Alexander M, *Feb. 9 2018*

Feb. 16 2018

ESTIMATION OF IBS AT CESR AT 500 AND 300 MeV (II)

Emittance growth defined by [see CBN 03-11 for details]

$$\frac{d\varepsilon_{x,y}}{dt} \cong \left\langle \left(H_{x,y} + \frac{\beta_{x,y}}{\gamma^2} \right) \frac{d(\Delta E/E)^2_{tot}}{dt} \right\rangle - 2\alpha_{x,y}\varepsilon_{x,y},$$

where dispersion invariant defined as

$$H_{x,y} = \frac{1}{\beta_{x,y}} \left(\eta^2_{x,y} + (\beta_{x,y}\eta'_{x,y} - \frac{1}{2}\beta'_{x,y}\eta_{x,y})^2 \right),$$

$\eta_{x,y}$ –are dispersion functions. Partial decrements $\alpha_{x,y,s}$ defined as $\alpha_i = \dfrac{J_i}{2\tau_s}$,
where $J_x \cong 1, J_y = 1, J_s \cong 2,\ J_x + J_s = 3$.
Partial decrement for energy spread is the same as for the emittance.
The rate of energy spread growth includes additive component from IBS .

$$\frac{d\left(\Delta E/E\right)^2_{tot}}{dt} = \frac{d\left(\Delta E/E\right)^2_{IBS}}{dt} + \frac{d\left(\Delta E/E\right)^2_{\gamma}}{dt} - \alpha_s \left(\frac{\Delta E}{E} \right)^2,$$

INTRA-BEAM SCATTERING

Collisions inside moving bunch equalize temperature; the same processes responsible for the shortening of a beam lifetime.

The temperature of electron gas can be expressed as the following

$$\tfrac{3}{2} Nk_B T \cong N \cdot mc^2 \gamma \left[\frac{\gamma\varepsilon_x}{\beta_x} + \frac{\gamma\varepsilon_y}{\beta_y} + \gamma \left(\frac{1}{\gamma^2} - \langle \psi \rangle \right) \left(\frac{\Delta p_\|}{p_0} \right)^2 \right], \qquad \psi = \frac{\gamma}{l} \frac{\partial l}{\partial \gamma}$$

Longitudinal part of temperature has this form because the longitudinal mass is

$$\frac{1}{m_\|} = \frac{1}{m\gamma} \left(\frac{1}{\gamma^2} - \alpha \right), \; \alpha = \left\langle \frac{\gamma}{l} \frac{\partial l}{\partial \gamma} \right\rangle, \; \alpha = \langle \psi \rangle$$

For values as

$$\beta_{x,y} \cong 10m \, , l_b \cong 1cm \, , \Delta p / p \cong 5 \cdot 10^{-4} , \gamma\varepsilon_s \cong 310^{-4} cm \cdot rad \, , \; \gamma\varepsilon_y \cong 3 \cdot 10^{-6} cm \cdot rad$$

$$\left| \tfrac{3}{2} k_B T \right| \cong mc^2 \gamma \left| 3 \cdot 10^{-7} + 3 \cdot 10^{-9} - 4 \cdot 10^{-11} \right|.$$

One can see that the longitudinal temperature is the lowest one and it is negative above the critical energy (for CESR α≈0.011→γ_{cr}≈10).

Other important moment is that during equalizing the vertical emittance becomes rising even without coupling associated with imperfections of magnetic structure.

In a moving frame, the velocity of transverse motion is dominant and the speed of diffusion can be expressed by simple formula

$$\frac{dp'^2}{dt'} = \frac{4\pi e^4 n' Ln_C}{v'},$$

where $Ln_C = ln\frac{a_{max}}{a_{min}} \cong ln\sqrt{\frac{(v'/c)^6}{4\pi r_0^3 n'}}$ is Coulomb's integral, n' is the density in the moving frame, v' stands for speed of transverse motion in moving frame.
Transforming in the Lab frame

$$\frac{d(\Delta p_\parallel / p)^2}{dt} \cong \frac{d(\Delta E/E)^2}{dt} = \sqrt{\frac{2}{\pi}} \frac{Ln_C N r_0^2 c}{\gamma^3 \varepsilon_x \cdot \sqrt{\varepsilon_y \beta_y} \sigma_s \cdot \sqrt{1 + \frac{(\eta \Delta p_\parallel / p)^2}{\varepsilon_x \beta_x}}}.$$

For simplest FODO structure solution of this equation can be expressed as

$$\varepsilon_x \cong l \cdot \left(\frac{N r_0^2 c \tau_x Ln_C}{4\kappa_0 \gamma^3 \sigma_s R^2} \right)^{0.4}, \qquad (2l\text{- is period of FODO})$$

where coupling $\kappa_0 = \sqrt{\varepsilon_y / \varepsilon_x}$ defined the the square root of emittance ratio. In some publications under this name now in use the square of this value.

So the IBS generates coupling what is

$$\kappa_{IBS}^2 \cong \frac{\langle \beta_y \rangle}{\gamma^2 \left\langle H_x \cdot \sqrt{\frac{1}{\beta_y}} \right\rangle} .$$

For FODO structure this can be estimated as $\kappa_{IBS} \cong \frac{R}{\gamma l}$. Geometrical coupling defined by rotation of quads by random angle within amplitude ϑ_0. So resulting coupling coefficient comes to

$$\kappa^2 = \kappa_0^2 + \kappa_{IBS}^2 .$$

For vertical emittance

$$\varepsilon_y \cong \left(\frac{N r_0^2 c \tau_x L n_C}{4 \gamma^3 \sigma_s R^2} \right)^{0.4} \gamma \cdot l \cdot \left(\kappa_0^2 + \kappa_{IBS}^2 \right)^{0.8} =$$

$$= \left(\frac{N r_0^2 c \tau_x L n_C}{4 \gamma^3 \sigma_s R^2} \right)^{0.4} \gamma \cdot l \cdot \left(\vartheta_0^2 \, 2\pi R l^{1/4} + \frac{R^2}{\gamma l^{3/4}} \right)^{0.8} .$$

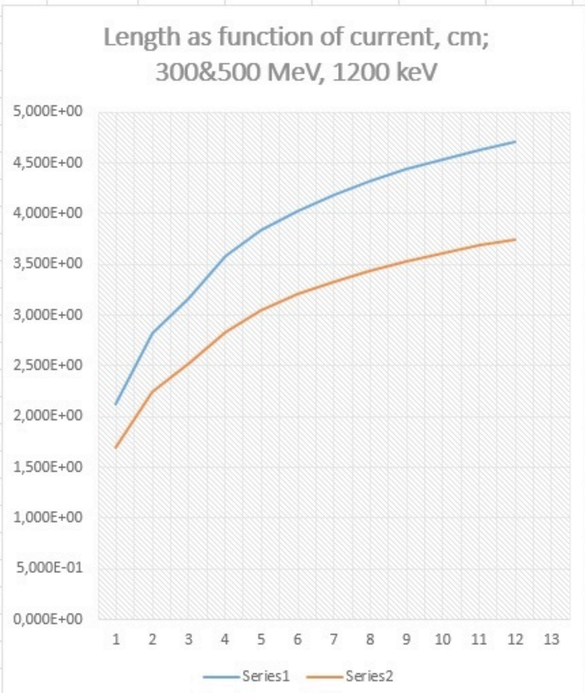
After reminding these formulas, on the next page there are results of numerica calculations. CESR approximated just by regular structure.
Numerical calculation carried with beta functions corresponding the regular structure.

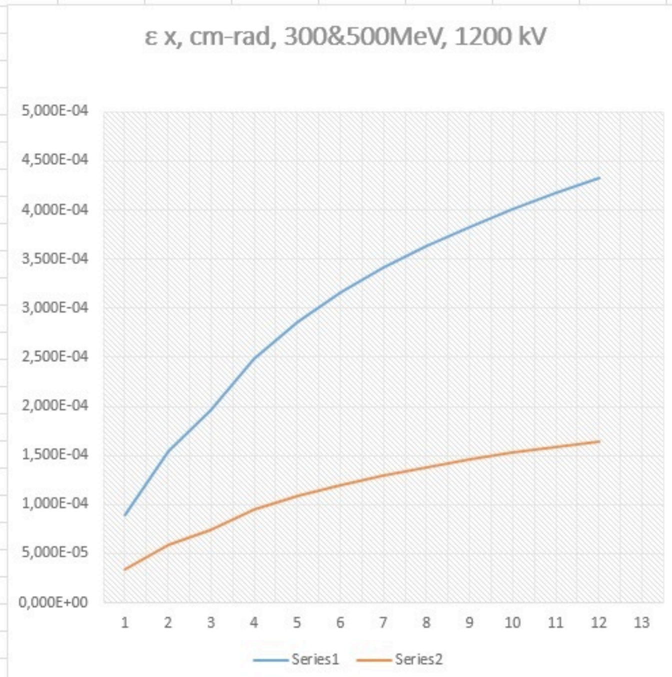
	A	B	C	D
1	0,1	2,130E+00	8,880E-05	2,25E-08
2	0,5	2,820E+00	1,545E-04	3,92E-08
3	1	3,170E+00	1,960E-04	4,98E-08
4	2	3,580E+00	2,488E-04	6,31E-08
5	3	3,840E+00	2,859E-04	7,26E-08
6	4	4,030E+00	3,157E-04	8,01E-08
7	5	4,180E+00	3,408E-04	8,65E-08
8	6	4,320E+00	3,628E-04	9,21E-08
9	7	4,440E+00	3,826E-04	9,71E-08
10	8	4,540E+00	4,005E-04	1,02E-07
11	9	4,630E+00	4,171E-04	1,06E-07
12	10	4,710E+00	4,324E-04	1,10E-07

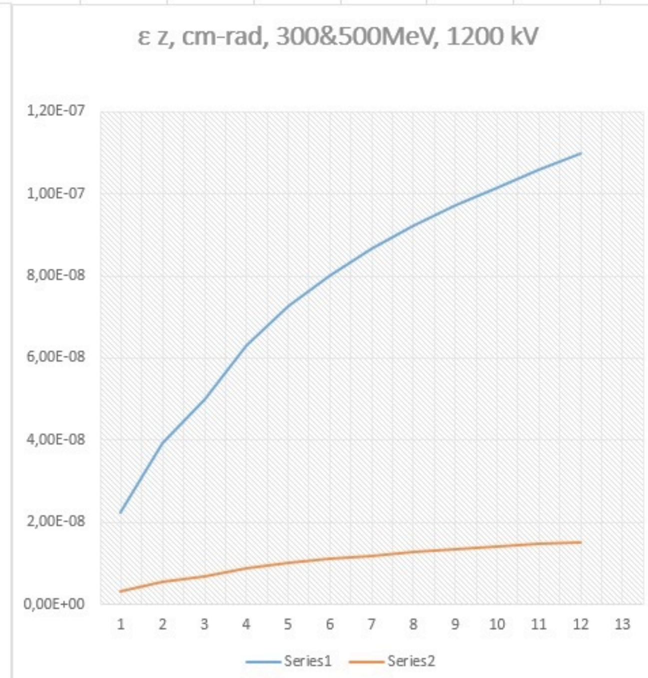
Copling at zero current is 0.001

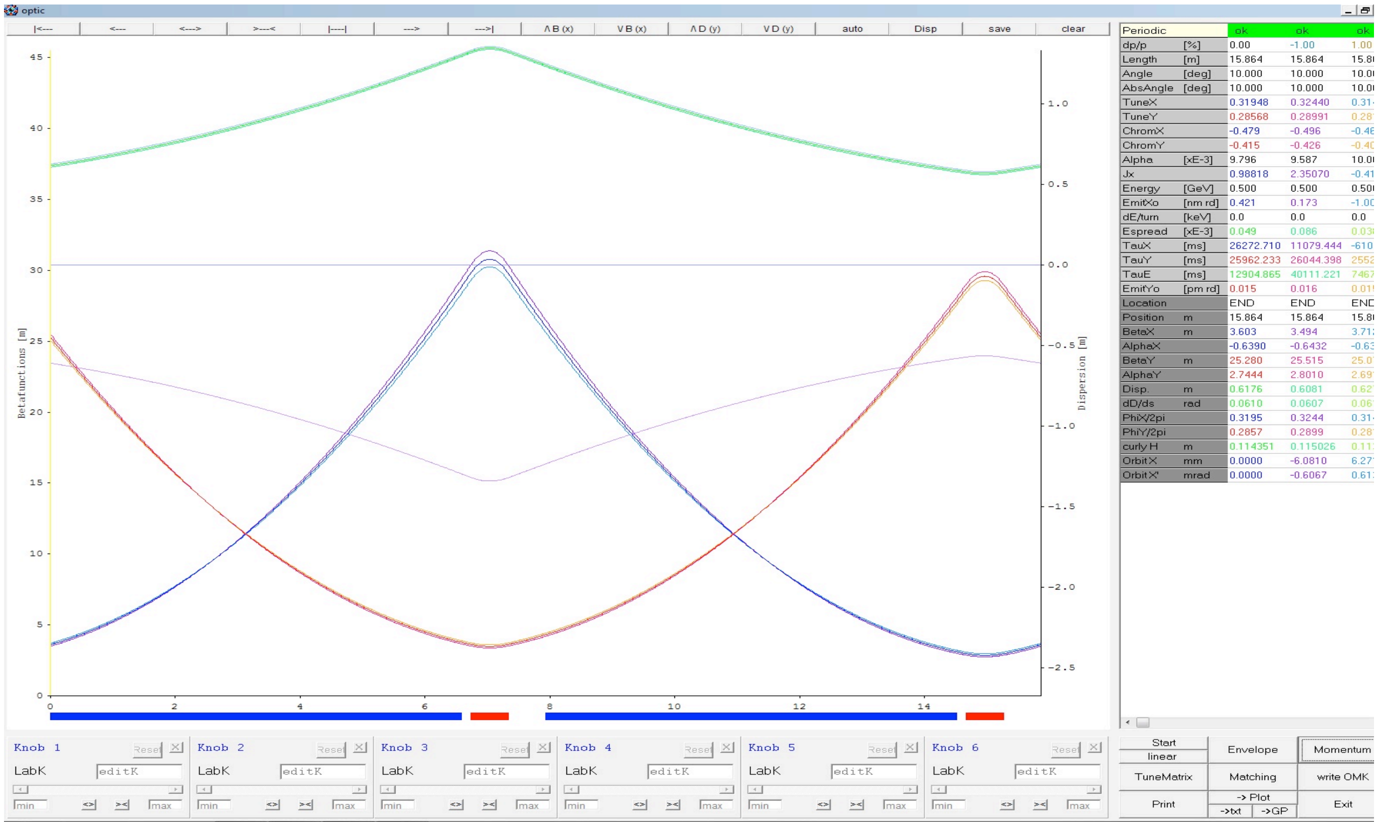
Current in mA; RF voltage is 1200kV

	S	T	U
1	1,700	3,37E-05	3,11E-09
2	2,240	5,87E-05	5,41E-09
3	2,520	7,45E-05	6,86E-09
4	2,840	9,46E-05	8,71E-09
5	3,050	1,09E-04	1,00E-08
6	3,207	1,20E-04	1,11E-08
7	3,330	1,30E-04	1,19E-08
8	3,440	1,38E-04	1,27E-08
9	3,530	1,46E-04	1,34E-08
10	3,610	1,52E-04	1,40E-08
11	3,690	1,59E-04	1,46E-08
12	3,750	1,65E-04	1,51E-08



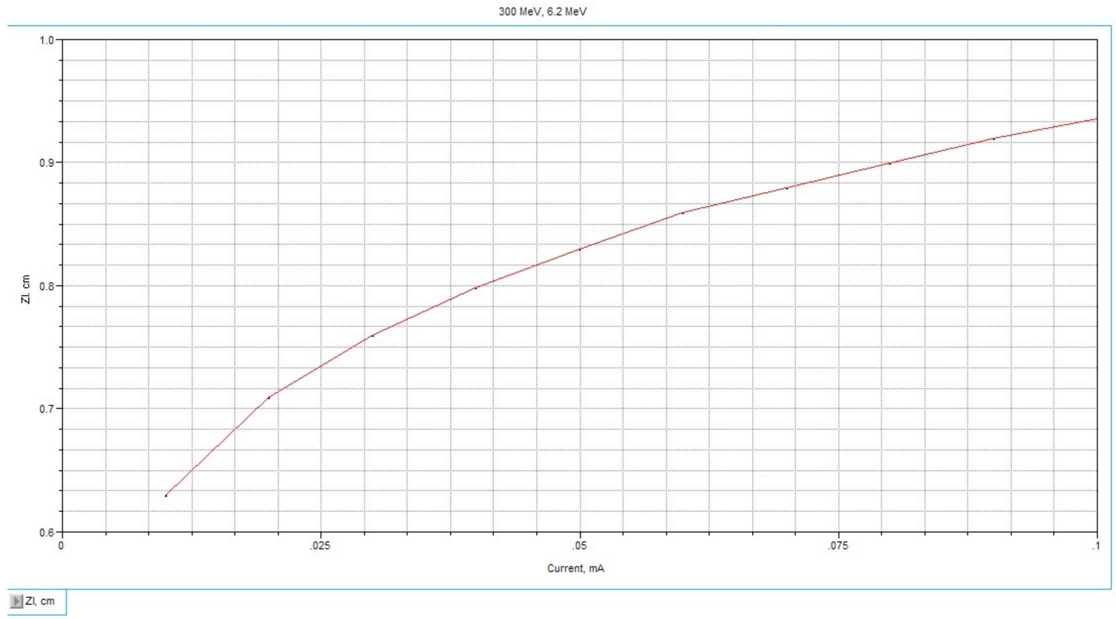


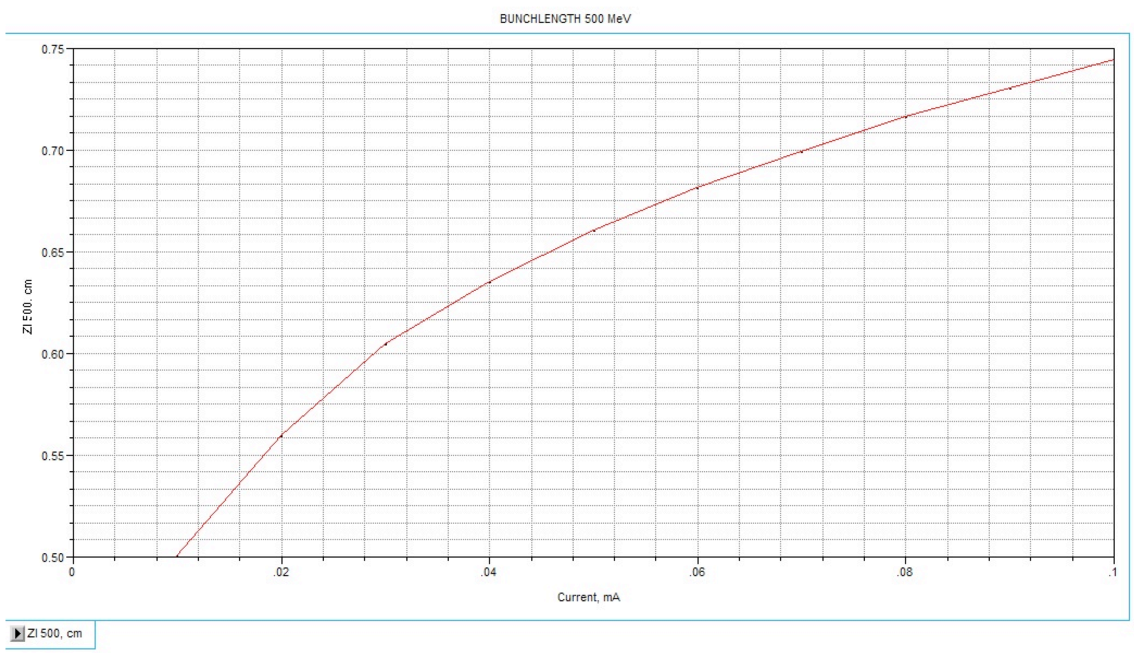




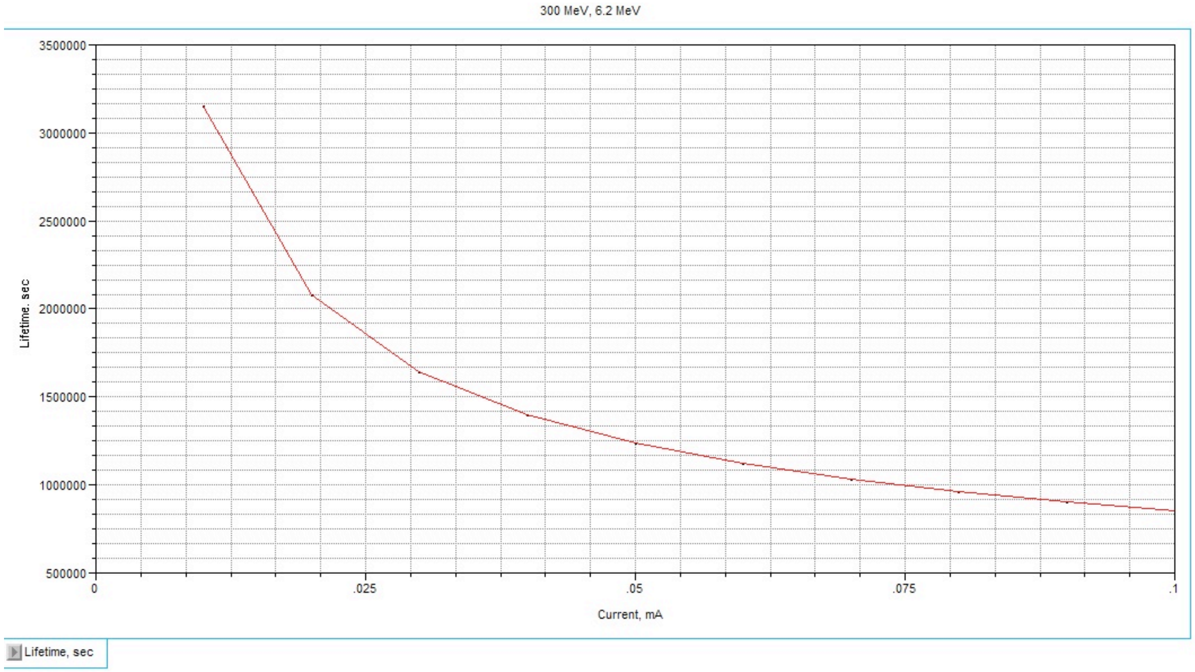
This slide reminds that calculations carried just with regular part of CESR

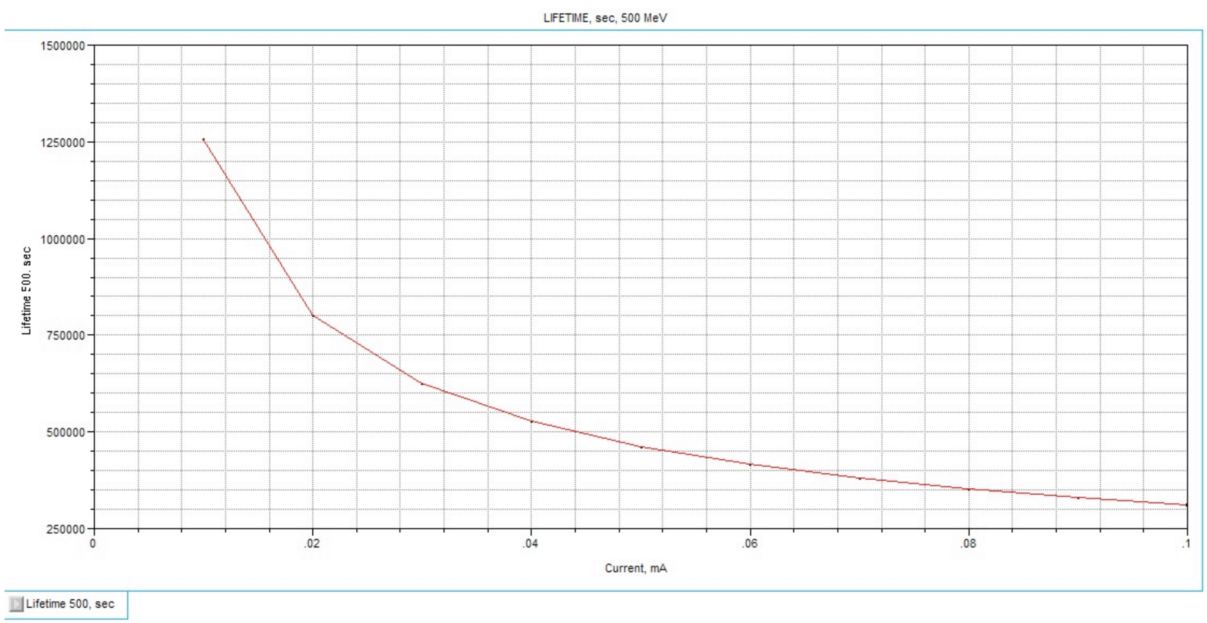
BUNCHLENGTH, cm



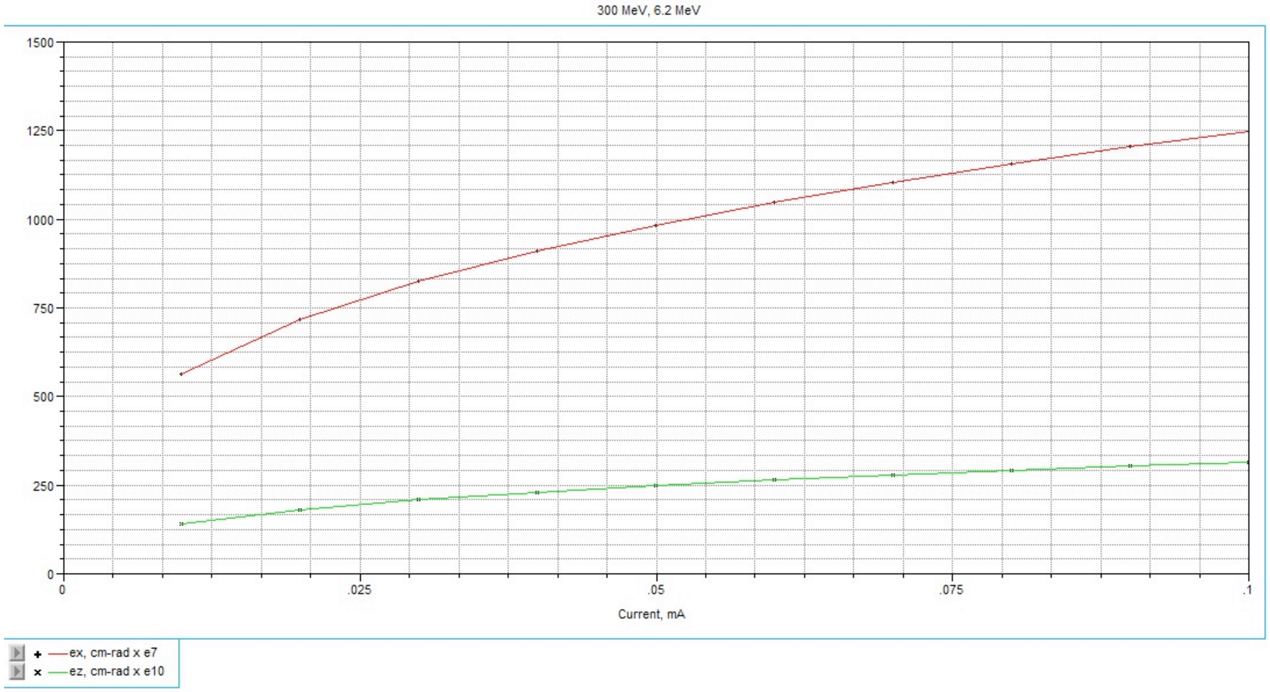


LIFETIMES, sec





EMITTANCES cm x rad



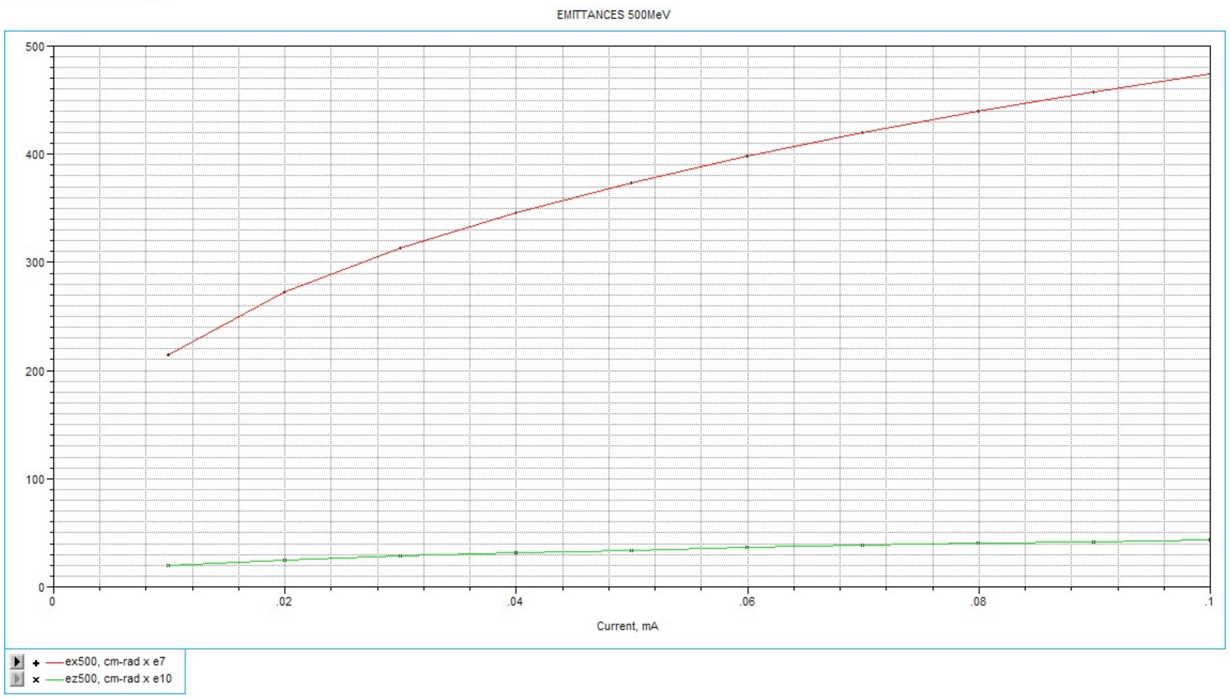


TABLE FOR ALL ABOVE

TUSHEK_OSC

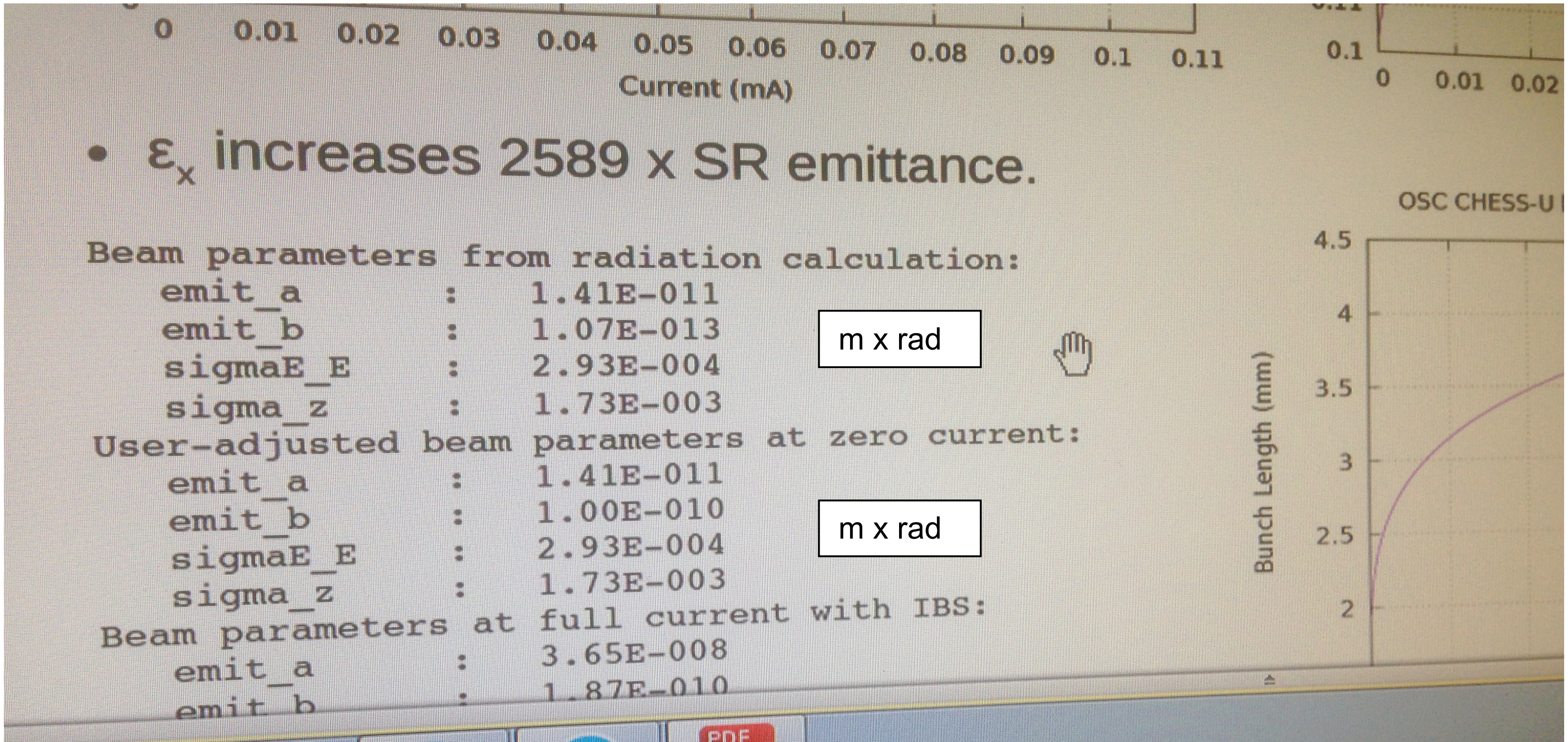
9 Cols / 10 Rows	Current, mA	Zl, cm	Lifetime, sec	ex, cm-rad x e7	ez, cm-rad x e10	Zl 500, cm	Lifetime 500, sec	ex500, cm-rad x e7	ez500, cm-rad x e10
1	0.01	0.63	3154571	566.34	143.98	0.501	1259831	214.81	19.8
2	0.02	0.71	2082425	719.05	182.75	0.56	803185	272.83	25.14
3	0.03	0.76	1648384	826.76	210.1	0.605	627709	313.77	28.91
4	0.04	0.799	1404929	912.81	231.94	0.636	529597	346.47	31.93
5	0.05	0.83	1241752	985.63	250.45	0.661	464029	374.16	34.48
6	0.06	0.86	1125202	1049.45	266.64	0.682	417987	398.44	36.71
7	0.07	0.88	1034873	1106.66	281.14	0.7	382954	420.16	38.72
8	0.08	0.9	964908	1158.68	294.35	0.717	355407	439.96	40.54
9	0.09	0.92	905094	1206.6	306.51	0.731	332798	458.17	42.22
10	0.1	0.936	856710	1251.08	317.82	0.745	314062	475.1	43.78

0 Selected

Current in mA
Zl , cm - bunch length @ 300 MeV
Zl 500,cm - bunch length @ 500 MeV

Ex, cm-rad x e7 –emittance in 10^{-7} cmxrad @ 300 MeV
Ex500, cm-rad x e7 –emittance in 10^{-7} cmxrad @ 500 MeV

Addendum



TUSHEK_OSC

9 Cols / 10 Rows	Current, mA	Zl, cm	Lifetime, sec	ex, cm-rad x e7	ez, cm-rad x e10	Zl 500, cm	Lifetime 500, sec	ex500, cm-rad x e7	ez500, cm-rad x e10
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