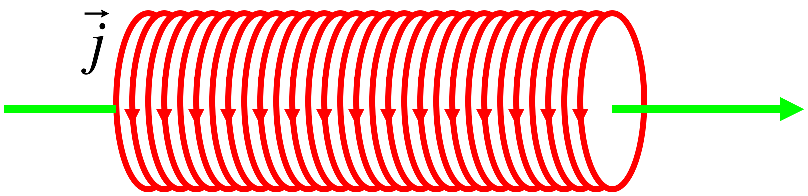
Multipoles in Accelerators ν=0: Solenoids



$$m\gamma\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} = q\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix}\times\begin{pmatrix}-\frac{x}{2}B_z'\\ -\frac{y}{2}B_z'\\ B_z\end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = \frac{qB_z}{m\gamma}\begin{pmatrix}\dot{y}\\ -\dot{x}\end{pmatrix} + \frac{qB_z'\dot{z}}{2m\gamma}\begin{pmatrix}y\\ -x\end{pmatrix}$$

$$\Downarrow$$

$$\ddot{w} = -i\frac{qB_z}{m\gamma}\dot{w} - i\frac{q\dot{B}_z}{2m\gamma}w$$

$$\psi = \Psi_0(z) - \frac{w\bar{w}}{4}\Psi_0''(z) \pm \dots$$

$$\vec{B} = \begin{pmatrix}\frac{x}{2}\Psi_0''\\ \frac{y}{2}\Psi_0''\\ -\Psi_0'\end{pmatrix} \Rightarrow \vec{\nabla}\cdot\vec{B} = 0$$

$$g = \frac{qB_z}{2m\gamma} \quad , \quad w_0 = w\, e^{i\int_0^t g\,dt}$$

$$\ddot{w}_0 = (\ddot{w} + i2g\dot{w} + i\dot{g}w - g^2 w)\, e^{i\int_0^t g\,dt}$$
$$= -g^2 w_0$$

$$\ddot{x}_0 = -g^2 x_0$$
$$\ddot{y}_0 = -g^2 y_0$$

Focusing in a rotating coordinate system

Solenoid vs. Strong Focusing

If the solenoids field was perpendicular to the particle's motion,

its bending radius would be $\rho_z = \frac{p}{qB_z}$

$$\ddot{\vec{r}} = -\left(\frac{qB_z}{2m\gamma}\right)^2 \vec{r} = -\frac{qv_z}{m\gamma} B_z \frac{\vec{r}}{4\rho_z}$$

Solenoid focusing is weak compared to the deflections created by a transverse magnetic field.

Transverse fields: $\vec{B} = B_x\,\vec{e}_x + B_y\,\vec{e}_y$

$$m\gamma \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = q \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \frac{qv_z}{m\gamma} \begin{pmatrix} -B_y \\ B_x \end{pmatrix}$$

Strong focusing

Weak focusing < Strong focusing by about r/ρ

Solenoid Focusing

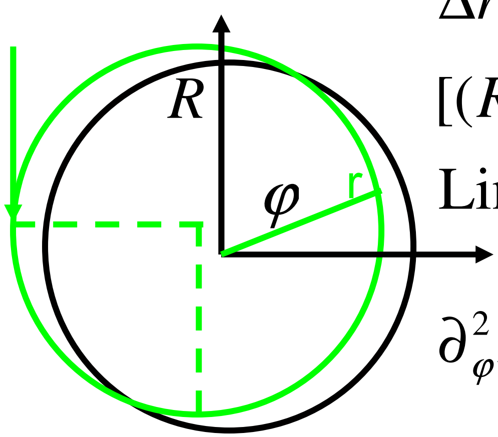
Solenoid magnets are used in detectors for particle identification via $\rho = \frac{p}{qB}$

The solenoid's rotation $\dot{\varphi} = -\frac{qB_z}{2m\gamma}$ of the beam is often compensated by a reversed solenoid called compensator.

Solenoid or Weak Focusing:

Solenoids are also used to focus low γ beams: $\ddot{w} = -\left(\frac{qB_z}{2m\gamma}\right)^2 w$

Weak focusing from natural ring focusing:



$$\Delta r = r - R$$

$$[(R+\Delta r)\cos\varphi - \Delta x_0]^2 + [(R+\Delta r)\sin\varphi - \Delta y_0]^2 = R^2$$

$$\text{Linearization in } \Delta: \quad \Delta r = (\cos\varphi\,\Delta x_0 + \sin\varphi\,\Delta y_0)$$

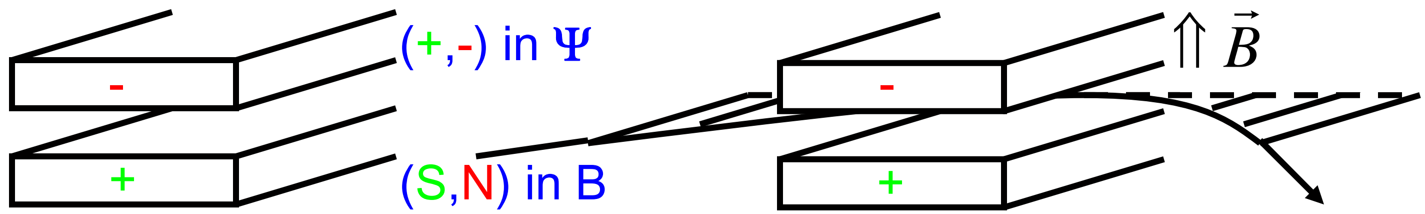
$$\partial_\varphi^2 \Delta r = -\Delta r \quad \Rightarrow \quad \Delta\ddot{r} = -\dot{\varphi}^2 \Delta r = -\left(\frac{v}{\rho}\right)^2 \Delta r = -\left(\frac{qB}{m\gamma}\right)^2 \Delta r$$

Multipoles in Accelerators   ν=1: Dipoles

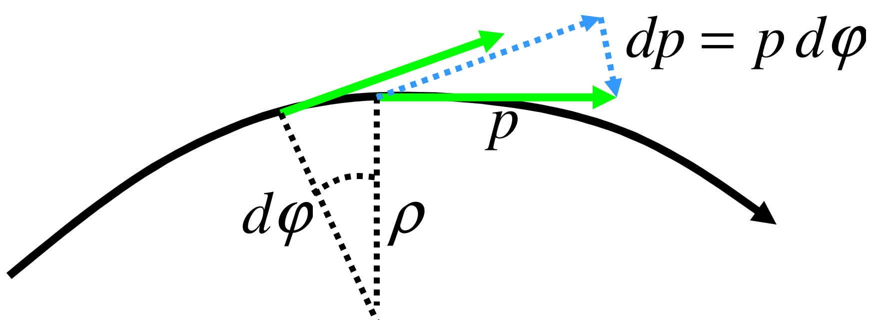
$$\psi = \Psi_1 \, \mathrm{Im}\{x - iy\} = -\Psi_1 \cdot y \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_1 \, \vec{e}_y$$

Equipotential
$y = \text{const.}$

C_1 Symmetry



Dipole magnets are used for steering the beams direction



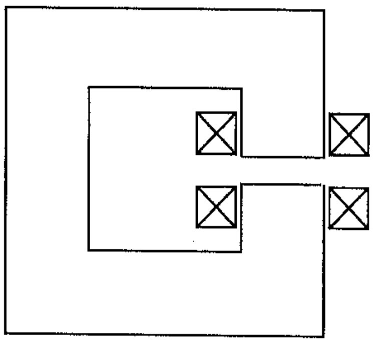
$$\frac{d\vec{p}}{dt} = q\,\vec{v} \times \vec{B} \quad \Rightarrow \quad \frac{dp}{dt} = qvB_\perp \quad \Rightarrow \quad \rho = \frac{dl}{d\varphi} = \frac{vdt}{dp/p} = \frac{p}{qB_\perp}$$

Bending radius: $\rho = \dfrac{p}{qB}$

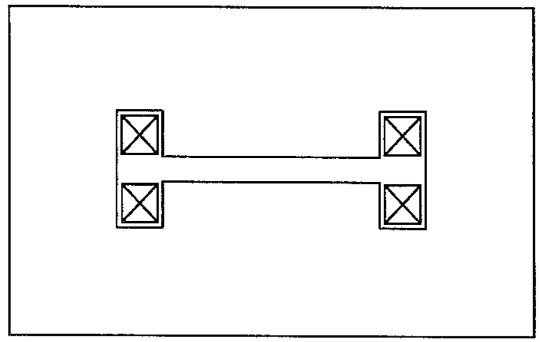
Different Dipoles

CHESS & LEPP

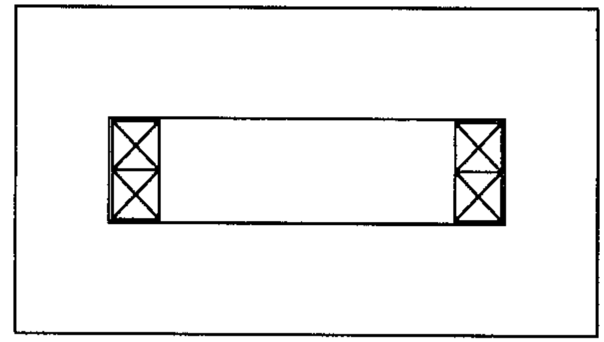
C-shape magnet:

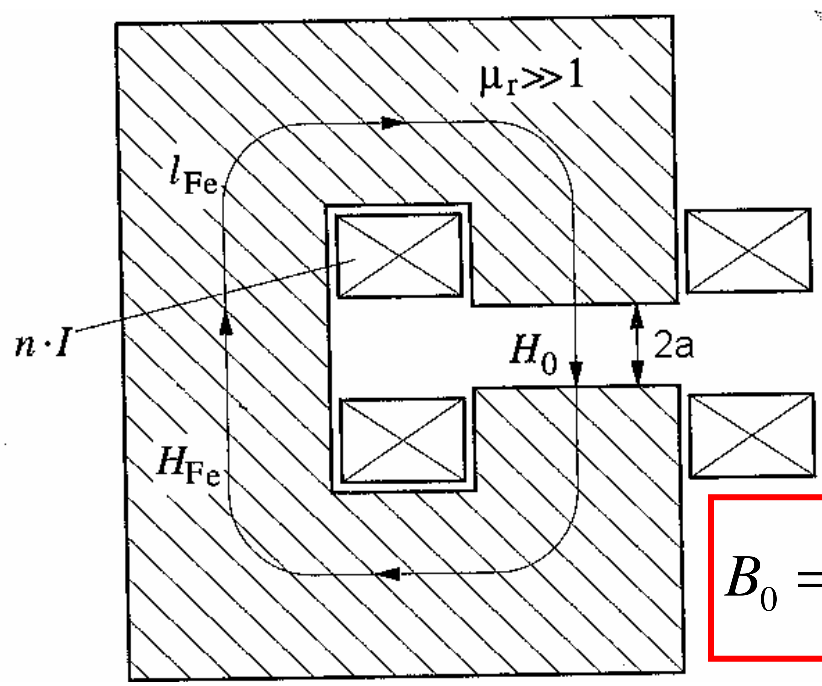


H-shape magnet:



Window frame magnet:





$$\vec{B}_\perp(\text{out}) = \vec{B}_\perp(\text{in})$$

$$\vec{H}_\perp(\text{out}) = \mu_r \vec{H}_\perp(\text{in})$$

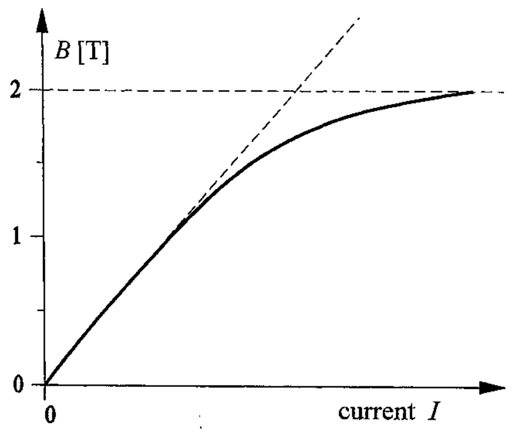
$$2nI = \oint \vec{H}\cdot d\vec{s} = H_{Fe} l_{Fe} + H_0 2a$$

$$= \frac{1}{\mu_r} H_0 l_{Fe} + H_0 2a \approx H_0 2a$$

$$B_0 = \mu_0 \frac{nI}{a}$$

Dipole strength: $\frac{1}{\rho} = \frac{q\mu_0}{p}\frac{nI}{a}$

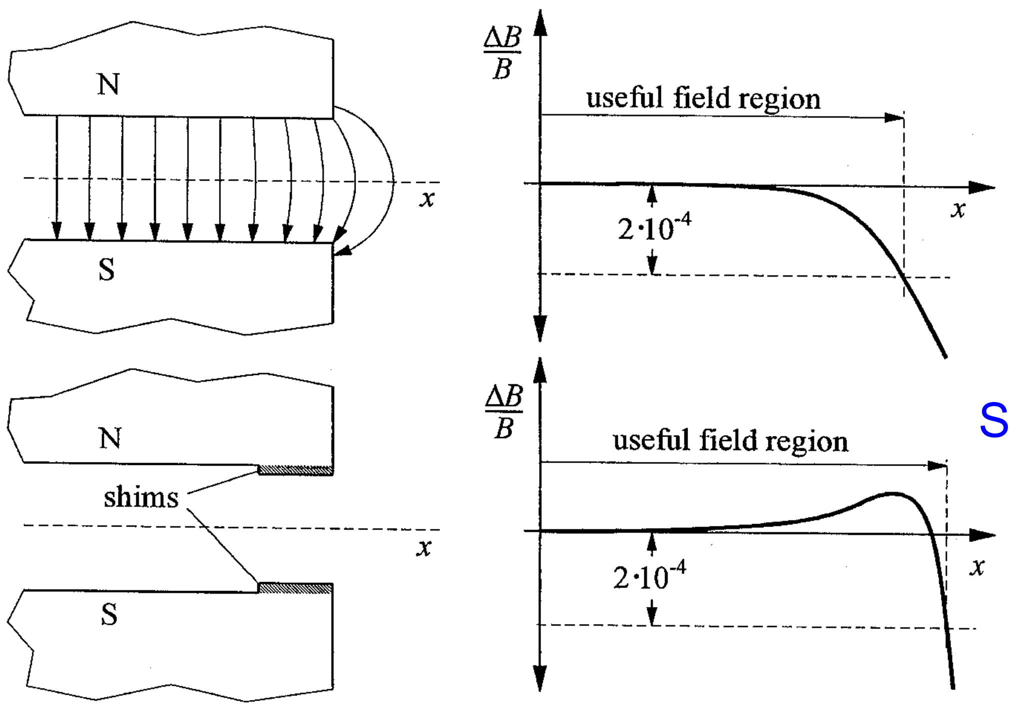
Dipole Fields



B = 2 T: Typical limit, since the field becomes dominated by the coils, not the iron. Limiting j for Cu is about 100A/mm²

B < 1.5 T: Typically used region

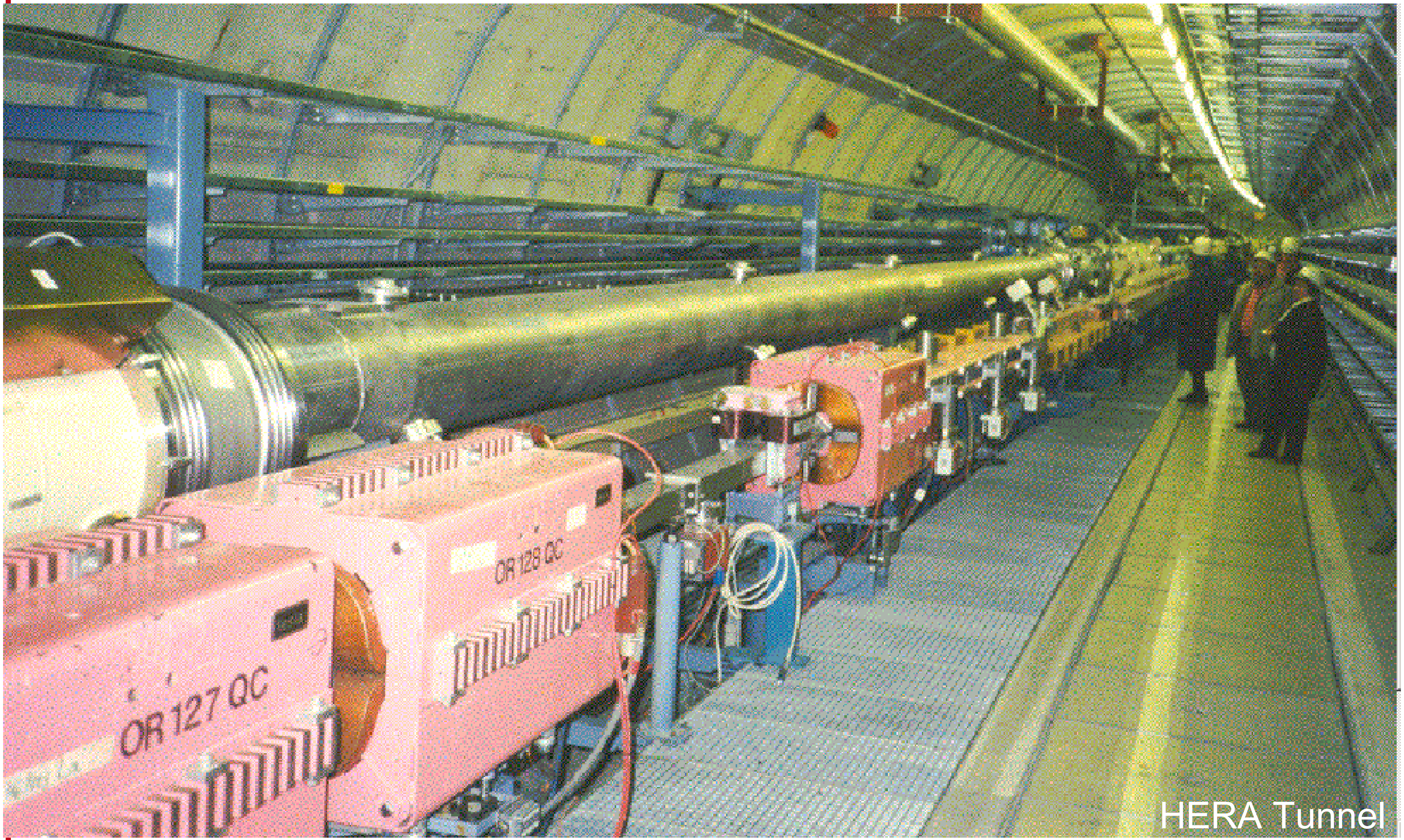
B < 1 T: Region in which $B_0 = \mu_0 \frac{nI}{a}$



Shims reduce the space that is open to the beam, but they also reduce the fringe field region.

Where is the vertical Dipole?

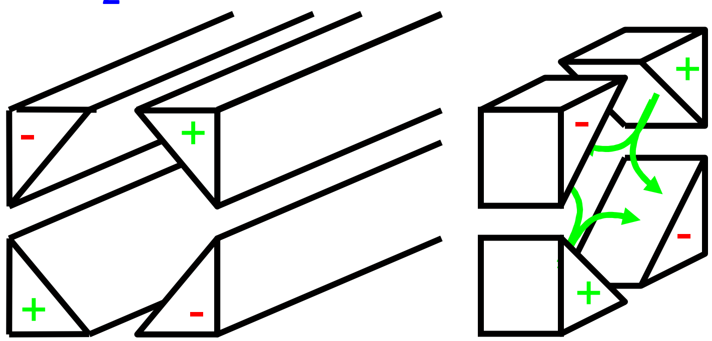
CHESS & LEPP

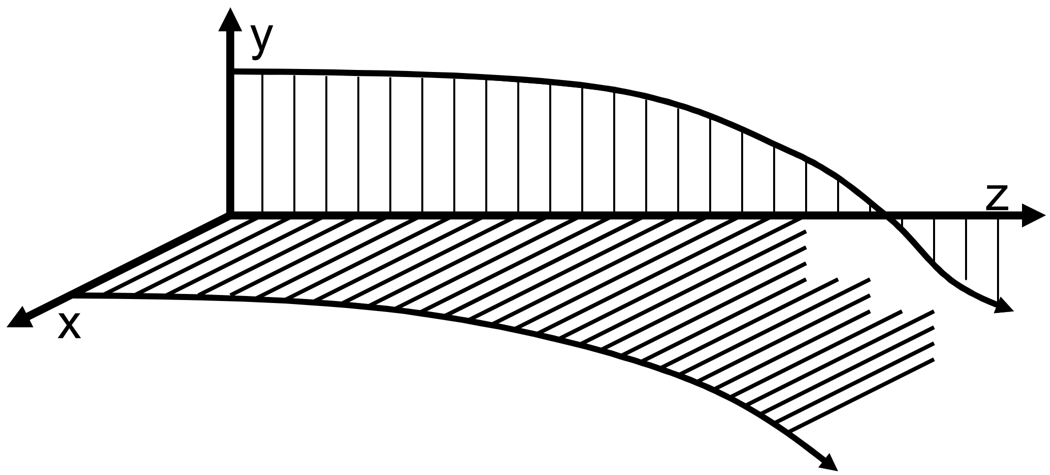


Multipoles in Accelerators ν=2: Quadrupoles

$$\psi = \Psi_2 \,\mathrm{Im}\{(x-iy)^2\} = -\Psi_2 \cdot 2xy \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_2 \, 2\begin{pmatrix} y \\ x \end{pmatrix}$$

C_2 Symmetry

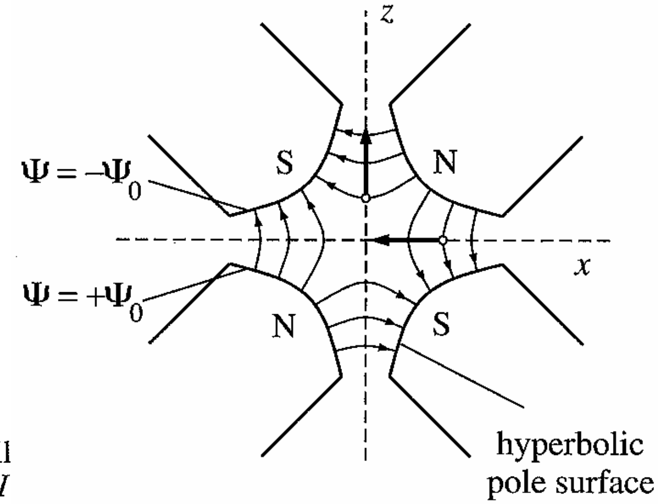


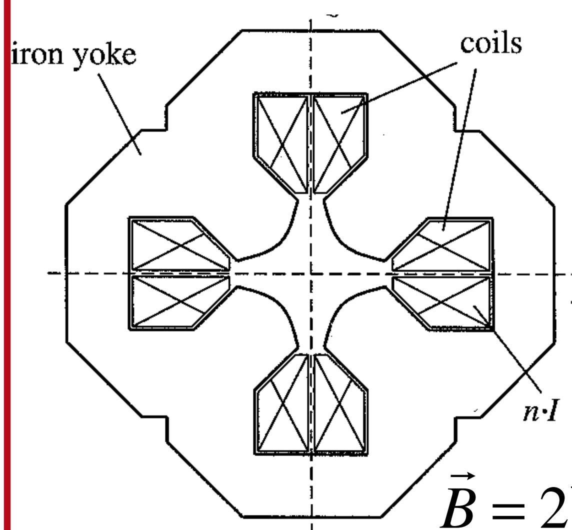


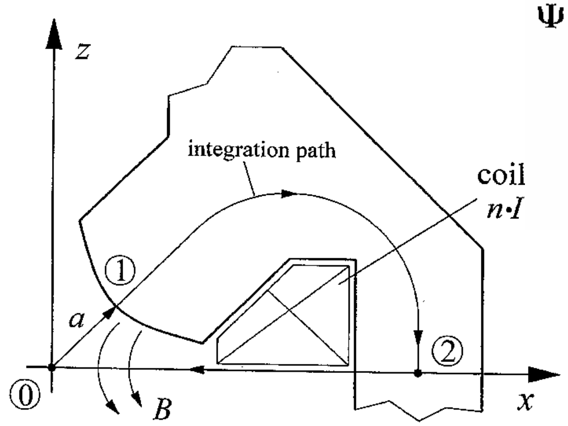
In a quadrupole particles are focused in one plane and defocused in the other plane. Other modes of strong focusing are not possible.

Quadrupole Fields

$$\psi = -\Psi_2 \cdot 2xy \quad \Rightarrow \quad \text{Equipotential: } x = \frac{\text{const.}}{y}$$







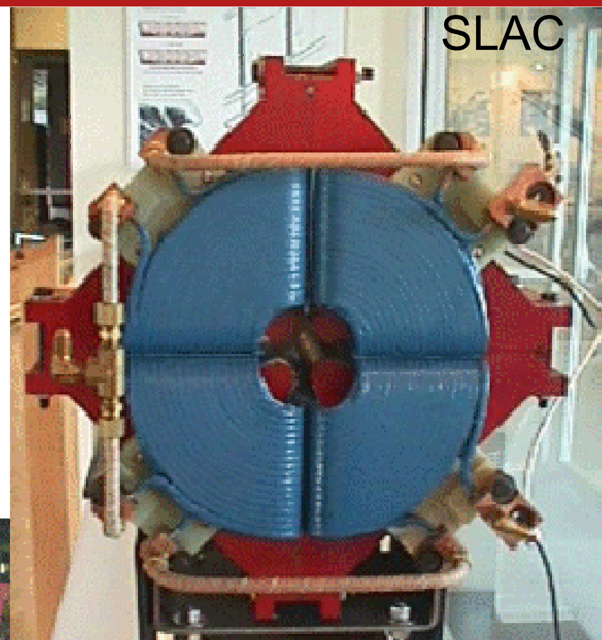
$$\vec{B} = 2\Psi_2 \begin{pmatrix} y \\ x \end{pmatrix} \quad \Rightarrow \quad \vec{B}(0 \mapsto 1) = 2\Psi_2 r \vec{e}_r$$

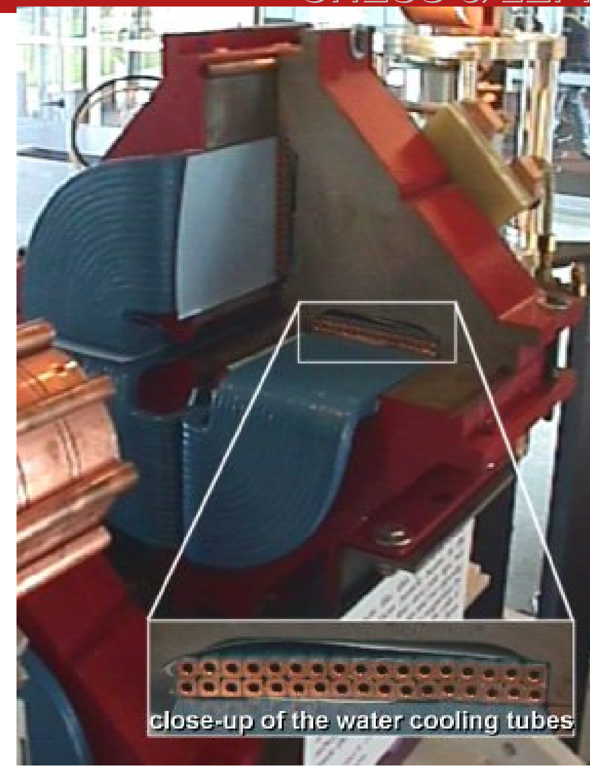
Quadrupole strength:

$$nI = \oint \vec{H} \cdot d\vec{s} \approx \int_0^a H_r dr = \Psi_2 \frac{a^2}{\mu_0}$$

$$k_1 = \frac{q}{p} \partial_x B_y \Big|_0 = \frac{q\mu_0}{p} \frac{2nI}{a^2}$$

Real Quadrupoles





The coils show that this is an upright quadrupole not a rotated or skew quadrupole.