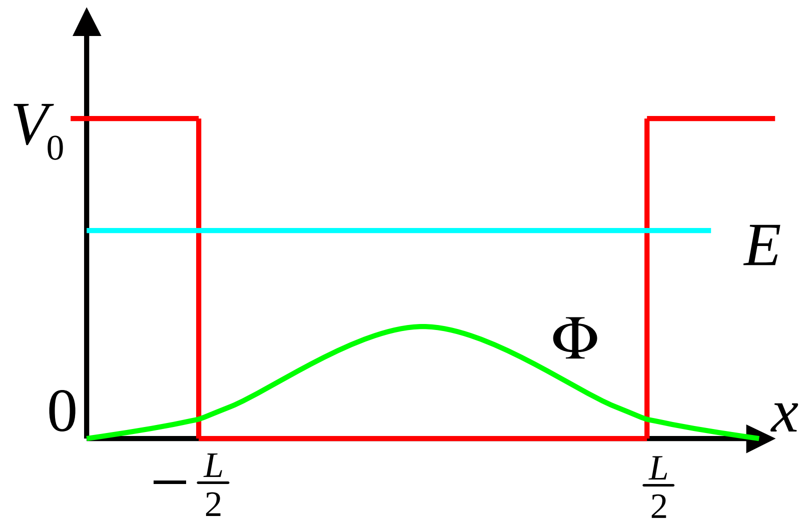
Stationary states in the square well

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

$$k = \sqrt{\frac{2m}{\hbar^2}E}\,, \quad k_V = \sqrt{\frac{2m}{\hbar^2}V_0}$$



$$z = \tfrac{L}{2}: \quad \left.\begin{aligned} kAe^{-\alpha\frac{L}{2}} &= kB\sin(k\tfrac{L}{2}+\varphi) \\ -\alpha Ae^{-\alpha\frac{L}{2}} &= kB\cos(k\tfrac{L}{2}+\varphi) \end{aligned}\right\} \rightarrow \left\{\begin{aligned} A^2e^{-\alpha L}(k^2+\alpha^2) &= k^2B^2 \\ \frac{A^2}{B^2}e^{-\alpha L} = \frac{E}{V_0} &= \frac{k^2}{k_V^2} \end{aligned}\right.$$

$$\left.\begin{aligned} \pm\frac{k}{k_V} &= \sin(k\tfrac{L}{2}+\varphi) \\ \mp\alpha\frac{k}{k_V} &= k\cos(k\tfrac{L}{2}+\varphi) \end{aligned}\right\} \rightarrow \left\{\begin{aligned} \frac{k}{k_V} &= |\sin(\tfrac{kL}{2}+\varphi)| \\ 0 &> \tan(\tfrac{kL}{2}+\varphi) \end{aligned}\right.$$

$$\boxed{\frac{k}{k_V} = -\mathrm{sign}[\cos(\frac{k}{k_V}\frac{k_V L}{2}+\varphi)]\sin(\frac{k}{k_V}\frac{k_V L}{2}+\varphi)}$$

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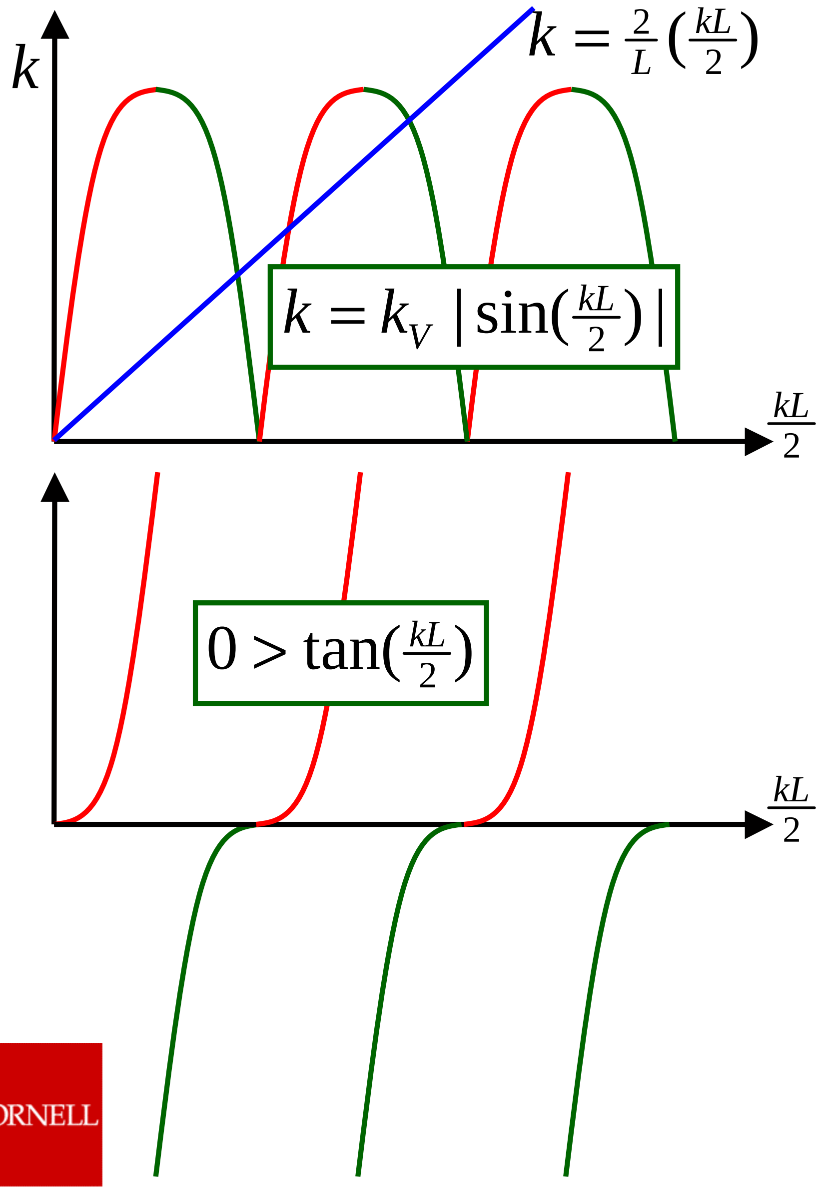
Graphical determination of possible wave numbers



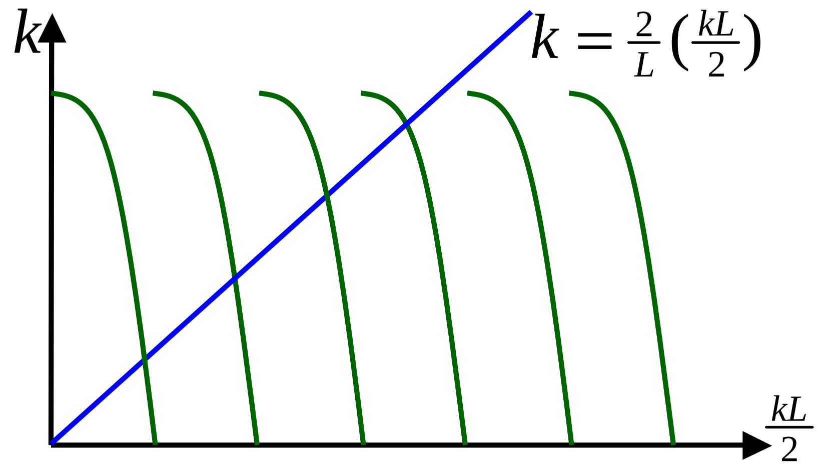
$$\frac{k}{k_V} = |\sin(\frac{kL}{2} + \varphi)|$$

and

$$0 > \tan(\frac{kL}{2} + \varphi)$$

with

$$\varphi = 0 \quad \text{or} \quad \varphi = \frac{\pi}{2}$$



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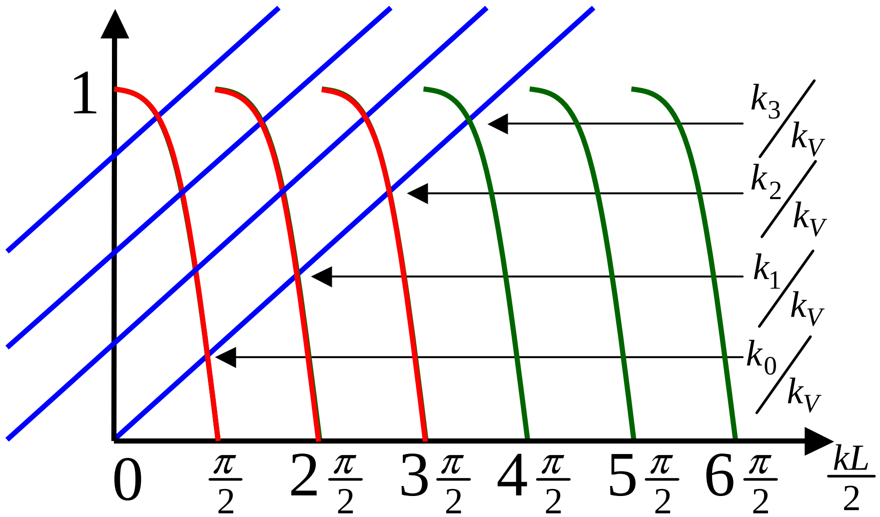
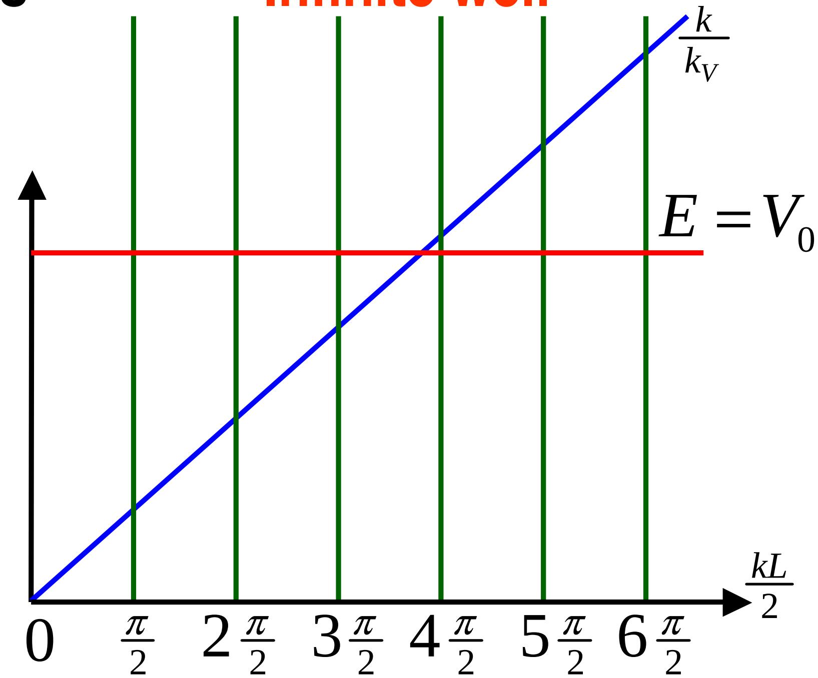
Square well versus Infinite well





Stationary states for the finite well:

$$\frac{k}{k_V} = \cos(\mathrm{mod}[\tfrac{kL}{2}, \tfrac{\pi}{2}])$$

Stationary states for the infinite well:

$$k = n\frac{\pi}{L}, \quad \mathrm{mod}[\tfrac{kL}{2}, \tfrac{\pi}{2}] = 0$$

- The **finite potential** well with height $\mathbf{V_0}$ has one stationary state more then the **infinite potential** well has states with energies below $\mathbf{V_0}$.
- Number of solutions = $\mathrm{int}[\frac{k_V L}{\pi}] + 1 = \mathrm{int}[\frac{L}{\pi}\sqrt{\frac{2m}{\hbar^2}V_0}] + 1$
- There is at least one bound state, and it is symmetric.

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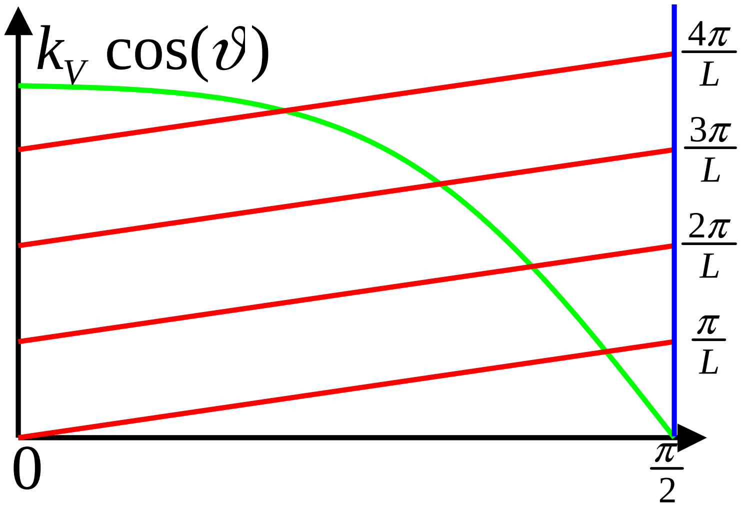
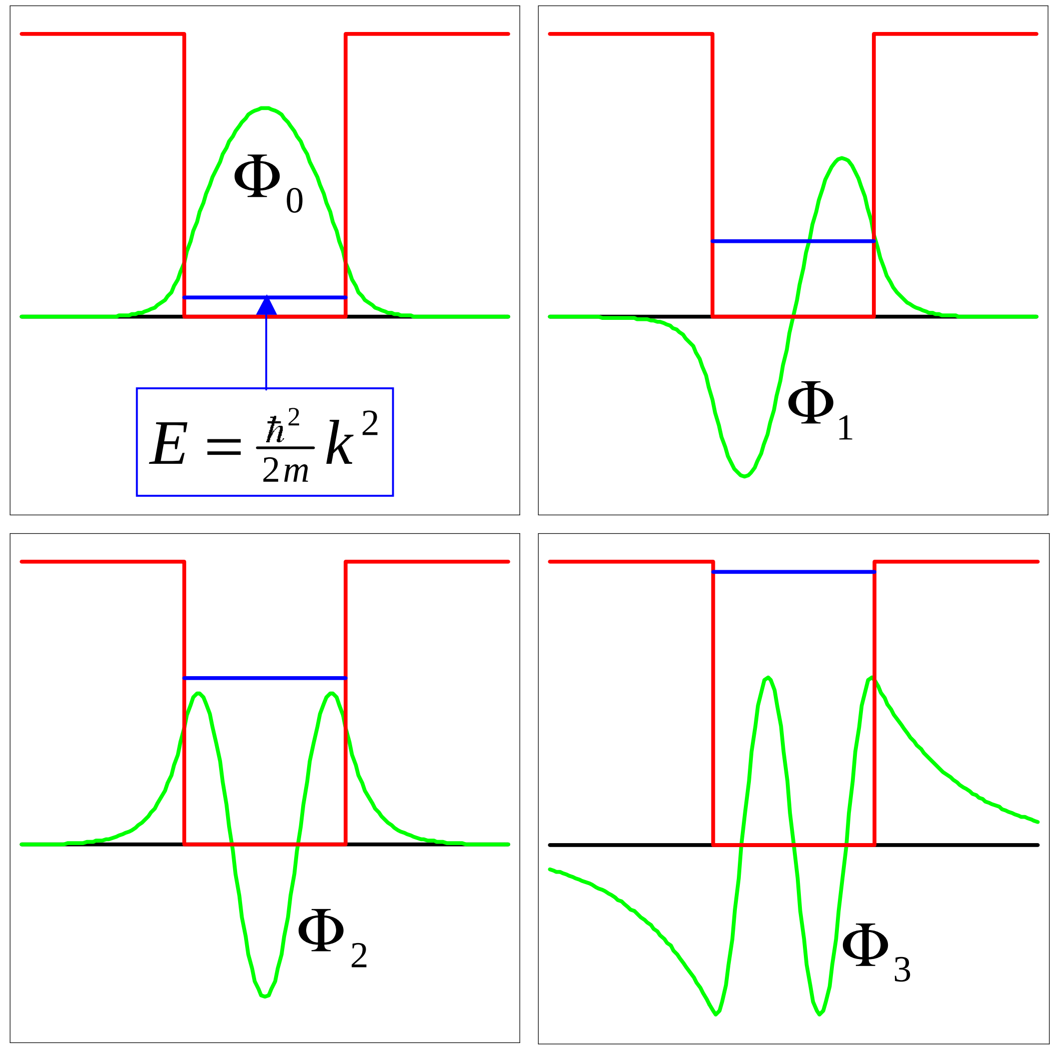
Graphic determination of states for the square well

Finite well:

$$k = k_V \cos(\text{mod}[\tfrac{kL}{2}, \tfrac{\pi}{2}])$$





Infinite well:

$$k = n\frac{\pi}{L}$$

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