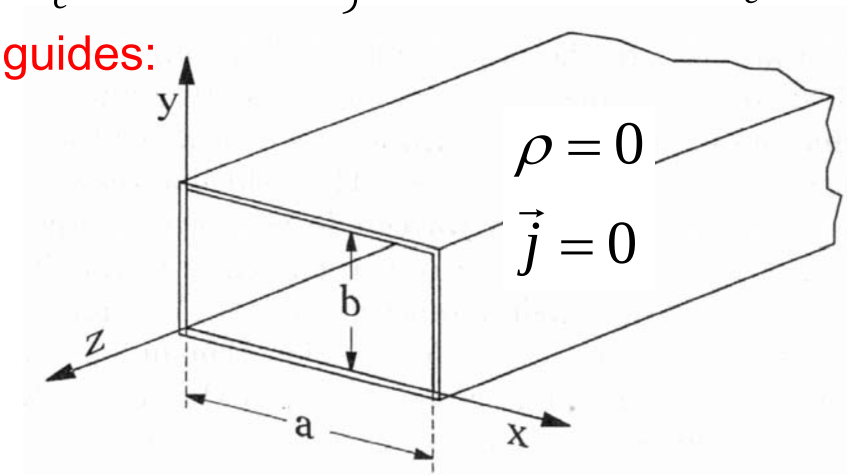
RF in Accelerators

$$\left.\begin{aligned}\vec\nabla\times\vec E &= -\partial_t\vec B\\ \vec\nabla\times\vec B &= \tfrac{1}{c^2}\partial_t\vec E+\mu_0\vec j\end{aligned}\right\}\begin{aligned}\vec\nabla\times\left(\vec\nabla\times\vec E\right) &= -\tfrac{1}{c^2}\partial_t^2\vec E-\mu_0\partial_t\vec j\\ \vec\nabla\times\left(\vec\nabla\times\vec B\right) &= -\tfrac{1}{c^2}\partial_t^2\vec B-\mu_0\vec\nabla\times\vec j\end{aligned}$$

Wave guides:



$$\rho = 0$$
$$\vec j = 0$$

Wave equation for all components

$$\vec\nabla^2\vec E = \tfrac{1}{c^2}\partial_t^2\vec E$$
$$\vec\nabla^2\vec B = \tfrac{1}{c^2}\partial_t^2\vec B$$

$$\left.\begin{aligned}\vec\nabla\times\vec E &= -\partial_t\vec B\\ \vec\nabla\times\vec B &= \tfrac{1}{c^2}\partial_t\vec E\end{aligned}\right\}\begin{aligned}\vec\nabla_\perp\times\vec E_\perp &= -\partial_t\vec B_z\\ \vec\nabla_\perp\times\vec B_\perp &= \tfrac{1}{c^2}\partial_t\vec E_z\end{aligned}$$

$$\left.\begin{aligned}\vec\nabla\cdot\vec E &= 0\\ \vec\nabla\cdot\vec B &= 0\end{aligned}\right\}\begin{aligned}\vec\nabla_\perp\cdot\vec E_\perp+\partial_z E_z &= 0\\ \vec\nabla_\perp\cdot\vec B_\perp+\partial_z B_z &= 0\end{aligned}$$

Search for simple modes:

Transverse electric and magnetic (TEM) waves cannot exists, since:

$$E_z = 0 \text{ and } B_z = 0 \;\Rightarrow\; \vec E_\perp = 0 \text{ and } \vec B_\perp = 0$$

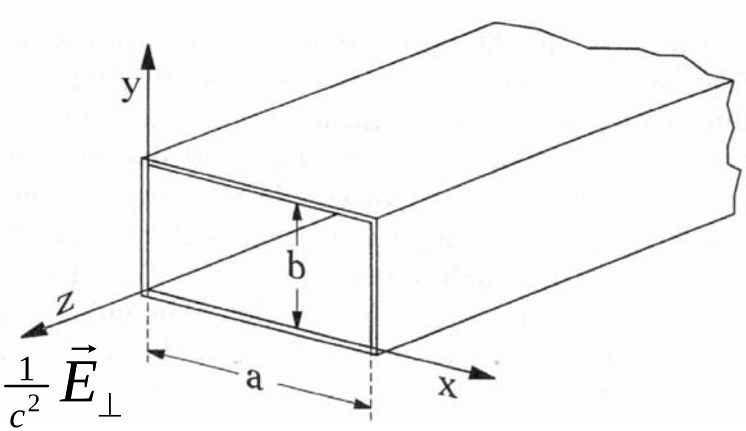
TE and TM Modes

Fourier expansion of the z-dependence: $\vec{E}(x,y,z,t)=\int \vec{E}_{k_z\omega}(x,y)e^{ik_z z - i\omega t}dk_z\, d\omega$

$$\vec{\nabla}^2\vec{E}=\tfrac{1}{c^2}\partial_t^2\vec{E} \qquad \Rightarrow \qquad \vec{\nabla}_\perp^2 E_z = -[(\tfrac{\omega}{c})^2 - k_z^2]E_z$$

$$\vec{\nabla}^2\vec{B}=\tfrac{1}{c^2}\partial_t^2\vec{B} \qquad\qquad \vec{\nabla}_\perp^2 B_z = -[(\tfrac{\omega}{c})^2 - k_z^2]B_z$$

Eigenvalue equation with boundary conditions:



Walls:

$\vec{E}_{//}=0 \quad \vec{B}_r=0$

$E_z=0 \quad \partial_r B_z=0$

$$\vec{\nabla}\times\vec{B}=\tfrac{1}{c^2}\partial_t\vec{E}$$

$$\vec{\nabla}_\perp\times B_z + ik_z\vec{e}_z\times\vec{B}_\perp = -i\omega\tfrac{1}{c^2}\vec{E}_\perp$$

$$\vec{\nabla}_r\times B_z + ik_z\vec{e}_z\times\vec{B}_r = -i\omega\tfrac{1}{c^2}\vec{E}_\varphi \Rightarrow \partial_r B_z = 0$$

Solutions for E or B only exist for a discrete set of eigenvalues: $(\tfrac{\omega}{c})^2 - k_z^2 = k_n^{(E)2}$

$$(\tfrac{\omega}{c})^2 - k_z^2 = k_n^{(B)2}$$

Due to different boundary conditions, E_z and B_z cannot simultaneously be nonzero.

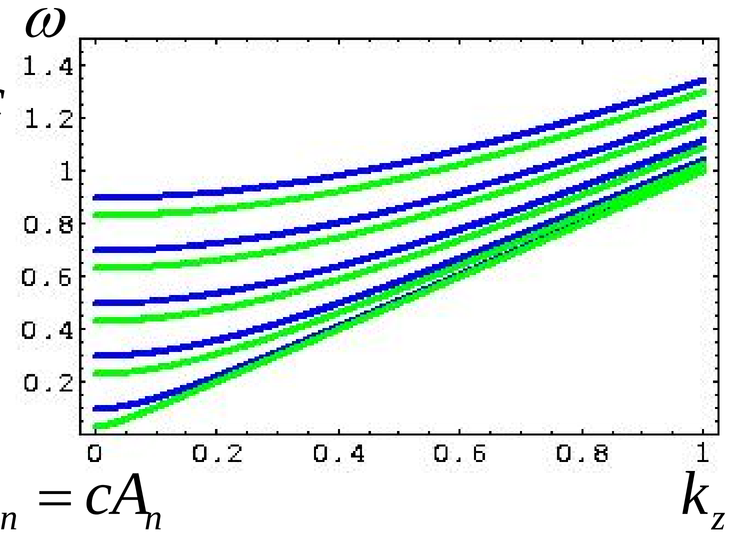
TE modes have $E_z=0$ TM modes have $B_z=0$

Dispersion relation

$$\omega(k_z) = c\sqrt{A_n^2 + k_z^2}$$

Phase velocity $v_{ph} = \omega / k_z = c\sqrt{1+\left(\frac{A_n}{k_z}\right)^2} > c$

Group velocity $v_{gr} = d\omega / dk_z = c / \sqrt{1+\left(\frac{A_n}{k_z}\right)^2} < c$



For each excitation frequency ω one obtains a propagation in the wave guide of

$$e^{ik_z z}\ , \quad k_z = \sqrt{(\tfrac{\omega}{c})^2 - A_n^2}$$

Transport for ω above the cutoff frequency $\omega > \omega_n = cA_n$

Damping for ω below the cutoff frequency $\omega < \omega_n = cA_n$

Rectangular Wave Guide

CHESS & LEPP

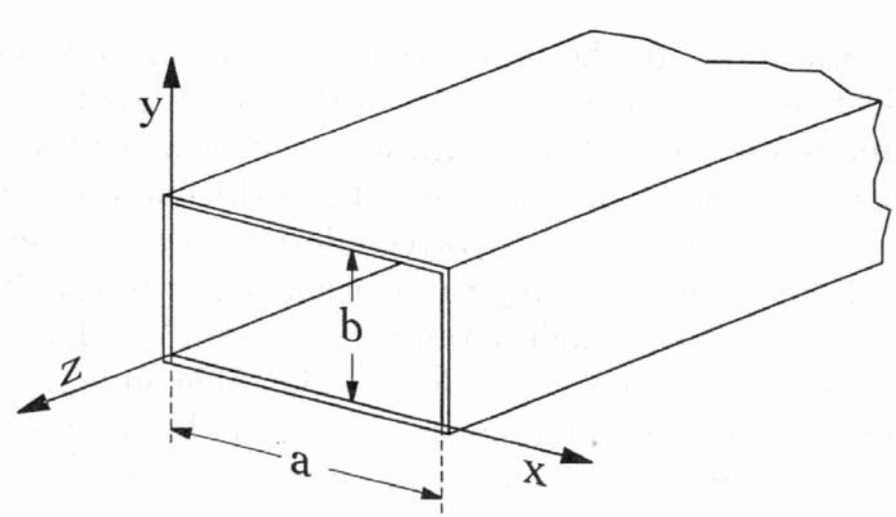
Boundary conditions:

$$E_z(\vec{x}_0)=0 \quad \vec{\nabla}_\perp^2 E_z=[k_z^2-(\tfrac{\omega}{c})^2]E_z$$

$$E_z(\vec{x})=E_z\sin(\tfrac{n\pi}{a}x)\sin(\tfrac{m\pi}{b}y)$$

$$(\tfrac{\omega}{c})^2-k_z^2=k_{nm}^{(B)2}=\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2$$

$$\partial_r B_z(\vec{x}_0)=0 \quad \vec{\nabla}_\perp^2 B_z=[k_z^2-(\tfrac{\omega}{c})^2]B_z$$



$$B_z(\vec{x})=B_z\cos(\tfrac{n\pi}{a}x)\cos(\tfrac{m\pi}{b}y)$$

$$(\tfrac{\omega}{c})^2-k_z^2=k_{nm}^{(E)2}=\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2$$

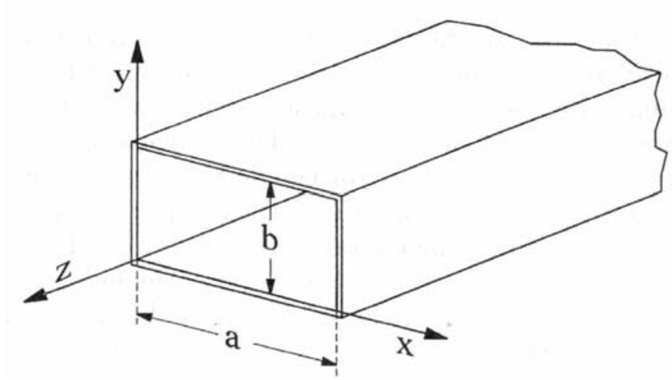
TE and TM modes happen to have the same eigenvalues.

For simplicity one still looks at TE and TM modes separately.

Rectangular TE Modes

Boundary conditions: $E_z(\vec{x}) = 0$

$$\vec{E}_{//}(\vec{x}_0) = 0$$



$$E_x(\vec{x}) = [A\cos(\tfrac{n\pi}{a}x) + B\sin(\tfrac{n\pi}{a}x)]\sin\tfrac{m\pi}{b}y)$$

$$E_y(\vec{x}) = \sin(\tfrac{n\pi}{a}x)[C\cos(\tfrac{m\pi}{b}y) + D\sin(\tfrac{m\pi}{b}y)]$$

$$\vec{\nabla}_\perp \cdot \vec{E}_\perp = 0 \quad \Rightarrow \quad D = 0\,, \quad B = 0\,, \quad C = -A\tfrac{n}{a}\tfrac{b}{m}$$

$$\vec{\nabla}_\perp \times \vec{E}_\perp = i\omega B_z \cos(\tfrac{n\pi}{a}x)\cos(\tfrac{m\pi}{b}y) \quad \Rightarrow \quad A\tfrac{b}{m\pi}\underbrace{[(\tfrac{m\pi}{b})^2 + (\tfrac{n\pi}{a})^2]}_{k_{nm}^{(E)2}} = -i\omega B_z$$

$$\vec{B}_r(\vec{x}_0) = 0 \qquad B_x(\vec{x}) = \sin(\tfrac{n\pi}{a}x)[C'\cos(\tfrac{m\pi}{b}y) + D'\sin(\tfrac{m\pi}{b}y)]$$

$$B_y(\vec{x}) = [A'\cos(\tfrac{n\pi}{a}x) + B'\sin(\tfrac{n\pi}{a}x)]\sin(\tfrac{m\pi}{b}y)$$

$$\vec{\nabla}_\perp \times \vec{B}_\perp = 0 \quad \Rightarrow \quad D' = 0\,, \quad B' = 0\,, \quad C' = A'\tfrac{n}{a}\tfrac{b}{m}$$

$$\vec{\nabla}_\perp \cdot B_\perp = -ik_z B_z \cos(\tfrac{n\pi}{a}x)\cos(\tfrac{m\pi}{b}y) \quad \Rightarrow \quad A'\tfrac{b}{m\pi}k_{nm}^{(E)2} = -ik_z B_z$$

Rectangular TE and TM Modes

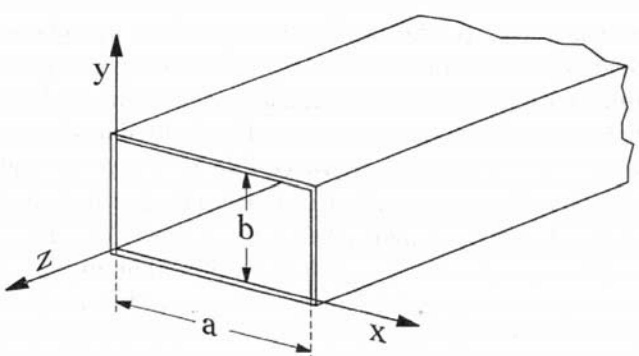
TE Modes

$$\vec{B}(\vec{x}) = B_z \begin{pmatrix} \frac{n\pi}{a}\frac{k_z}{k_{nm}^{(E)2}}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ \frac{m\pi}{b}\frac{k_z}{k_{nm}^{(E)2}}\cos(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ \cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\cos(k_z z-\omega t) \end{pmatrix}$$

TM Modes:
Exchange of
E and B

$$\vec{E}(\vec{x}) = \frac{\omega}{k_{nm}^{(E)2}} B_z \begin{pmatrix} \frac{m\pi}{b}\cos(\frac{n\pi}{a}x)\sin(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ -\frac{n\pi}{a}\sin(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)\sin(k_z z-\omega t) \\ 0 \end{pmatrix}$$

Notation: TE_{nm} Mode

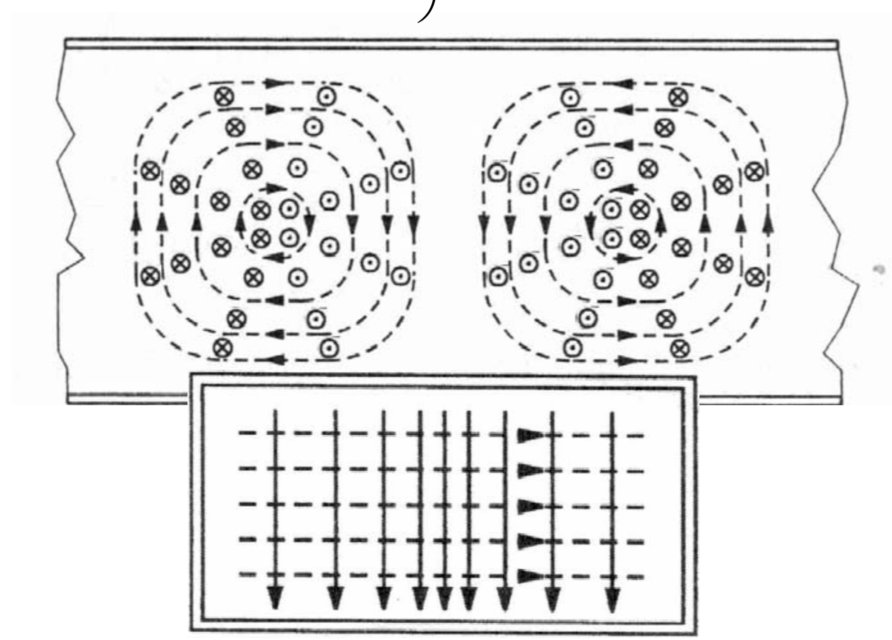


$\vec{E}$ ——→

$\vec{B}$ - - - →

$n = 1$

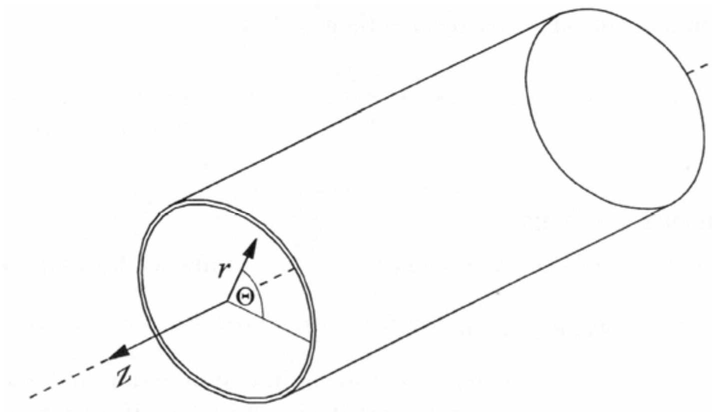
$m = 0$



Cylindrical Wave Guides

TM Modes:

$$E_z(\vec{x}_0)=0 \qquad \vec{\nabla}_\perp^2 E_z=[k_z^2-(\tfrac{\omega}{c})^2]E_z$$



$$(\partial_r^2+\tfrac{1}{r}\partial_r+\tfrac{1}{r^2}\partial_\varphi^2)E_z=[k_z^2-(\tfrac{\omega}{c})^2]E_z$$

$$(\xi^2\partial_\xi^2+\xi\partial_\xi+\xi^2-n^2)E_z=0\,,\quad \xi=k_{nm}^{(E)}\,r$$

$$E_z(\vec{x})=E_zJ_n(k_{nm}^{(B)}r)e^{in\varphi}$$

$k_{nm}^{(B)}$ is the mth 0 of the nth Bessel function over r.

TE Modes:

$$\partial_r B_z(\vec{x}_0)=0 \qquad \vec{\nabla}_\perp^2 B_z=[k_z^2-(\tfrac{\omega}{c})^2]B_z$$

$$B_z(\vec{x})=B_zJ_n(k_{nm}^{(E)}r)e^{in\varphi}$$

$k_{nm}^{(E)}$ is the mth extremeum of J_n over r.

Notation: TE_{nm} Mode

Fundamental Mode

Mode for particle acceleration: TM_{01}

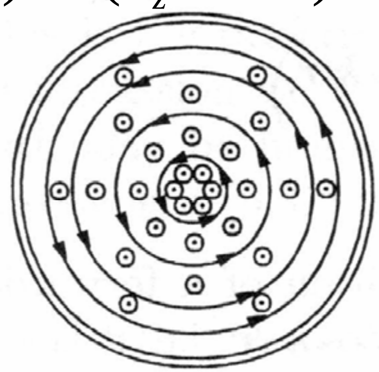
$$E_z(\vec{x}) = E_z J_0(\tfrac{r}{r_0})\cos(k_z z - \omega t)$$

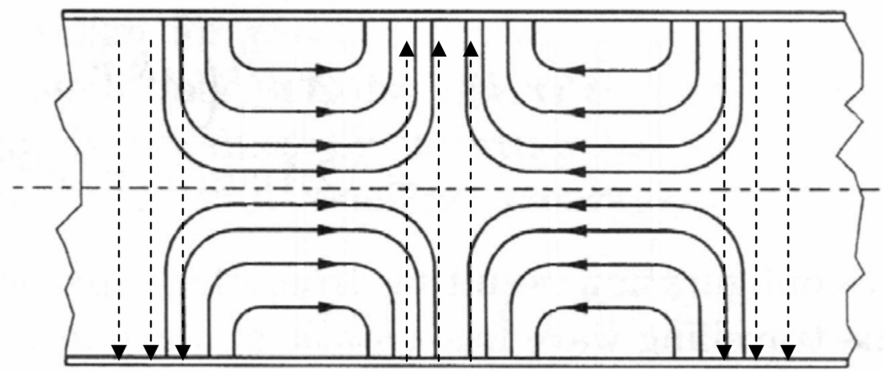
$$E_r(\vec{x}) = -E_z r_1 k_z J_0'(\tfrac{r}{r_1})\sin(k_z z - \omega t)$$

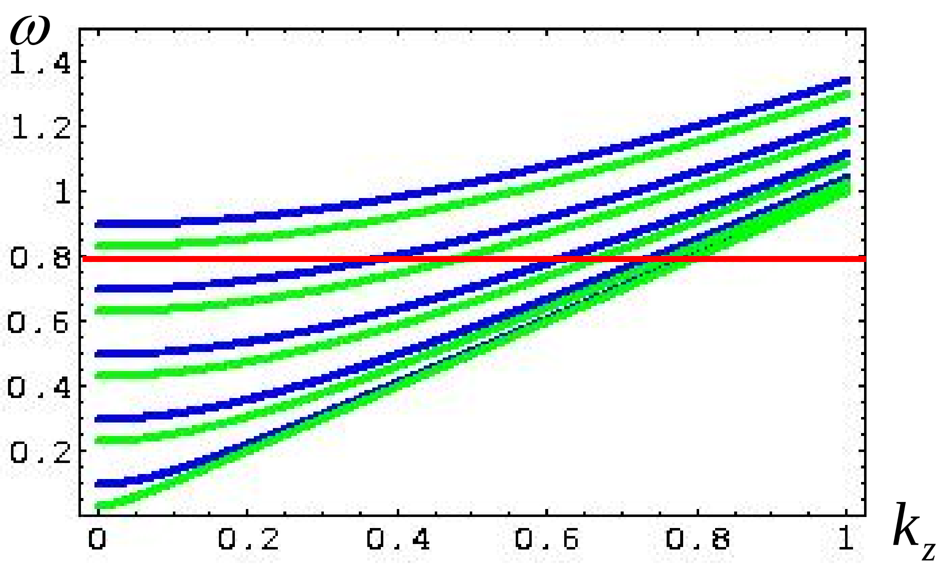
$$E_\varphi(\vec{x}) = 0$$

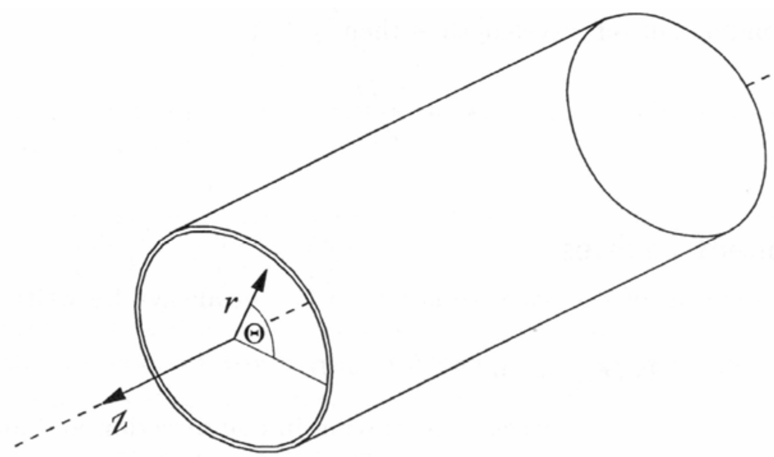
$$B_r(\vec{x}) = 0$$

$$B_\varphi(\vec{x}) = -E_z r_1 \tfrac{\omega}{c^2} J_0'(\tfrac{r}{r_1})\sin(k_z z - \omega t)$$

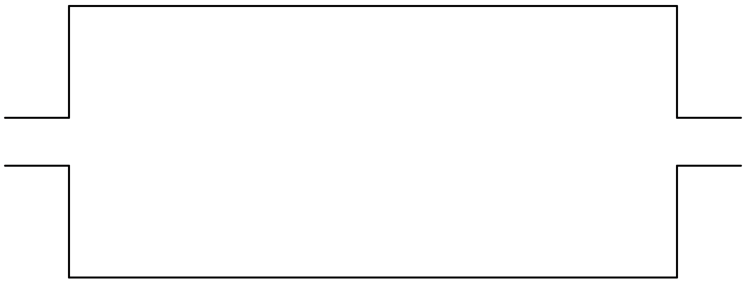








Resonant Cavities

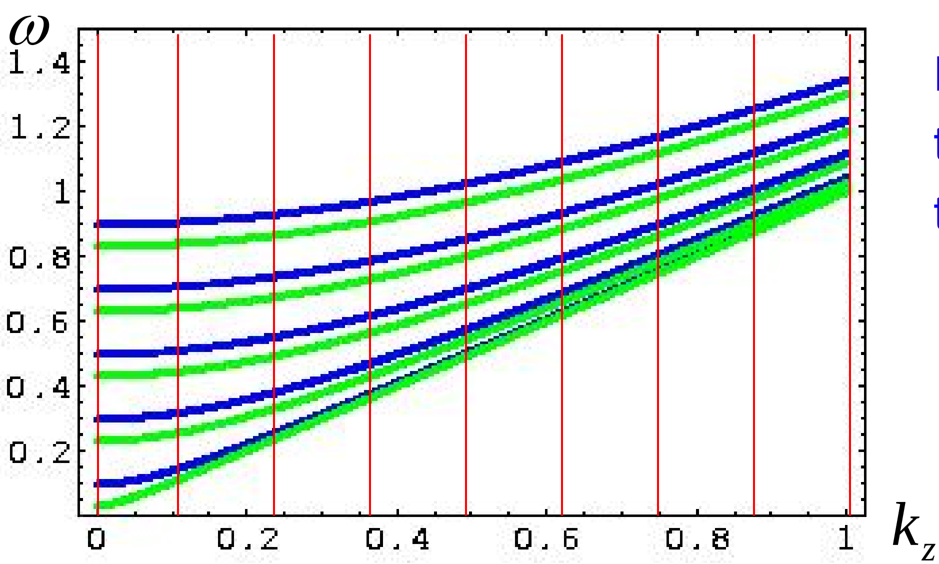


TE Modes: Standing waves with nodes

$$B_z(\vec{x}) \propto \sin(k_z z)\sin(\omega t)\,, \quad k_z = \frac{l\pi}{L}$$
$$l > 0$$

TM Modes: Standing waves with nodes

$$E_z(\vec{x}) \propto \cos(k_z z)\cos(\omega t)\,, \quad k_z = \frac{l\pi}{L}$$
$$l \geq 0$$

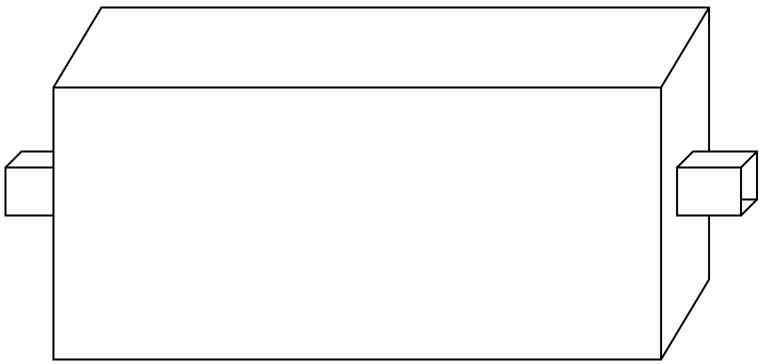


For each mode TE_{nm} or TM_{nm} there is a discrete set of frequencies that can be excited.

$$\omega_{nm}^{(E/B)} = c\sqrt{k_{nm}^{(E/B)2} + \left(\frac{l\pi}{L}\right)^2}$$

Resonant Cavities Examples

Rectangular cavity:

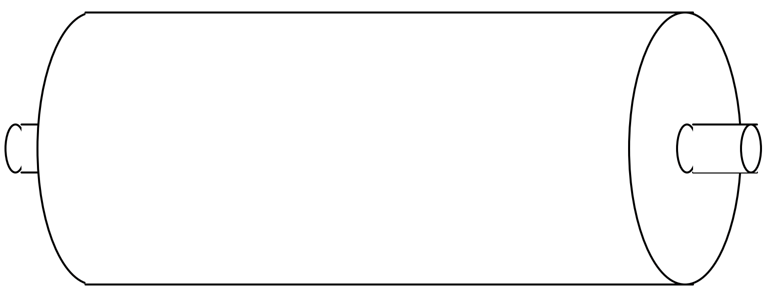


$$\omega_{nml}^{(E/B)} = c\sqrt{\left(\frac{n\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{l\pi}{L_z}\right)^2}$$

Fundamental acceleration mode: $\omega_{110}^{(B)} = c\frac{\pi}{L}\sqrt{2}$

$$L_x = L_y = 22cm \quad \Rightarrow \quad f_{110}^{(B)} = 1.0\text{GHz}$$

Pill Box cavity:



$$\omega_{nm}^{(E/B)} = c\sqrt{k_{nm}^{(E/B)2} + \left(\frac{l\pi}{L}\right)^2}$$

$k_{nm}^{(B)}r$ is the m^{th} 0 of the n^{th} Bessel function.

$k_{nm}^{(E)}r$ is the m^{th} extremeum of J_n

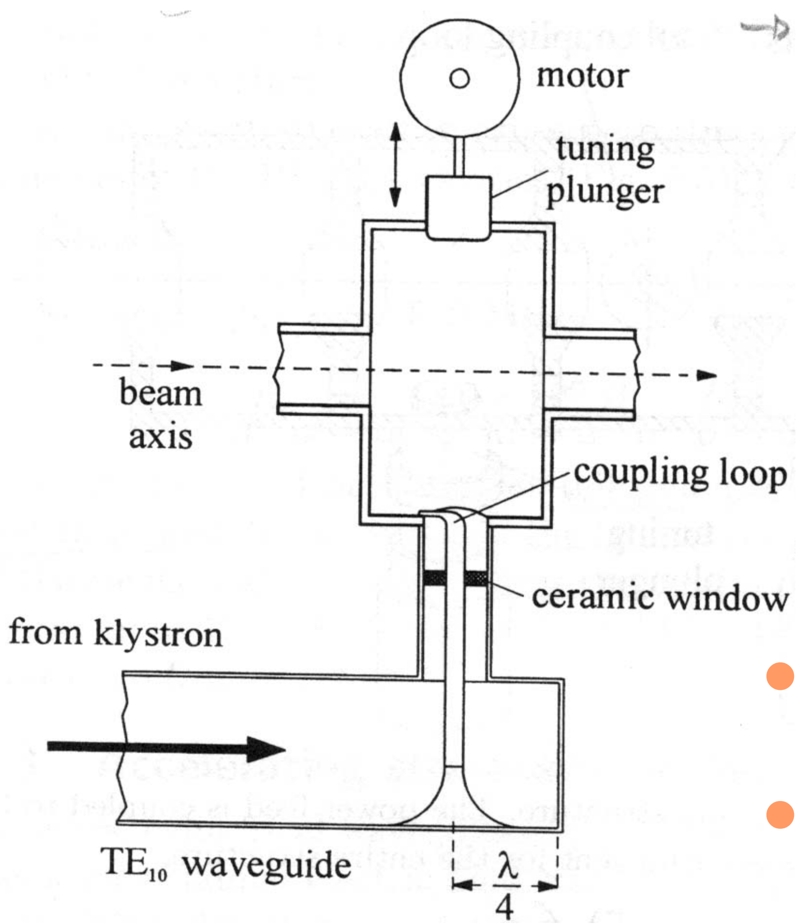
Fundamental acceleration mode: $\omega_{010}^{(E)} = c\frac{2.4}{r}$

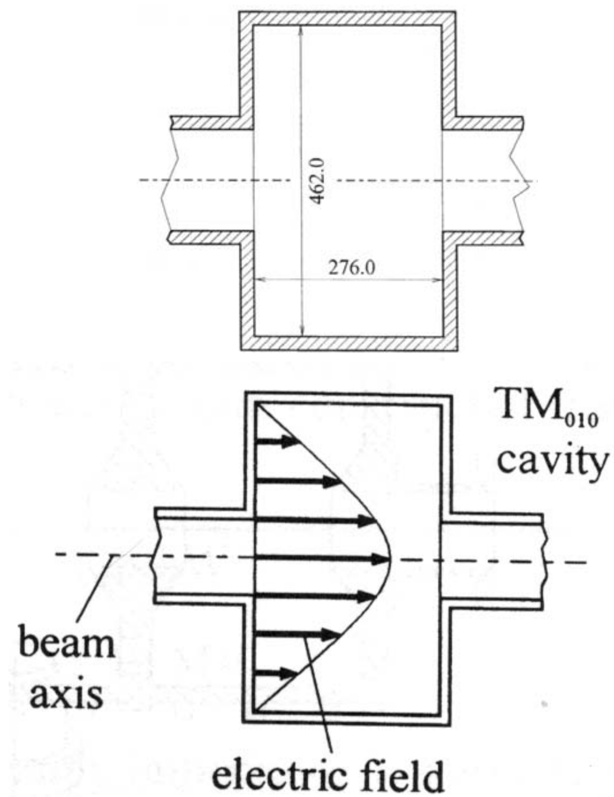
$$r = 11cm \quad \Rightarrow \quad f_{010}^{(M)} = 1.0\text{GHz}$$

Cavity Operation

500MHz Cavity of DORIS:

$$r = 23.1cm \quad \Rightarrow \quad f_{010}^{(M)} = 0.4967\text{GHz}$$

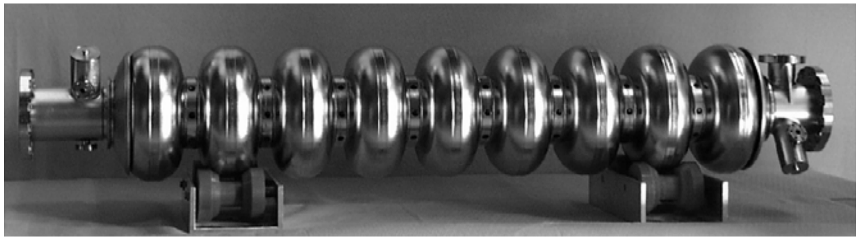




- The frequency is increased and tuned by a tuning plunger.
- An inductive coupling loop excites the magnetic field at the equator of the cavity.

Superconducting Cavities

CHESS & LEPP



Q = 10^{10}

E = 20MV/m

A bell with this Q would ring for a year.

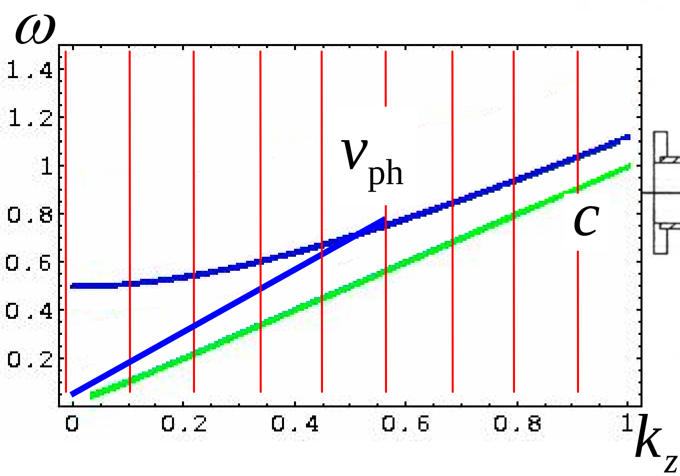
- Very low wall losses.
- Therefore continuous operation is possible.
- Energy recovery becomes possible.

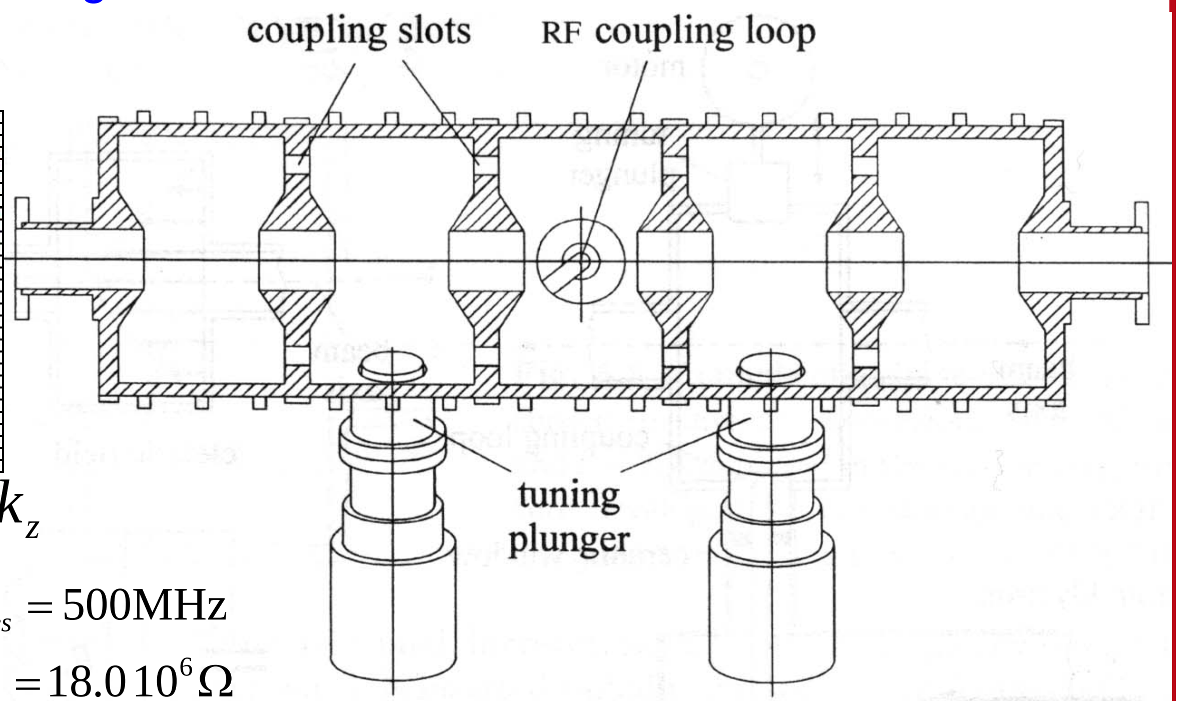
Normal conducting cavities

- Significant wall losses.
- Cannot operate continuously with appreciable fields.
- Energy recovery was therefore not possible.

Multicell Cavities

The filed in many cells can be excited by a single power source and a single input coupler in order to have the voltage of several cavities available.





Example: PETRA Cavity

$f_{res} = 500\text{MHz}$

$R_s = 18.0\, 10^6\,\Omega$

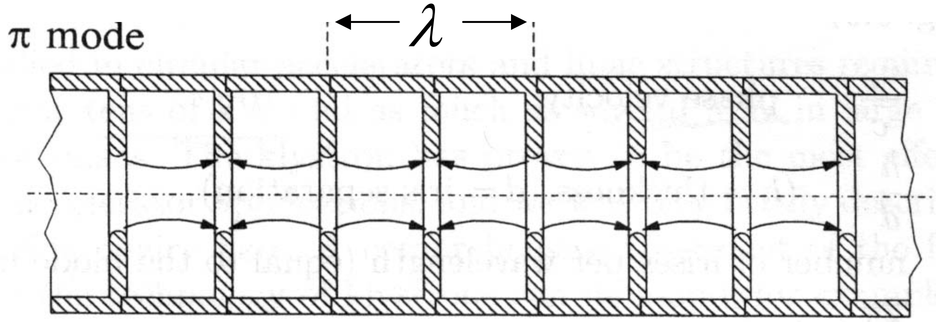
$125\text{kW} \rightarrow 2.12\text{MV}$

Without the walls: Long single cavity with too large wave velocity. $v_{\text{ph}} = \dfrac{\omega}{k}$

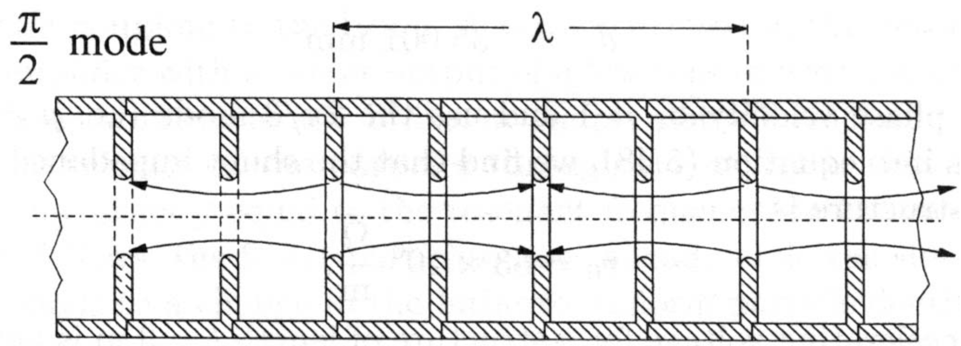
Thick walls: shield the particles from regions with decelerating phase.

Modes in Waveguides

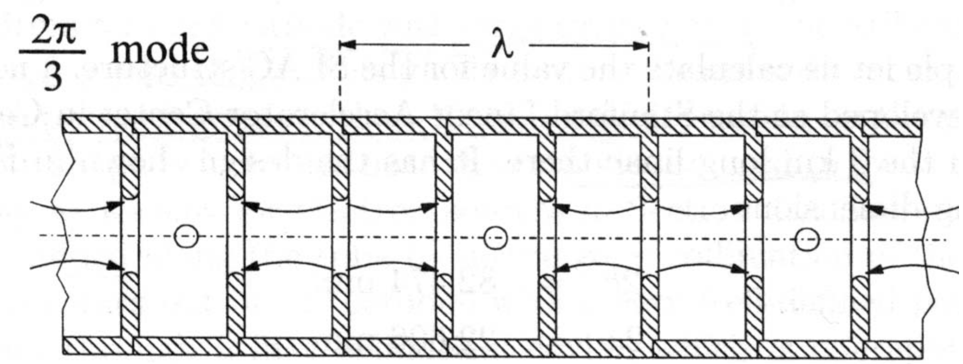
The iris size is chosen to let the phase velosity equal the particle velocity.



Long initial settling or filling time, not good for pulsed operation.



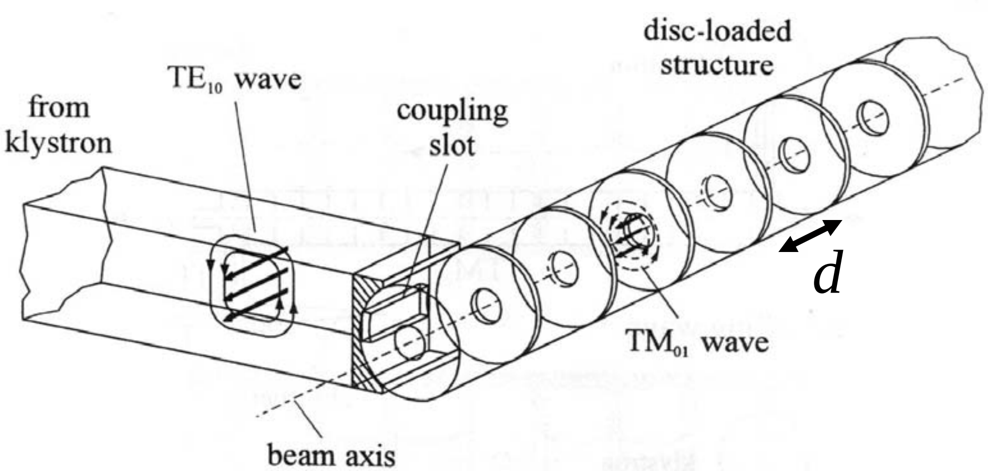
Small shunt impedance per length.

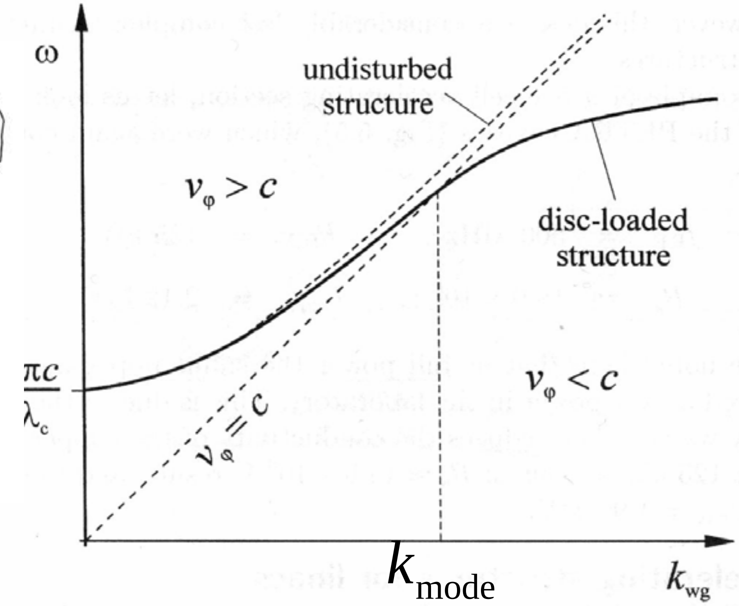


Common compromise.

Disc Loaded Waveguides

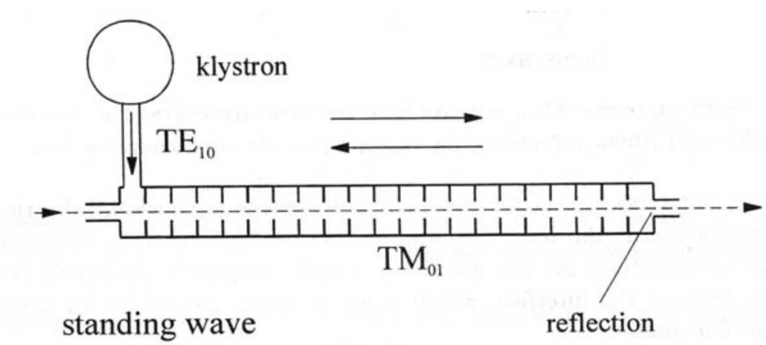
The iris size is chosen to let the phase velocity equal the particle velocity.



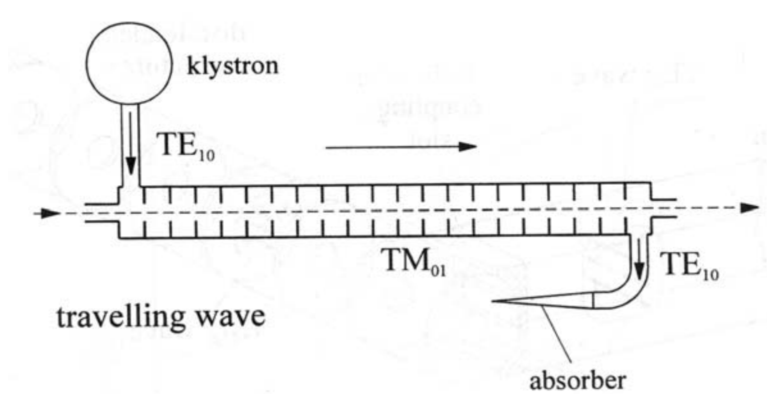


Loss free propergation: $k = \frac{2\pi}{nd}$

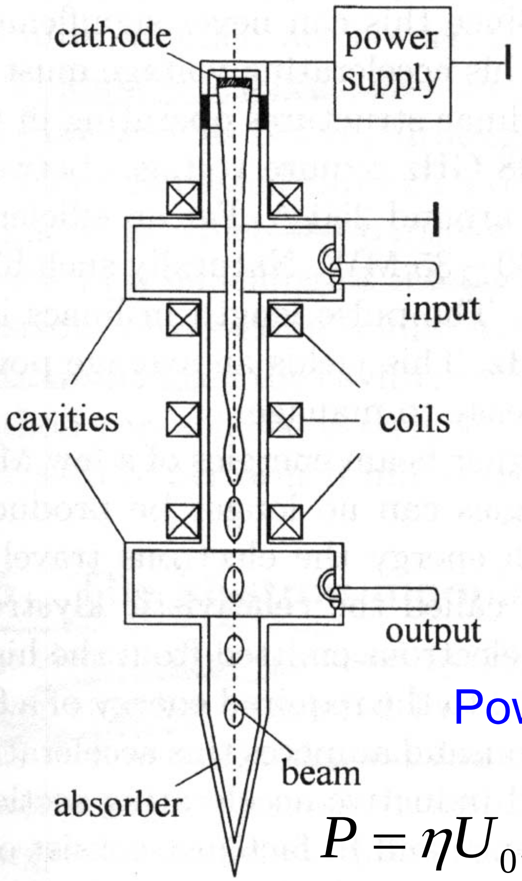
Standing wave cavity.



Traveling wave cavity (wave guide).



The Klystron as Power Source



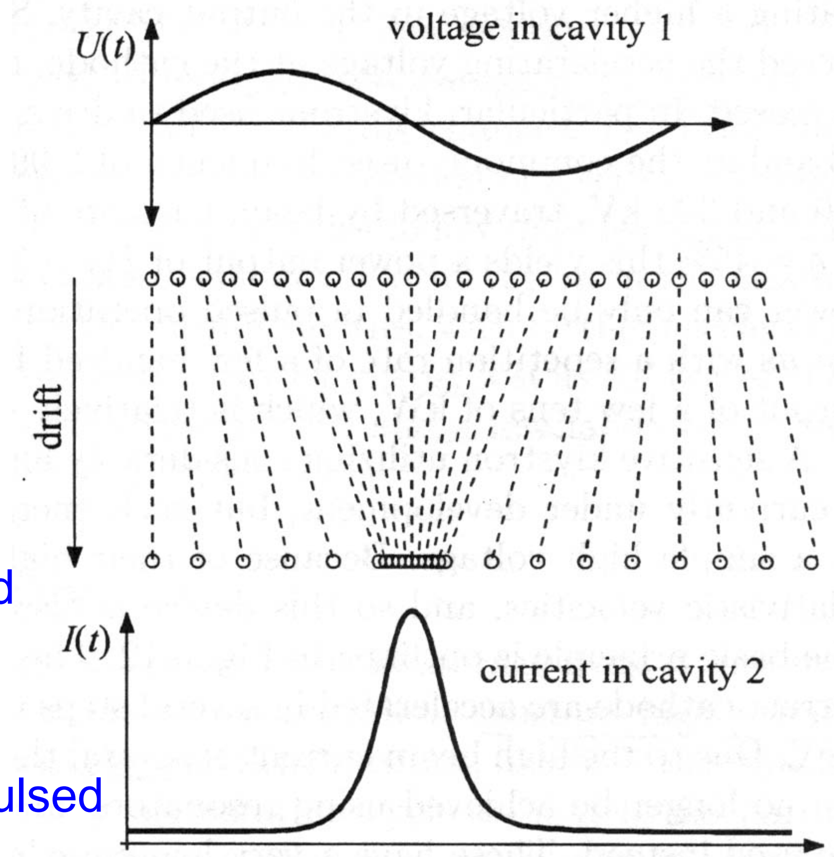
I up to > 10A

Power < 1.5MW

Power < 40MW pulsed

$$P = \eta U_0 I_{\text{beam}}\,, \quad \eta \le 65\%$$

Time of flight bunching



- DC acceleration to several 10kV, 100kV pulsed
- Energy modulation with a cavity
- Time of flight density modulation
- Excitation of a cavity with output coupler

Only works for non-relativistic electrons