# The Coulomb potential

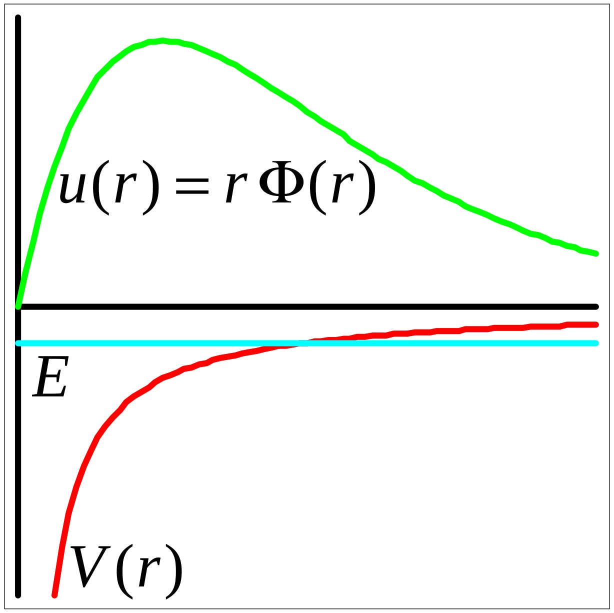
04/15/2005

$$\frac{\partial^2}{\partial r^2}u(r)=\frac{2m}{\hbar^2}[-\frac{Ze^2}{4\pi\varepsilon_0 r}-E]u(r), \quad E<0, \quad u(0)=0$$

$$a_0=\frac{1}{Z}\frac{4\pi\varepsilon_0\hbar^2}{me^2}, \quad E_1=-\frac{Ze^2}{8\pi\varepsilon_0 a_0}$$

$$\frac{\partial^2}{\partial r^2}u(r)=[(\frac{1}{a_0}\sqrt{\frac{E}{E_1}})^2-\frac{2}{a_0 r}]u(r)$$

$$\Rightarrow \quad u(r) \text{ goes to } 0 \text{ as } e^{-\frac{r}{a_0}\sqrt{\frac{E}{E_1}}} \text{ for large } r$$




$$u(\xi)=A\,w(r)\,e^{-\frac{r}{a_0}\sqrt{\frac{E}{E_1}}}$$

$$\boxed{w_1(r)=r}$$

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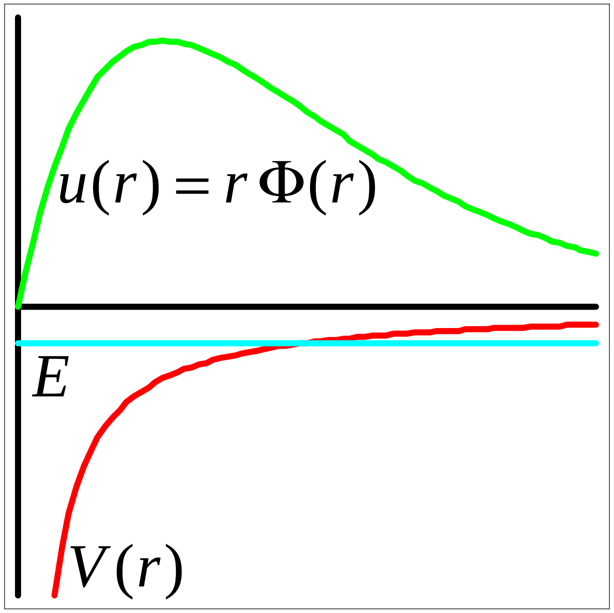
# Energies for the Coulomb potential

$$w(r) = \sum_{n=1}^{\infty} A_n r^n, \quad w(0) = 0$$




As with the Harmonic oscillator this leads to an iteration equation for the **An**. This iteration has to terminate at some finite **n**, which leads to energies of

$$\frac{E_n}{E_1} = \frac{1}{n^2}$$

This leads to a wave function $u_n(r) = w_n(r) e^{-\frac{r}{a_0}\sqrt{\frac{E}{E_1}}}$ with **n nodes**.

Therefore this leads to **all possible wave functions.**

$$E = \frac{1}{n^2} E_1\,, \quad E_n = -\frac{1}{n^2}\frac{Ze^2}{8\pi\varepsilon_0 a_0}$$

as in Bohr's theory

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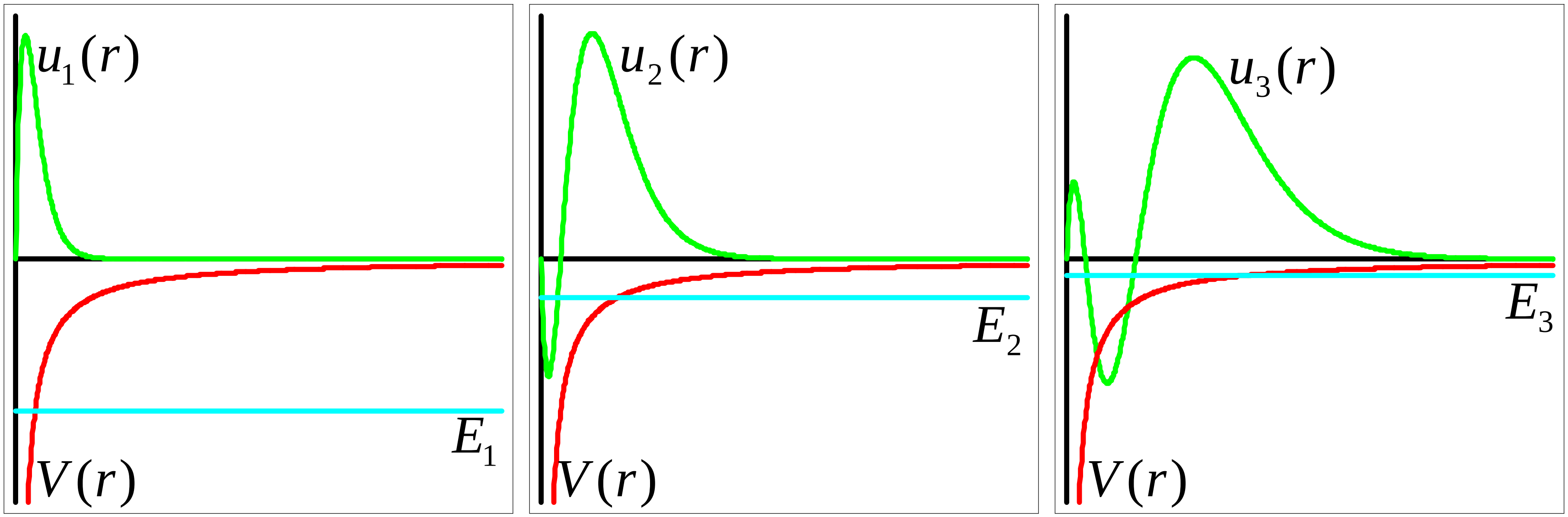
# Graphical properties of spherical symmetric wave functions

$E = E_1$ $\qquad$ $E_2 = \frac{1}{4} E_1$ $\qquad$ $E_3 = \frac{1}{9} E_1$

$w_1(r) \propto \frac{r}{a_0}$ $\qquad$ $w_2(r) \propto \frac{r}{a_0} - \frac{1}{2}\left(\frac{r}{a_0}\right)^2$ $\qquad$ $w \propto \frac{r}{a_0} - \frac{2}{3}\left(\frac{r}{a_0}\right)^2 + \frac{2}{27}\left(\frac{r}{a_0}\right)^3$




**Features of u(r):**

1) Larger values for shallower parts of the potential
2) Larger wavelength for shallower part of the potential
3) n-nodes for $n^{st}$ wave function above the ground state
4) Infinitely many stationary bound states

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