Math Career Day *11 October 2007*

Corning Community College

Microcosmic Math

A simple math application

for elementary-particle physics

From the discovery of the nucleus to that of the quark, the simple mathematical properties of elementary objects have been used to reveal unexpected physical structure at unimaginably (?!) small distance scales.

Jim Crittenden

Laboratory for Elementary-Particle Physics

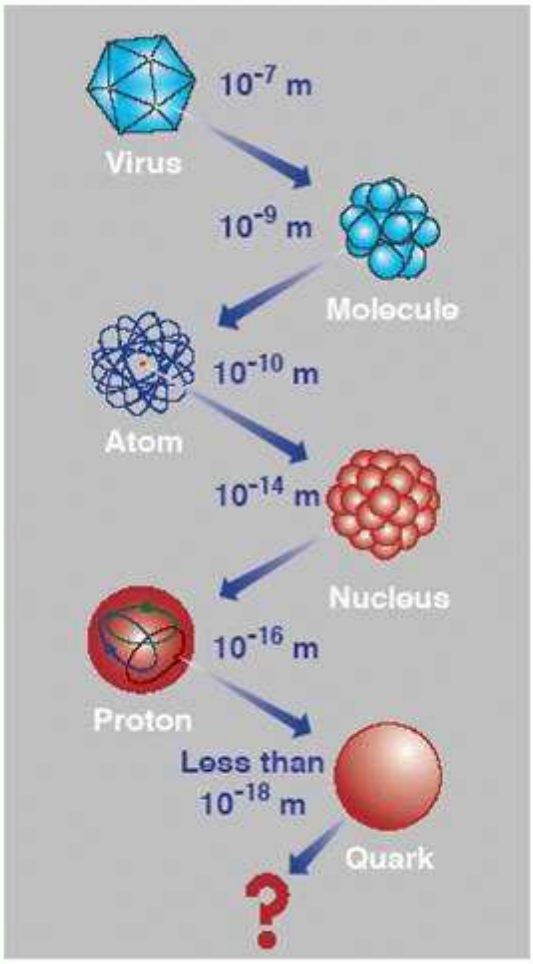
Cornell University

What is the meaning of "elementary" ?

"Elementary" is an expression of ignorance !

(and therefore time-dependent)

(About 1/1000 of a human hair diameter)



Rutherford (1911)

Hofstadter (1953)

(Nobel Prize, 1961)

Taylor, Friedman and Kendall (1967)

(Nobel Prize, 1990)

Present limit of our knowledge:

10^{-18} *m*

($\simeq \frac{1}{1000}$ *Proton radius*)

limited by presently attainable particle energies

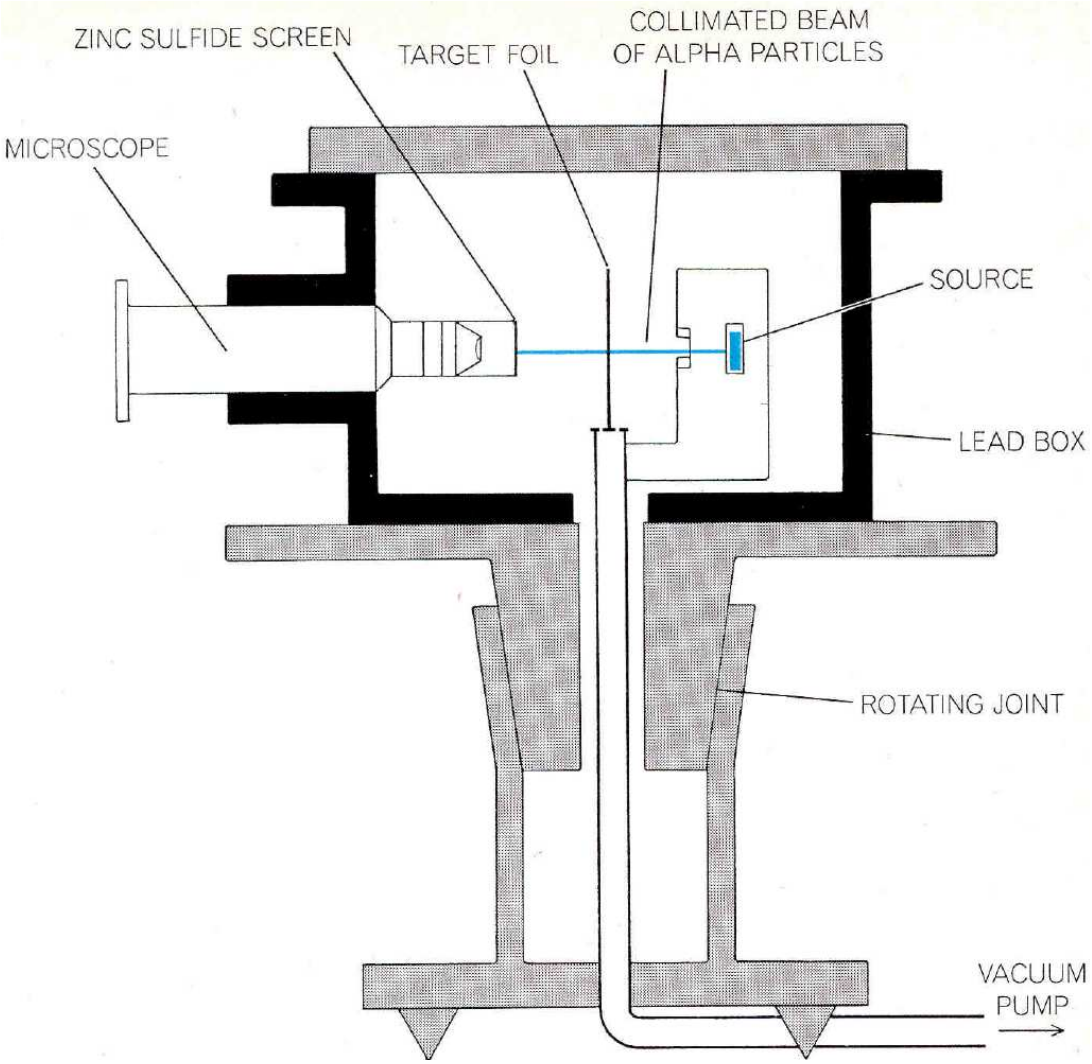
(i.e. limited by available technology)

Ernest Rutherford (1871 – 1937)

„All science is either physics or stamp-collecting“

(Nobel prize for Chemistry 1908)

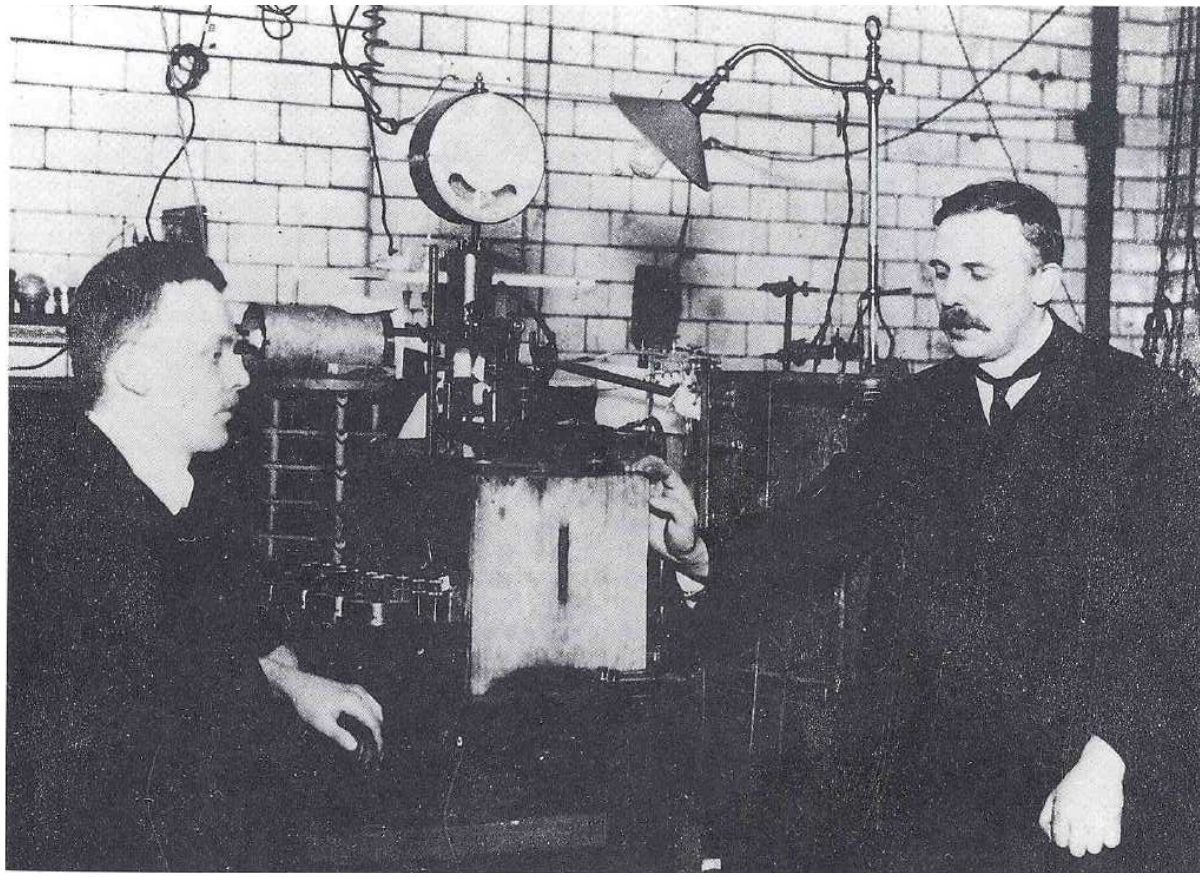
Rutherford's Experiment



The Apparatus for Measuring
α-Scattering on Gold Foils

($E_\alpha \simeq 0.005$ GeV)

The Data Acquisition System

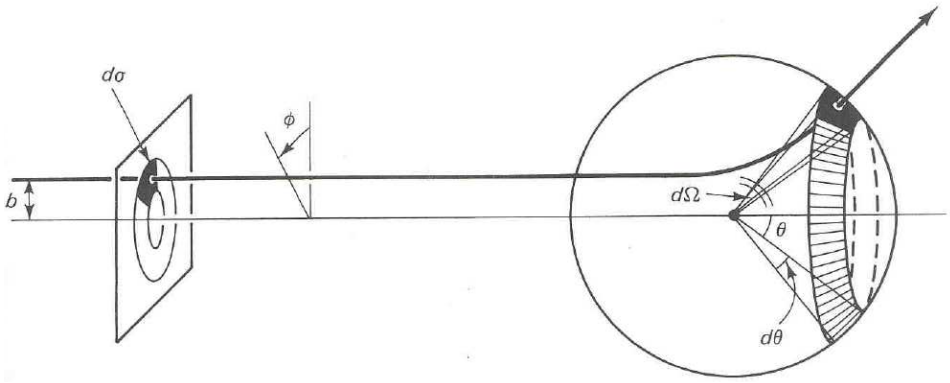


Hans Geiger and Ernest Rutherford count α-Particles in Manchester

„Geiger and Marsden found, for example, that a small fraction of the α particles, about 1 in 20,000, were turned through an average angle of 90° ...“

E. Rutherford, Phil. Mag. Vol. xxi (1911) 669

Scattering Experiments



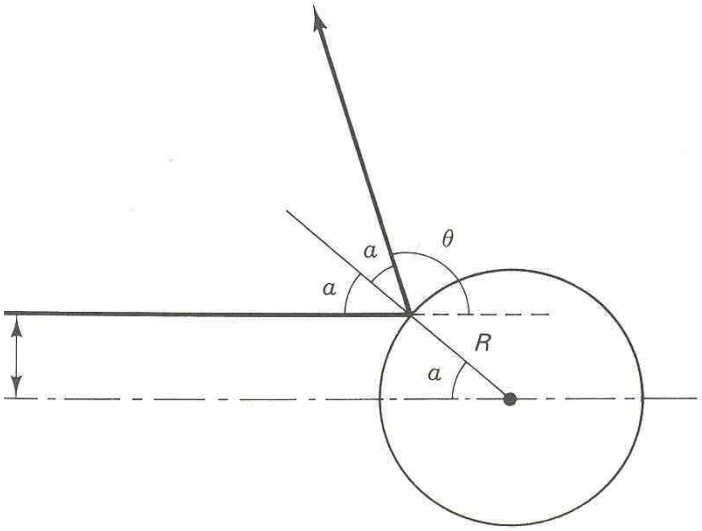
Differential Cross Section per Unit Solid Angle

$d\sigma = F(\theta, \phi) d\Omega$ where $d\Omega = d(\cos\theta) d\phi$

„Probability of single deflexion through any angle“

(E. Rutherford, Phil. Mag. Vol. xxi (1911) 669)

Example: Scattering on a hard sphere



$$\frac{d\sigma}{d\Omega} = R^2/4$$

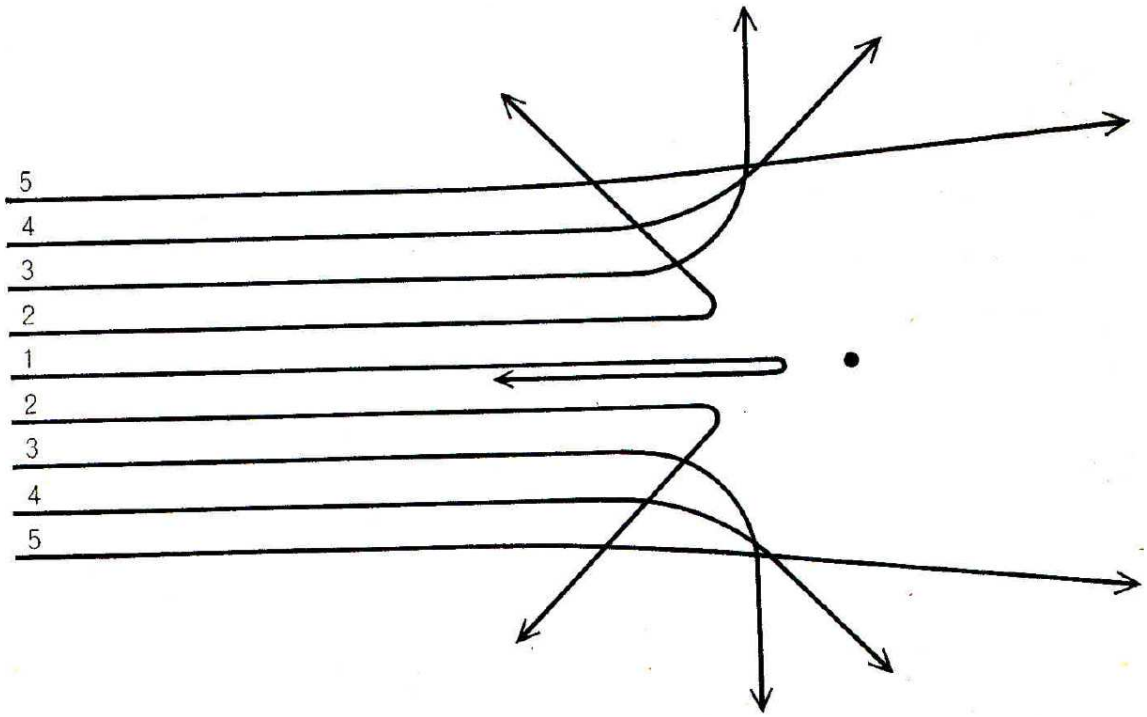
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \pi R^2$$

"Geometrical"
Cross Section

If incident flux known, then scattering rate gives the cross section

Units: 1 barn (100 fm^2) $\simeq$ Cross section of the largest nucleus

Scattering on a Heavy Point Charge (Rutherford Scattering)



Rutherford's Scattering Formula

$$\frac{d\sigma}{d\Omega} \propto \frac{(Q_1 Q_2)^2}{p^4 \sin^4 \theta/2}$$

Momentum transfer: $q = p \sin \theta/2$

$$\frac{d\sigma}{d(q^2)} \propto \frac{1}{q^4}$$

The Discovery of "Inner Space" !

„It seems certain that these large deviations of the α particle are produced by a single atomic encounter."

„A simple calculation shows that the atom must be a seat of an intense electric field in order to produce such a large deflexion at a single encounter."

– E. Rutherford (1911)

"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

– Transcript of one of his last lectures

Niels Bohr's Atomic Model (1913)

The atom is empty!

The Bohr Radius

$$a_0 = \frac{\hbar^2}{e^2\, m_e} \simeq 50,000 \text{ fm}$$

From Atom to Proton

Elastic Electron-Proton Scattering

$$e + p \longrightarrow e + p \quad (E_e = 0.2 \text{ GeV})$$

Robert Hofstadter, Stanford (1953)

Nobel Prize for Physics 1961

$$\frac{d\sigma}{dq^2} \propto \frac{1}{q^4} F(q)$$

Finding

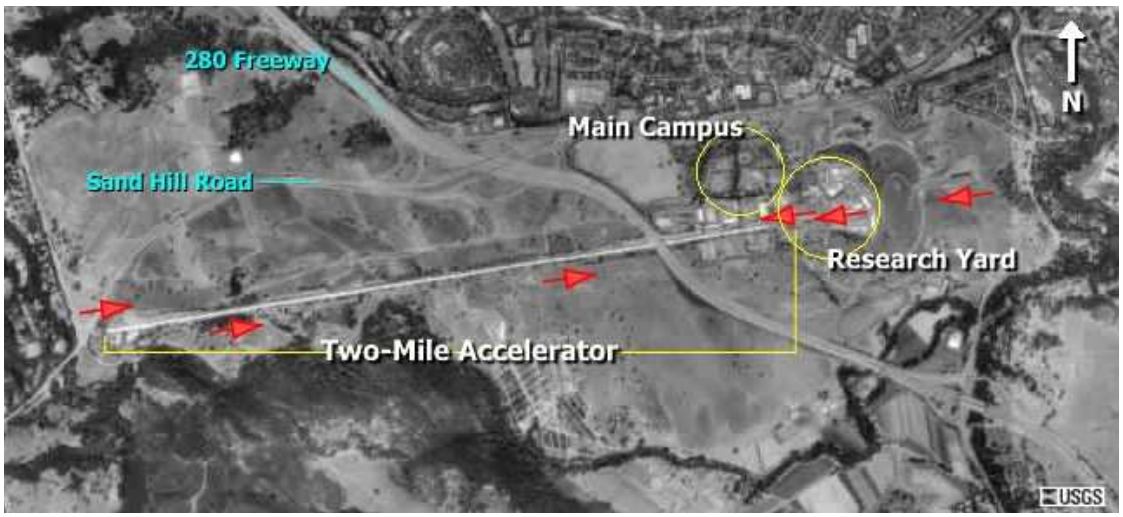
The form factor $F(q)$ falls steeply with q

$$\left(\text{“Dipole Form Factor”} : F(q) \propto \frac{1}{(m^2 - q^2)^2} \right)$$

The charge of a proton
is of finite extent !

$\Rightarrow$ Proton radius: $R_p = 0.8$ fm

The Linear Accelerator at the Stanford Linear Accelerator Laboratory



The Two-Mile-Long Electron Accelerator

Commissioning: 1967

$E_e = 20$ GeV

The Spectrometers

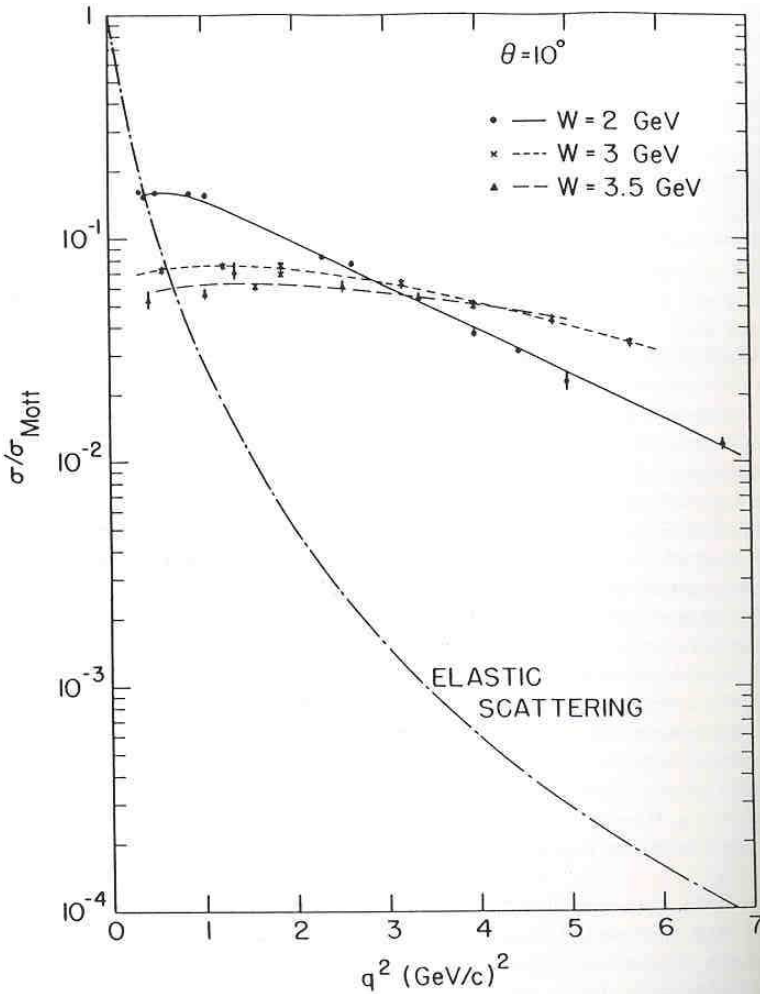
Two Spectrometers in End-Station A

Deep Inelastic Electron Scattering

$$e + p \rightarrow e + X$$

An "inclusive" reaction: $\dfrac{d\sigma}{dq^2} \propto \dfrac{1}{q^4} F(q, x)$

"Before these results were obtained it had been assumed that the inelastic-continuum cross sections would decrease as rapidly as the elastic cross sections when the momentum transfer was raised." –Kendall und Panofsky, Sci. Am., Vol. 224 (1971)



$F(q, x)$ is nearly independent of q !

Last Year's Nobel Prize for Physics

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross

Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA

b. 1941

H. David Politzer

California Institute of Technology
Pasadena, CA, USA

b. 1949

Frank Wilczek

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1951

The strong interaction can be described as a quantum field theory as for the electromagnetic force, where the electric charge is replaced by a *color* charge. (1973)

"Asymptotic Freedom"

The strength of the reaction *increases* with the distance between the color charges (?!)

This is like no other known force!

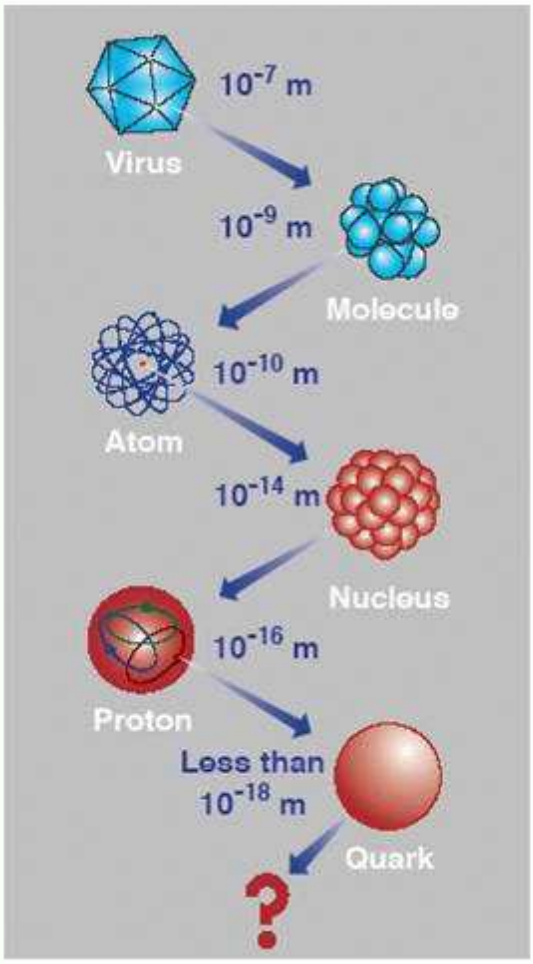
It's why Rutherford-type scattering was observed for strongly-bound quarks !!

What is the meaning of "elementary" ?

"Elementary" is an expression of ignorance !

(and therefore time-dependent)

(About 1/1000 of a human hair diameter)



Rutherford (1911)

Hofstadter (1953)

(Nobel Prize, 1961)

Taylor, Friedman and Kendall (1967)

(Nobel Prize, 1990)

Present limit of our knowledge:

10^{-18} *m*

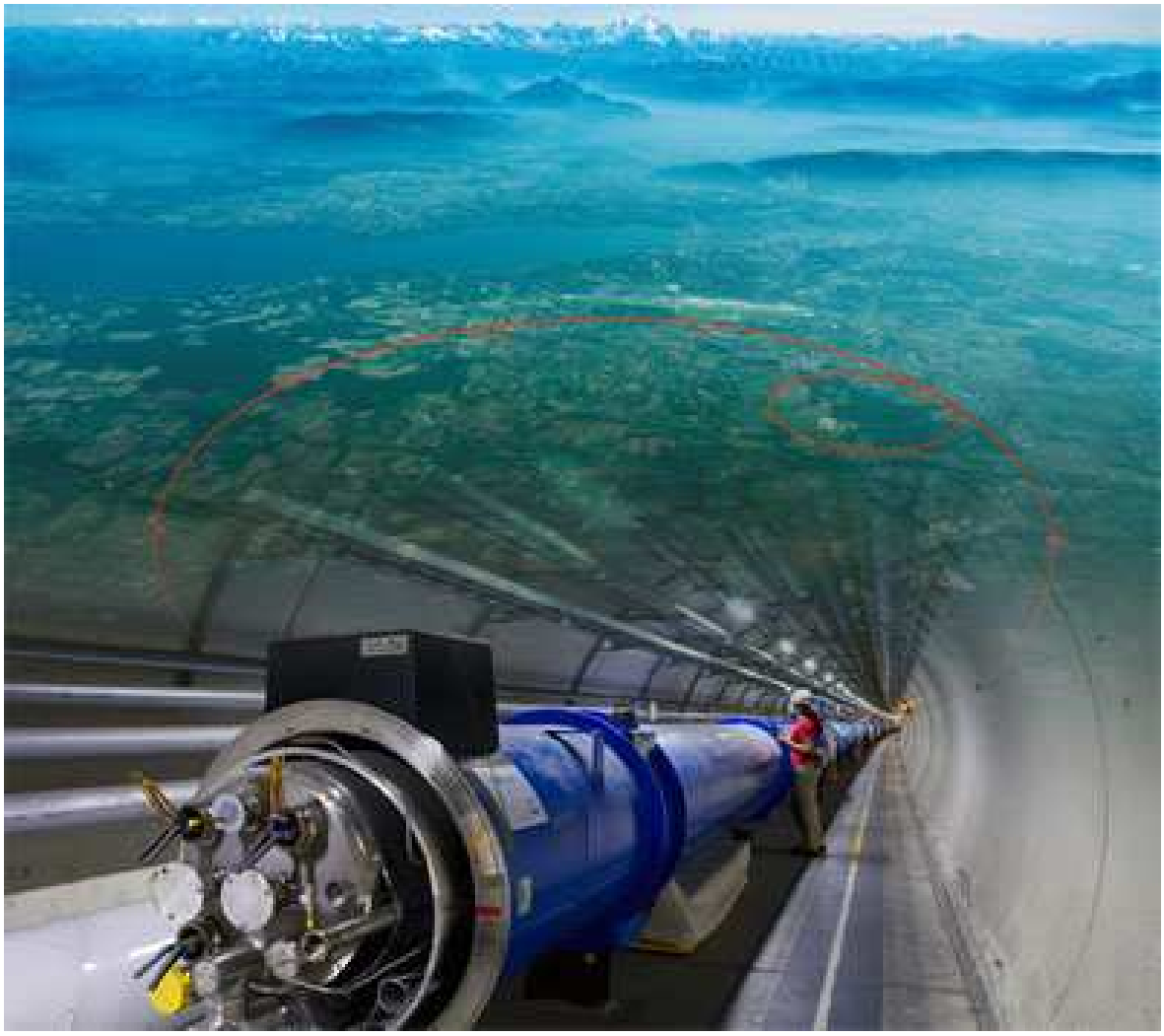
($\simeq \frac{1}{1000}$ *Proton radius*)

limited by presently attainable particle energies

(i.e. limited by available technology)

Present-Day Energy Frontier

The "Large Hadron Collider" (LHC) in Switzerland at the European Organization for Nuclear Research

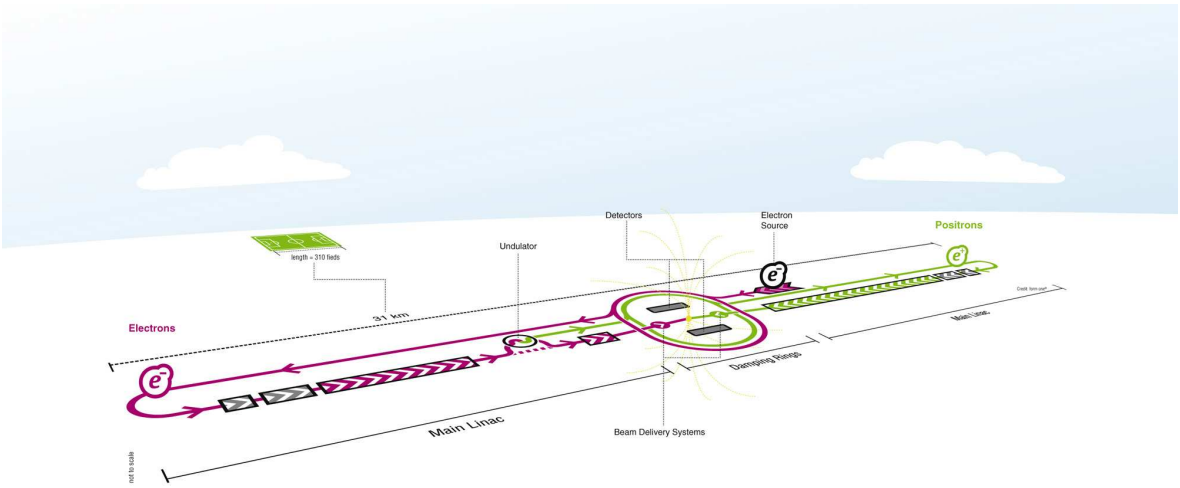


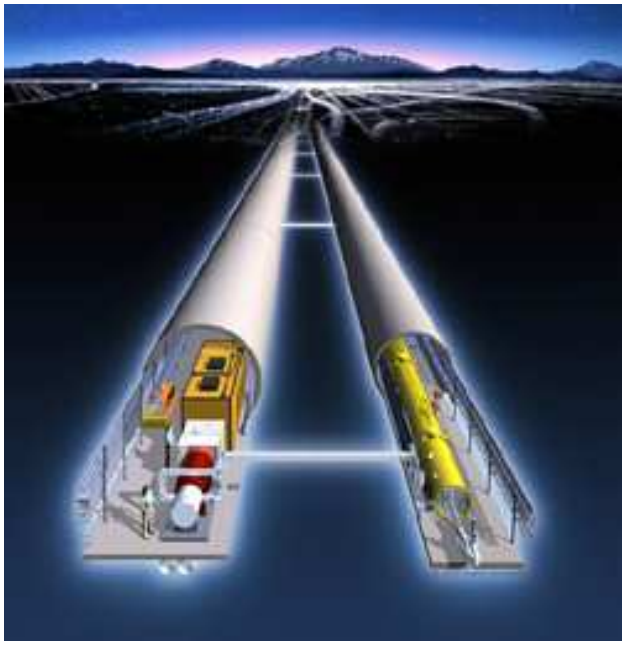
17 miles in circumference, commissioning 2008

Extends the previous energy limit by a factor of 7

A Plan for the Future

The International Linear Collider





22 miles long, now in engineering phase

Investigates LHC energy region, but with electron-positron interactions