

Experimental Study of Horizontal-longitudinal Coupling at CEsrTA *

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Abstract

In storage rings, the presence of horizontal dispersion in the RF cavities introduces x - z coupling. The result is that the beam is skewed in the horizontal-longitudinal plane. The skew angle is proportional to the V_{15} term of the 6×6 coupling matrix which is proportional to the RF cavity voltage and the horizontal dispersion in the cavity. Here we report experiments at CEsrTA where x - z coupling was explored using three distinct lattice configurations with different V_{15} coupling terms. We explore x - z coupling for each of these lattices by measuring the horizontal projection of the beam with a beam size monitor as the RF voltage is varied. The first lattice has about 1 m dispersion in the RF cavities, resulting in a V_{15} term at the beam size monitor source point corresponding to 16 mrad x - z tilt. In the second, the V_{15} generated in one pair of cavities is compensated at the second pair by adjusting the horizontal betatron phase advance between the cavity pairs. In the third, the optics are adjusted so that the RF cavity region is dispersion-free, eliminating the coupling entirely. Additionally, intra-beam scattering is evident in our measurements of beam size vs. RF voltage.

INTRODUCTION

Just as coupling of horizontal and vertical motion can result in a bunch profile that is tilted in the transverse plane, coupling of horizontal and longitudinal motion will in general produce a tilt in the horizontal/longitudinal plane. The requisite horizontal/longitudinal coupling can be generated by dispersion in RF accelerating cavities.

In the CESR ring, the RF straights are in close proximity to the interaction region/damping wiggler straight. Because of the intervening hard bend magnets, there is no practical lattice solution with zero dispersion in both straights. We generally opt for zero dispersion in the wiggler straight, in order to minimize the horizontal emittance. The result is horizontal dispersion ~ 1 meter in the RF cavities. This horizontal dispersion results in a tilt of the beam in the x - z by an amount that depends on the total RF accelerating voltage.

The horizontal beam size monitor at CEsrTA measures the projection of the beam into the horizontal lab frame coordinate. A tilt of the beam in the x - z plane is seen at the instrumentation as a larger horizontal beam size. Bunch lengths in CEsrTA are typically 1 cm and bunch widths are typically $150 \mu\text{m}$. Even small amounts of tilt can result in a significantly larger measured horizontal size.

In this paper we present formalism for calculating the amount of tilt at the instrumentation source point and for calculating the projected beam size. Using the formalism, it is possible to accurately calculate projected beam sizes in a ring with horizontal-longitudinal coupling.

We also present two methods for eliminating the tilt. One, we cancel the tilt by adjusting the horizontal phase advance between the two pairs of RF cavities. Two, we develop a new lattice with 6 of the 12 wigglers powered off. This frees the optics such that the horizontal dispersion can be set to zero in the RF straight.

We test our formalism for calculating beam sizes, and also our lattices for eliminating the tilt, by conducting beam size versus RF voltage experiments.

Adjusting the RF voltage also changes the particle density, which in turn changes the amount of IBS blow up. In addition to measuring the effect of beam tilt, we observe IBS effects.

THEORY

After corrections, transverse coupling is measured to be very small $< 0.2\%$. We assume there is no x - y or y - z coupling and write the full turn 4×4 x - z transfer matrix as,

$$\mathbf{T} = \begin{pmatrix} \mathbf{M} & \mathbf{m} \\ \mathbf{n} & \mathbf{N} \end{pmatrix}, \quad (1)$$

where,

$$\mathbf{T} \begin{pmatrix} x \\ x' \\ z \\ \frac{\delta p}{p} \end{pmatrix}_i = \begin{pmatrix} x \\ x' \\ z \\ \frac{\delta p}{p} \end{pmatrix}_{i+1}. \quad (2)$$

\mathbf{T} is symplectic and can be decomposed into normal modes, $\mathbf{T} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}$, where \mathbf{U} is block diagonal and,

$$\mathbf{V} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^\dagger & \gamma\mathbf{I} \end{pmatrix}, \quad (3)$$

where the symplectic conjugate is

$$\mathbf{C}^\dagger = \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix}. \quad (4)$$

Note that C_{11} in the 4×4 x - z formalism discussed here, corresponds to V_{15} from the full 6×6 formalism.

\mathbf{V} takes normal mode coordinates \vec{u} to lab frame coordinates \vec{x} , $\mathbf{V}\vec{u} = \vec{x}$. Evidently, the x - z tilt is given by C_{11} . It is straightforward to show that [1],

$$\mathbf{C} = -\frac{(\mathbf{m} + \mathbf{n}^\dagger) \operatorname{sgn}(\operatorname{Tr}[\mathbf{M} - \mathbf{N}])}{\sqrt{\operatorname{Tr}(\mathbf{M} - \mathbf{N})^2 + 4|\mathbf{m} + \mathbf{n}^\dagger|}}. \quad (5)$$

* Work supported by NSF and DOE contracts PHY-0734867, PHY-1002467, PHYS-1068662, DE-FC02-08ER41538, DE-SC0006505.

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Consider a ring with an RF cavity located at point 1. We wish to determine the x - z tilt at point 0. The full turn matrix is given by,

$$\mathbf{T} = \mathbf{T}_{10}\mathbf{T}_{RF}\mathbf{T}_{01}, \quad (6)$$

where \mathbf{T}_{01} is the map from 0 to 1 and \mathbf{T}_{RF} is the map for the RF cavity. For simplicity, assume β_x is uniform around the ring and $\alpha_x = 0$, then populate Eqn. 1 with,

$$\mathbf{M}_{01} = \begin{pmatrix} \cos(\Delta\phi_{01}) & \beta_x \sin(\Delta\phi_{01}) \\ -\frac{1}{\beta_x} \sin(\Delta\phi_{01}) & \cos(\Delta\phi_{01}) \end{pmatrix}, \quad (7)$$

where $\Delta\phi_{01} = \phi_1 - \phi_0$ is the horizontal phase advance from 0 to 1, and

$$\mathbf{m}_{01} = \begin{pmatrix} 0 & \eta_1 \\ 0 & \eta'_1 \end{pmatrix} - \mathbf{M}_{01} \begin{pmatrix} 0 & \eta_0 \\ 0 & \eta'_0 \end{pmatrix}, \quad (8)$$

where η and η' are the dispersion and its derivative, and

$$\mathbf{N}_{01} = \begin{pmatrix} 1 & L_{01}\alpha_{01}^p \\ 0 & 1 \end{pmatrix}, \quad (9)$$

where L_{01} and α_{01}^p are the fraction of the total circumference from 0 to 1 and effective momentum compaction be-

tween 0 and 1. From the symplecticity of \mathbf{T}_{01} we have,

$$\mathbf{n}_{01}^T = \mathbf{M}_{01}^T \mathbf{s} \mathbf{m}_{01} \mathbf{N}_{01}^{-1} \mathbf{s}, \quad (10)$$

where,

$$\mathbf{s} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (11)$$

The transfer matrix for an RF cavity with peak voltage V and frequency ω is

$$\mathbf{T}_{RF} = \begin{pmatrix} 1 & 0 \\ \tilde{V} & 1 \end{pmatrix}, \quad (12)$$

where

$$\tilde{V} = \frac{e\omega V}{cE_{beam}}. \quad (13)$$

$\omega = 2\pi 500$ MHz is the RF frequency, V is the peak RF voltage, and E_{beam} is the beam energy. e and c are the electric charge and speed of light.

Writing the 1-turn matrix using Eqn. 6 and calculating \mathbf{C} using Eqn. 5, we find the coupling parameters at the observation point, location 0,

$$C_{11}^0 = \frac{\tilde{V}}{\chi} [(\cos(\mu_x - \Delta\phi_{01}) - \cos \Delta\phi_{01}) \eta_1 - \beta_{x0} \eta'_1 (\sin \Delta\phi_{01} + \sin(\mu_x - \Delta\phi_{01}))] \quad (14)$$

$$C_{12}^0 = \frac{2}{\chi} (\cos \mu_x - 1) \eta_0 - \frac{\tilde{V}}{\chi} [(L_{01}\alpha_{01}^p + L_{10}\alpha_{10}^p) \eta_0 + (L_{10}\alpha_{10}^p \cos \Delta\phi_{01} + L_{01}\alpha_{01}^p \cos \Delta\phi_{10}) \eta_1 + (-L_{10}\alpha_{10}^p \sin \Delta\phi_{01} + L_{01}\alpha_{01}^p \sin \Delta\phi_{10}) \beta_{x0} \eta'_1], \quad (15)$$

where μ_x is the horizontal tune and $\chi = \sqrt{\text{Tr}(\mathbf{M} - \mathbf{N})^2 + |\mathbf{m} + \mathbf{n}^\dagger|}$.

C_{11}^0 is the tilt in the x - z plane at point 0, and C_{12}^0 is the x - p_z coupling, which is dominated by the dispersion at the observation point. We see that C_{11} is proportional to the RF cavity voltage and the dispersion at the RF cavity.

LATTICE DESIGN

In CESR, there are two pairs of RF cavities, separated by about 1.5 betatron wavelengths. By adjustment of the horizontal phase advance between the cavities it is possible to compensate the tilt generated by one pair of cavities, with the second pair. This is done in practice by minimizing C_{11} at the instrumentation source point using an optimizer.

Shown in Fig. 1 are the model x - z C_{11} values for the base lattice, the lattice with C_{11} minimized, and a lattice with zero dispersion in the RF cavities.

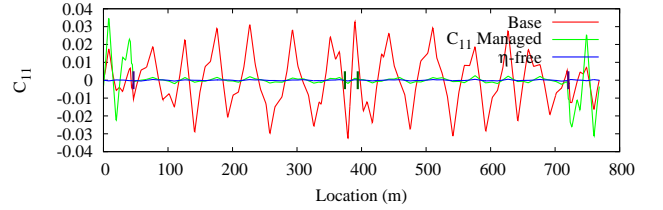


Figure 1: x - z C_{11} along CESR for standard lattice (red), a lattice where C_{11} is mitigated by adjusting the phase advance between the RF cavities (green), and a lattice with zero dispersion in the RF cavities (blue). Indigo bars show location of RF cavities, dark green bars are location of horizontal beam size monitors.

EXPERIMENT

Measurements are taken with each of the three lattices at 2.1 GeV using a single bunch of positrons. The experiment is conducted by setting RF voltage, then taking several bunch size measurements at both 0.5 and 1.0 mA.

Base 0.5 mA	+
Base 1.0 mA	×
C_{11} Managed 0.5 mA	*
C_{11} Managed 1.0 mA	□
η -free 0.5 mA	■
η -free 1.0 mA	○

Figure 2: Key to plots in Fig. 3

Horizontal and vertical beam size and bunch length are recorded from 6.3 MV down to 1.0 MV in roughly 1 MV increments. The total RF voltage is split equally among the 4 RF cavities.

DATA

Plotted in Fig. 3 are the horizontal, vertical, and bunch length data along with simulation results.

The simulation includes intrabeam scattering (IBS) calculated using the Kubo-Oide formalism [3]. The implementation of this formalism at CesrTA is discussed in [2]. The model lattices used are ideal, with no vertical dispersion or transverse coupling. The result is that the simulation predicts negligible IBS blow up in the vertical dimension.

The projection of the beam envelop into the lab frame coordinates is calculated by generating the beam Σ -matrix using the normal mode emittances and the eigen-decomposition of the 1-turn transfer matrix at the instrumentation source point.

CONCLUSION

The data from the C_{11} Managed and η -free lattices agree well with the simulation results. Evidently, our method for mitigating x - z tilt is effective.

The increase in the measured horizontal beam size from 0.5 mA to 1.0 mA is due to IBS. The change in beam size is accurately predicted by our simulation.

The measured tilt is somewhat greater than predicted by the model for the Base lattice at the highest RF voltages. Possible explanations include error in our model of the CESR optics or some subtle effect of x - z coupling that is not properly incorporated in the IBS formalism.

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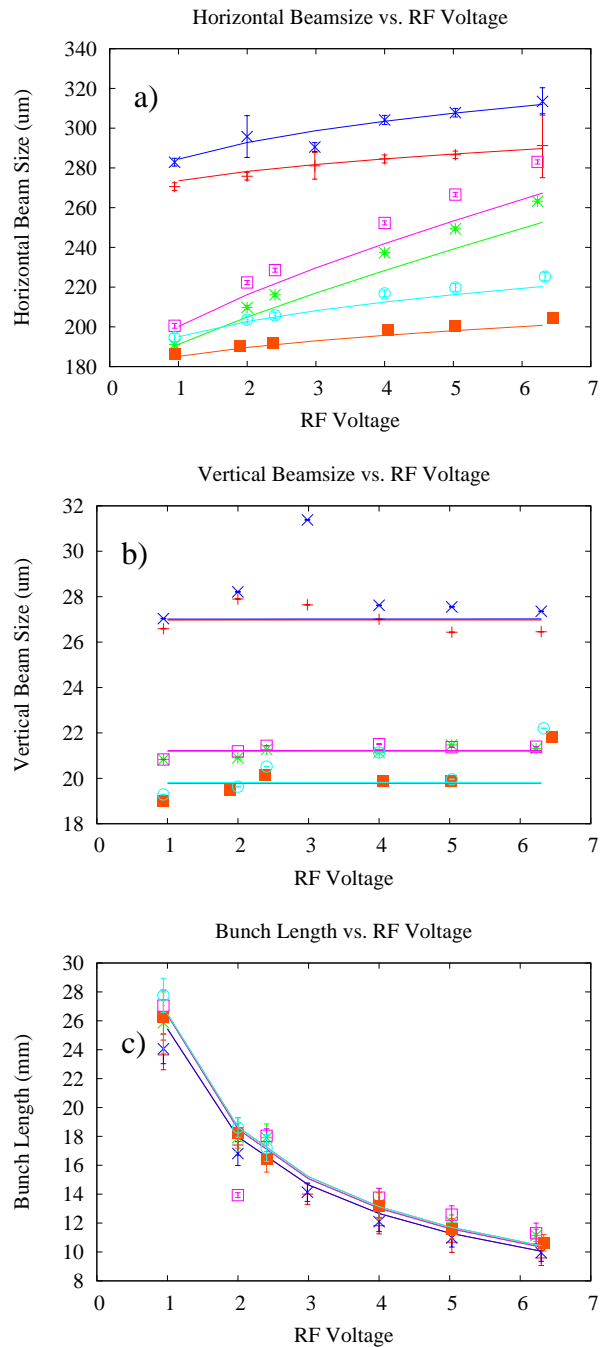


Figure 3: (a) horizontal, (b) vertical and (c) bunch length measurements and simulation results. Points are data and curves are simulation result.