

# Operation of an Optical Klystron with Small Dispersion

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## Abstract

The IR Upgrade design at Jefferson Lab uses an optical klystron in order to enhance the flexibility of the free-electron laser system and to match the efficiency to the energy recovery lattice. Most optical klystrons operate with a strong dispersion section [1]. The IR upgrade design requires operation with a dispersion of only a few periods in order to allow the full range of efficiency of the FEL to be explored. This paper will study the details of an optical klystron in this small dispersion limit. The peak gain vs. dispersion section strength has an oscillatory behavior, suggesting that the dispersion section strength should be adjusted in unit steps rather than continuously. The gain vs. the effective number of periods is calculated and found to be, on average, in good agreement with theory. Finally, some comments on the relative merits of using an optical klystron or a uniform wiggler in a high power FEL will be presented.

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## 1. Introduction

The design specifications for the IR upgrade FEL at Jefferson Lab called for the use of a wiggler capable of operating to very long wavelengths at high electron beam energy and the need for a small Rayleigh range to maximize the spot size on the mirrors. In addition, the number of wiggler periods had to be on the order of 40, since the IR Demo proved that the energy spread produced by a 40 period wiggler could be efficiently energy recovered. It is nearly impossible to meet all these requirements with a uniform wiggler so the option of using an optical klystron was explored and accepted. This allowed the length to be less than 6 meters while allowing operation at wavelengths up to 25 microns at an electron beam energy of over 130 MeV. The effective number of periods can be adjusted from 25 to 60 by varying the dispersion section field. This provides an efficiency knob for the FEL user.

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Simulations of the energy recovery system for the IR upgrade have indicated that operation with energy spread much larger than in the IR Demo might be acceptable [2]. This implies that operation with an effective number of periods much smaller than 40 is possible. McGinnis et al. have shown [3] that the power output is not enhanced for strong dispersion. The best area to operate is for  $N_D < N_W$  where  $N_W$  is the number of wiggler periods in each wiggler of the optical klystron and  $N_D$  is the number of periods of effective length in the dispersion section. It is useful to look in detail at the behavior of the optical klystron in this atypical range of dispersion strength.

## 2. The IR Upgrade Optical Klystron

The optical klystron for the IR Upgrade consists of two wigglers with a period of 20 cm. The *rms* wiggler parameter can be varied from 0.4 to 4, yielding a very large tuning range. Each wiggler has 23 full strength poles and two half-strength poles, providing an effective number of periods of 12.5. The two wigglers are separated by 90 cm. The dispersion section is a 3 pole design providing up to 40 periods of dispersion for the strongest wiggler field. For weaker wiggler fields the dispersion can be much higher. Since the magnet is an electromagnet, the magnet quality is nearly ideal, with *rms* phase noise of less than  $1.5^\circ$  over the full operating range. The specifications for the optical klystron are summarized in table 1.

## 3. Details of the modeling

Theoretical models of optical klystrons assume two ideal wigglers and an ideal dispersion section. Since effects such as end fields and the details of the dispersion section might be important for small dispersion, it is useful to study the behavior of the optical klystron using the measured field of the optical klystron and Madey's gain-spread theorem [4]. The spectrum and phase shift can be calculated from the first integral of the magnetic field using the computer code B2E [5]. The measured field from the optical klystron was scaled to set the strength of the optical klystron wigglers and dispersion section. The phase shift was calculated by integrating the square of the angular deviation (this is the added path length in the wiggler vs. a straight-ahead path) and subtracting off the linear phase shift of a resonant pondermotive potential. The calculated on-axis spectrum was then differentiated to find the gain curve. The gain curves were normalized to the peak gain for zero dispersion. The field for  $K=2.1$ , and  $N_D = 12$  is shown in figure 1. The regions around the wiggler ends were adjusted to zero the first and second integrals.

How much dispersion is necessary to keep the two wigglers in phase? In figure 2 the phase of the electrons with respect to a co-propagating pondermotive potential vs. distance is shown for  $N_D = 0, 2, \dots, 12$ . The oscillation with a 10 cm period is the figure-8 motion of the electrons in the wiggler. In the wigglers, the electrons' longitudinal velocity is slowed by the magnetic field so that they stay in phase with the pondermotive potential. In the space between the wigglers the electrons speed up and move ahead with respect to the pondermotive potential wells. This is the equivalent of a negative dispersion in the gap between the wigglers. It is easy to show that the number of periods of dispersion in a field free gap is

$$\Delta N_{\lambda} = -\frac{L}{\lambda_w} \frac{K^2}{1+K^2} \quad (1)$$

where  $L$  is the effective distance between the wigglers, including the effect of the wiggler ends,  $\lambda_w$  is the wiggler period, and  $K$  is the rms wiggler parameter. The quantity  $L$  can be found empirically by solving equation (1) for a calculated dispersion and a given value of  $K$ . For the IR upgrade wiggler,  $L$  is found to be 85.7 cm.

The peak gain is highest when the phase slip is an odd number of half periods (-3.5, -2.5, ...). This is due to the fact that the wigglers are separated by 4.5 wiggler periods. If one inserts 10 cm of extra drift into the magnetic field map, the peak gain is highest for an even number of half periods. From equation (1), the dispersion in the drift is a half integral value for  $K=2.105$ , 1.183, and 0.73.

The equation for the gain of an optical klystron derived using the gain-spread theorem is given by [1]

$$G \propto \left(1 - \frac{\nu}{2\pi N_w}\right) \frac{\partial}{\partial \nu} \left[ \left( \frac{\sin(\nu/2)}{\nu/2} \right)^2 \left(1 + \cos((\nu - 2\pi N_w)(1 + \delta))\right) \right] \quad (2)$$

where  $\nu$  is the usual detuning parameter, and  $\delta = N_D/N_w$ . Dattoli and Ottaviani have derived an approximate form of the peak of this curve vs.  $\delta$  [6]

$$g_{o.k.} = 8g_0 \left(1 + 0.913 \frac{\delta}{1 + 0.057/\delta}\right) \quad (3)$$

where  $g_0$  is the small signal gain parameter for one wiggler.

Equation (2) does not take into account the dispersion of the drift. This can be fixed by inserting equation 1 into equation (2).

$$G \propto \left(1 - \frac{\nu}{2\pi N_w}\right) \frac{\partial}{\partial \nu} \left[ \left( \frac{\sin(\nu/2)}{\nu/2} \right)^2 \left(1 + \cos\left(\left(\nu - 2\pi N_w\right)(1 + \delta) - \frac{2\pi K^2}{\lambda_w (1 + K^2)}\right)\right) \right] \quad (4)$$

#### 4. Calculations vs. Theory

In figure 3, I show the gain vs. dispersion section strength using equations (3) and (4) as well as that calculated from the measured fields for the case of  $K=2.105$ . The basic behavior of the calculated gain compared to the results of equation (2) is the same. An oscillatory variation is seen that arises from the movement of the fringes in the optical klystron spectrum with respect to the resonant wavelength. This is shown clearly in figure 4 where I have plotted the gain vs. dispersion vs. photon energy. The peaks move to lower energy as the dispersion is increased. For integer dispersion the gain is maximized while for half integer values the two peaks are equal and the peak value switches from one peak to the other. Equation 3 is derived from a form of equation (2) that is always anti-symmetric about the resonant wavelength. Thus, there is no oscillation in the gain. Note however that the peaks in the gain curve agree quite well with Dattoli's formula. If the dispersion is changed in unit steps, equation (3) is quite a good approximation.

There is also an oscillation from peak to peak in the gain calculated from the field so that odd values agree quite well with the theory but even values are higher. For the case of  $K = 1.183$ , the odd values again agree well

with theory and the even values are lower. This may be due to an interference effect between the ends of the wigglers. They can add constructively or destructively when the dispersion is even but cancel out for  $N_D$  odd.

## 5. Conclusions

Except for some possible interference effects for even values of the dispersion, the gain curves calculated from the actual wiggler fields agree well with theory. How then does an optical klystron compare with a uniform wiggler? It is clear from equation (3) that the gain increases only linearly with the dispersion. If we instead increase the number of wiggler periods in a uniform wiggler, the gain of a plane wave will increase as  $N_w^3$ . For a Gaussian cavity mode, the mode cross-sectional area grows linearly with the length of a wiggler so the gain varies as the square of the number of periods. In both cases the gain bandwidth is inversely proportional to the number of effective periods. Since the saturation efficiency tends to be proportional to the gain bandwidth, this means that the gain-efficiency product is constant for the optical klystron but grows with period number for the uniform wiggler.

Since we now believe that our energy recovery lattice can accept a much larger fractional energy spread, we now know that one can tolerate a very small number of wiggler periods in a uniform wiggler. Increasing the peak current of the electron beam can compensate the decrease in gain. This leads to a larger energy spread but that is not a problem when the number of periods is small. The main remaining advantage of the optical klystron is its flexibility of power variation and the ability to step taper the wigglers. If one wants the most power however, it is not the best choice.

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## 6. Figure Captions

Figure 1. Measured field for the IR upgrade optical klystron. The field has been set to produce an *rms K* of 2.1. The dispersion section strength is 12.

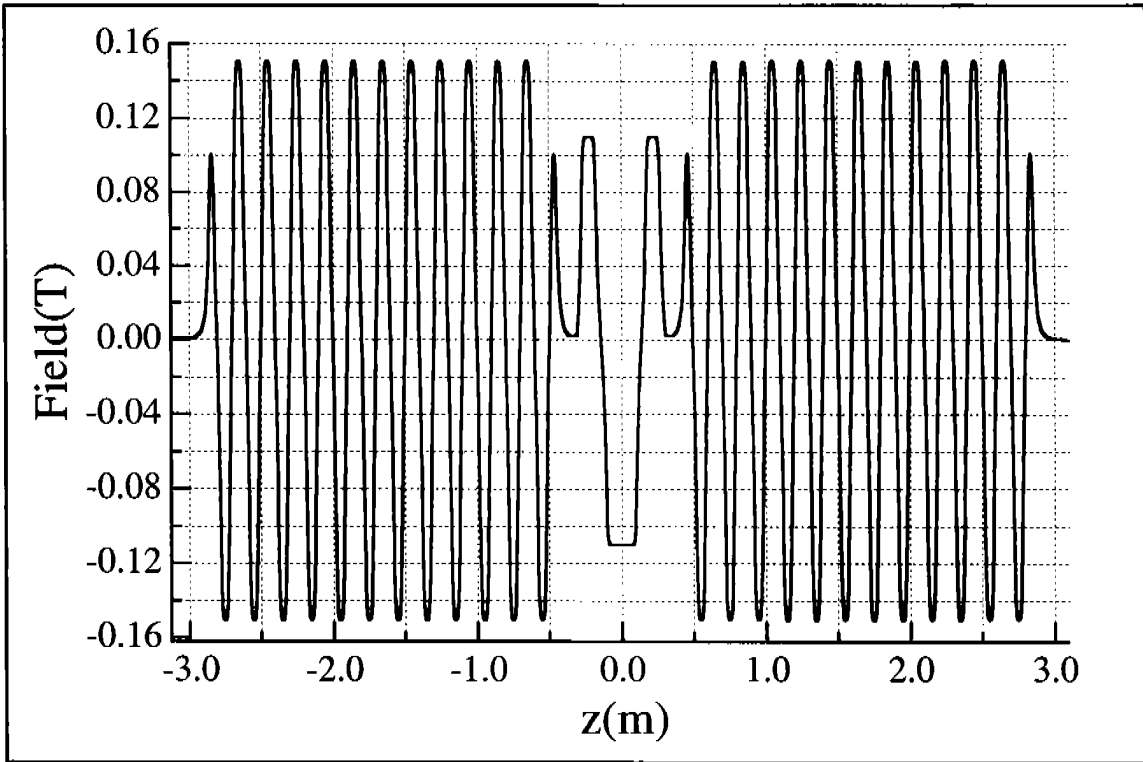
Figure 2. Phase of the electrons with respect to the pondermotive phase space for  $N_D=0, 2, 4, 6, 8, 10,$  and 12. See text for explanation.

Figure 3. Gain vs.  $N_D$  calculated from the measured field, and from equation (3) and (4).

Figure 4. Contour plot of gain vs. photon energy vs. dispersion over two periods of dispersion.

Table 1. Optical klystron specifications. The dispersion strength did not take into account the negative dispersion of the drift..

|                                  | Specification    | Achieved    |
|----------------------------------|------------------|-------------|
| Wavelength(cm)                   | 20               | 20±0.01     |
| <i>rms</i> wiggler parameter $K$ | 0.5-4            | 0.5-4.2     |
| <i>rms</i> phase noise           | <5°              | <1.5°       |
| Periods/Wiggler                  | 12               | 12.5        |
| Polarization                     | Linear, vertical |             |
| Disp. Streng. $N_D$              | 40 at $K=4$      | 36 at $K=4$ |
| Total length(m)                  | <6               | 5.8         |
| gap (mm)                         | >26              | 26.5        |



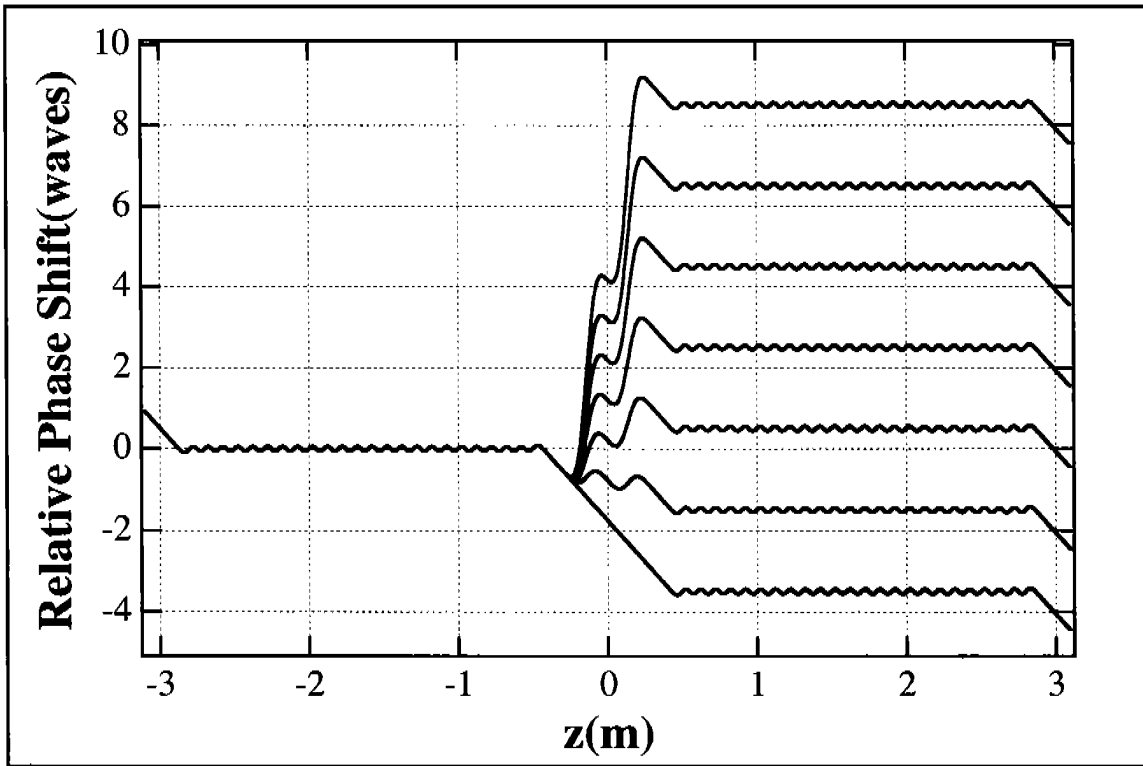


Figure 2



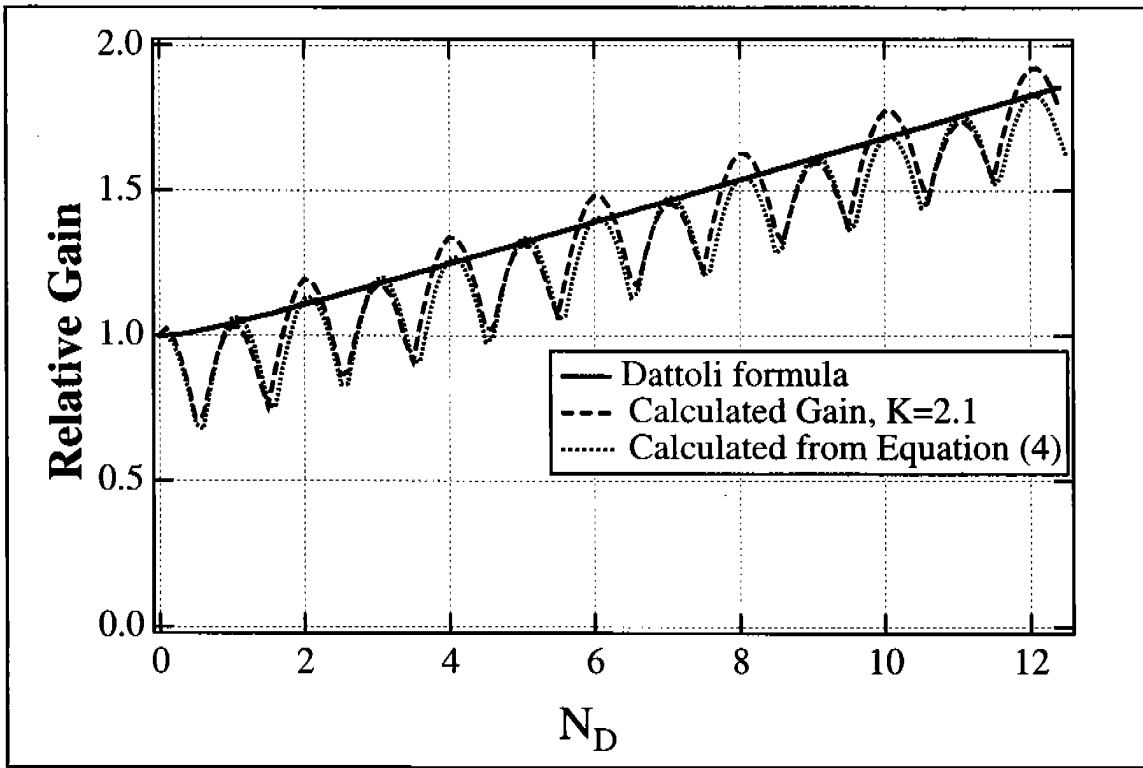


Figure 3

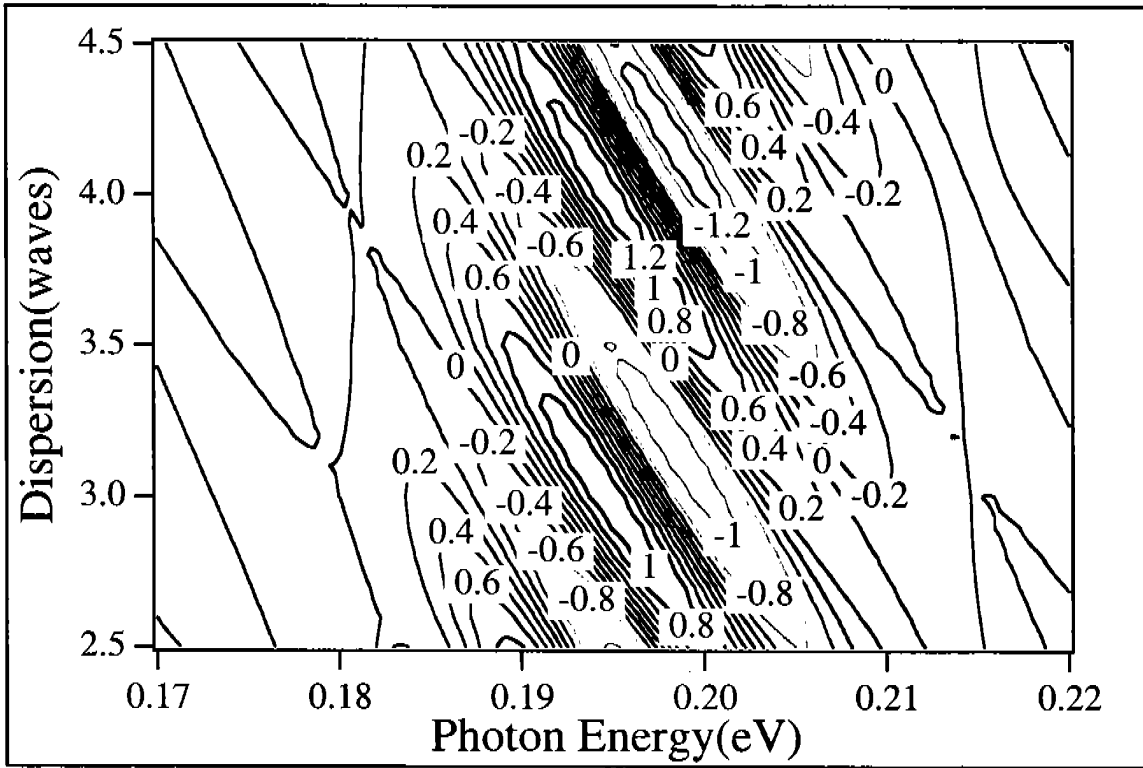


Figure 4