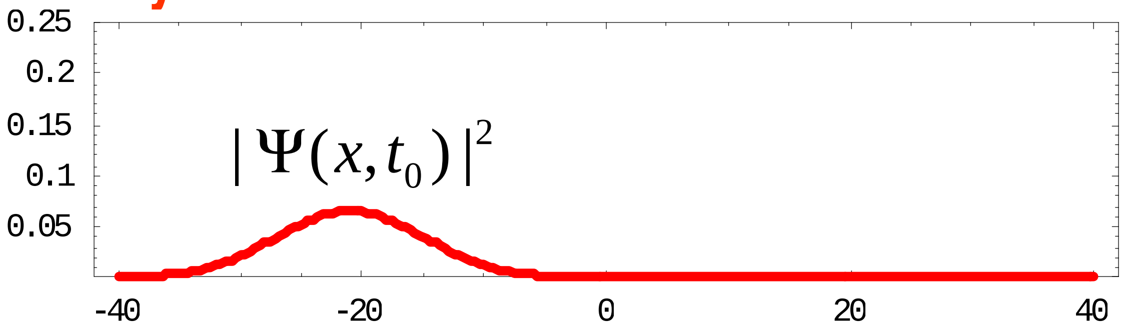
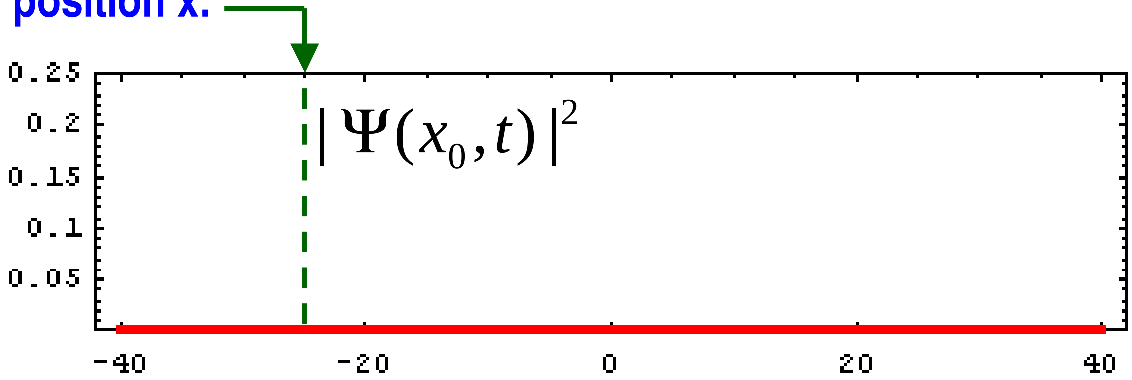
The energy time uncertainty relation

04/06/2005

Wave at fixed time t:



Wave traveling passed a fixed position x:



$$\left.\begin{aligned} \Delta t &\approx \frac{\Delta x}{v_{\text{group}}} \\ \Delta\omega &\approx \Delta k \tfrac{d}{dk}\omega = \Delta k\, v_{\text{group}} \end{aligned}\right\} \quad \Delta t \Delta\omega \approx \Delta x \Delta k \geq 1/2$$

$$\Delta t \Delta E \geq \tfrac{\hbar}{2}$$

This uncertainty relation is again due to a Fourier transform, here between time and frequency:

$$\Psi(x,t) = \int_{\text{all }\omega} A_\omega(x) \frac{e^{-i\omega t}}{\sqrt{2\pi}} d\omega \quad \Leftrightarrow \quad A_\omega(x) = \int_{\text{all }t} \Psi(x,t) \frac{e^{i\omega t}}{\sqrt{2\pi}} dt$$

Georg.Hoffstaetter@Cornell.edu

The shape and width of energy levels

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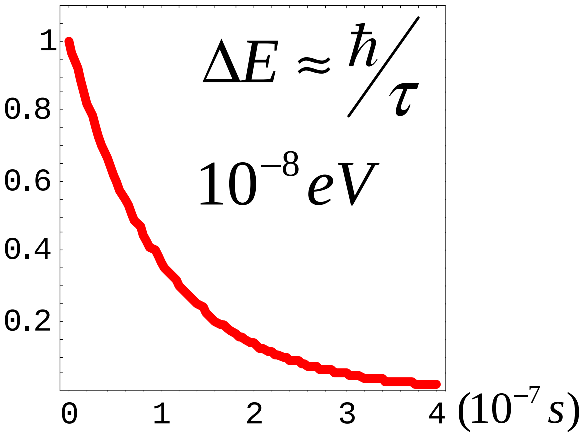
The time-energy uncertainty relation relates more things than the simple picture suggest:

1) **(a)** Lifetime of excited states of atoms and nuclei τ with $N(t) = N_0 e^{-t/\tau}$

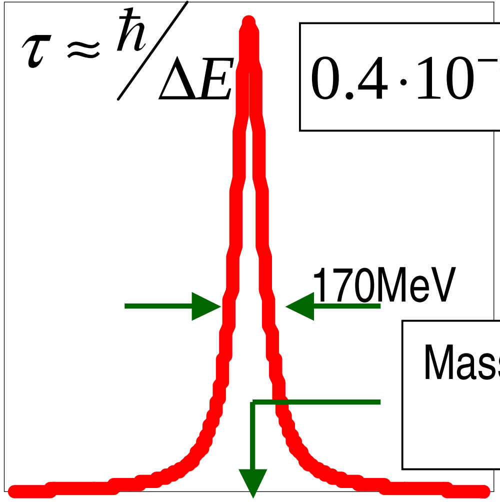
(b) Frequency spread $\Delta\omega$ of the emitted radiation or energy spread ΔE of the emitted particles

Activity of a radiating sample



$\Delta E \approx \hbar/\tau$

$10^{-8}\,eV$



$\tau \approx \hbar/\Delta E$

$0.4 \cdot 10^{-23}\,s \approx \frac{10^{-15}\,m}{c}$

170MeV

Mass of the ρ meson

770MeV

2) **(a)** The probability of the excited particle to have dropped back to its ground state energy, and therefore the probability to measure an emitted photon is $|\psi(t)|^2 \propto e^{-t/\tau}$

(b) The probability to find a photon with frequency ω is $|A_\omega|^2$ where A_ω is related to $\psi(t) = e^{-\frac{t}{2\tau}} e^{-i\omega_0 t}$ by a **Fourier transform**:

$$A_\omega \propto \int_0^\infty e^{-\frac{t}{2\tau} - i\omega_0 t} e^{i\omega t}\,dt = \left[\frac{1}{i(\omega-\omega_0) - \frac{1}{2\tau}} e^{t\{i(\omega-\omega_0) - \frac{1}{2\tau}\}}\right]_0^\infty = \frac{1}{\frac{1}{2\tau} - i(\omega-\omega_0)}$$

$$|A_\omega|^2 \propto \frac{1}{\frac{1}{4\tau^2} + (\omega-\omega_0)^2}$$

This is called a Lorentz curve around ω_0

with full width half max of $\Delta\omega = \frac{1}{2\tau}$

Georg.Hoffstaetter@Cornell.edu

9) Particle scattering and barrier penetration

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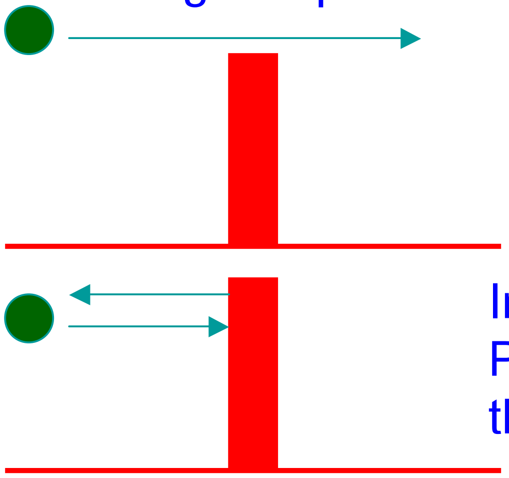
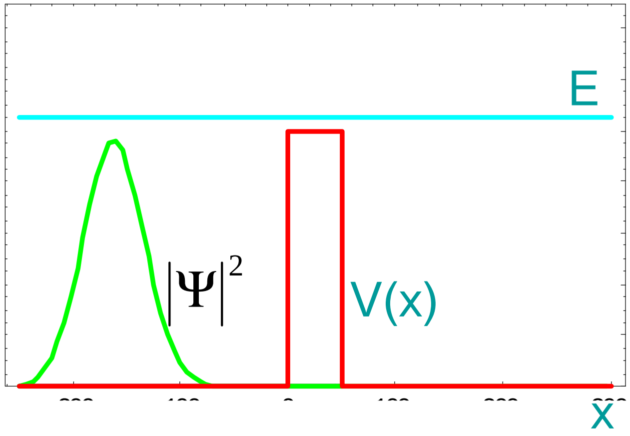
Scattering of a particle



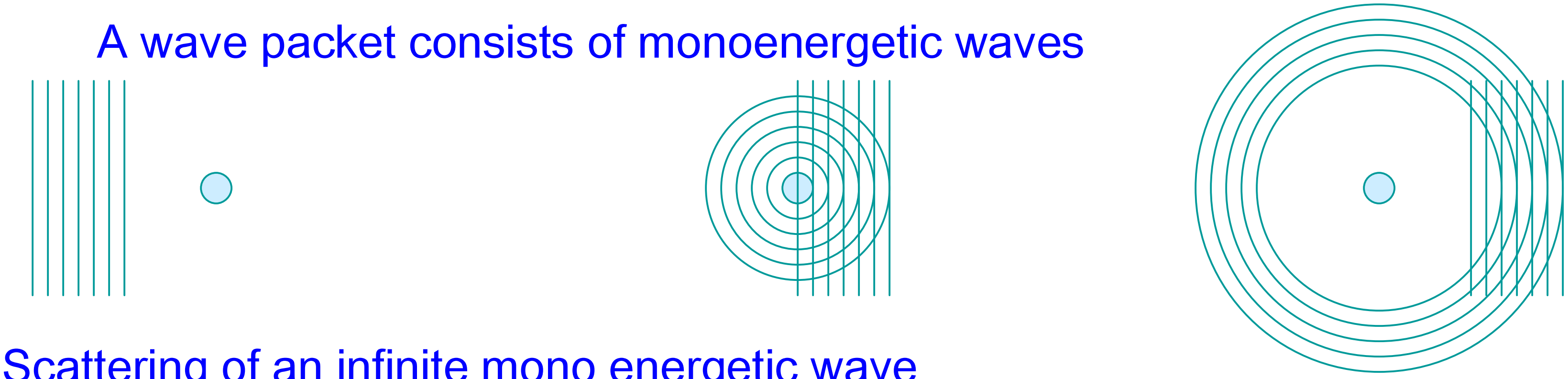
The particle is described by a wave packet.

In analogy to water waves:
Part of the wave passes
the barrier, part is reflected

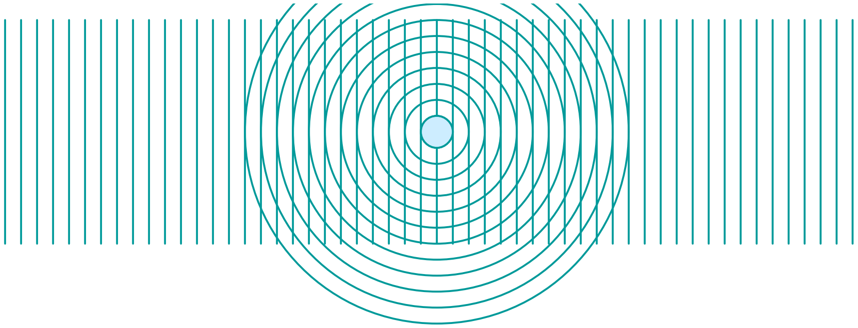
Scattering of a wave packet



A wave packet consists of monoenergetic waves



Scattering of an infinite mono energetic wave



Rather than to look at time dependent wave packets, one finds the time independent scattering result of a mono energetic wave.

Georg.Hoffstaetter@Cornell.edu