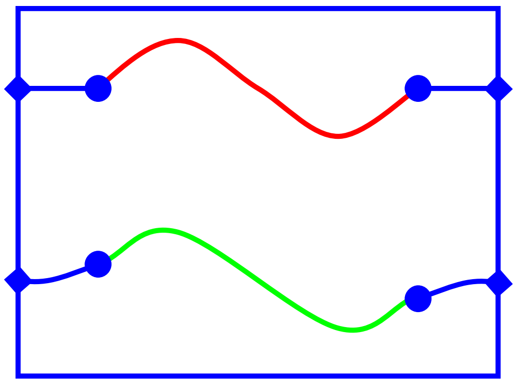
Particles in nonrigid boxes

05/18/2004

String attached with infinite force: only string vibrates with **sin(wt)**.

String attached at a realistic fixture: fixture also vibrates a little bit.

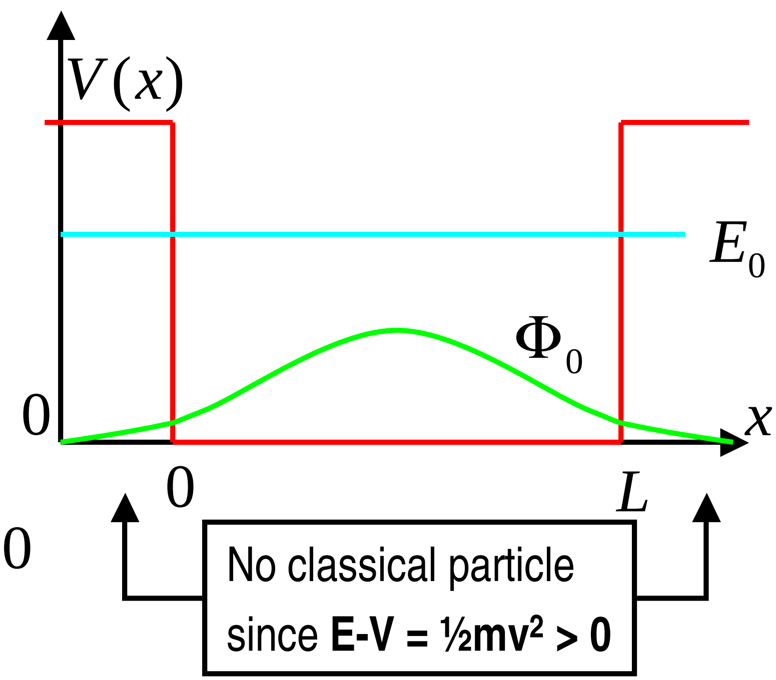


Infinite potential well: Wave function is 0 outside well.

Finite potential well: There is a wave function with **small amplitude outside the well**.

$$-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}\frac{\partial}{\partial x}\Phi(x)+V(x)\Phi(x)=E\,\Phi(x)$$

$$\Phi''(x)=-\frac{2m(E-V)}{\hbar^2}\Phi(x)$$



E > V: Curvature **towards x-axis**

E < V: Curvature **away from x-axis** with $\lim_{x\to\pm\infty}\Phi(x)=0$

to have $\int_{-\infty}^{\infty}|\Phi(x)|^2\,dx=1$

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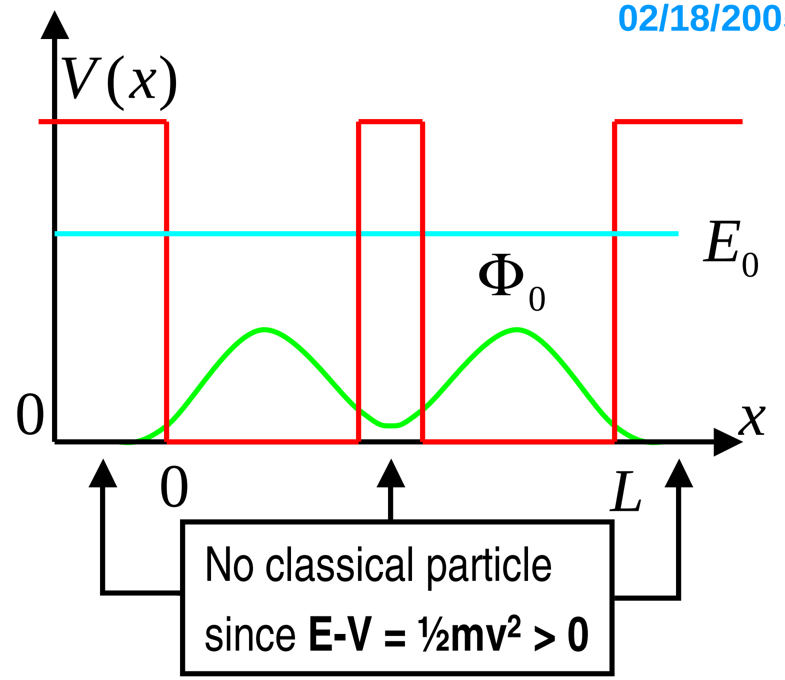
Tunneling

How can there be a probability of finding the particle on the left as well as a probability of finding it on the right ?
The particle can come from the left to the right by **tunneling**.



Properties of the wave function:

- Stationary Schrödinger equation is satisfied: $$\Phi''(x) = -\frac{2m(E-V)}{\hbar^2}\Phi(x)$$
- The first derivatives are continuous: $$\lim_{x\to +0}\Phi'(x) = \lim_{x\to -0}\Phi'(x)$$
- The wave function is normalized: $$\int_{-\infty}^{\infty}|\Phi(x)|^2\,dx = 1$$

and the wave function is therefore bounded $$\lim_{x\to\pm\infty}\Phi(x) = 0$$

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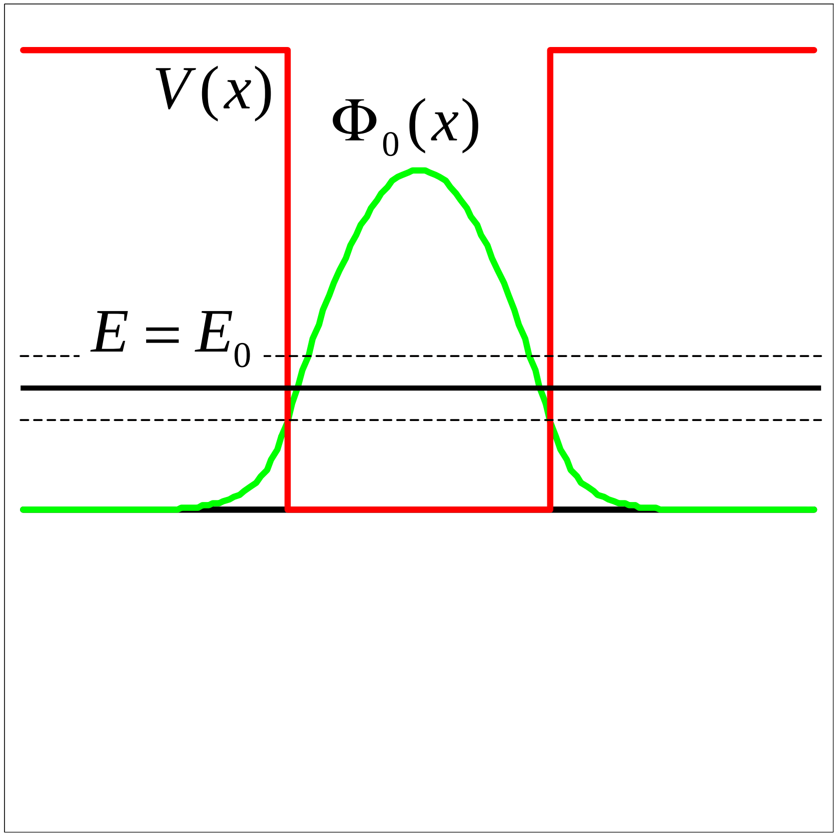
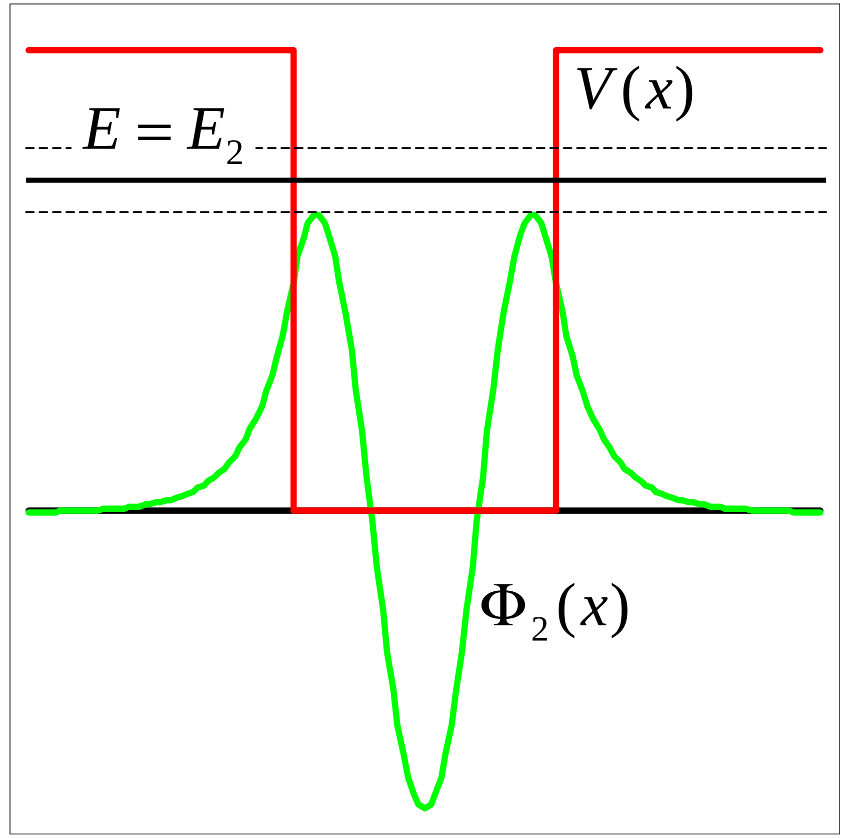
Fulfilling the boundary conditions

02/18/2005

$$\lim_{x\to\pm\infty} \Phi(x) = 0$$





$$\Phi''(x) = -\frac{2m(E-V)}{\hbar^2}\Phi(x)$$

The normalization requirement leads to boundary conditions at infinity and they lead to energy quantization.

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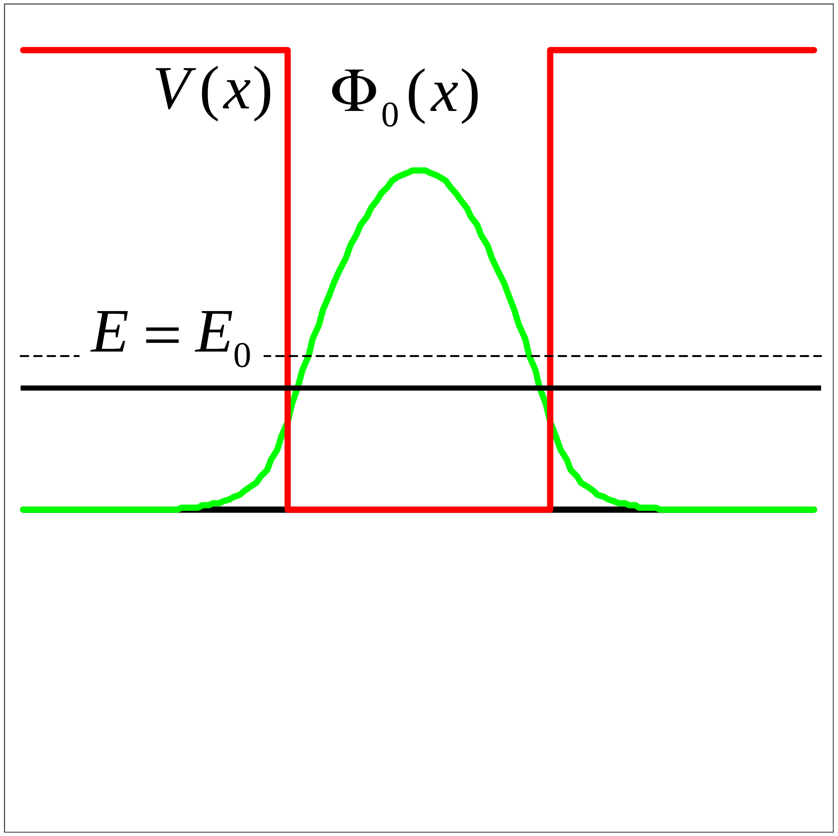
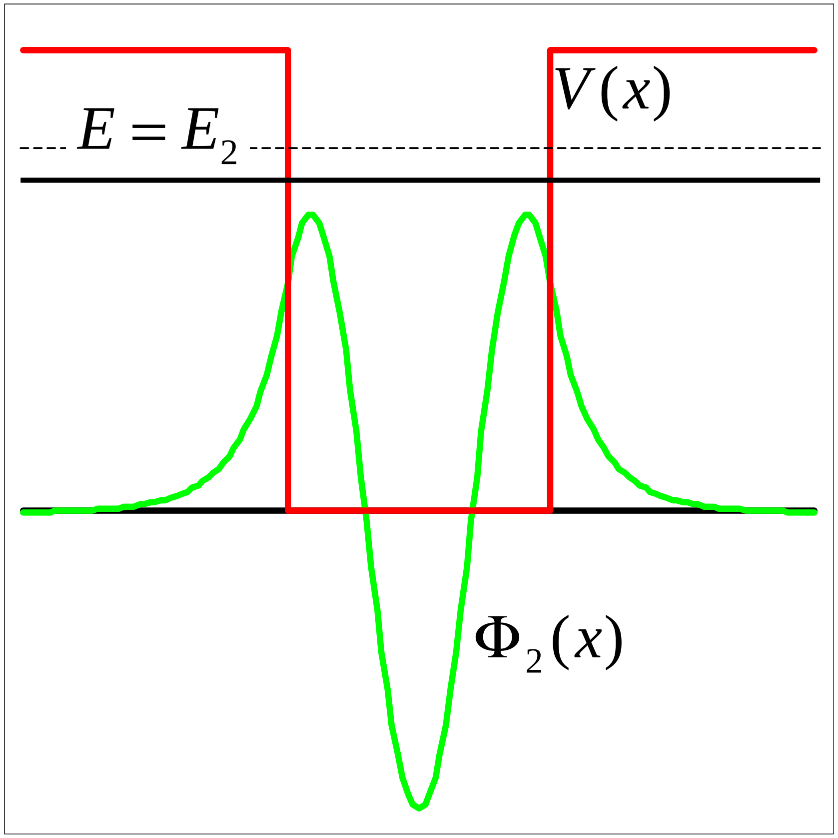
Energy of a stationary wave function





The "curvature" $\Phi''(x)/\Phi(x) \propto -(E-V)$ of the wave function is less for the finite potential well and therefore the energy levels E_n of stationary states are lower.

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