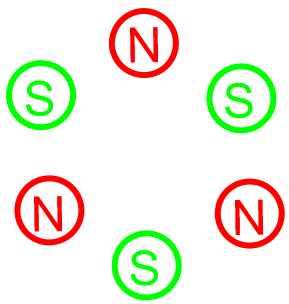
Multipoles in Accelerators   ν=3: Sextupoles

$$\psi = \Psi_3 \,\mathrm{Im}\{(x-iy)^3\} = \Psi_3 \cdot (y^3 - 3x^2 y) \quad \Rightarrow \quad \vec{B} = -\vec{\nabla}\psi = \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

C_3 Symmetry



$$\vec{B} = -\vec{\nabla}\psi = \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix}$$

$$x \mapsto \Delta x + x$$

$$\vec{B} \approx \Psi_3 \, 3 \begin{pmatrix} 2xy \\ x^2 - y^2 \end{pmatrix} + 6\Psi_3 \Delta x \begin{pmatrix} y \\ x \end{pmatrix} + O(\Delta x^2)$$

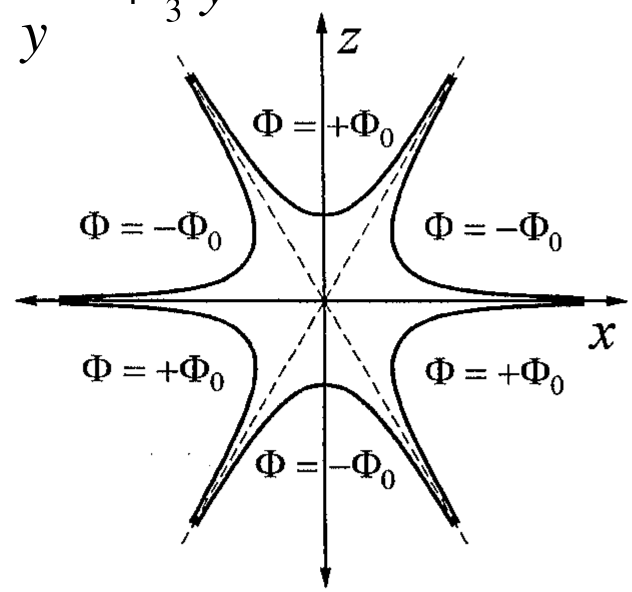
i) Sextupole fields hardly influence the particles close to the center, where one can linearize in x and y.

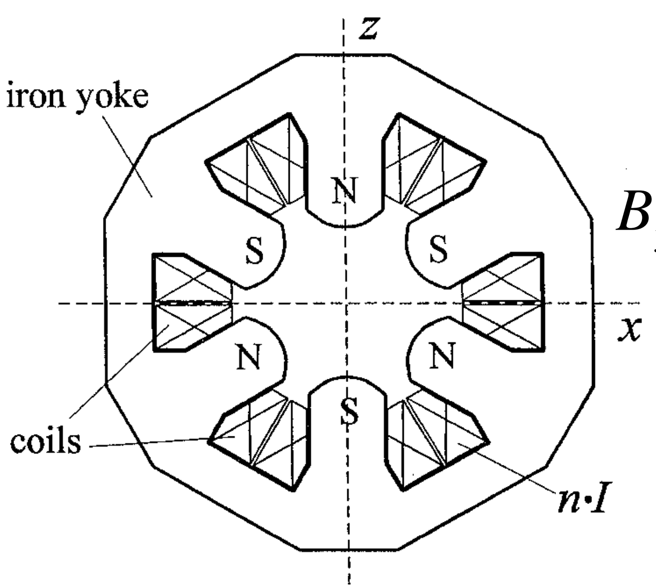
ii) In linear approximation a by Δx shifted sextupole has a quadrupole field.

iii) When Δx depends on the energy, one can build an energy dependent quadrupole.

Sextupole Fields

$$\psi = \Psi_2 \cdot (y^3 - 3x^2 y) \quad \Rightarrow$$ Equipotential: $$x = \sqrt{\frac{\text{const.}}{y} + \tfrac{1}{3} y^2}$$





$$B_y\Big|_{x=0} = -\Psi_3 3y^2$$

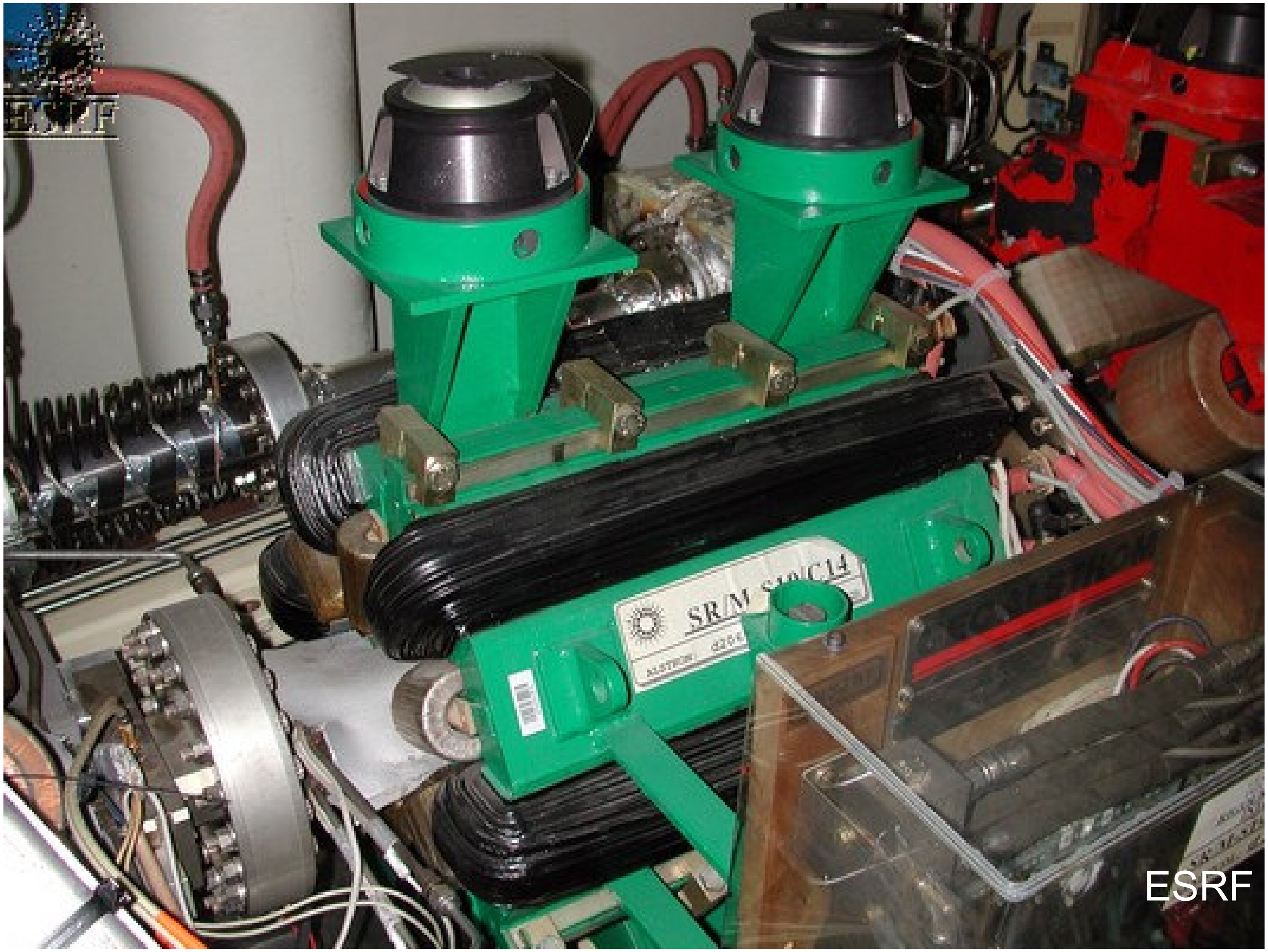
$$nI = \oint \vec{H} \cdot d\vec{s} \approx \int_0^a H_r dr = \Psi_3 \frac{a^3}{\mu_0}$$

Quadrupole strength:

$$k_2 = \frac{q}{p} \partial_x^2 B_y \Big|_0 = \frac{q\mu_0}{p} \frac{6nI}{a^3}$$

Real Sextupoles

CHESS & LEPP



The CESR Tunnel

Higher order Multipoles

CHESS & LEPP

$$\psi = \Psi_n \,\mathrm{Im}\{(x-iy)^n\} = \Psi_n \cdot (\ldots - i\,n\,x^{n-1}y) \quad \Rightarrow \quad \vec{B}(y=0) = \Psi_n\, n \begin{pmatrix} 0 \\ x^{n-1} \end{pmatrix}$$

Multipole strength: $k_n = \frac{q}{p}\partial_x^n B_y\Big|_{x,y=0} = \frac{q}{p}\Psi_{n+1}\,(n+1)!$ units: $\frac{1}{\mathrm{m}^{n+1}}$

p/q is also called Bρ and used to describe the energy of multiply charge ions

Names: dipole, quadrupole, sextupole, octupole, decapole, duodecapole, …

Higher order multipoles come from

- Field errors in magnets
- Magnetized materials
- From multipole magnets that compensate such erroneous fields
- To compensate nonlinear effects of other magnets
- To stabilize the motion of many particle systems
- To stabilize the nonlinear motion of individual particles

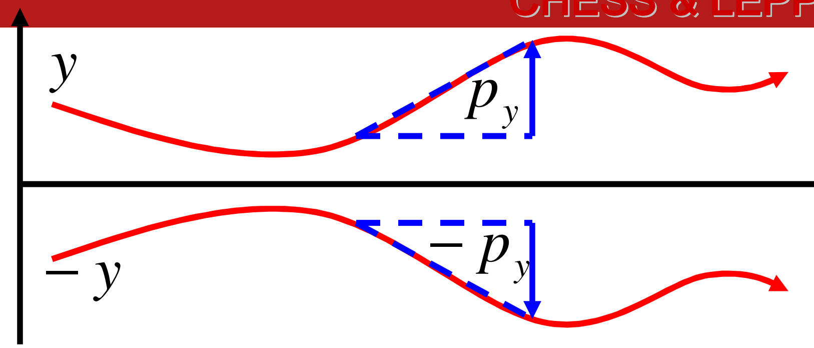
Midplane Symmetric Motion

CHESS & LEPP

$$\vec{r}^{\oplus} = (x, -y, z)$$

$$\vec{p}^{\oplus} = (p_x, -p_y, p_z)$$

$$\frac{d}{dt}\vec{p} = \vec{F}(\vec{r}, \vec{p}) \quad \Rightarrow \quad \frac{d}{dt}\vec{p}^{\oplus} = \vec{F}(\vec{r}^{\oplus}, \vec{p}^{\oplus})$$



$$v_y B_z - v_z B_y = -v_y B_z(x,-y,z) - v_z B_y(x,-y,z) \qquad B_x(x,-y,z) = -B_x(x,y,z)$$
$$v_z B_x - v_x B_z = -v_z B_x(x,-y,z) + v_x B_z(x,-y,z) \Rightarrow \quad B_y(x,-y,z) = B_y(x,y,z)$$
$$v_x B_y - v_y B_x = v_x B_y(x,-y,z) + v_y B_x(x,-y,z) \qquad B_z(x,-y,z) = -B_z(x,y,z)$$

$$\boxed{\psi(x,-y,z) = -\psi(x,y,z)}$$

$$\Psi_n \operatorname{Im}\{e^{in\vartheta_n}(x+iy)^n\} = -\Psi_n \operatorname{Im}\{e^{in\vartheta_n}(x+iy)^n\}$$
$$\Rightarrow \quad \Psi_n \operatorname{Im}[e^{in\vartheta_n} 2\operatorname{Re}\{(x+iy)^n\}] = 0 \quad \Rightarrow \boxed{\vartheta_n = 0}$$

The discussed multipoles

produce midplane symmetric motion. When the field is rotated by $\pi/2$, i.e $\vartheta_n = \pi/2n$, one speaks of a skew multipole.

Superconducting Magnets

CHESS & LEPP

Above 2T the field from the bare coils dominate over the magnetization of the iron.

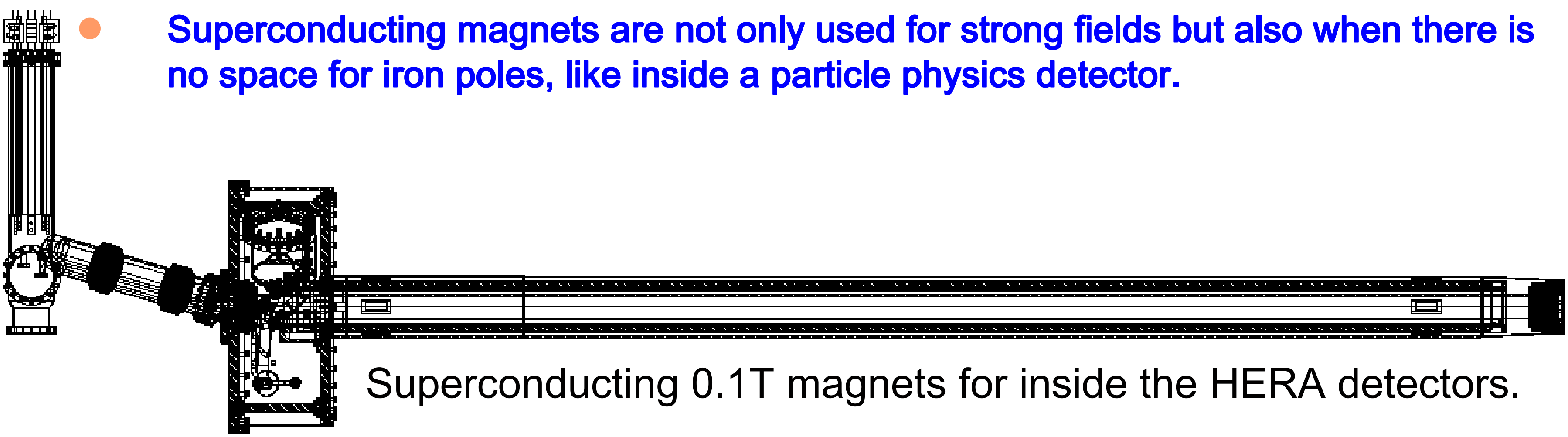
But Cu wires cannot create much filed without iron poles:

5T at 5cm distance from a 3cm wire would require a current density of

$$j = \frac{I}{d^2} = \frac{1}{d^2}\frac{2\pi\, r\, B}{\mu_0} = 1389\frac{\text{A}}{\text{mm}^2}$$

Cu can only support about 100A/mm^2.

- Superconducting cables routinely allow current densities of 1500A/mm^2 at 4.6 K and 6T. Materials used are usually Nb aloys, e.g. NbTi, Nb_3Ti or Nb_3Sn.
- Superconducting magnets are not only used for strong fields but also when there is no space for iron poles, like inside a particle physics detector.

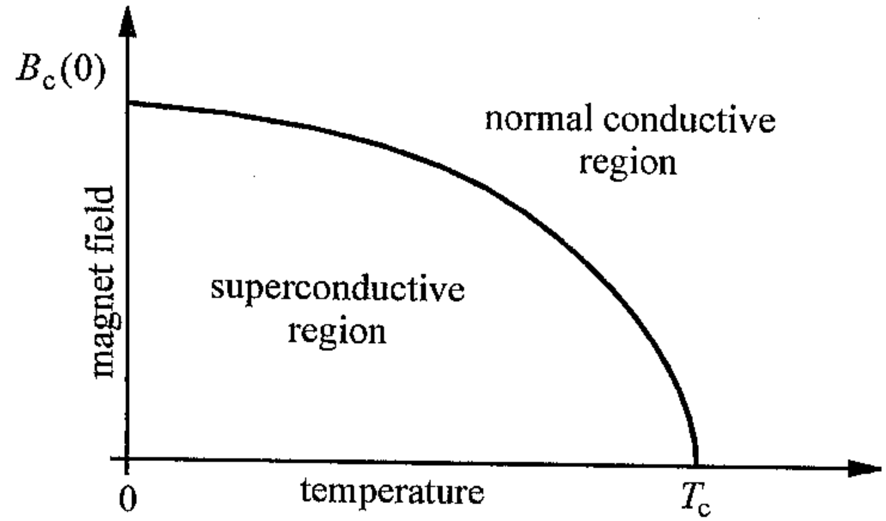


Superconducting 0.1T magnets for inside the HERA detectors.

Superconducting Magnets

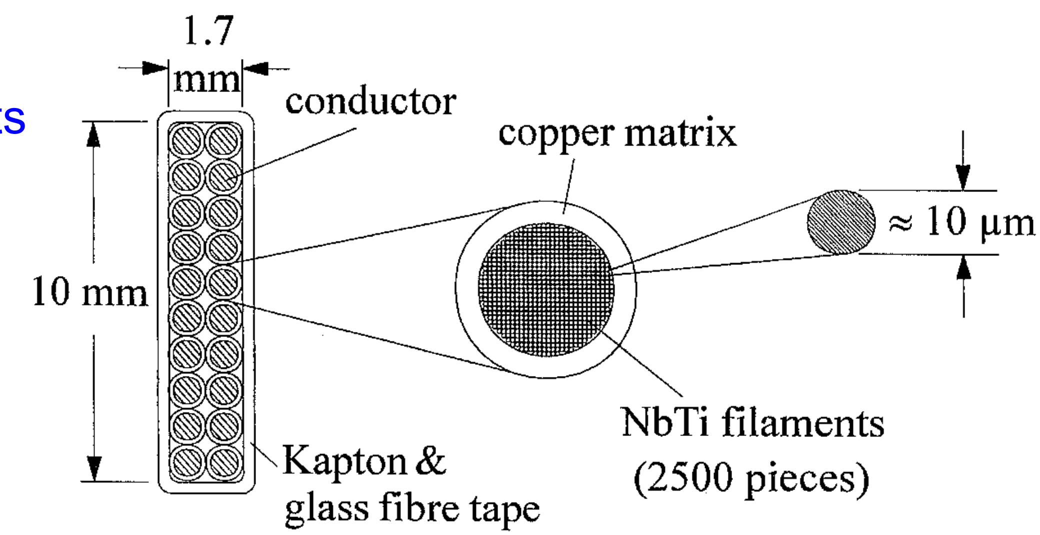
Problems:

- Superconductivity brakes down for too large fields
- Due to the Meissner-Ochsenfeld effect superconductivity current only flows on a thin surface layer.



Remedy:

- Superconducting cable consists of many very thin filaments (about 10µm).



Complex Potential of a Wire

Straight wire at the origin: $\vec{\nabla}\times\vec{B}=\mu_0\,\vec{j} \quad\Rightarrow\quad \vec{B}(r)=\frac{\mu_0\,I}{2\pi\,r}\vec{e}_\varphi=\frac{\mu_0\,I}{2\pi\,r}\begin{pmatrix}-y\\ x\end{pmatrix}$

Wire at $\vec{a}$:

$$\vec{B}(x,y)=\frac{\mu_0\,I}{2\pi\,(\vec{r}-\vec{a})^2}\begin{pmatrix}-[y-a_y]\\ x-a_x\end{pmatrix}$$

This can be represented by complex multipole coefficients Ψ_ν

$$\vec{B}(x,y)=-\vec{\nabla}\Psi \quad\Rightarrow\quad B_x+iB_y=-\left(\partial_x+i\partial_y\right)\psi=-2\partial_{\overline{w}}\psi$$

$$B_x+iB_y=\frac{\mu_0 I}{2\pi}\frac{-i(w_a-w)}{(w_a-w)(\overline{w}_a-\overline{w})}=i\,\frac{\mu_0 I}{2\pi}\frac{-\frac{w_a}{a^2}}{1-\frac{\overline{w}w_a}{a^2}}$$

$$=i\,\frac{\mu_0 I}{2\pi}\partial_{\overline{w}}\ln(1-\tfrac{\overline{w}w_a}{a^2})=-2\partial_{\overline{w}}\,\mathrm{Im}\left\{\frac{\mu_0 I}{2\pi}\ln(1-\tfrac{\overline{w}w_a}{a^2})\right\}$$

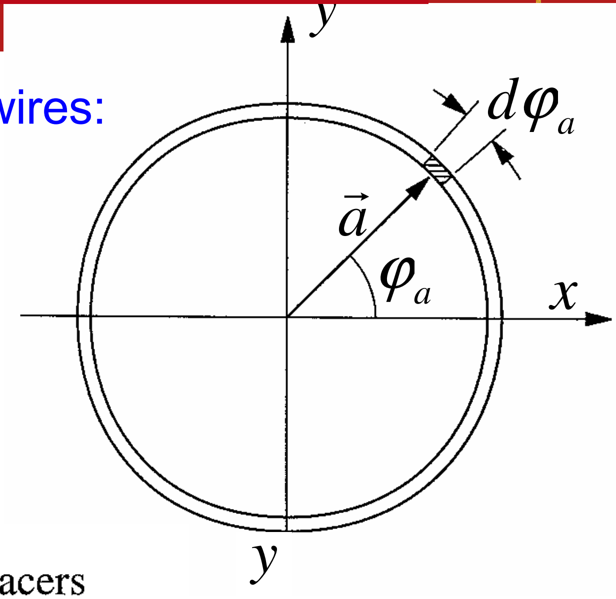
$$\psi=\mathrm{Im}\left\{\frac{\mu_0 I}{2\pi}\ln(1-\tfrac{\overline{w}w_a}{a^2})\right\}=-\mathrm{Im}\left\{\frac{\mu_0 I}{2\pi}\sum_{\nu=1}^{\infty}\frac{1}{\nu}\left(\frac{w_a}{a^2}\right)^\nu\overline{w}^\nu\right\} \quad\Rightarrow\quad \Psi_\nu=\frac{\mu_0 I}{2\pi}\frac{1}{\nu}\frac{1}{a^\nu}e^{i\nu\varphi_a}$$

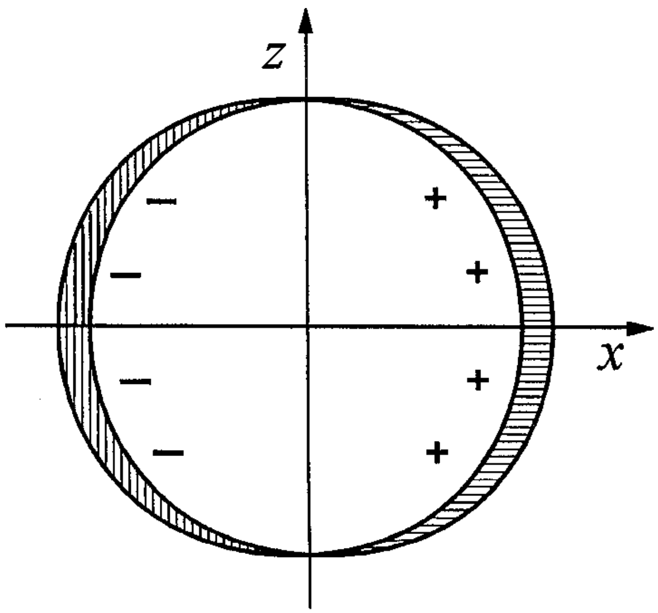
Air-coil Multipoles

Creating a multipole be created by an arrangement of wires:

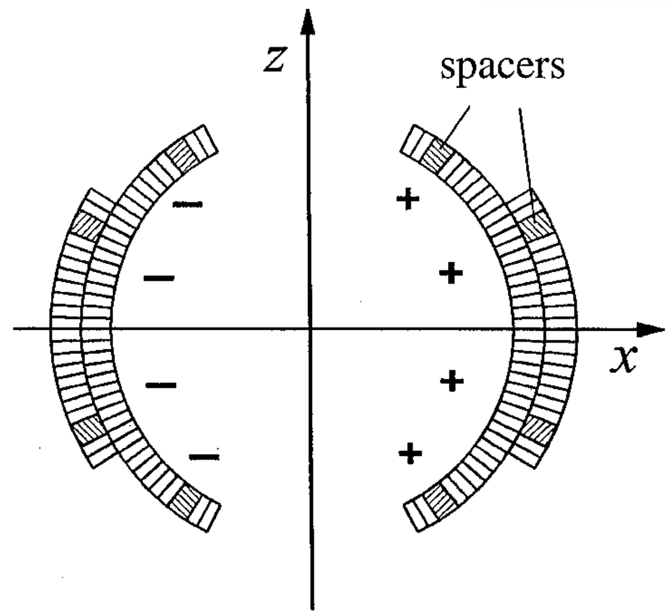
$$\Psi_\nu = \int_0^{2\pi} \frac{\mu_0}{2\pi} \frac{1}{\nu} \frac{1}{a^\nu} e^{i\nu\varphi_a} \frac{dI}{d\varphi_a} d\varphi_a$$

$$\Psi_\nu = \delta_{\nu n} \frac{\mu_0}{2} \frac{1}{n} \frac{1}{a^n} \hat{I} \quad \text{if } I(\varphi_a) = \hat{I} \cos n\varphi_a$$





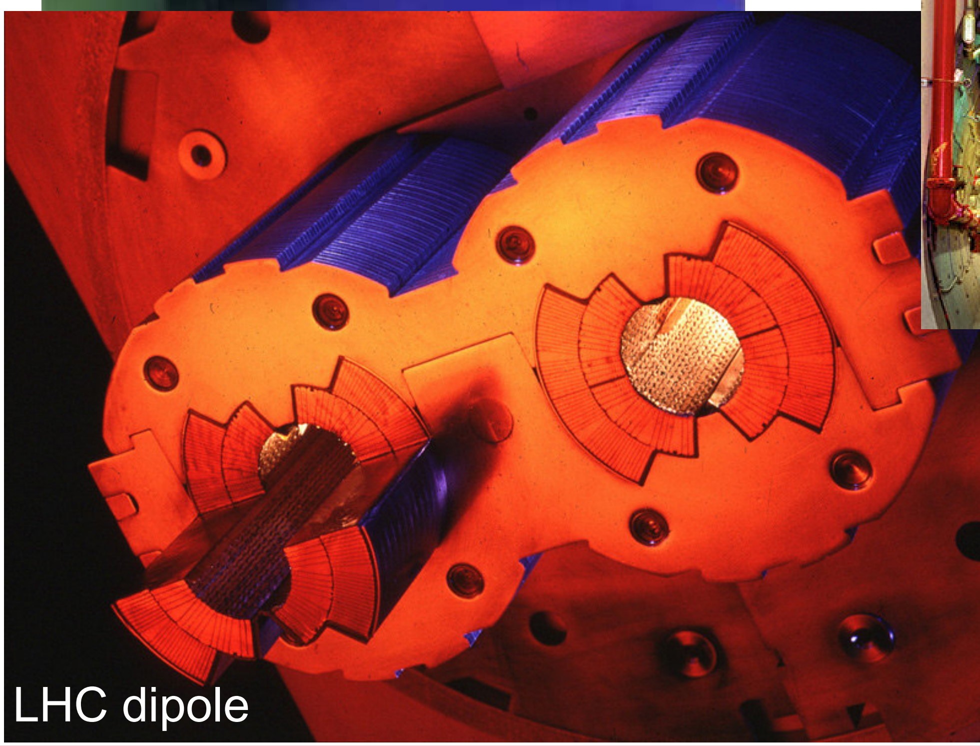
Ideal multipole



Approximate multipole

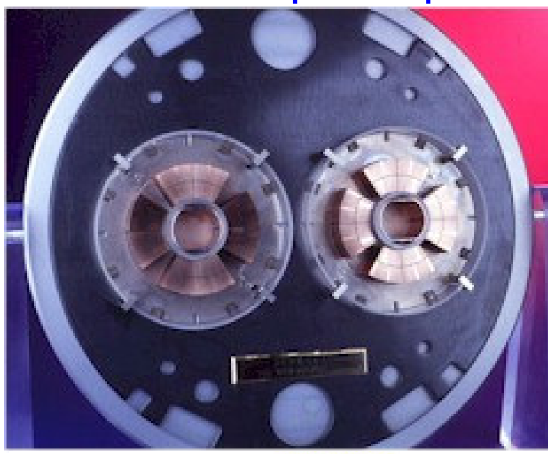
Real Air-coil Multipoles

Quadrupole corrector

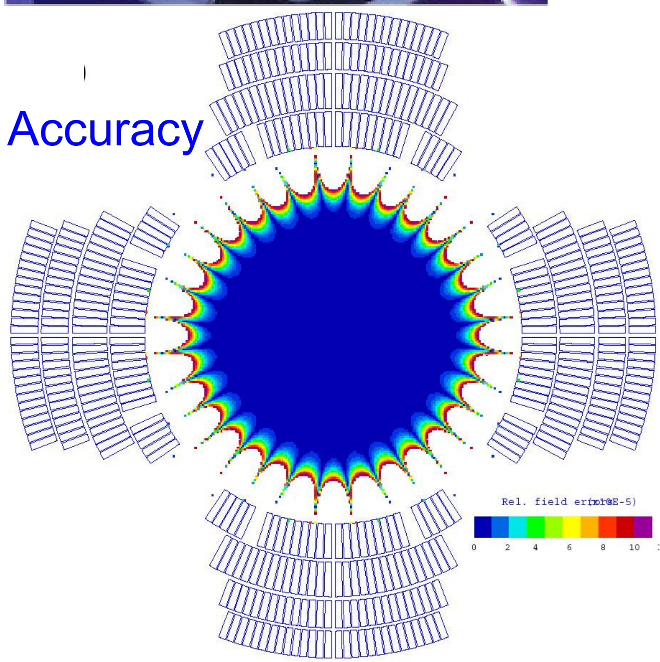


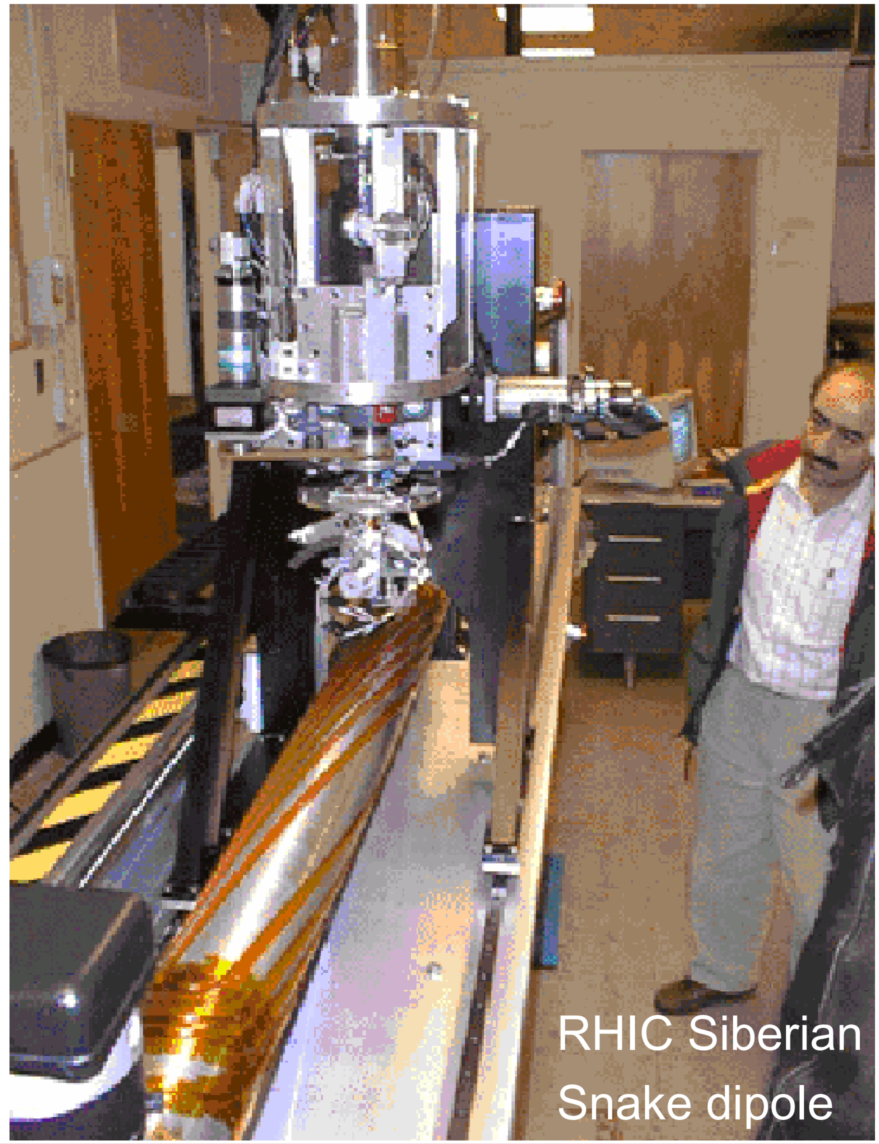
Special SC Air-coil Magnets

LHC double quadrupole



Accuracy





RHIC Siberian
Snake dipole