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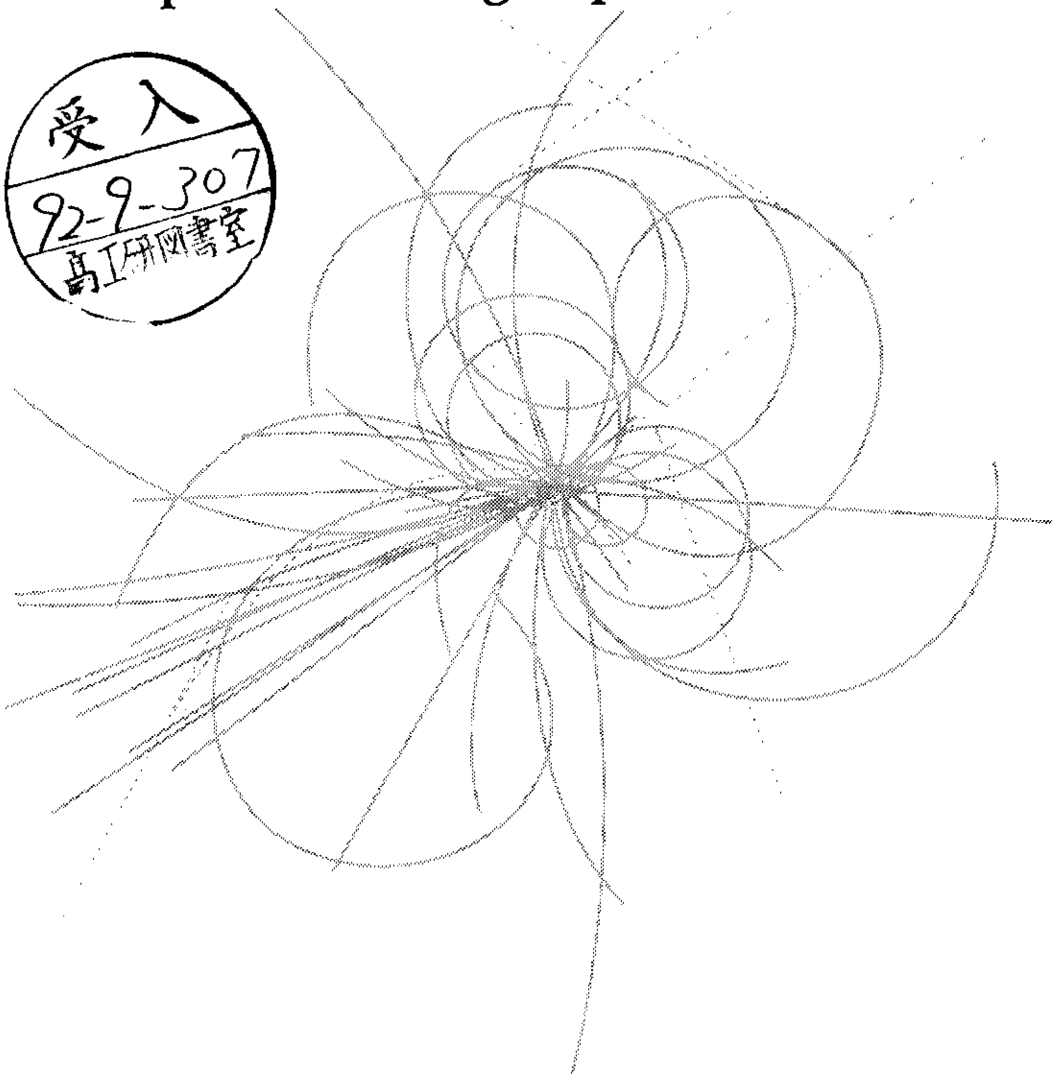

Echo Effect in Hadron Colliders

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ABSTRACT

It is shown that due to the reversible nature of decoherence one can restore the memory of the beam betatron oscillations in a hadron accelerator well after they have completely damped out (decohered) due to a tune spread. An analytical theory is developed that predicts an echo effect in the case when the beam is sequentially exposed to a dipole kick and a quadrupole kick. The echo is represented by a train of coherent dipole oscillations of the beam that builds up at a particular moment after the quadrupole kick.

1.0 INTRODUCTION

The effect of decoherence—or Landau damping—has many applications in accelerator physics. (For examples, see References 1 and 2.) In its simplest form, applied to betatron oscillations, it predicts that due to tune spread, coherent betatron oscillations damp to extinction on a time scale that is equal to the inverse spread of the betatron frequencies in the beam. However, a nondissipative nature of decoherence manifests itself in that even after the oscillations are completely damped out, the system keeps some "recollection" about them. This memory, in principle, can be visualized at a later time if one makes a quadrupole kick on the beam. A similar effect of memory for Landau damping of plasma oscillations is well known in plasma physics, where it bears the name of plasma echo.[3]

In this paper, we show how the echo effect can be observed in hadron accelerators, where synchrotron damping does not play a role. The experimental arrangement needed for that observation is shown schematically in Figure 1. It consists of two kickers, one of which produces a dipole (D) kick and the other a quadrupole (Q) kick on the beam. Assume that at time $t = 0$ the beam is exposed to a dipole kick. This kick drives betatron oscillations that will damp due to tune dependence on the amplitude. After these oscillations have completely damped out, let us make a quadrupole kick at the time $t = \tau$. This kick does not produce any visible displacement of the beam at that time, but close to the time $t = 2\tau$ one will find that the beam experiences betatron oscillations, with the amplitude increasing from zero to some maximum value and then decreasing back to zero (see Figure 2).

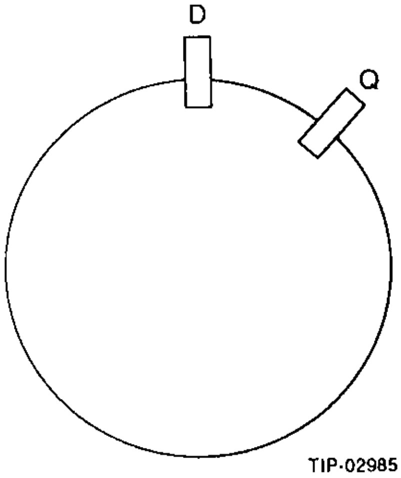

Figure 1. Accelerator ring with kickers needed for observation of the echo effect.

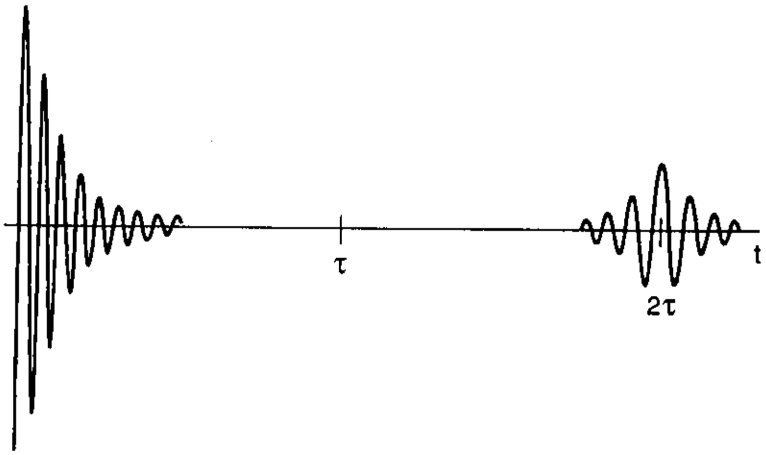

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Figure 2. Beam deviation as a function of time. The beam is deflected by the dipole kicker at $t = 0$, and the quadrupole kick occurs at $t = \tau$. The signal at $t \approx 2\tau$ is the echo.

In the next two sections, we will give a formal description of this phenomenon and show how the form and the amplitude of the echo are related to the strengths of the kicks.

2.0 GENERAL EQUATIONS

To describe the particle motion in an accelerator, we will use the following variables:

$$\eta = \frac{y}{\sqrt{\beta}}, \quad p = \frac{1}{\nu\Omega}\frac{d\eta}{dt}, \tag{1}$$

where y stands for the particle deviation with respect to the closed orbit, β is the beta function, Ω is the revolution frequency, and ν is the tune. A kick is characterized by a change in the particle momentum p by an amount Δp:

$$p' = p + \Delta p, \tag{2}$$

where p' is the new momentum. If the kicks are generated by perturbing the magnetic field on the orbit, then the deviations of the particle momentum produced by the dipole and quadrupole kicks are, respectively,

$$\Delta p_{dip} = \sqrt{\beta_{dip}}\,\frac{\delta B l_{dip}}{B\rho} \equiv \varepsilon, \tag{3}$$

$$\Delta p_{quad} = -\eta\,\frac{\delta B' l_{quad}\beta_{quad}}{B\rho} \equiv -q\eta, \tag{4}$$

where δB is the magnetic field in the dipole kicker, $\delta B'$ is the gradient of the magnetic field in the quadrupole kicker, β_{dip} and β_{quad} are the values of the beta function at the positions of the kickers, l_{dip}

and l_{quad} are their lengths, B is the bending magnetic field, and ρ is the bending radius. The coefficient q introduced in Eq. (4) is equal to the ratio β_{quad}/f_{quad}, where f_{quad} is the focal length of the quadrupole kicker. In what follows we assume that the kicks are weak; that is,

$$\varepsilon,\ q << 1. \tag{5}$$

As is seen in Figure 1, between the successive kicks the beam performs free oscillations. They are related to the following transformation in the phase space for the time interval, t:

$$\eta' = \eta\cos\nu\Omega t + p\sin\nu\Omega t, \quad p' = -\eta\sin\nu\Omega t + p\cos\nu\Omega t, \tag{6}$$

where p, η are the initial phase coordinates and p', η' are the final phase coordinates.

We will describe the beam dynamics with the use of the distribution function $\psi(p,\eta,t)$, normalized so that

$$\int \psi(p,\eta,t)dpd\eta = 1. \tag{7}$$

Initial distribution function is assumed to be Gaussian:

$$\Psi = \frac{1}{2\pi J_0}\exp\left(-\frac{p^2+\eta^2}{2J_0}\right), \tag{8}$$

where J_0 is a constant. It is convenient to transform from p and η to the action-angle variables J and ϕ in accordance with

$$\eta = \sqrt{2J}\cos\phi, \quad p = -\sqrt{2J}\sin\phi\,. \tag{9}$$

With these variables, the initial distribution function (Eq. (8)) takes the form

$$\Psi = \frac{1}{2\pi J_0}\exp(-J/J_0), \tag{10}$$

and the transformation (Eq. (6)) corresponding to free oscillations conserves the action J:

$$J' = J, \quad \phi' = \phi + \nu\Omega t. \tag{11}$$

In what follows, we will assume (an assumption that is crucial for the Landau damping effect) that the tune depends on the amplitude of the oscillations:

$$\nu = \nu(J). \tag{12}$$

Most of our results are not sensitive to the particular form of the dependence of ν versus J. However, in order to simplify further calculations (and to be in good agreement with the real situation), we will assume a simple linear dependence:

$$\nu = \nu_0 + \Delta\nu \frac{J}{J_0}, \tag{13}$$

where $\Delta\nu$ has a meaning of the tune spread in the beam.

Given the distribution function ψ, one can calculate the evolution of the averaged displacement $\overline{\eta}$ utilizing a simple integration:

$$\overline{\eta} = \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} \eta \psi(p,\eta,t) d\eta = \sqrt{2} \int_{0}^{\infty} \sqrt{J} dJ \int_{0}^{2\pi} \cos(\phi) \psi(J,\phi,t) d\phi. \tag{14}$$

The Hamiltonian motion defines a mapping of canonical variables from the initial values p, η (at $t = 0$) to the final values p', η' (at the time t): $\eta, p \to \eta', p'$. The evolution of the distribution function corresponding to this motion can be found from the Vlasov equation. We adopt here a different approach, based on the following principle (one that is, in fact, equivalent to Vlasov equation): in order to obtain the distribution function at the time t, one has to express p and η in the initial distribution function in terms of p' and η'. In other words, Hamiltonian mapping induces the following transformation of the distribution function:

$$\psi_{init}(p,\eta) \to \psi_{fin}(p',\eta') = \psi_{init}(p(p',\eta'), \eta(p',\eta')), \tag{15}$$

which links the initial distribution function ψ_{init} given at $t = 0$ with the distribution function ψ_{fin} at time t. Eq. (15) will allow us to follow the evolution of ψ step by step, starting from its initial value, $\psi_0 = \Psi(p,\eta)$.

3.0 CALCULATION OF THE ECHO

The first dipole kick induces the following transformation of ψ:

$$\psi_0(p,\eta) \to \psi_1(p,\eta) = \psi_0(p - \Delta p_{dip}, \eta), \tag{16}$$

where ψ_1 refers to the distribution function after the kick. Taking into account that Δp_{dip} is small, we expand ψ, keeping the linear term in perturbation:

$$\psi_1 \approx \psi_0(p,\eta) - \Delta p_{dip} \frac{\partial \psi_0}{\partial p} = \Psi(J) + \varepsilon\sqrt{2J} \sin\phi \frac{d\Psi(J)}{dJ}, \tag{17}$$

where we used Eq. (3) and transformed to J and ϕ. The dipole kick is followed by betatron oscillations on time interval τ. According to Eq. (11), they result in the following transformation:

$$\psi_1(J,\phi) \to \psi_2(J,\phi) = \psi_1(J, \phi - \nu\Omega\tau). \tag{18}$$

Putting Eq. (17) into Eq. (18) one finds

$$\psi_2 = \Psi(J) + \varepsilon\sqrt{2J}\sin(\phi - \nu(J)\Omega\tau)\frac{d\Psi(J)}{dJ}. \tag{19}$$

Using Eqs. (19), (13), and (14) one can find the averaged displacement as a function of time after the dipole kick:

$$\overline{\eta} = \varepsilon\left[\frac{1-\Delta\nu^2\Omega^2\tau^2}{(1+\Delta\nu^2\Omega^2\tau^2)^2}\sin\nu_0\Omega\tau + \frac{2\Delta\nu\Omega\tau}{(1+\Delta\nu^2\Omega^2\tau^2)^2}\cos\nu_0\Omega\tau\right]. \tag{20}$$

The plot of this function for $\Delta\nu/\nu_0 = 0.1$ is shown in Figure 3. It illustrates the effect of decoherence in our model. Note that, for large τ, $\overline{\eta}$ decreases as τ^{-2}.

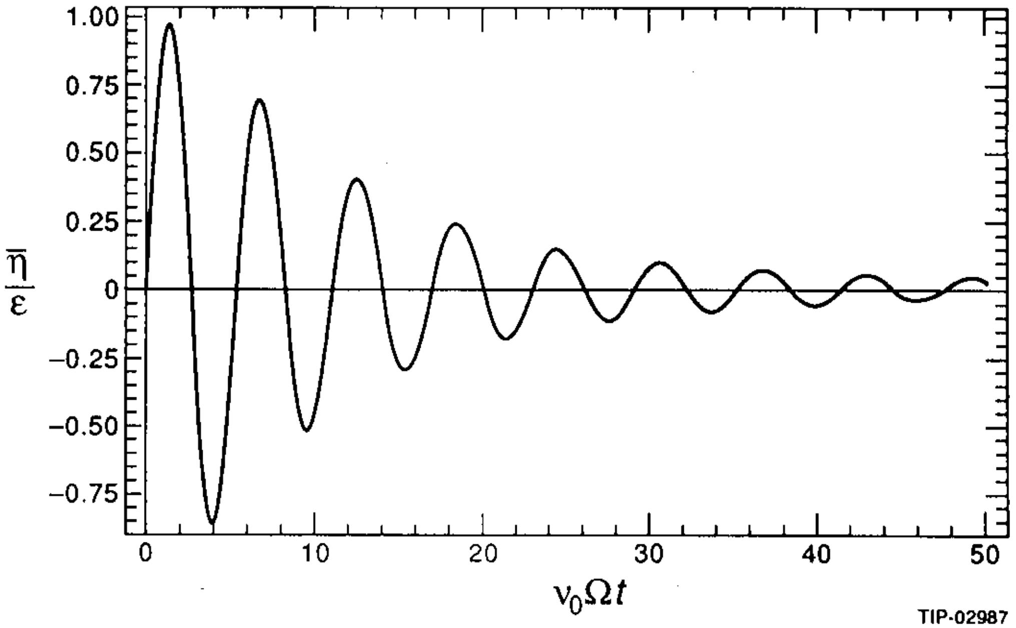

Figure 3. Averaged displacement of the beam produced by a dipole kick.

Now, make a quadrupole kick at $t = \tau$. In order that initial oscillations be damped at this moment, we have to assume that

$$\Delta\nu\Omega\tau >> 1. \tag{21}$$

The kick produces the following transformation of the distribution function:

$$\psi_2\ (p,\eta) \rightarrow \psi_3\,(p,\eta) = \psi_2(p - \Delta p_{quad},\eta). \tag{22}$$

To perform Taylor's expansion in Δp_{quad} in Eq. (22), in addition to the inequalities of Eq. (5), we will also assume that

$$q\Delta\nu\Omega\tau << 1. \tag{23}$$

This allows us to write down approximately

$$\psi_2(p,\eta) \approx \psi_2(p,\eta) - \Delta p_{quad}\frac{\partial\psi_2}{\partial p} = \tag{24}$$

$$= \Psi(J) + \varepsilon\sqrt{2J}\sin(\phi - \nu(J)\Omega\tau)\frac{d\Psi(J)}{dJ} + q\eta\frac{\partial}{\partial p}\left[\Psi(J) + \varepsilon\sqrt{2J}\sin(\phi - \nu(J)\Omega\tau)\frac{d\Psi(J)}{dJ}\right].$$

As we can show, the echo effect is contained in the last term on the right-hand side of Eq. (21). The other three terms either do not contribute to $\overline{\eta}$ or they make a contribution that damps in a manner similar to that shown in Figure 3. Using the relation

$$\frac{\partial}{\partial p} = -(2J)^{1/2}\sin\phi\frac{\partial}{\partial J} - (2J)^{-1/2}\cos\phi\frac{\partial}{\partial\phi},$$

one can find that the largest term that produces echo comes from the differentiation of $\sin(\phi - \nu(J)\Omega\tau)$ with respect to J in Eq. (24). Denoting it by $\psi_3^{(echo)}$ one finds

$$\psi_3^{(echo)} = 2\varepsilon q\eta\Delta\nu\Omega\tau\sin(\phi)\cos(\phi - \nu(J)\Omega\tau)\frac{J}{J_0}\frac{d\Psi(J)}{dJ}. \tag{25}$$

Free oscillations on time interval s transform this term according to Eq. (11):

$$\psi_3^{(echo)}(J,\phi) \rightarrow \psi_4^{(echo)}(J,\phi) = \psi_3^{(echo)}(J,\phi - \nu\Omega s). \tag{26}$$

Putting $\psi_4^{(last)}$ into Eq. (14) and performing the integration, one finds

$$\overline{\eta}^{echo} = q\varepsilon\Delta\nu\Omega\tau\left[\frac{A(A^2-3)}{(1+A^2)^3}\cos\nu_0\Omega(\tau - s) + \frac{3A^2-1}{(1+A^2)^3}\sin\nu_0\Omega(\tau - s)\right], \tag{27}$$

where $A = \Delta\nu\Omega(\tau - s)$. Figure 4 illustrates the dependence of $\overline{\eta}^{echo}$ versus the difference $(\tau - s)$ for $\Delta\nu/\nu_0 = 0.1$.

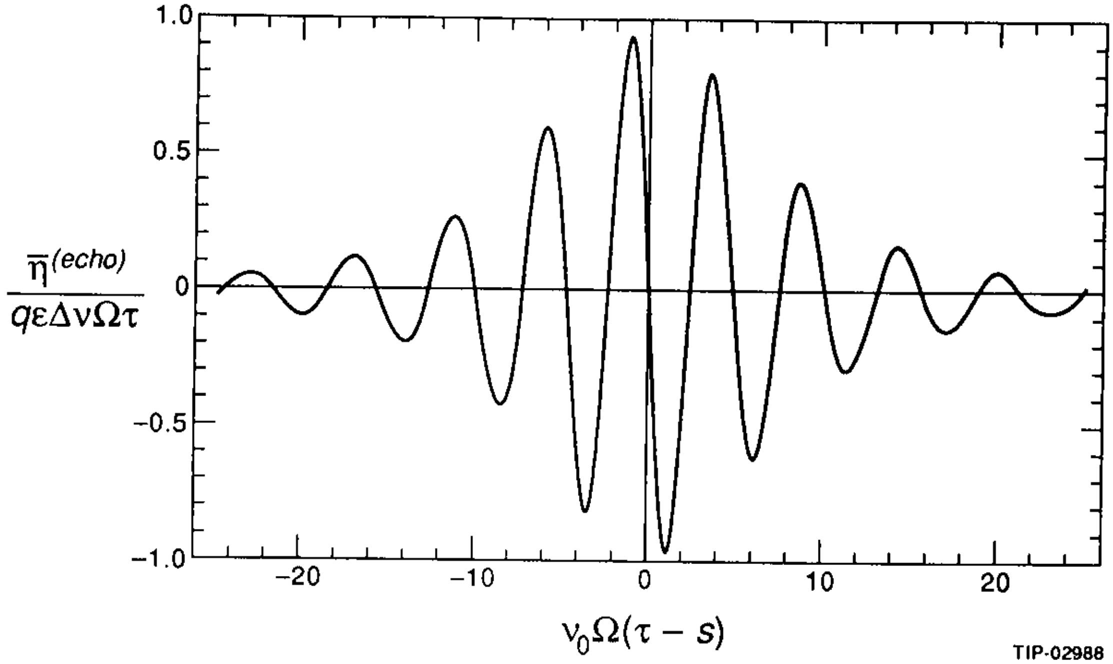

Figure 4. Echo signal of the beam.

Formally, it follows from Eq. (27) that the echo amplitude grows linearly with q. Note, however, that our consideration is restricted by the requirement of Eq. (23), which sets the upper limit on $\overline{\eta}^{(echo)}$. A more detailed study of the maximum attainable echo is performed elsewhere.[4]

4.0 DISCUSSION

In this note we predicted the existence of the echo effect using a simple model that takes into account one degree of freedom of the beam. The physical mechanism of the echo effect is based on a subtle phasing of the distribution function that occurs at a particular time during evolution of a beam subjected to successive dipole and quadrupole kicks. Our result shows that the amplitude of the echo is related only to the strength of the kicks and does not depend on the time gap between them. Typically the echo amplitude is smaller than the original oscillations produced by the first (dipole) kick; however, for a strong quadrupole kick, their amplitudes can be roughly comparable.*

In reality, there exist effects that could deteriorate observation of the echo. Any mechanism that smoothes out oscillations of the distribution function in the phase space (such as diffusion due to gas scattering or particle chaotic motion) would result in echo suppression, especially on a large time scale. On the other hand, high sensitivity of the echo to particle diffusion may be used as a diagnostic tool for studying stochastic dynamics of the beam. Possible applications of the echo require additional analysis.

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*This case lies beyond the scope of the present theory.

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