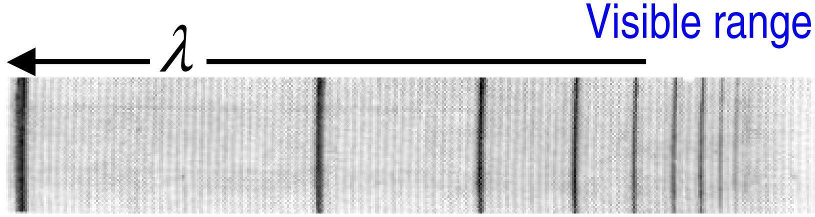
The Rutherford-Bohr Atom

Visible range: 430-750 THz

The Balmer lines of Hydrogen

$$\omega_n = 2\pi c R_H \left(\tfrac{1}{4} - \tfrac{1}{n^2}\right)$$



700-400 nm

1.8 –3.1 eV photons

Rutherfords scattering experiment (1909) lead to the insight that an atom has a positively charged core in a negatively charged surrounding.

Bohrs posutlates (1913):

1) Electrons orbit around the core with Z protons according to classical mechanics but do not radiate.

$$F = \frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2} \quad \rightarrow \quad E = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

2) Electrons can change their orbit only by radiating a photon with the energy hν.

$$E_{photon} = -[E(r_f) - E(r_i)] = \hbar\omega = \frac{Ze^2}{8\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

3) Observation of Balmer lines then leads to the conclusion that only certain radii are possible:

$$r_n = a_0 n^2 \rightarrow \omega_{j,n} = \frac{Ze^2}{8\pi\varepsilon_0 \hbar a_0}\left(\frac{1}{j^2} - \frac{1}{n^2}\right), \qquad Z = 1 \text{ for Hydrogen}$$

Balmer lines: j=2, lines for j=1,4,5 were found with Bohr’s prediction.

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Correspondence principle

For large quantum numbers, the predictions of quantum theory must agree with the predictions of classical mechanics and electrodynamics.

Dropping to the next lower energy level for quantum number N:

$$\omega_{N-1,N} = \frac{Ze^2}{8\pi\varepsilon_0\hbar a_0}\left[\frac{1}{(N-1)^2} - \frac{1}{N^2}\right] = \frac{Ze^2}{8\pi\varepsilon_0\hbar a_0}\frac{2N-1}{(N-1)^2 N^2} \approx \frac{Ze^2}{4\pi\varepsilon_0\hbar a_0}\frac{1}{N^3} = \frac{Ze^2}{4\pi\varepsilon_0\hbar a_0}\left(\frac{a_0}{r_N}\right)^{\frac{3}{2}}$$

Classical radiation of a charge which oscillates with frequency ν:

$$\omega^2 = \frac{1}{mr}\frac{mv^2}{r} = \frac{1}{mr}\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{Ze^2}{4\pi\varepsilon_0 m a_0^3}\left(\frac{a_0}{r}\right)^3$$

$$\lim_{N\to\infty} \omega^2_{N-1,N} / \omega^2 = 1 \rightarrow \frac{Ze^2 m}{4\pi\varepsilon_0\hbar^2} a_0 = 1, \quad a_0 = \frac{4\pi\varepsilon_0\hbar^2}{Ze^2 m} = \frac{1}{Z} 0.529\cdot 10^{-10}\, m$$

Niels Bohr
(1885-1962)
1922 Nobel prize

Leads to the correct Rhydberg constant and **ionization energy**:

$$\omega_{2,n} = \frac{Z^2 e^4 m}{2(4\pi\varepsilon_0)^2\hbar^3}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = 2\pi c Z^2 R_H \left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$\Delta E_{ion} = \hbar\omega_{1,\infty} = 2\pi c Z^2 R_H \hbar = 13.6\ eV\, Z^2$$

Leads to the **He**$^+$ spectrum with $Z = 2$

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