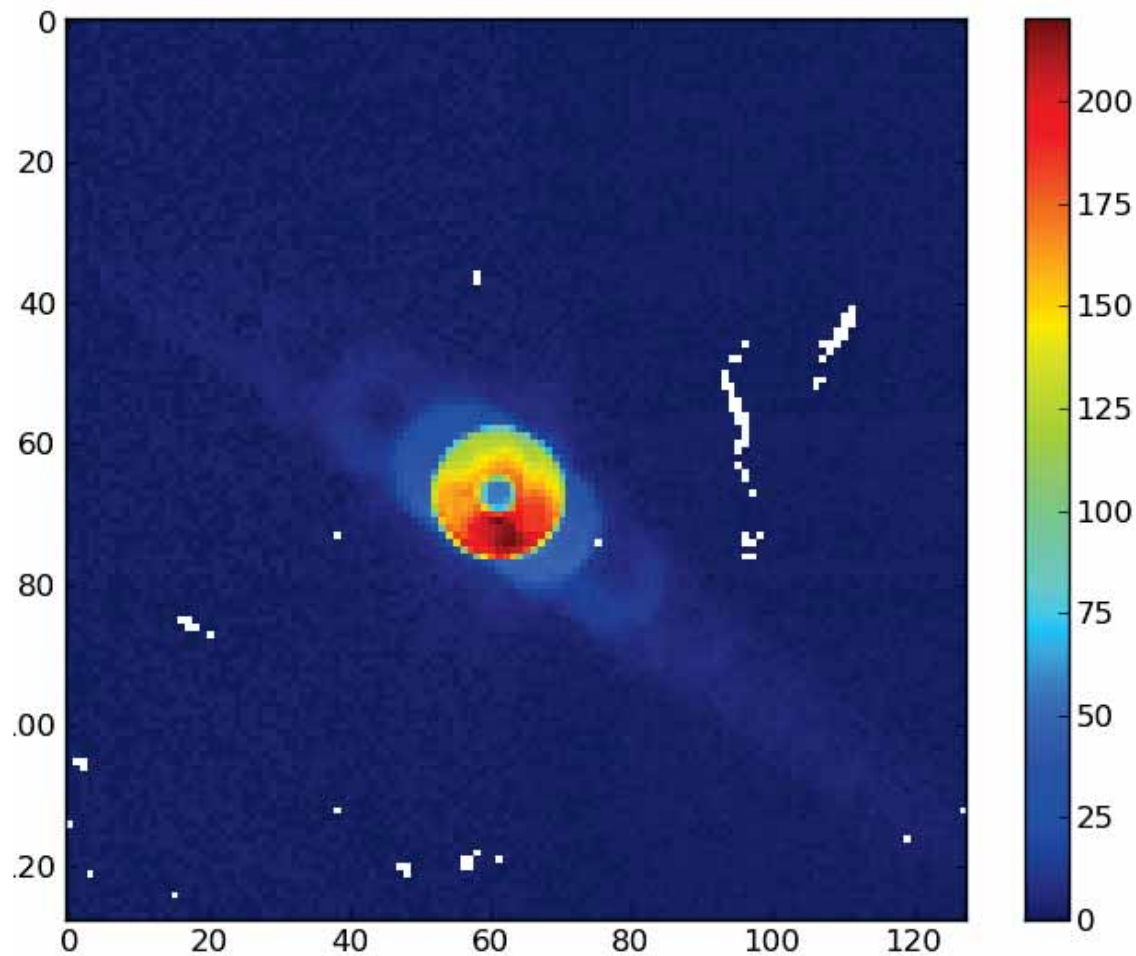
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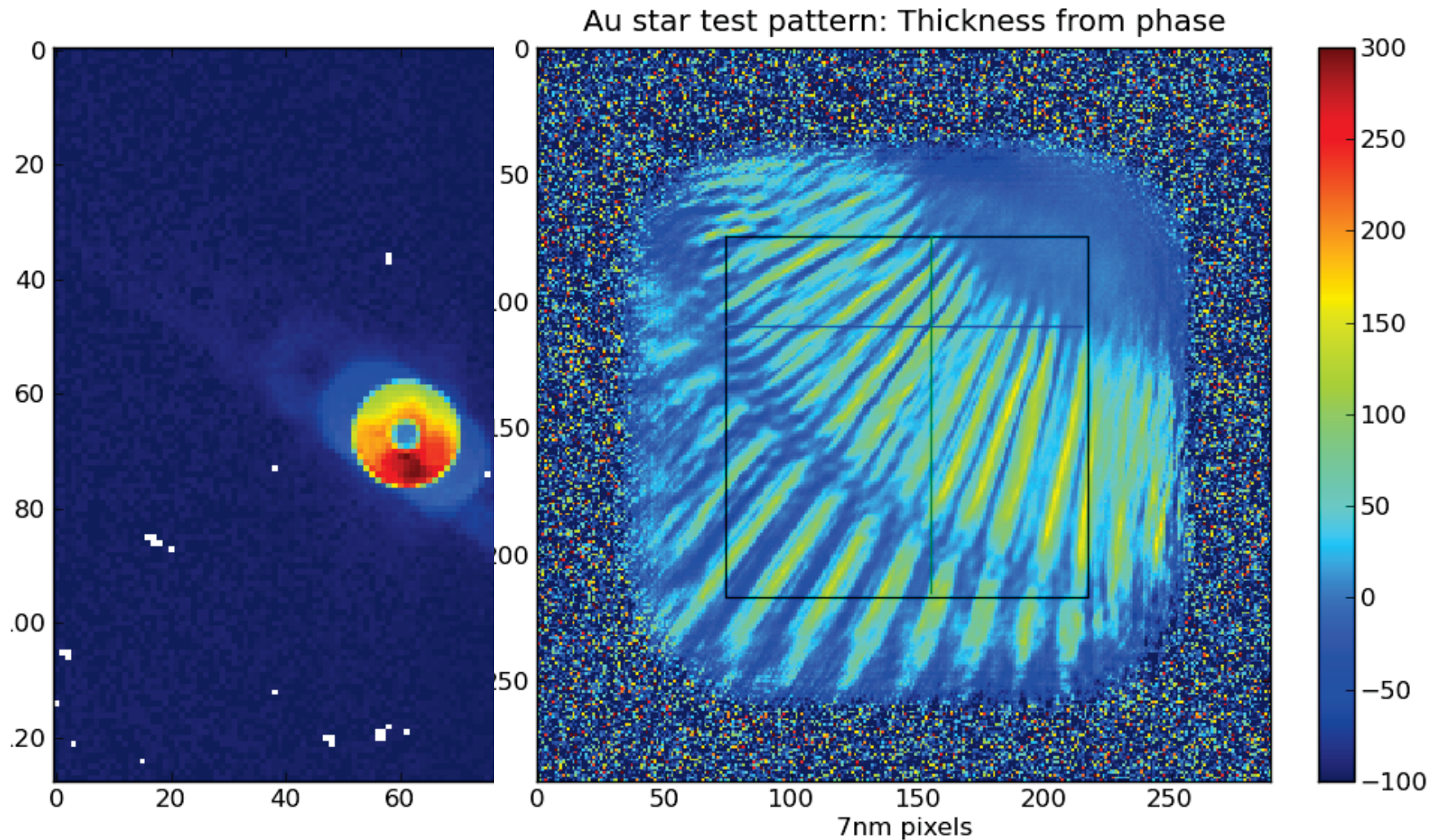
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# Signal to noise

Lensless  
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theory

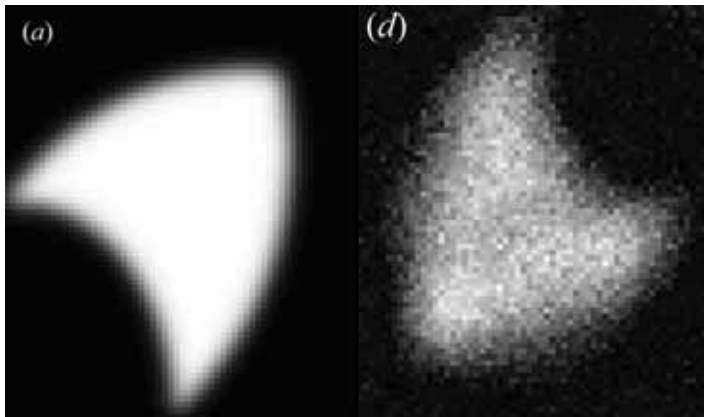
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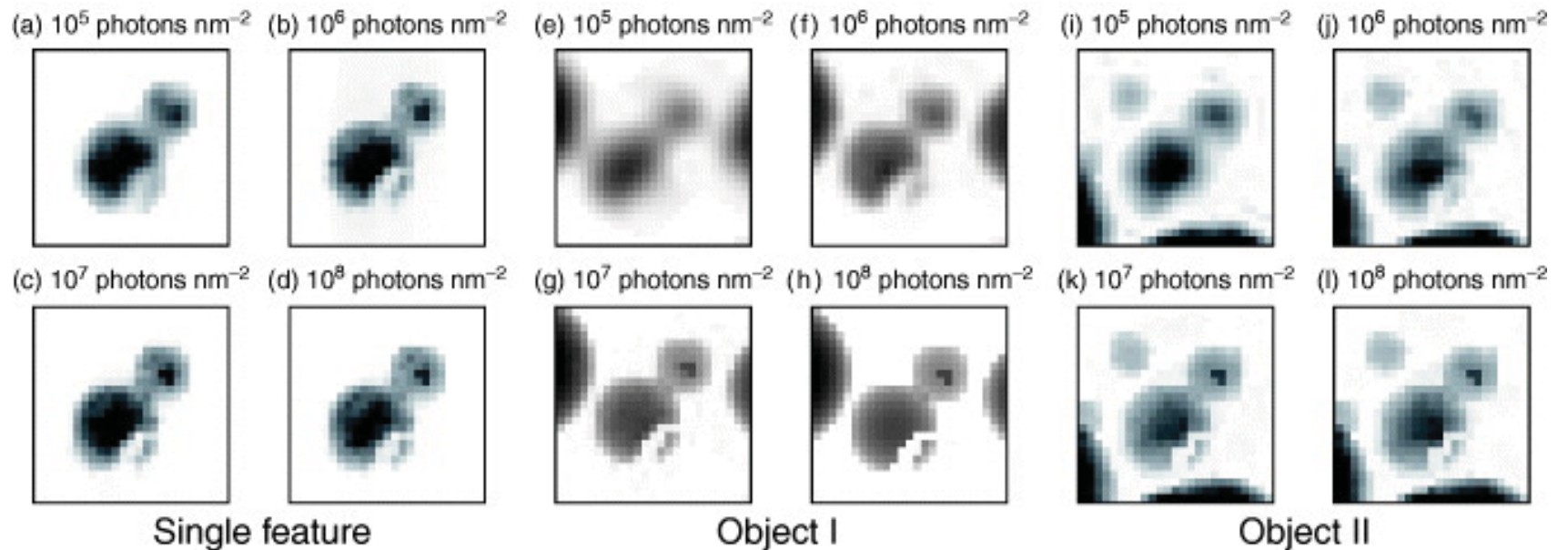
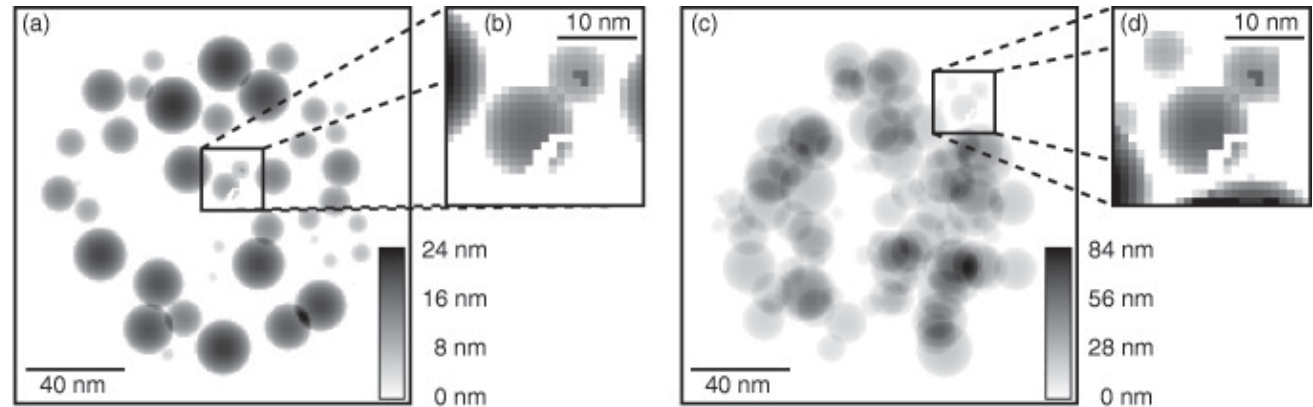


Garth Williams *Acta Cryst A*  
2006.

Effect of Poisson noise in  
diffraction pattern similar to  
Poisson noise in image.

# Signal to noise

Andreas  
Schropp and  
Christian G  
Schroer *NJP*  
2010.



resolution for isolated feature same (or better) than same feature as part of larger sample

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# X-ray microscopy comparison

Lensless  
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	NA	efficiency	mode	contrast
TXM	$\frac{\lambda}{2\Delta r}$	$e_{lens} \times e_{det}$	full field	absorption, phase
FCDI	$\sin\left(\frac{N\lambda}{4a}\right)$	$e_{det}$	full-field	absorption, phase
ptycho- graphy	$\sin\left(\frac{N\lambda}{4a}\right)$	$e_{det}$	scanning	abs., phase, fluorescence, photoelec- trons

# X-ray microscopy comparison

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	NA	efficiency	mode	contrast
TXM	$\frac{\lambda}{2\Delta r} = 0.01$	$e_{lens} \times e_{det}$	full field	absorption, phase
FCDI	$\sin\left(\frac{N\lambda}{4a}\right) = 0.5$	$e_{det}$	full-field	absorption, phase
ptycho- graphy	$\sin\left(\frac{N\lambda}{4a}\right) = 0.5$	$e_{det}$	scanning	abs., phase, fluorescence, photoelec- trons

assuming

- $\lambda = 0.1nm$  wavelength
- $\Delta r = 5nm$  finest zone width in zone plate
- $N = 1000$  number of rows/columns in area detector
- $a = 50nm$  illumination size

# Multi-modal diffraction imaging

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## Fresnel CDI

- Full field imaging with variable zoom
- No fast scanning
- Dosing more even – take multiple exposures of same illuminated area and cross correlate – no partially overlapping illumination

## Ptychography

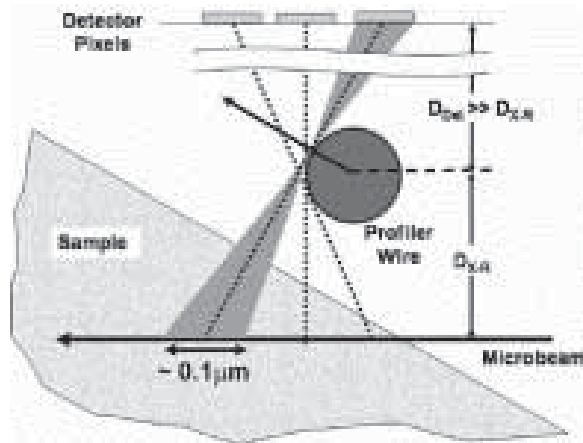
- X-ray fluorescence
- Photoelectrons

Potential for z-scanning instead of tomography (poorer resolution)

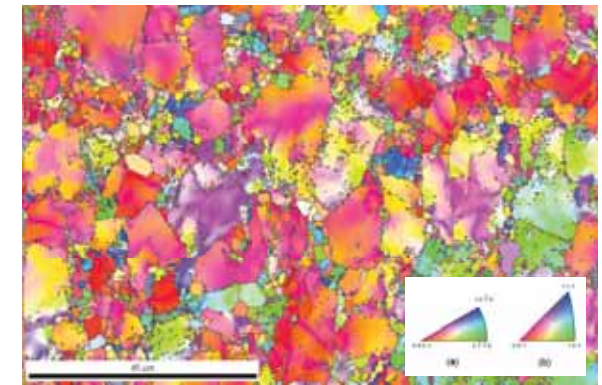


# Strain mapping within polycrystalline film

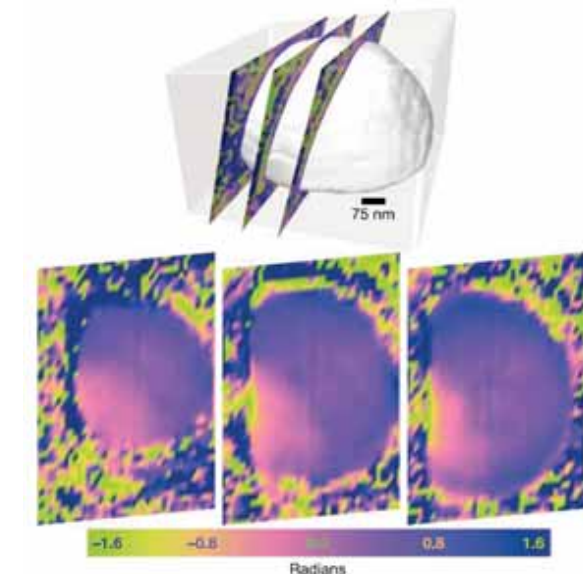
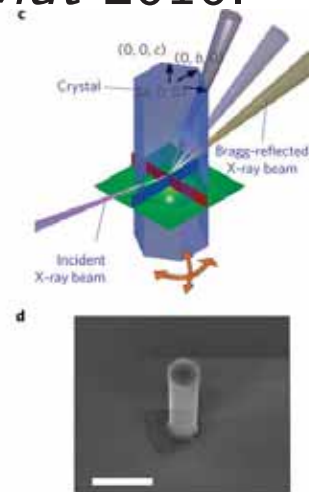
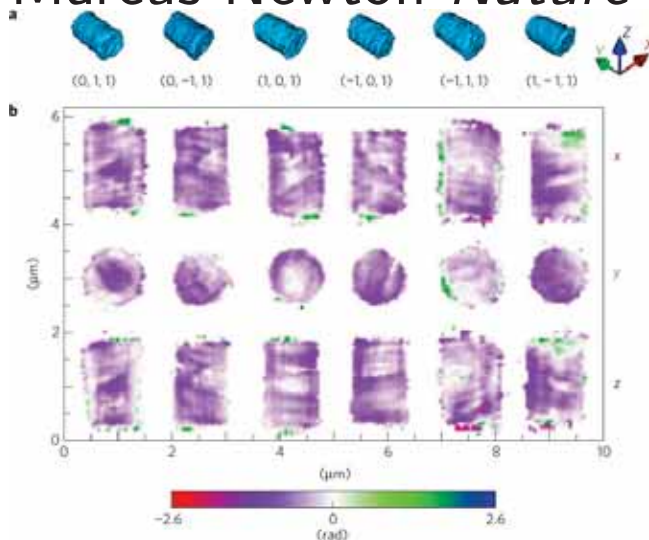
Diff. aperture diffraction imaging.  
Wang, *Micron* 2004.



Matt Miller Group



Marcus Newton *Nature Mat* 2010.



Lensless imaging with lenses  
M. Pfeifer  
Coherent diffraction – theory  
Coherent diffraction – theory  
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# Multi-energy Bragg CDI

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theory

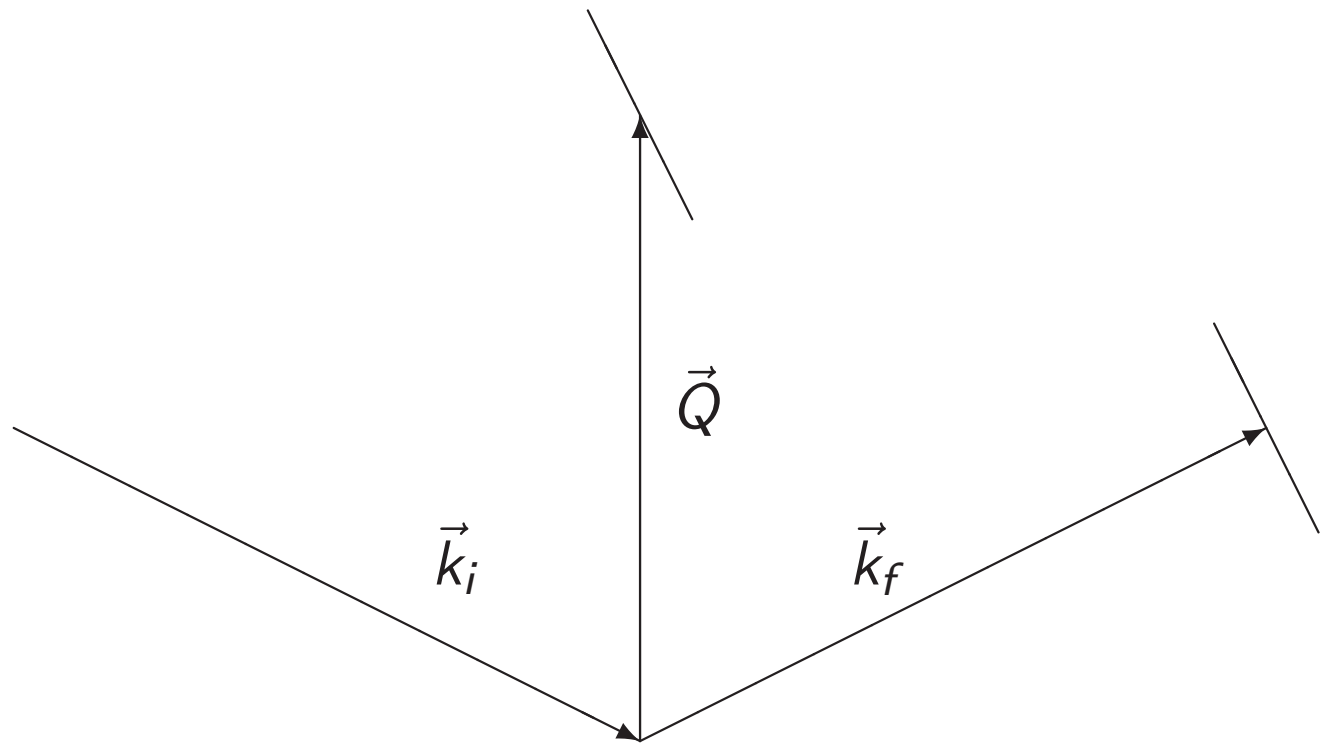
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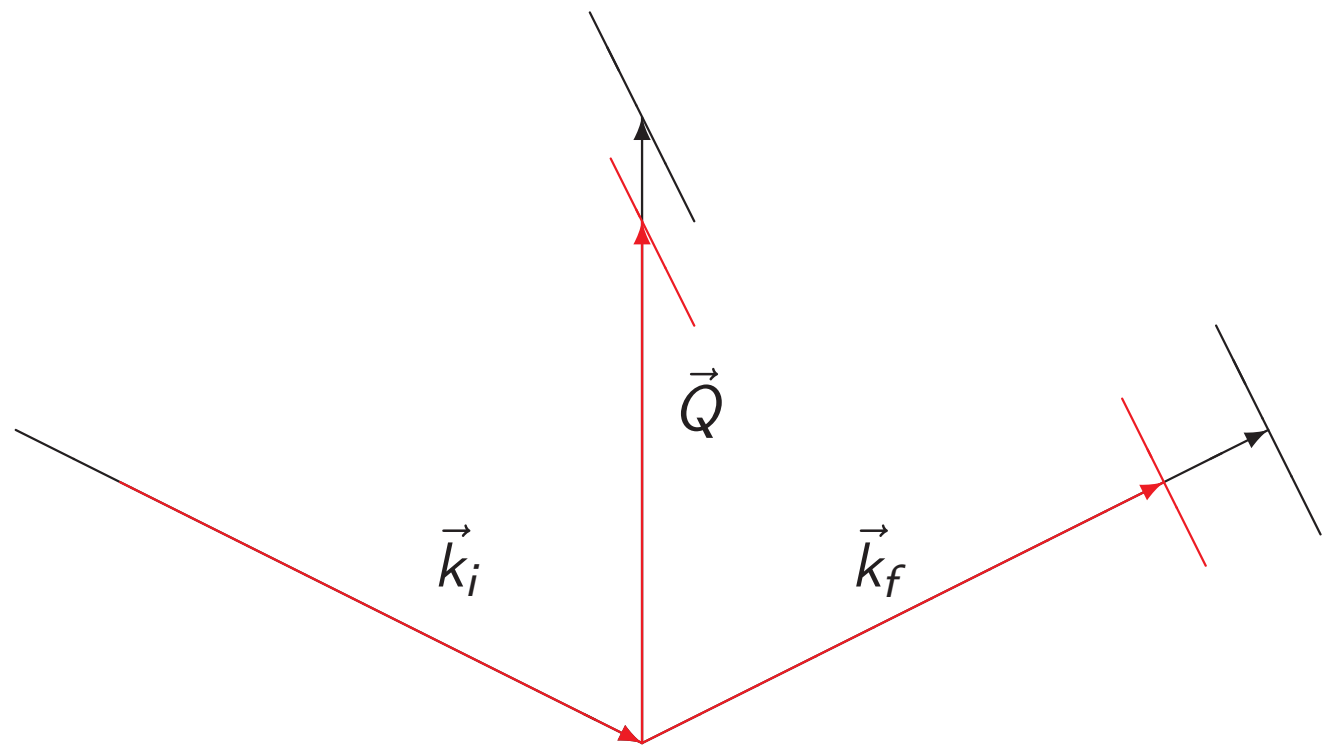
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# Single shot three dimensional imaging of 3D strain

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Consider a detector at a distance  $l$  from a sample, centered about a Bragg peak,  $G$ , at an energy  $E$ .

Measurements made over values  $p_x$ ,  $p_y$ , and  $\frac{\delta E}{E}$ .

$$\vec{q} = \frac{k_o}{l} p_x \hat{q}_x + \left( \frac{k_o}{l} p_y - \frac{\delta E}{E} G \cos \theta \right) \hat{q}_y + \frac{\delta E}{E} G \sin \theta \hat{q}_z$$

For 5nm resolution imaging of 500nm particle at detector distance  $l = 2\text{m}$ , and energy  $E = 6\text{keV}$ , require:

- $200^2$  pixels of  $(100\mu\text{m})^2$
- 480eV range at 2.4keV resolution

If an appropriate energy discriminating detector were possible, then a full three dimensional mapping of the strain a nanocrystal could be made *in situ* in a single measurement, with no scanning.

If three detectors could be located at three Bragg peaks (*i.e.* three  $\{200\}$  peaks when the incident beam is co-axial with the  $\{111\}$ ), then a three dimensional mapping of three different strain projections could be made in real time.

# Summary

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- STXM without phase retrieval is throwing away information.
- Coherent diffraction will be able to stay ahead of purely lens based imaging - but *high efficiency* lenses help.
- Fresnel CDI and ptychography have same basic geometry and optics, and can recover your focal spot complex wave field.
- Efficiency of optic (or optical system) and detector as important as source properties.