Homework 488/688

Advanced Topics in Accelerator Physics (Hoffstaetter)

Due Date: Monday, 10/04/04 - 14:45 in 132 Rockefeller Hall

Exercise 1: Determine how the beta function changes in a drift space by propagating Twiss parameters with the transport matrix. Then check that your solutions satisfies the nonlinear differential equation for the beta function, i.e. $\alpha' + \gamma = k\beta$. Do the same for the horizontal beta function in a horizontally focusing quadrupole.

Exercise 2: Given the Twiss parameters α, β, γ, and Ψ in an initial plane at s_0 and at a final plane s. Use the transformation from the J and ϕ_0 coordinates to x and x' given by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \sqrt{2J} \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \sin(\Psi + \phi_0) \\ \cos(\Psi + \phi_0) \end{pmatrix} \tag{1}$$

to express the transport matrix $\mathbf{M}$ that transports (x_0, x_0') at s_0 to (x, x') at s.

Exercise 3:

A) Use your result from exercise 2 to show that the one turn matrix of a ring at s can be written as

$$\mathbf{M} = \mathbf{1}\cos\mu + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin\mu \tag{2}$$

when α, β, and γ are the Twiss parameters that are periodic with the length L of the ring and $\mu = \Psi(L) - \Psi(0)$ is the one turn phase advance.

B) Show that the matrix before $\sin\mu$ in this equation has a characteristic of the complex i in that squaring it leads to $-\mathbf{1}$.

C) Use this to compute $\mathbf{M}^n$.

Exercise 4: If the one turn matrix

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{3}$$

is known, specify how the periodic Twiss parameters and the one turn phase advance can be computed. Under what conditions is the one turn phase advance real? What does this mean for the long term motion in phase space which is described by $\mathbf{M}^n \vec{z}_0$ for large n.