Interaction rate

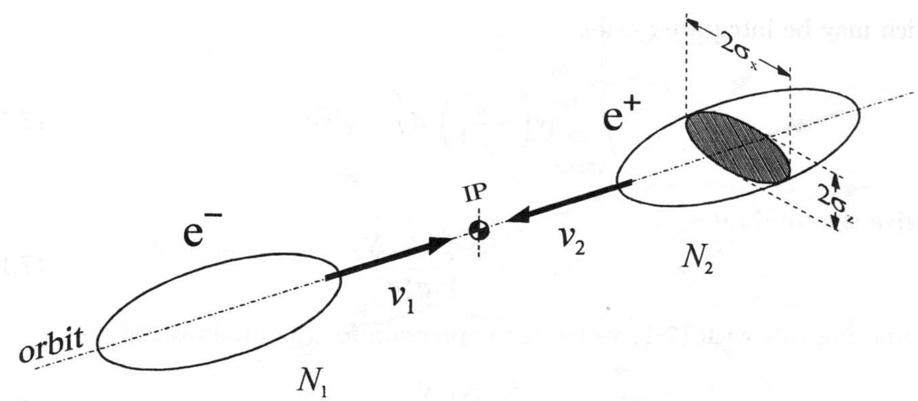
When the cross section for a process is known, the number of events per time is

$\dot{N}_{\text{events}} = L \cdot \sigma_{\text{cross section}}$ where the luminosity L is independent of the process.

$$L[\frac{1}{\text{cm}^2\text{s}}] = L\,10^{33}[\frac{1}{nb\ s}]$$

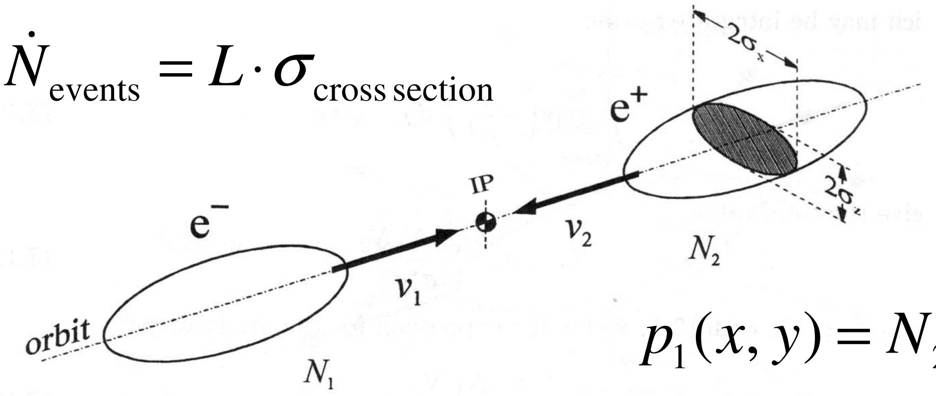
Integrated Luminosity: $\int L\,dt = N_{\text{events}} / \sigma_{\text{cross section}}$

Colliding beams:



Luminosity

$$\dot{N}_{\text{events}} = L \cdot \sigma_{\text{cross section}}$$



$$p_1(x,y) = N_2 \int \rho_2(x,y,\tau)\,d\tau \cdot \sigma_{\text{cross section}}$$

$$N_{events} = \sum p_1 = N_1 \int \rho_1(x,y,\tau)\, p_1(x,y) dx\,dy\,d\tau$$

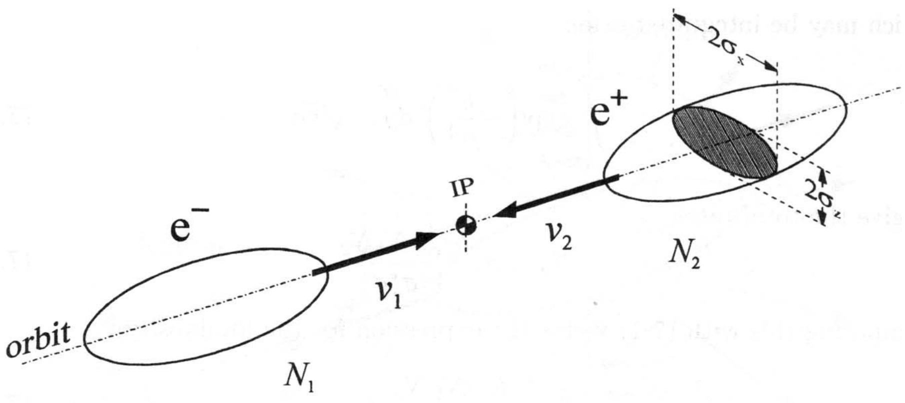
$$L = \frac{N_1 N_2}{\Delta t} \int [\int \rho_1(x,y,\tau)\,d\tau \int \rho_2(x,y,\tau)\,d\tau]\,dx\,dy$$

Gaussian beams:

$$L = \frac{N_{\text{bunch}} f_0 N_1 N_2}{4\pi^2 \sigma_{1y}\sigma_{1x}\sigma_{2y}\sigma_{2x}} \int e^{-\frac{x^2}{2\sigma_{1x}^2}} e^{-\frac{x^2}{2\sigma_{2x}^2}} e^{-\frac{y^2}{2\sigma_{1y}^2}} e^{-\frac{y^2}{2\sigma_{2y}^2}} dxdy$$

$$= \frac{N_{\text{bunch}} f_0 N_1 N_2}{2\pi\sqrt{\sigma_{1x}^2+\sigma_{2x}^2}\sqrt{\sigma_{1y}^2+\sigma_{2y}^2}} = \frac{N_{\text{bunch}} f_0 N_1 N_2}{2\pi\Sigma_x\Sigma_y} = \frac{1}{N_{\text{bunch}} f_0}\frac{I_1 I_2}{2\pi e^2 \Sigma_x \Sigma_y}$$

Luminosity



$$\dot{N}_{\text{events}} = L \cdot \sigma_{\text{cross section}}$$

$$p_1(x_0, y_0, z_0) = N_2 \int \rho_2(x_0 + v_x^{(1)} t, y_0 + v_y^{(1)} t, z_0 + v_z^{(1)} t, t) | \vec{v}_1 + \vec{v}_2 | \, dt \, \sigma_{cross}$$

$$N_{events} = \sum p_1 = N_1 \int \rho_1(x_0, y_0, z_0, 0) \, p_1(x_0, y_0, z_0) dx_0 dy_0 dz_0 \sigma_{cross}$$

$$L = \frac{N_1 N_2}{\Delta t} \int \rho_1(x, y, z, 0) \rho_2(x + v_x^{(1)} t, y + v_y^{(1)} + v_z^{(1)} t, t) | \vec{v}_1 + \vec{v}_2 | \, dt \, d^3 \vec{r}$$

$$L = \frac{N_1 N_2}{\Delta t} | \vec{v}_1 + \vec{v}_2 | \int \rho_1(\vec{r}, t) \rho_2(\vec{r}, t) dt \, d^3 \vec{r}$$

Gaussian beams with crossing angle

$$g(\vec{r}) = \frac{1}{\sqrt{2\pi}^3 \sigma_x \sigma_y \sigma_z} e^{-(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}+\frac{z^2}{2\sigma_z^2})}$$

$$\rho_1(\vec{r},0) = g_1(\underline{R}(\tfrac{\theta}{2})\vec{r}) \,, \quad \rho_1(\vec{r},t) = g_1(\underline{R}(\tfrac{\theta}{2})\vec{r} - \vec{e}_z ct)$$

$$L = \frac{2N_1N_2}{\Delta t}\int g_1(\underline{R}(\tfrac{\theta}{2})\vec{r} - \vec{e}_z\xi) g_2(\underline{R}(-\tfrac{\theta}{2})\vec{r} + \vec{e}_z\xi) d\xi d^3\vec{r}$$

$$L = \frac{2N_1N_2}{\Delta t}\frac{1}{2\pi\sigma_{y1}\sigma_{y2}}\int e^{-\frac{y^2}{2}(\frac{1}{\sigma_{y1}^2}+\frac{1}{\sigma_{y2}^2})} dy$$

$$\times \int g_1(\begin{pmatrix} \cos(\frac{\theta}{2})x + \sin(\frac{\theta}{2})z \\ \cos(\frac{\theta}{2})z - \sin(\frac{\theta}{2})x - \xi \end{pmatrix}) g_2(\begin{pmatrix} \cos(\frac{\theta}{2})x - \sin(\frac{\theta}{2})z \\ \cos(\frac{\theta}{2})z + \sin(\frac{\theta}{2})x + \xi \end{pmatrix}) d\xi dx dz$$