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# Uniform Magnetic Fields and Double-Wrapped Coil Systems:
## Improved Techniques for the Design of Bioelectromagnetic Experiments

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A common mistake in biomagnetic experimentation is the assumption that Helmholtz coils provide uniform magnetic fields; this is true only for a limited volume at their center. Substantial improvements on this design have been made during the past 140 years with systems of three, four, and five coils. Numerical comparisons of the field uniformity generated by these designs are made here, along with a table of construction details and recommendations for their use in experiments in which large volumes of uniform intensity magnetic exposures are needed. Double-wrapping, or systems of bifilar windings, can also help control for the non-magnetic effects of the electric coils used in many experiments. In this design, each coil is wrapped in parallel with two separate, adjacent strands of copper wire, rather than the single strand used normally. If currents are flowing in antiparallel directions, the magnetic fields generated by each strand will cancel and yield virtually no external magnetic field, whereas parallel currents will yield an external field. Both cases will produce similar non-magnetic effects of ohmic heating, and simple measures can reduce the small vibration and electric field differences. Control experiments can then be designed such that the only major difference between treated and untreated groups is the presence or absence of the magnetic field. Double-wrapped coils also facilitate the use of truly double-blind protocol, as the same apparatus can be used either for experimental or control groups.

**Key words: experimental controls, biomagnetic effects, coil designs, Helmholtz coils**

## INTRODUCTION

Virtually all laboratory-based experiments which seek to assess the effect of static or oscillating magnetic fields on living systems use electric coils of some sort to generate the electromagnetic exposure. In addition to the altered magnetic

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field, however, these coils introduce other changes into the experimental environment which might also have some influence on the results. These include, in no particular order of importance, (1) magnetic field gradients, (2) electrical (ohmic) heating effects within the coils, (3) variable-frequency noise and/or vibrations produced by magnetomechanical interaction between adjacent wires within the coils, (4) small electric fields produced by the voltage drop between loops within the coils, and (5) the presence of enamel and other insulating materials on the wires, with unknown biological effects. Depending upon current strength, frequency, and coil design, some or all of these side effects will probably be present in almost any experimental apparatus.

As noted recently by Adair [1991], most claims that weak magnetic fields can produce biological effects at the cellular level are greeted with intense skepticism by biophysicists, principally due to the problems associated with coupling these fields to non-ferromagnetic biological materials. (Biogenic magnetite is the only known substance present in living systems which interacts strongly with earth-strength magnetic fields.) Hence, biomagnetists must take seriously the old adage that incredible claims require incredible proof, and strive to control more precisely the magnetic fields and artifacts within their experiments. Furthermore, as biomagnetic experimentation turns increasingly towards understanding and testing the underlying mechanism of interaction with the magnetic field, it will be necessary to produce electromagnetic stimulae as free as possible of the other artifacts mentioned above. For example, biogenic magnetites could be sensitive to mechanical dislocation produced by spatial gradients in the applied field [e.g., Kirschvink and Gould, 1981]; experiments using poorly-designed coil systems that introduce such gradients could not be used to distinguish an effect resulting from a uniform field from that with gradients. The recent discovery of biogenic magnetite in the soft tissues of the human brain [Kirschvink et al., 1992] argues that precise control of the field is indeed warranted.

In this short paper, I will review briefly some fairly simple procedures which can be used to minimize the sometimes subtle, non-magnetic differences between control and experimental groups, particularly the thermal, vibrational, and electric field differences. In the last section, I present results from detailed numerical comparisons of different coil designs which are far superior than the classic Helmholtz design at producing large volumes of uniform, gradient-free magnetic fields. The suggestions recommended here should be easy to implement, and may make results of the research more palatable to a skeptical scientific community. They could also control better for subtle cues which sometimes have a profound impact on the experimental results [e.g., Prasad et al., 1990].

## DOUBLE-WOUND COILS AS AN EXPERIMENTAL CONTROL PROCEDURE

In his attempt to condition pigeons to magnetic stimuli, Beaugrand [1976] was apparently the first to introduce the use of double-wrapped coils to geomagnetic behavioral experiments, and it has been used extensively ever since [e.g., Phillips 1985; Walker and Bitterman 1989a,b; Kirschvink and Kobayashi-Kirschvink 1991]. Examples where it has been used in other biomagnetic experiments are rare [e.g., Wolpaw et al., 1989]. The basic idea is shown schematically in Figure 1. Rather than a single strand of insulated wire being used to wrap a coil, two wires are wrapped together simultaneously on the coil frame, care being taken to ensure that the wires are laid down side by side. Considered as a pair, magnetic fields generated by equal and opposite currents flowing through them will cancel, producing no measurable fields a few wire diameters away. Connecting the coils in series guarantees that the currents are equal. On the other hand, if the current directions are parallel, their magnetic fields will add together and produce fields in the surrounding space. A double-pole, double-throw (DPDT) relay or switch configured as shown in Figure 1 provides an easy way to reverse the direction of current flowing in one of the strands. This double-wrapping technique can be used along with any of the coil designs for producing uniform magnetic fields discussed below.

For experiments involving two groups of laboratory animals, an experimental group being exposed to the magnetic stimulus and a non-exposed control, the best configuration is to have two identical, double-wrapped chambers wired with opposite polarity but sharing the same power source as shown in Figure 1. With this configuration, one setup will always provide a magnetic stimulus and the other the proper non-magnetic control. If truly double-blind procedures are desired, a noiseless solid-state relay can be controlled by a computer-generated (secret) ran-

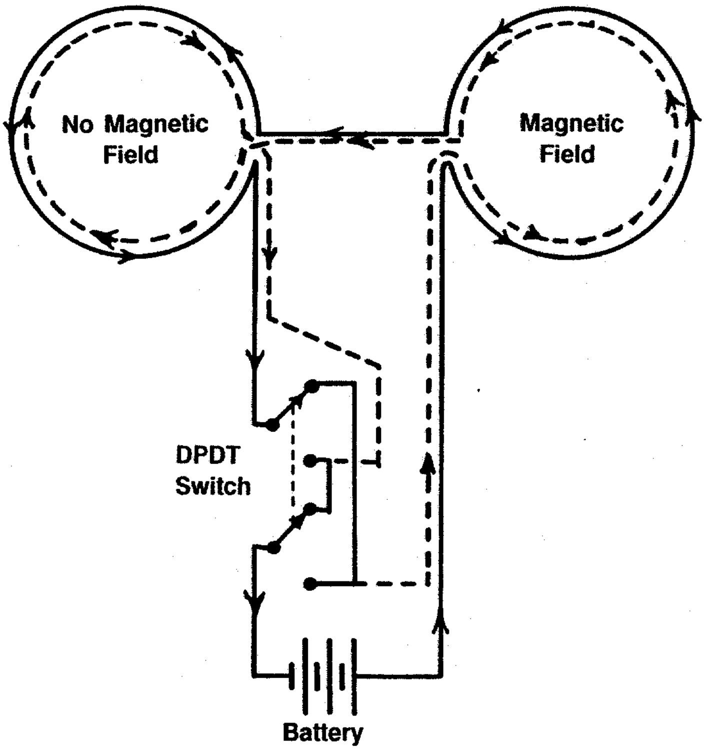


Fig. 1. Schematic diagram of a double-wrapped coil system configured to produce a magnetic field in one apparatus, but not in the other. Current always flows in the same direction in the solid loop, but the DPDT switch can change the direction of current flow in the dashed loop. With the switch positioned as shown, a magnetic field will be generated in the right-hand coil, but not the left. Changing the switch setting will reverse the field and no-field positions. Both coil systems, if wrapped with the same gauge wire, should produce the same heating and other non-magnetic effects which are desirable traits for experimental controls in biomagnetic experiments. If the four wires which run to each coil set, and from them to the DPDT switch and power supply, are positioned together in a bundle, their magnetic fields will also cancel.

dom number such that no human knows which chamber is receiving the magnetic treatment until after the experiment is over. Although not foolproof (a detective with a compass or a fluxgate could probably locate the magnetically active system), this design eliminates completely any thermal differences which might be present.

For experiments in which low-frequency alternating-fields are used, vibrations or noise generated by motions of the wires may also be a problem. Unfortunately, in the double-wrapped coil system discussed above, the pattern of magnetostatic force acting on the wires is not the same in the two current modes (parallel and antiparallel). Separating the pair of wires into two separate coil systems is not a good solution, as the bulk field from each system will still interact strongly, producing vibrations in the supporting structures. Furthermore, separate coil systems will have to be displaced slightly in space from each other, and then one is faced with the problem of nulling precisely the resulting field in the antiparallel case. My recommendation is to bind the double-wrapped coils together tightly with cement or epoxy so as to inhibit the mechanical motions between adjacent wires. Use of industrial grade epoxy with high thermal conductivity may be necessary in some cases to prevent overheating. Physical isolation of the coil system from the biological materials is also a good idea.

Finally, the possibility of electrical fields produced by voltage drops across the coils can be eliminated by using electrically-grounded shielding of either copper or aluminum. Note, however, that for alternating fields a small gap should be left to stop the flow of induced eddy currents.

## ALTERNATE COIL DESIGNS FOR PRODUCING UNIFORM MAGNETIC FIELDS

Numerous improvements on the Helmholtz [1849] design have been devised in attempts to produce larger volumes of space with uniform magnetic fields. In the classic Helmholtz design, the coils (either square or circular) are separated at a distance such that the first and second spatial derivatives of the applied field are zero at the center of the coil system. Subsequent work has shown that several higher-order derivatives can be zeroed using assemblies of three, four, or five coils, yielding much larger volumes of uniform field space. Lee-Whiting [1957], in particular, provides a comprehensive but little-known analysis of four-coil systems (both circular and square) that provide up to 8-order field uniformity.

Of the various coil designs which have been discussed in the literature, five of the easiest to construct are compared with the standard Helmholtz design in Table 1 and Figures 2 and 3 here. Table 1 gives design specifications for each of these, as well as a relationship for estimating the field strength produced at the center. Figure 2 presents schematic illustrations for the various designs, whereas Figure 3 shows results from detailed calculations of the field uniformity. In general, the square coil systems are easier to construct and use where large volumes of uniform field space are needed, whereas for systems of relatively small volume the coil forms for circular systems are easier to machine. Most of the 4 and 5 coil systems are relatively tolerant of small design imperfections. Although an infinitely-long solenoid will yield the best field uniformity, these coil designs are usually easier to build and use.

**Table 1. Coil Spacings and Design Specifications for Systems Discussed in the Text.***

| Coil Design | Coil Shape | No. of Coils | Coil length or diameter (meters) | Coil spacing w.r.t center of system | Ampere-turn Ratios | Central field (μT/ampere) |
|---|---|---|---|---|---|---|
| A. Helmholtz | circular | 2 | d,d | -.2500d, +.2500d | 1/1 | 1.798/d |
| | square | 2 | d,d | -.2726d, +.2726d | 1/1 | 1.629/d |
| B. Lee-Whiting | circular | 4 | d,d,d,d | -.4704d, -.1216d, +.1216d, +.4704d | 9/4/4/9 | 17.96/d |
| C. Merritt et al. | square | 3 | d,d,d | -.4106d, 0, +.4106d | 39/20/39 | 68.21/d |
| D. Alldred and Scollar | square | 4 | .95552d, d, d, .95552d | -.5254d, -.1441d, +.1441d, +.5254d | 21/11/11/21 | 40.29/d |
| E. Merritt et al. | square | 4 | d,d,d,d | -.5055d, -.1281d, +.1281d, +.5055d | 26/11/11/26 | 46.65/d |
| F. Rubens | square | 5 | d,d,d,d,d | -.5d, -.25d, 0, +.25d, +.5d | 19/4/10/4/19 | 35.69/d |

*Coil spacings are measured along the axis of the system in units of the diameter (d) of the largest coils. The ampere-turn ratios are the recommended ratio of wire turns to use in each coil, as discussed in the text, and are given in the same order as the coil diameter and spacing numbers are. The strength of the magnetic field produced at the center of each system is inversely proportional to the size, d, of the coils. Calibration values in the final column give the field strength expected at the center of a d = 1 meter system wrapped with one set of turns in the recommended ratio with one ampere of current flowing from the power supply. Wire should be wrapped in series in the same direction for all coils in the set. For the construction of a double-wrapped coil system, the pair of wires should follow the recommended turn ratio, and since twice the number of turns are present, only half of the specified current will be needed to produce the field value.

## Numerical Methods for Uniformity Calculations

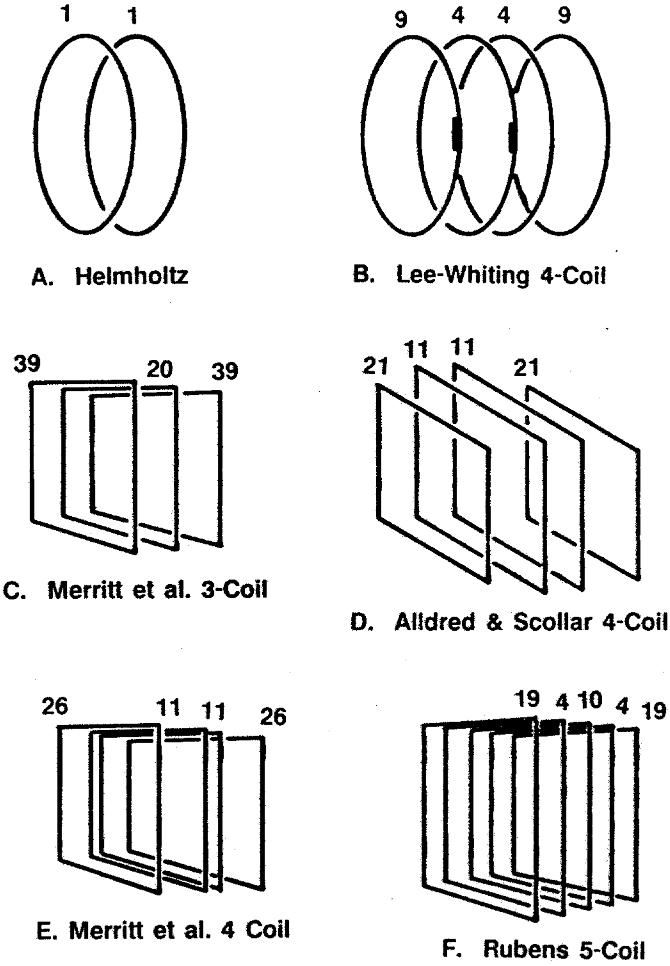
For the square coil systems, Cartesian components of the magnetic field vector produced at an arbitrary point in space around the coil system can be calculated by using the Biot-Savart law for each linear segment. However, the problem for calculating off-axis values of the magnetic field produced by circular coils is somewhat more complex. Using cylindrical coordinates for a point in space of (r, $\theta$, z), with a circular coil of radius a in the r - $\theta$ plane having its center at the origin, the vector potential of the magnetic field produced at (r, $\theta$, z) by a current of I amperes is given by:

$$A(r,\ \theta,\ z) = \frac{a\mu I}{2\pi}\int_{o}^{\pi} \frac{\cos\ \Phi\ d\Phi}{\left(a^2 + r^2 + z^2 - 2ar\cos\ \Phi\right)^{\frac{1}{2}}} \tag{1}$$

where $\mu = 47\ \pi \times 10^{-7}$ weber/amp•m is the permeability constant of free space. Eq. (1) is not expressible in terms of analytic functions, but Stratton [1941] notes that this can be expressed as:

$$A(r,\ \theta,\ z) = \frac{\mu I}{k\pi} \bullet \frac{a^{1/2}}{r}\left[\left(1 - \frac{k^2}{2}\right)K\left(k^2\right) - E\left(k^2\right)\right] \tag{2}$$

where $k^2 = 4ar/[(a + r)^2 + z^2]$, and K and E are complete elliptic integrals of the first and second kinds, respectively. The functions K and E have simple poly-



A. Helmholtz

B. Lee-Whiting 4-Coil

C. Merritt et al. 3-Coil

D. Alldred & Scollar 4-Coil

E. Merritt et al. 4 Coil

F. Rubens 5-Coil

Fig. 2. Configuration of coil designs for generating uniform magnetic fields. The integers near the coils are proportional to the recommended number of ampere-turns in the loop. Coil size and spacing information is given in Table 1. In all cases the central magnetic field is generated parallel to the axis of the coil systems, perpendicular to the plane of the coils.

nomial approximations accurate to a few parts in $10^{-8}$, which makes them quick and easy to evaluate numerically. The components of the magnetic field can then be found by noting that B = curl(A), leading to the radial and axial components (in tesla) of:

$$B_r = \frac{\mu I}{2\pi} \bullet \frac{z}{r\left[(a+r)^2 + z^2\right]^{1/2}} \left[ -K + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2} E \right] \qquad (2)$$

and

$$B_z = \frac{\mu I}{2\pi} \bullet \frac{1}{\left[(a+r)^2 + z^2\right]^{1/2}} \left[ K + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} E \right] \qquad (3)$$

The third component, $B_\theta$, is obviously zero due to the symmetry in the cylindrical coordinate system.

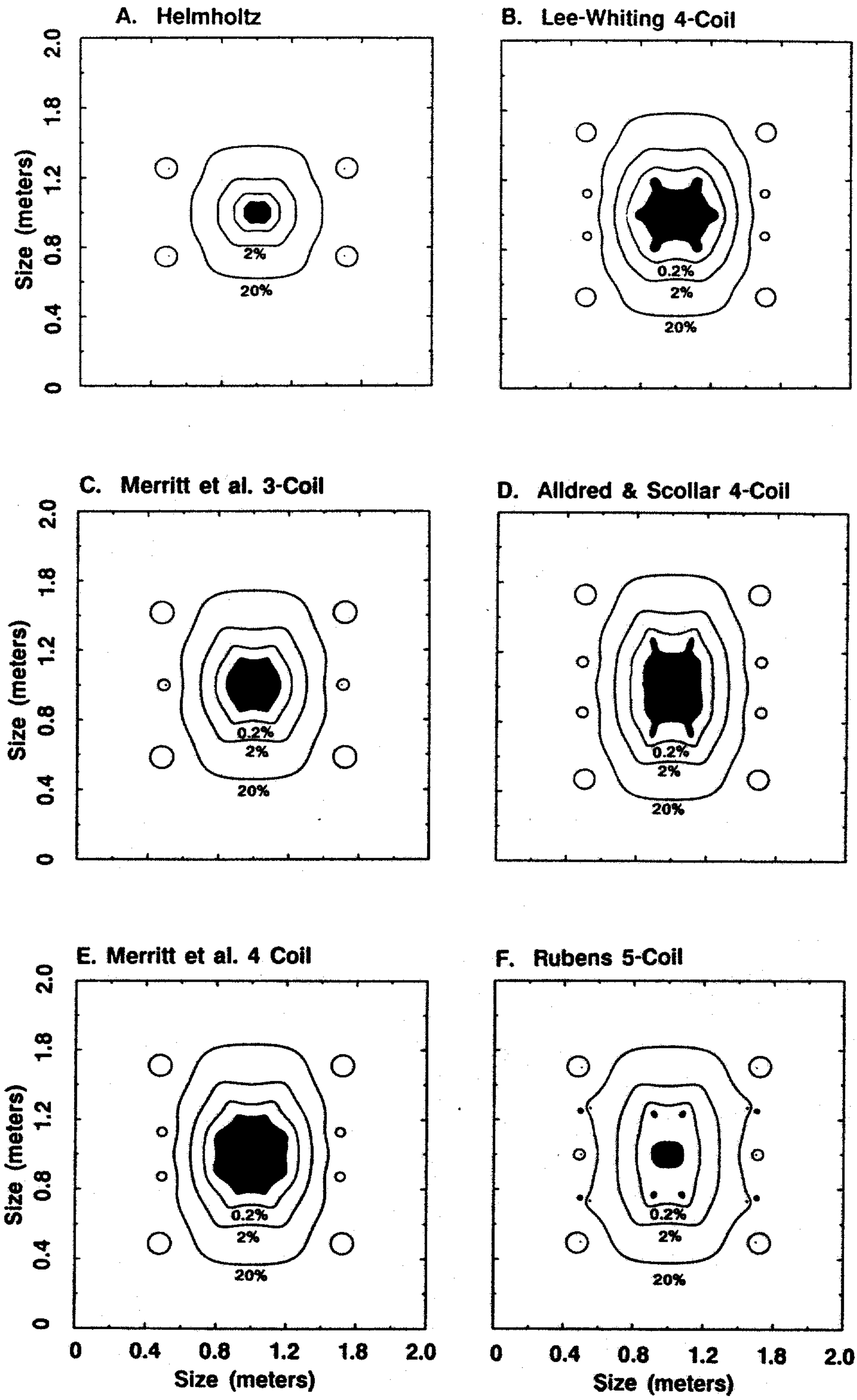


Fig. 3. Results of numerical calculations of field uniformity. For each coil design, calculations were made on a 2 × 2 m$^2$ plane passing through the center of the system parallel to the axis. For the square coil systems, the calculation plane is parallel to one set of linear edges and is thus perpendicular to the other set. Contour lines show the boundaries for the 0.02%, 0.2%, 2%, 20%, and 200% variations from uniform field conditions. Dark areas in the center of the figures show the area where the field uniformity is better than 0.02%. The small closed circles are the 200% contours, which show the location of the locally strong fields which surround each coil loop where they pass perpendicularly through the calculation plane. Contours above the 200% level are not shown. Note that this diagram does not show the flux lines for the magnetic field which are normally depicted in the space surrounding electric coils.

Equations (3) and (4) above can be used to calculate the field components produced at any arbitrary point in space surrounding a circular coil, and contributions from multiple coils in the design can be summed at each point to produce the total field generated from the coil design.

Each diagram in Figure 1 shows results of these field computations on a 2 × 2 $m^2$ plane through the center of the coil system, parallel with the axis, for each of the 6 systems shown in Figure 2. For the calculations of total intensity, the dimension for the largest coil has been set at 1 $m^2$, either in diameter or edge length, and calculations were made at each point on a 512 × 512 grid spanning this 2 $m^2$. Contours of equal field intensity at the 0.02%, 0.2%, 2%, and 20% levels were calculated by mathematically imposing a uniform 50 microtesla field along the coil axis, and adjusting the current flowing in the systems to exactly cancel the total field at their center. Hence, the 10 nanotesla (nT) contour at the center corresponds to the 0.02% uniformity level, 100nT the 0.2% level, and so forth. The area within the 0.02% contour levels are shaded on each diagram in Figure 3 for easy comparison of the most uniform regions. Note that the relative geometry of the contours does not change with size; they scale in the same fashion as the coils do. Hence, these maps can be used graphically to predict the uniform regions for any size coil system under consideration. Brief notes, suggestions, and design tips for each of these systems are presented next, along with simple scaling relationships for estimating the field strength which will result in the central area. Designs utilizing both circular and square coils are also presented; the circular systems are easiest to build for small experiments, whereas square coils are best where large volumes are required.

## Circular and Square Helmholz Pairs

The classic design of Helmholtz [1849] consists of two circular coils, separated by their radius as shown in Figure 2A. As mentioned in the Introduction, neither these nor their square counterparts [e.g., Lee-Whiting, 1957; Firester, 1966] are recommended for use in most experiments where uniform exposures to magnetic fields are desired, as very large coil sizes are needed to provide useful working volumes. Hence, the uniformity calculations for the circular Helmholtz [1849] design are presented here in Figure 3A mainly for comparison with the other 5 coil designs. (For coils of the same diameter, the square design provides only slightly better uniformity, largely as a result of the greater effective area of the coils).

## Four-Concentric, Equidiameter Circular Coils

Lee-Whiting [1957] provides numerical and graphical solutions for systems composed of two pairs of circular coils, configured so as to give eighth-order uniformity. The best of these designs is the system of four coils of equal radius wound on the same cylindrical surface as shown in Figure 2B. For experiments which require only a small volume of uniform field space, this design is recommended highly. Grooves spaced at the proper distances (given in Table 1) for winding the coils are easy to machine on the surface of a cylinder with a standard lathe. Although the ideal current ratio between the outer and inner pair of coils is 2.2604 [Lee-Whiting, 1957), a simple turn ratio of 9/4/4/9 is within 0.5% of this ideal value and works quite well, yielding the field uniformity pattern shown in Figure 3B. Note that the central, shaded region with field uniformity better than 0.02% has a volume roughly equal to that within the 2% Helmholtz contour, and that its 2% contour encloses the same volume as does that of the 20% contour for the Helmholtz system.

### Three Equal Size Square Coils

Merritt et al. [1983] describe a system of 3 equal diameter square coils configured so as to produce maximum field uniformity at its center, as shown in Figure 2C. The ideal current ratio (outer coils to inner) is 1.950; this can be approximated to within 0.5% by a turn ratio of 33/17/33, or almost exactly by 39/20/39 [Merritt et al., 1983]. Although the field uniformity shown in Figure 3C is far better than for the Helmholtz design, it is not as good as either the 4 or 5 coil designs. Merritt et al. also note that its field uniformity is somewhat less tolerant of imperfections in construction than are the 4 or 5 coil systems. It might be of use, however, for experiments which require greater ease of access from the sides than is permitted by the other designs.

### Four-Square Coils

This design [Alldred and Scollar, 1967] extended the approach of Lee-Whiting [1957] to search for square coil systems in which the current ratios would be rational integers. Their "practical" design differs from others here in that the outer pair of square coils is slightly smaller than the inner one. In practice, this has the effect of slightly reducing the field uniformity compared with systems where all coils are of the same size. McElhinny et al. [1971], however, realized that having the outer coils slightly smaller was ideal for constructing nested coil systems capable of controlling the central magnetic field along all 3 axes of a Cartesian coordinate system. Field uniformity for this design, shown in Figure 3D, is better than for either the 3-coil or Rubens designs, but is not as good as the 4-coil Merritt et al. [1983] design discussed next. Hence, this system is recommended only when multiple coil sets are to be nested, and the McElhinny et al. [1971] paper is recommended as an easy to follow construction guide.

### Four-Square Coils of Equal Size

Merritt et al. [1983] designed the system of 4 square coils of equal dimension shown schematically in Figure 2E. Numerical calculations for this design, shown in Figure 3E, yield the largest volume of uniform field space of any considered here. The optimum current ratio in the coils is 2.3612, which is approximated to within 0.1% by the turn ratio of 26/11/11/26, to within 0.05% by 59/25/25/59, or within .004% by the ratio 85/36/36/85 [Merritt et al., 1983]. Uniformity calculations done here, however, show no appreciable difference between the 0.1% and lower ratios, in agreement with Merritt et al. (1983), who concluded that minor errors in coil positions are more important to worry about. This is probably the best coil design to use to produce large volumes of uniform field space.

### Rubens 5 Cube-Centered Coil Design

The previous five coil designs were achieved by trying to select configurations which would zero as many high-order spatial derivatives of the magnetic field as possible at the center of the coil configurations. Rubens [1945] took a different approach, in which he began with a set of 5 cube-centered coils (each coil spaced a distance of d/4 apart on the surface of a cube as shown in Figure 2F), and then solved for sets of integral turn ratios which would produce a large volume of relatively uniform field in the center. The formulation which led to his 19/4/10/4/19 ratio was that which brought the fields at five points along the axis to nearly the same values (at the center, at +/- 0.15*d*, and at +/- 0.25*d*). Although this approach succeeds relatively well at the 0.2% uniformity level as shown in Figure 3F, the volume with highest uniformity (< 0.02%) is located in five separate patches within an elongate rectangular volume in the center of the coil system. In comparison, both the Merritt et al. and Alldred and Scollar 4-coil designs have far better uniformity at all contour levels than does that of Rubens. Although this is much better than a Helmholtz system, it should not be used in situations where very precise controls of the field and field gradients are required [e.g., Kirschvink, 1989].

## CONCLUSIONS AND RECOMMENDATIONS

(1). All biomagnetic experiments should use double-wrapped coil systems wherever possible, both to provide controls for the non-magnetic side effects as well as to facilitate running the experiments in a double-blind fashion.

(2). Stabilizing the coils with epoxy or other cements can reduce the vibration problem.

(3). Electric fields can be removed by using copper or aluminum shielding.

(4). The simple Helmholtz design for coils does not provide adequate field uniformity for many applications in biomagnetics. Of the alternate coil designs, the set of four circular coils of equal radii designed by Lee-Whiting [1957] is best and easiest to make for experiments using relatively small volumes, whereas the Merritt et al. [1983] 4-square coil design is recommended for larger volumes.

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