The beam-beam force

$$\vec{E}(x,y,z) \approx \frac{Q\rho_z}{2\pi\varepsilon_0}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x}\\ \frac{y}{\sigma_y}\end{pmatrix} \qquad \vec{B}(x,y,z)=0$$

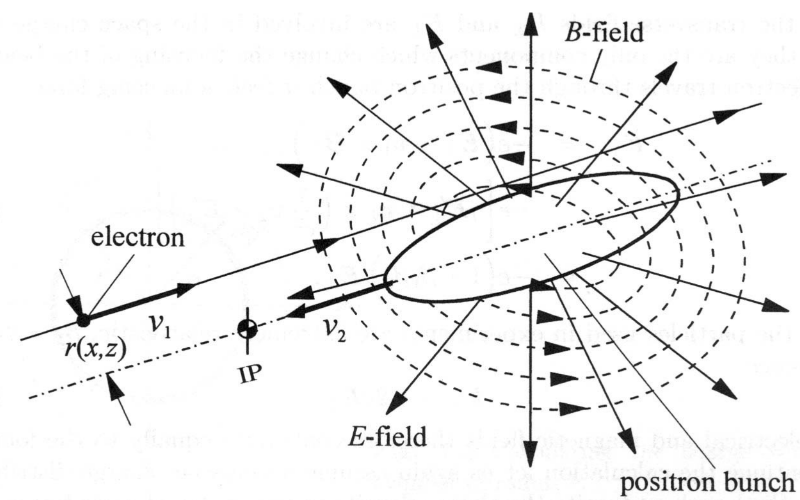
$$\left\{\begin{array}{l}\vec{E}_{\text{lab}}(x,y,z) \approx \frac{Q\rho_{\text{lab}\,z}}{2\pi\varepsilon_0}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x}\\ \frac{y}{\sigma_y}\end{pmatrix}\\ \vec{B}_{\text{lab}}(x,y,z)=\frac{1}{c}\vec{\beta}\times\vec{E}_{\text{lab}}(x,y,z)\end{array}\right.$$

$$\Delta\vec{p}(x,y)=\int\vec{F}(x,y,v^{(1)}t)dt$$

$$\approx \frac{Q}{2\pi\varepsilon_0}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x}\\ \frac{y}{\sigma_y}\end{pmatrix}\int\rho_{\text{lab}\,z}(v^{(1)}t+v^{(2)}t)dt(1+\beta^{(1)}\beta^{(2)})$$

$$=\frac{Q}{2\pi\varepsilon_0}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x}\\ \frac{y}{\sigma_y}\end{pmatrix}\frac{1+\beta^{(1)}\beta^{(2)}}{v^{(1)}+v^{(2)}}$$

$$\approx \frac{Q}{2\pi\varepsilon_0 c}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x}\\ \frac{y}{\sigma_y}\end{pmatrix}$$



The beam-beam tune shift

CHESS & LEPP

$$\begin{pmatrix}\Delta x' \\ \Delta y'\end{pmatrix} = \frac{\Delta \vec{p}}{p} \approx \frac{q^{(1)}Q}{2\pi\varepsilon_0 p^{(1)}c}\frac{1}{\sigma_x+\sigma_y}\begin{pmatrix}\frac{x}{\sigma_x} \\ \frac{y}{\sigma_y}\end{pmatrix} = \begin{pmatrix}k_x x \\ k_y y\end{pmatrix}$$

$$\Delta\nu_x^{(1)} \approx \frac{q^{(1)}Q}{8\pi^2\varepsilon_0 p^{(1)}c}\frac{\beta_x^{(1)}}{\sigma_x(\sigma_x+\sigma_y)} \approx \frac{r_{cl}^{(1)}N_{cpb}^{(2)}}{2\pi}\frac{\beta_x^{(1)}}{\sigma_x^{(2)}(\sigma_x^{(2)}+\sigma_y^{(2)})}$$

Tune spread over the beam,
amplitude dependent tune shift
and the tune shift cravat