OSC Radiation - Coherent and Incoherent Effects

W. Bergan

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I. Overview

The analysis presented here is based on the methods employed by Zolotorev (Phys. Rev. E, Volume 50, No. 4, October 1994). Briefly, each time the electron passes through the OSC kicker, it will interact with its own radiation (the coherent field - described in detail in David Rubin's November 16, 2017 note) and the radiation of the nearby electrons (which will give a much larger, random kick). We also include damping and excitation due to the usual radiative effects in the rest of the storage ring. Balancing the coherent and incoherent kicks each electron sees in the OSC kicker gives us an optimal value for the strength of the radiation for a given set of optics and beam current, which in turn lets us determine the equilibrium emittance of the beam. Finally, reasonable values for the parameters are included to get estimates for the size of the effects we may expect to see.

II. Definitions

The emittance of the beam in the kicker will be given by $\epsilon = \beta x'^2 + 2\alpha x x' + \gamma x^2$.

In the pickup, a single electron will produce an electromagnetic wave with amplitude such that, after amplification and focusing, it will give an in-phase electron in the kicker a fractional energy change of ξ.

I will refer to the electron phase space coordinates and emittance as x, x', and ϵ before a kick and x_k, x'_k, and ϵ_k after a kick.

There are N other electrons contributing to the radiation seen by any given electron. (If there are M undulator periods, of undulator period λ, then N will be the longitudinal electron density of the bunch multiplied by $M\lambda$.)

Unless stated otherwise, all parameters will be evaluated in the kicker.

III. Incoherent Effects

The incoherent radiation field seen by our electron will give it a fractional energy kick of $\xi\sqrt{N}\sin(\phi)$, where ϕ is a random phase. (The intensities of the N electrons add, so the incoherent field scales as $\sqrt{N}$.) We then find that the new phase-space coordinates of our electron are

$$x_k = x + \eta\xi\sqrt{N}\sin(\phi)$$

$$x'_k = x' + \eta'\xi\sqrt{N}\sin(\phi)$$

This implies that the new emittance will be

$$\epsilon_k = \beta(x' + \eta'\xi\sqrt{N}\sin(\phi))^2 + 2\alpha(x + \eta\xi\sqrt{N}\sin(\phi))(x' + \eta'\xi\sqrt{N}\sin(\phi)) + \gamma(x + \eta\xi\sqrt{N}\sin(\phi))^2$$

Since ϕ is random, $\langle\sin(\phi)\rangle = 0$, but $\langle\sin^2(\phi)\rangle = \frac{1}{2}$. It then follows that the change is emittance is given by

$$\Delta\epsilon = \epsilon_k - \epsilon = \frac{N}{2}\beta(\eta'\xi)^2 + N\alpha\eta\eta'\xi^2 + \frac{N}{2}\gamma(\eta\xi)^2$$

$$\Delta\epsilon = \frac{N}{2}\xi^2(\beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2)$$

$$\Delta\epsilon = \frac{N}{2}\xi^2 H$$

where $H \equiv \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2$.

IV. Coherent Effects

The coherent radiation field seen by our particle will be produced only by itself. From David Rubin's note on November 16, 2017, we have that, in the best case,

$$\Delta\epsilon = -\xi(\gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2)^{1/2}\frac{\mu_1}{\sqrt{\epsilon_{max}}}\epsilon$$

where $\mu_1 \approx 3.8317$ is the first root of the Bessel function $J_1(x)$. Using H defined above, this simplifies to

$$\Delta\epsilon = -\xi H^{1/2}\frac{\mu_1}{\sqrt{\epsilon_{max}}}\epsilon$$

V. Equilibirum Emittance Calculation

Combining the above equations, we find that, each time the electron passes through the OSC insertion, the change in its emittance will be

$$\Delta\epsilon = -\xi H^{1/2}\frac{\mu_1}{\sqrt{\epsilon_{max}}}\epsilon + \frac{N}{2}\xi^2 H$$

Since we are operating this device in a storage ring with significant radiation damping, we must also consider those effects. Using the standard model of radiation damping, we will see both heating and cooling from the radiation, so that

$$\Delta\epsilon = -2\alpha\epsilon + K$$

where α is the horizontal damping coefficient and K represents the heating from the synchrotron radiation. (I could not find any simple standard variable to represent the latter.) In the 500 MeV CESR lattice, these parameters have values $2\alpha = 7.16 \times 10^{-7}$ and $K = 1.88 \times 10^{-16}$m. Solving for the equilibrium ($\Delta\epsilon = 0$) in the case with no OSC insertion gives $\epsilon = \frac{K}{2\alpha} = 263$ pm-rad, in agreement with the emittance claimed by cesrv. If we now include all known effects on the beam, we find that, in one turn,

$$\Delta\epsilon = -(2\alpha + \xi H^{1/2}\frac{\mu_1}{\sqrt{\epsilon_{max}}})\epsilon + (\frac{N}{2}\xi^2 H + K)$$

It folllows that the equilibrium emittance will be

$$\epsilon = \frac{K + \frac{N}{2}H\xi^2}{2\alpha + \frac{\mu_1}{\sqrt{\epsilon_{max}}}\sqrt{H}\xi}$$

To find the optimal value of ξ for some given lattice, we may set the derivative of ϵ with respect to ξ equal to zero. This gives us an equation for the optimal value of ξ:

$$0 = \frac{N}{2}\frac{\mu_1}{\sqrt{\epsilon_{max}}}H\xi^2 + 2\alpha N\sqrt{H}\xi - K\frac{\mu_1}{\sqrt{\epsilon_{max}}}$$

VI. Physical Values

At this point, we may insert numbers into our formulas to see what sort of ξ values we need to obtain, and how much of a change in beam size we may expect to see. We first note that in neither of the last two equations do we see ξ or H alone, but only in the combination $\xi\sqrt{H}$, so adjusting H will only affect the requisite ξ value, but will not affect the final emittance. In order to get a scale for the ξ required, pick $H \sim 1$. Let us pick $\epsilon_{max} = 10\epsilon_{uncooled} = 2.63$ nm-rad. Then, the only parameter left to determine the emittance is N, which will be fixed by the beam current. To get the beam size, we note that the 500 MeV lattice has $\beta = 8.5$ m at the VBSM source point.

With one bunch at 0.1 mA, $N = 9.6 \times 10^5$. This gives us $\sqrt{H}\xi = 5.37 \times 10^{-11}$, which, if $H = 1$, corresponds to an energy shift of 6.2 meV, a reduction in emittance from 263 to 160 pm-rad, and a reduction in beam size from 47 to 37 microns. Given that unamplified radiation will, to first order, give us an energy kick of 290 meV, there does not seem to be much need for an amplifier, and we may in fact want to do the reverse, or reduce H.

Our assumption about using μ_1 as the maximum value of the Bessel function we wish to use is probably too optimistic, since David's paper also assumed that the argument of the Bessel function was small enough to be approximately linear. If we make our assumption more conservative, using 1 in place of μ_1, we find a beam size reduction of 1.5 microns, which is at the limit of what I could detect in the past. Reducing the current by a factor of 10 would bring the beam size reduction to over 8 microns, which should be a very clear signal.