Optimization of Elliptical SRF Cavities for $\beta < 1$

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INTRODUCTION

Elliptical superconducting radio-frequency (SRF) cavities are widely used for accelerating electrons and positrons that travel close to the speed of light ($\beta = v/c \approx 1$). For the acceleration of heavier particles, such as protons and ions, β can be considerably less than one. In these cases there are many different cavity designs used, with the choice depending upon the respective goals and constraints of the accelerator [1]. However, if β is not too low it is still effective to utilize an elliptical SRF cavity, which is desirable due to the simplicity of its shape, and, as a result, lower cost of production.

Our discussion will be limited to elliptical cavities, which some research groups have prototyped with β as low as 0.47 [1, 2]. Interest in these elliptical cavities for $\beta < 1$ has grown as they have been proven effective in accelerating protons onto neutron spallation sources in order to produce tritium for use in fusion reactions or to transmute nuclear waste into fissionable elements [3, 4].

In this publication we will further develop and improve upon previous results for optimizations of elliptical SRF cavities. First we will compare the optimization of multi-cell cavity inner cells for $\beta < 1$ with the results for $\beta = 1$ [5]. Then we will analyze the results from Jefferson Lab collaborations with the Bhabha Atomic Research Center (BARC) and the Istituto Nazionale di Fisica Nucleare (INFN) where the presented analysis seems insufficient [2, 3]. Additionally we will analyze the effect of scaling the length of a single-cell cavity on the ratio of the peak magnetic field to the accelerating field ($Hpk/Eacc$). Finally we will attempt to improve the geometry that resulted in the world-record accelerating gradient for a single-cell cavity [6] by optimizing for single-cell boundary conditions.

CELL GEOMETRY

Most current high-beta accelerating cavities are designed with an elliptical profile. Figure 1 represents the cross-section of a half-cell of such a cavity.

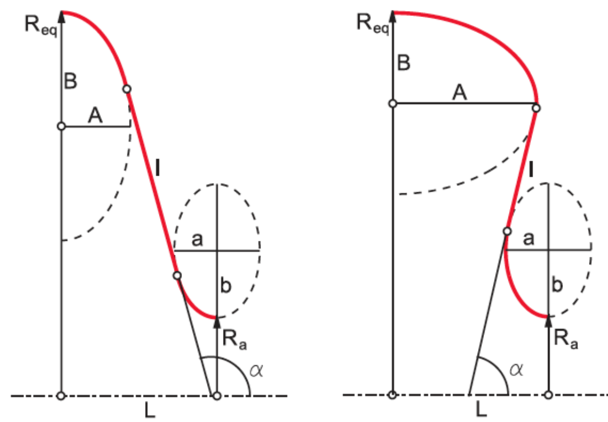

FIG. 1. Cross section of an elliptical accelerating cavity half-cell. Shown on the left is a non-reentrant cell, where $\alpha > 90°$, while on the right is a reentrant cell, where $\alpha < 90°$.

The profile of the cavity is made up of two elliptical arcs that are connected by a tangent segment. The half-length of the cavity, L, is chosen based on the driving frequency in order to make the transit time of the particles through the cell half of the period of field oscillation, resulting in a field that accelerates in the same direction the whole time the particles are in the cell. Therefore the cell half-length is given by the following formula.

$$L = \beta_g \frac{c}{4f} \tag{1}$$

In this formula we use geometrical beta, β_g, instead of β because it is often used as a scale factor for the half-cell length in a single-cell cavity, rather than being equal to v/c, because the electric field does not go to zero at the ends of the cavity, but decays exponentially as it reaches into the beam pipe. For a multi-cell cavity, typically $\beta = \beta_g$ to ensure acceleration in the same direction throughout the entire cavity. However, in some multi-cell cavities $\beta_g < \beta$ is used [2, 7].

The equatorial radius, Req, is adjusted to make the RF driving frequency equal to the frequency of the TM_{01} mode. For both single and multi-cell cavities the aperture radius, Ra, is chosen to allow for the propagation of higher-order modes (HOMs) out of the cavity where they can be removed by resistive loads. For the multi-cell cavity Ra also affects cell-to-cell coupling.

For the purpose of optimizing the cavity's electromagnetic properties the elliptical axes, $A, B, a,$ and b, are used as free parameters. In our case the purpose of optimization of the cell geometry is to allow for a large accelerating field without causing magnetic quenching in the superconductor. Therefore our goal is to minimize the ratio of the peak magnetic field at the surface of the superconductor to the accelerating electric field ($Hpk/Eacc$).

NUMERICAL SIMULATION

For our optimization the numerical simulation code SuperLANS was used, along with the wrapper code TunedCell. TunedCell adjusts Req to make the frequency of the TM_{01} mode equal to the RF driving frequency, creates the geometry files for use in SuperLANS and allows for linear variation of the free parameters. By using a spherical cavity with known analytical solutions SuperLANS has been shown to produce the most accurate results when compared to the 2D simulation codes SUPERFISH, URMEL, URMEL-T and URMESH [8].

Because TunedCell only allows for linear variation of free parameters it was beneficial to write a Matlab formula as a wrapper code for TunedCell that optimizes the cavity for minimum $Hpk/Eacc$. This was accomplished using the following algorithm.

1. Check $Hpk/Eacc$ for a central set of free parameters, then for all combinations where at least one free parameter is one step above or below its central value. For four free parameters this gives a four-dimensional, 81 point grid.

2. If the central value is the minimum, shrink the step size of each free parameter. If not, move in the direction that gives the minimum $Hpk/Eacc$ until $Hpk/Eacc$ begins to increase again.

3. Repeat from beginning until the step size is reduced to zero.

For several cases of optimization it was possible to reduce the number of free parameters by expressing some of them as functions of the other free parameters based on saturated geometrical constraints, such as minimum wall angle and minimum radius of curvature of the cell profile. When this was possible it was the preferred method because it reduced the chance that the optimizer would get stuck at an incorrect minimum due to step sizes that would break the constraints. Additionally this method reduced the run time of the Matlab optimizer.

OPTIMIZATION OF A MULTI-CELL CAVITY FOR $\beta = 1$

It can be seen in numerical simulations that attempting to alter the geometry of the cavity in order to reduce $Hpk/Eacc$ tends to cause an increase in the ratio of $Epk/Eacc$. This places a constraint on our optimization because the maximum electric field at the surface of the superconductor (Epk) should not be too large or it will result in field emission from the superconductor. However, the electric field strength required to cause field emission from the niobium is dependent on the mechanical, heat and chemical treatment applied to the cavity before operation, so this is not a hard limit of the superconducting material [9].

Additionally, decreasing the wall angle of the cell tends to reduce $Hpk/Eacc$. Yet performing surface treatment on cells where α is close to or less than 90 degrees is difficult because of the liquid-based methods used to treat the superconducting surface [9]. Therefore this angle should be limited to a minimum value based on fabrication concerns.

Shown in Figure 2 are results obtained in [5] using SuperLANS where $\beta = 1$ for various limiting values of $Epk/Eacc$.

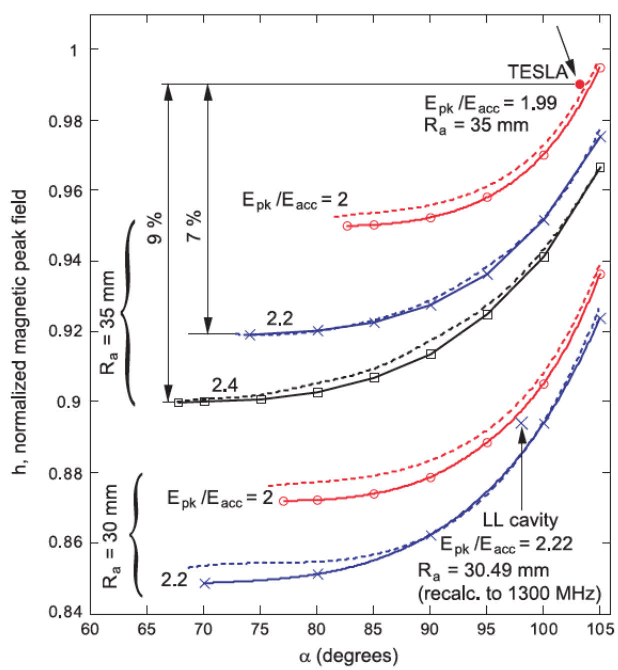

FIG. 2. Results of optimization of the inner cell of a multi-cell cavity where $\beta = 1$. Solid lines show optimization for minimum $h = Hpk/42Eacc$ (normalized for TESLA cavity where $Hpk/Eacc \approx 42$ Oe/(MV/m)) [5]. Dashed lines show optimization for maximum GR_{sh}/Q.

Figure 2 shows that optimizing the cavity for minimum h or minimal losses (maximum GR_{sh}/Q) lead to almost the same shape for the case where $\beta = 1$. However, in a high-current electron accelerator the shapes that produce these minimums are usually shaped such that HOMs are excited in the accelerating cavity. Therefore these shapes must be altered to allow for the HOMs to propagate out of the cavity, where they can be removed by resistive loads.

MINIMIZING $Hpk/Eacc$ IN A MULTI-CELL CAVITY FOR $\beta < 1$

SRF cavities where $\beta < 1$ are often designed for heavier particles with lower currents that do not excite HOMs. Therefore the shape that results in a minimum h or minimal losses for a $\beta < 1$ cavity may not need to be tweaked if HOMs are not excited by the particle beam.

In Figure 3 the results for minimum h where $\beta = 1$ and $Epk/Eacc = 2$ are compared to the results of optimization for minimum h where $\beta = 0.9$ with the same limitation on $Epk/Eacc$.

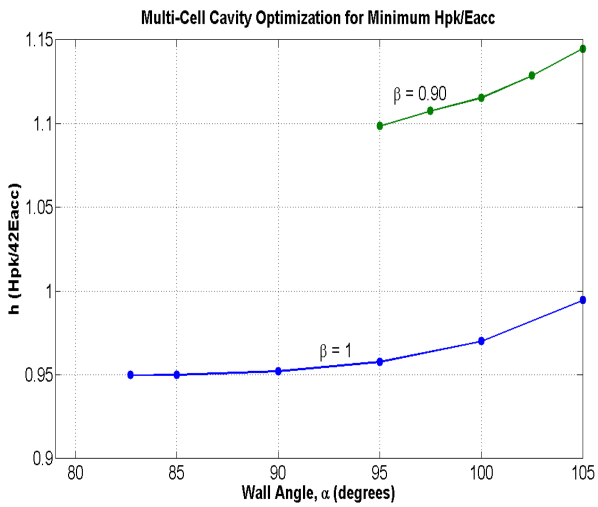

FIG. 3. Optimization for minimum h of an inner cell of a multi-cell cavity with $Ra = 35$ mm and maximum $Epk/Eacc = 2$ for $\beta = 1$ from [5] and new result for $\beta = 0.9$.

This data shows that h increases with wall angle similarly for $\beta = 0.9$ as it does when $\beta = 1$. Additionally it is shown that h is increased as β is decreased. This limits the accelerating field of low-beta, elliptical cavities, making the elliptical cell shape ineffective for accelerating particles where $\beta < 0.4$ [1].

VERIFICATION OF BARC AND JEFFERSON LAB COLLABORATION

In a paper published in 2011 by the Bhabha Atomic Research Center (BARC) in collaboration with Jefferson Lab an attempt was made at minimizing the value of $Epk/Eacc$ for a single cell accelerating cavity using the numerical simulation code SUPERFISH. The given results are for $\beta = 0.49$, $f = 1050$ MHz, $A = B = 20$ mm and $Ra = 39$ mm. The results from this article are given in Figures 4 and 5.

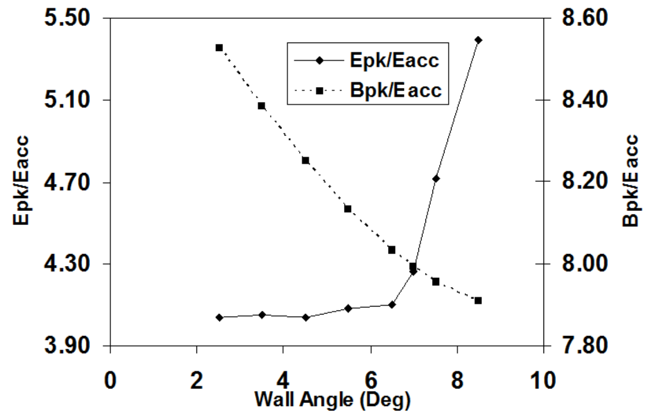

FIG. 4. Variation of $Epk/Eacc$ and $Bpk/Eacc$ with wall angle where $a/b = 0.7$ from [3]. Here the wall angle is measured from a line perpendicular to the beam axis, making this angle equal to $\alpha - 90°$.

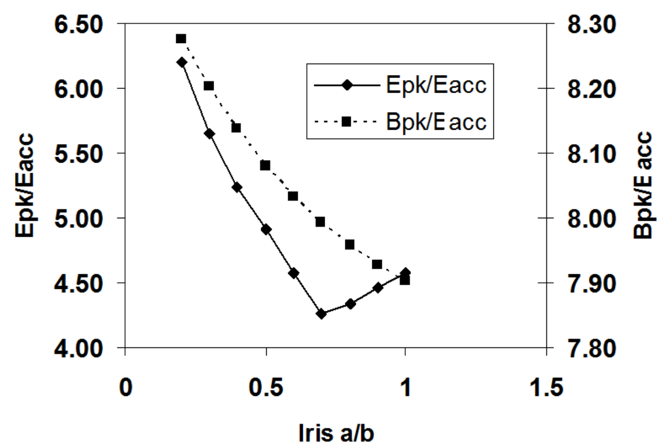

FIG. 5. Variation of $Epk/Eacc$ and $Bk/Eacc$ with iris ellipse ratio where $\alpha = 6.5°$ from [3].

The authors of this article claim that Figure 4 shows a minimum of $Epk/Eacc$ at a wall angle of 6.5°. However, this does not appear to be the case. It would seem that there is a minimum where the wall angle is approximately 4.3°, but this also appears to be the result of the limited accuracy of the results shown.

More concerning is the result obtained when trying to recreate this data. Figure 6 shows the simulation results obtained when using the boundary conditions for an inner cell of a multi-cell cavity compared with the results from the BARC article.

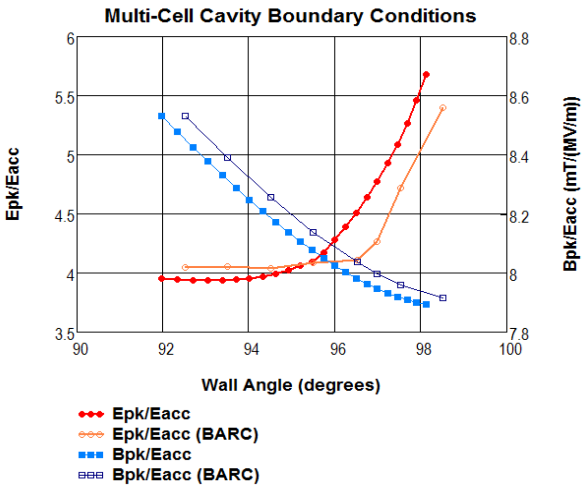

FIG. 6. BARC results using SUPERFISH compared with our results from SuperLANS with multi-cell boundary conditions.

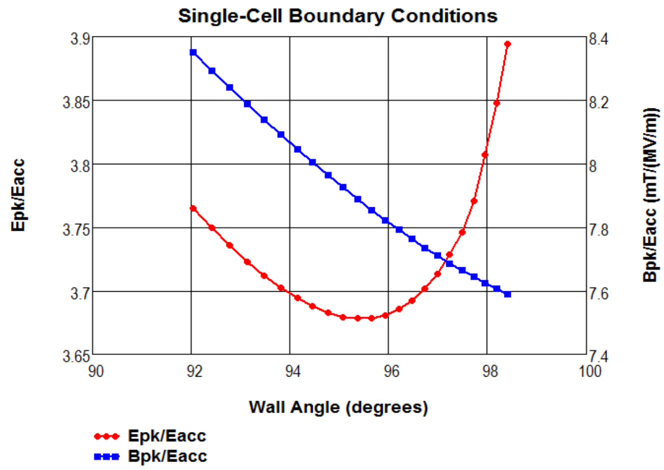

FIG. 7. SuperLANS results for single-cell boundary conditions using BARC constraints ($r = a/b = 0.7$, $\beta = 0.49$, $f = 1050$ MHz, $A = B = 20$ mm and $Ra = 39$ mm).

Although this data does not exactly agree the qualitative trends are the same, strongly suggesting that BARC utilized the wrong boundary conditions for their optimization. The discrepancies could be accounted for by different levels of accuracy in the results from SUPERFISH compared to SuperLANS or from different levels of accuracy in the free parameters.

Figure 7 shows the variations of the electromagnetic parameters with respect to wall angle for single-cell boundary conditions, which do not agree with the results from the BARC article. The data from our SuperLANS simulation has clear minimum for $Epk/Eacc$ at $\alpha \approx 95.4°$ that is lower than the minimum from the BARC article by 0.36 (BARC minimum $Epk/Eacc \approx 4.04$; Our single-cell minimum $Epk/Eacc \approx 3.68$). The values of $Bpk/Eacc$ are also considerably lower than those given by multi-cell boundary conditions.

SINGLE-CELL OPTIMIZATION

Because the results from BARC are based on incorrect boundary conditions we have chosen to complete the optimization for the single-cell cavity. However, we will minimize the ratio of $Hpk/Eacc$ in order to reduce the chance for magnetic quenching. As shown in [5] reducing $Hpk/Eacc$ will also serve to reduce losses in the cavity. The values of Ra, β, f and L are the same as those from the BARC article, with the optimization being done by varying our free parameters A, B, a, and b.

For the single-cell cavity the values of $Epk/Eacc$ tended to be considerably lower than those found utilizing the multi-cell boundary conditions. Based on the trend in results given by SuperLANS and the values of $Epk/Eacc$ used by the inner cells of the TRASCO-ASH and RIA cavities where $\beta = 0.49$ we chose a maximum of $Epk/Eacc = 3.5$ [2].

This optimization quickly led to values of a and b which result in an extremely small radius of curvature for sections of the cell profile. In order to allow for the fabrication of the cavity from niobium the minimum radius of curvature of the cavity profile should not be too small. We have restricted our radius to twice the thickness of the Niobium sheet from which the cavity is formed: 6mm [10]. Therefore, we used this minimum radius of curvature as an additional constraint in our optimization.

For the constraints given above the following single-cell cavity was found to have a minimum value of $Hpk/Eacc$.

TABLE I. Result of Minimization of $Bpk/Eacc$ for a Single-Cell Accelerating Cavity

Constraints	$L = 34.976$ mm	$Req = 131.899$ mm	$Epk/Eacc = 3.5$	$\alpha = 96.5°$
Free Parameters	$A = 20.811$ mm	$B = 51.3$ mm	$a = 10.510$ mm	$b = 18.41$ mm
Result	$Bpk/Eacc = 8.15$ mT/(MV/m)			

TABLE II. Mondal Single-Cell Optimization

Free Parameters	$A = 20$ mm	$B = 20$ mm	$a/b = 0.7$	$\alpha = 96.5°$
Result	$Bpk/Eacc = 8.02$ mT/(MV/m)		$Epk/Eacc = 4.26$	

Our results for the optimization of the single-cell cavity have a 1.6% higher value of $Bpk/Eacc$ but we have limited our value of $Epk/Eacc$ to be lower than the BARC optimization by 17.8%. However, the value of $Epk/Eacc$ quoted in the BARC article was obtained using their simulation which seemed to utilize multi-cell boundary conditions. For the BARC free parameters our results for the electromagnetic parameters using single-cell boundary conditions in Figure 7 are $Bpk/Eacc \approx 7.76$ mT/(MV/m) and $Epk/Eacc \approx 3.69$. This makes our optimization result 5% higher for $Bpk/Eacc$ and 5.1% lower for $Epk/Eacc$. However in the next section we will discuss how $Bpk/Eacc$ can be lowered further by adding a fifth free parameter: the cell length.

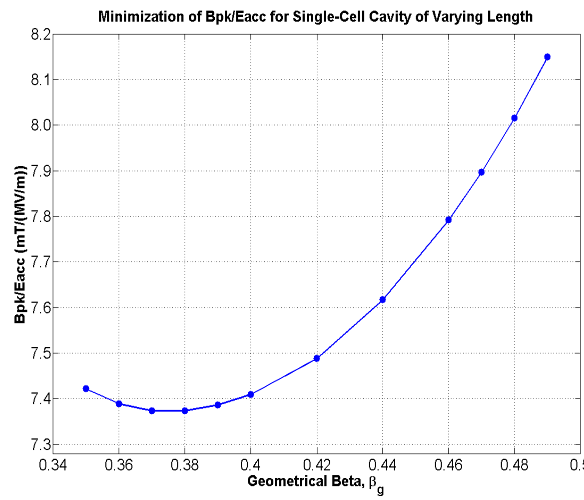

FIG. 8. Variation in $Hpk/Eacc$ with geometrical beta (β_g) ($Ra = 39$ mm, $\beta = 0.49$, $f = 1050$ MHz, minimum radius of curvature = 6 mm, minimum wall angle = 96.5° and maximum $Epk/Eacc = 3.5$).

VARYING LENGTH SCALE FACTOR, β_g

In a single-cell accelerating cavity the accelerating electric field does not reduce to zero at the ends of the cell, but rather it reaches into the surrounding sections of beam pipe for some distance as it undergoes an exponential decay. Because of this effect it seems that it would be beneficial to shrink the cavity to a smaller half-length than that given by the formula $L = \beta c/4f$ in order to expose the beam particles to a larger potential difference when passing through the cavity and surrounding sections of beam pipe. Based on this fact the single-cell cavity optimization from the previous section was extended to a fifth parameter: the cavity length scale factor, β_g.

Figure 8 shows the results of minimizing the ratio $Hpk/Eacc$ for different values of β_g using the BARC values of Ra, β and f while keeping the constraints on the minimum radius of curvature, wall angle and $Epk/Eacc$ enumerated in the previous section.

By shortening the cavity length we were able to reduce the value of $Bpk/Eacc$ to 7.37 mT/(MV/m) while keeping $Epk/Eacc = 3.5$. This is a reduction of 5% in $Bpk/Eacc$ from the single-cell boundary condition BARC value.

The data, from a SuperLANS simulation, shows a minimum value of h at approximately $\beta_g = 0.375$. This value is considerably lower than the $\beta_g = 0.47$ used in the RIA cavities where $\beta = 0.49$ [2]. However, these cavities are of the multi-cell variety, where the the inner cells have a field that ideally reduces to zero at the cell boundary when $\beta = \beta_g$. Other multi-cell cavities, such as those used in the TRASCO-ASH and SNS designs, also use a β_g that is 0.02 less than β [2].

VERIFICATION OF INFN AND JEFFERSON LAB COLLABORATION

In a paper published in 2001 by the Istituto Nazionale di Fisica Nucleare (INFN) in collaboration with Jefferson Lab a different set of free parameters is used to describe the cell geometry. Where we use the ellipse half-axes

(A, B, a, and b) the INFN article utilizes the equator ellipse ratio ($R = B/A$), iris ellipse ratio ($r = b/a$), wall distance (d), and wall angle (alpha, which is measured from a line perpendicular to the beam axis, making it equal to our $\alpha - 90°$). Although the INFN free parameters are different they use the same constraints ($D = R_{eq}$, $R_{iris} = R_a$ and L) and therefore they must still use four free parameters.

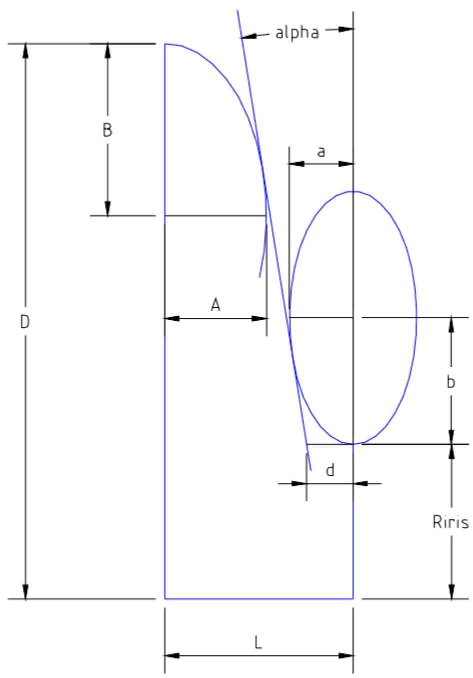

FIG. 9. Cell geometry with parameters used in [2]

By varying each one of the free parameters while keeping the other three fixed the INFN article hoped to attribute the variation of each free parameter to particular changes in the electromagnetic and mechanical parameters of an inner cell of a multi-cell cavity. Varying the free parameters turned out to cause monotonic or negligible (in the case of the equator ellipse ratio, R) changes in $Epk/Eacc$ and $Bpk/Eacc$ for all cases except for the effect of varying the iris ellipse ratio, r, on $Epk/Eacc$.

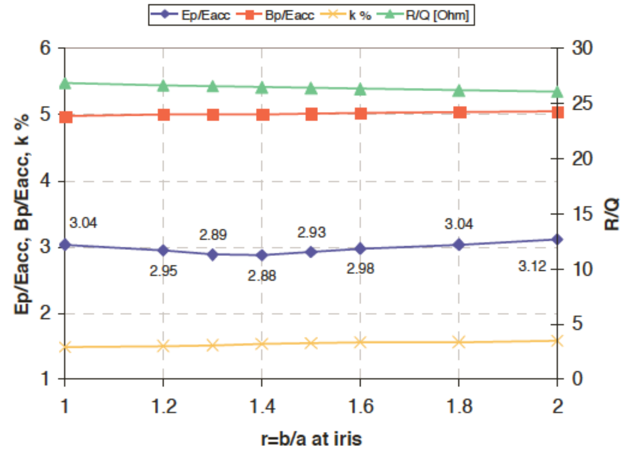

FIG. 10. Change in electromagnetic parameters of the inner cell of a multi-cell cavity with variation in iris ellipse ratio, r, from [2].

The authors of the INFN article note that this result shows a minimization of $Epk/Eacc$ by about 10% by choosing $r = 1.4$. Yet this result only shows the value of r that minimizes $Epk/Eacc$ for fixed values of R, d and alpha. By varying either of the other free parameters while also varying r we were able to check if the value of r which minimizes $Epk/Eacc$ changes. To accomplish this a Matlab formula wrapper for TunedCell was written which varied the INFN free parameters and used Matlab's "fsolve" function to convert them to the free parameters used by TunedCell. Using this scheme we found that changing R does not cause the optimal value of r to change, but this is not the case for d or alpha.

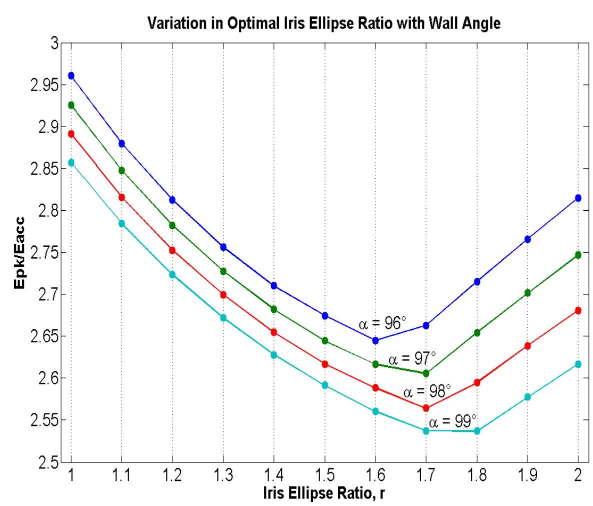

FIG. 11. $Epk/Eacc$ with variations in r and α.

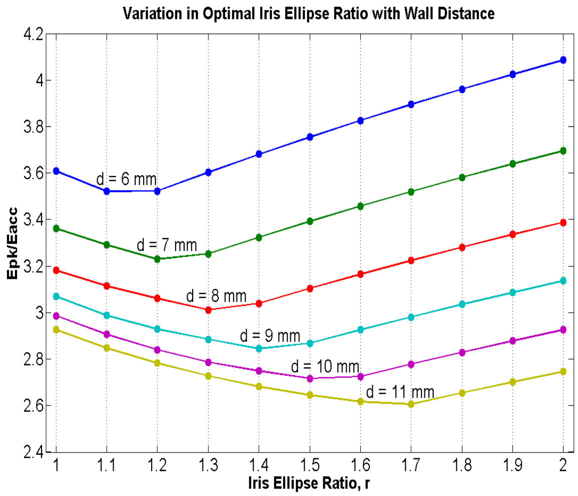

FIG. 12. $Epk/Eacc$ with variations in r and d.

Together, Figures 11 and 12 show that increasing either the wall angle or the wall distance increases the value of r that minimizes $Epk/Eacc$.

ACKNOWLEDGMENTS

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