The Single Resonance Model

$$\frac{d}{d\vartheta} J = \sum_{n,m=-\infty}^{\infty} m H_{nm}(J) \sin(n\vartheta + m\varphi + \Psi_{nm}(J))$$

$$\frac{d}{d\vartheta} \varphi = \upsilon + \partial_J \sum_{n,m=-\infty}^{\infty} H_{nm}(J) \cos(n\vartheta + m\varphi + \Psi_{nm}(J))$$

Strong deviation from: $J = J_0\ ,\quad \varphi = \upsilon\vartheta + \varphi_0$

Occur when there is coherence between the
perturbation and the phase space rotation: $n + m \frac{d}{ds}\varphi \approx 0$

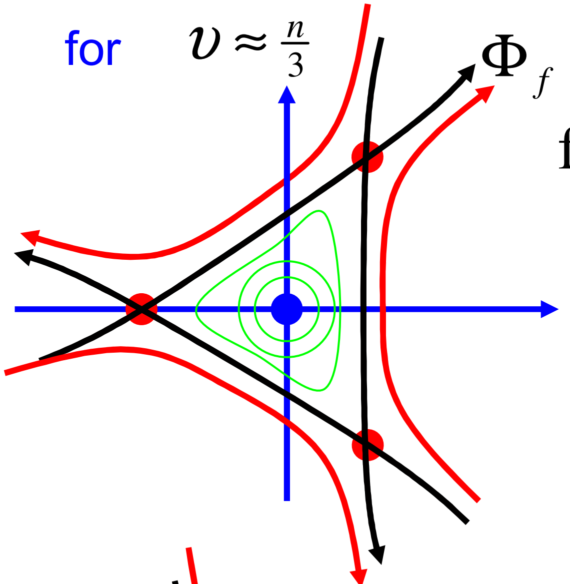
Resonance condition: tune is rational $\boxed{n + m\ \upsilon = 0}$

On resonance the integral would increases indefinitely !

Neglecting all but the most important term

$$H(\varphi, J, \vartheta) \approx \upsilon J + H_{00}(J) + H_{nm}(J) \cos(n\vartheta + m\varphi + \Psi_{nm}(J))$$

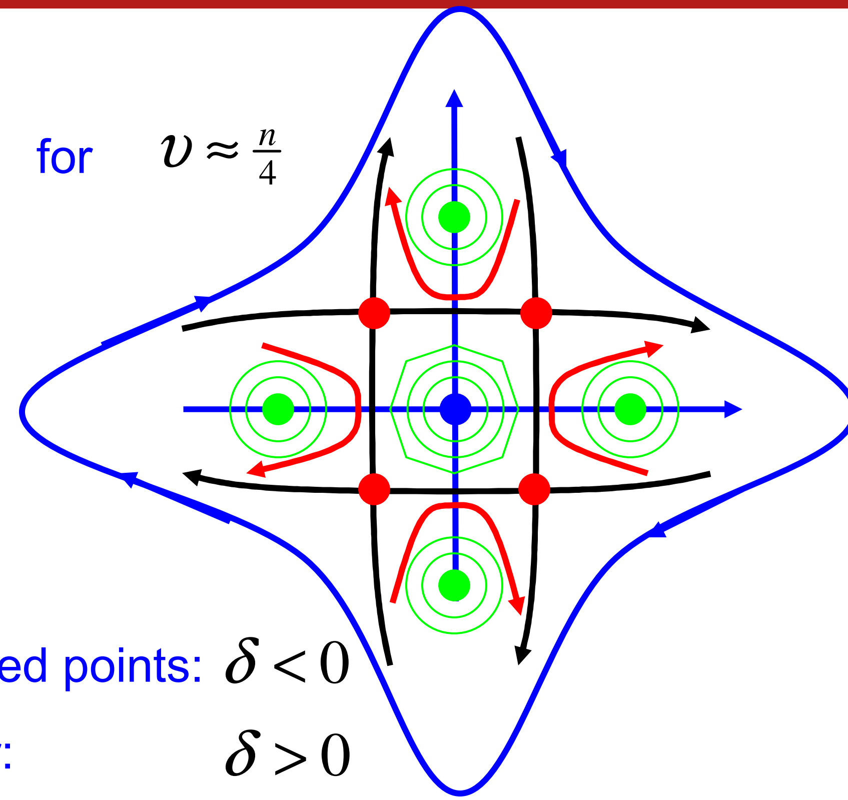
for $\upsilon \approx \frac{n}{3}$

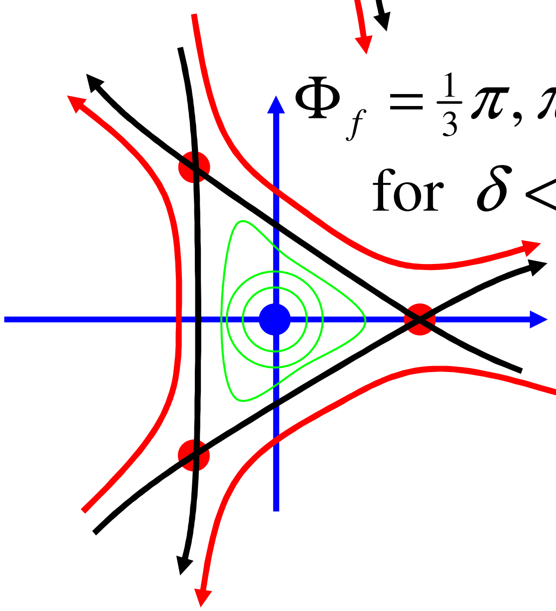


$$\Phi_f = \tfrac{1}{3}\pi, \pi, \tfrac{5}{3}\pi$$

for $\delta > 0$

for $\upsilon \approx \frac{n}{4}$





$$\Phi_f = \tfrac{1}{3}\pi, \pi, \tfrac{5}{3}\pi$$

for $\delta < 0$

Either 8 fixed points: $\delta < 0$

or none for: $\delta > 0$

How can the motion inside the fixed points be simplified for a real accelerator ?

→ Normal From Theory

Coupling Resonances

$$\frac{d}{d\vartheta}J_x = \cos(\tilde{\psi}_x + \varphi_x)\sqrt{2J_x\beta_x}\Delta f_x \frac{L}{2\pi} \quad , \quad \frac{d}{d\vartheta}\varphi_x = \upsilon_x - \sin(\tilde{\psi}_x + \varphi_x)\sqrt{\frac{\beta_x}{2J_x}}\Delta f_x \frac{L}{2\pi}$$

$$\frac{d}{d\vartheta}J_y = \cos(\tilde{\psi}_y + \varphi_y)\sqrt{2J_y\beta_y}\Delta f_y \frac{L}{2\pi} \quad , \quad \frac{d}{d\vartheta}\varphi_y = \upsilon_y - \sin(\tilde{\psi}_y + \varphi_y)\sqrt{\frac{\beta_y}{2J_y}}\Delta f_y \frac{L}{2\pi}$$

$$\frac{d}{d\vartheta}\vec{\varphi} = \vec{\partial}_J H \quad , \quad \frac{d}{d\vartheta}\vec{J} = -\vec{\partial}_\varphi H \quad , \quad H(\vec{\varphi},\vec{J},\vartheta) = \vec{\upsilon}\cdot\vec{J} - \frac{L}{2\pi}\int_0^{\vec{x}} \Delta\vec{f}(\hat{\vec{x}},s)d\hat{\vec{x}}$$

The integral form can be chosen since it is path independent. This is due to the Hamiltonian nature of the force: $\Delta f_{x,y}(x,y,s) = -\partial_{x,y}\Delta H(x,y,s)$

Single Resonance model for two dimensions means retaining only the amplitude dependent tune shift and one term in the two dimensional Fourier expansion:

$$H(\vec{\varphi},\vec{J},\vartheta) = \vec{\upsilon}\cdot\vec{J} + H_{00}(\vec{J}) + H_{n\vec{m}}(\vec{J})\cos(n\vartheta + m_x\varphi_x + m_y\varphi_y + \Psi_{n\vec{m}}(\vec{J}))$$

For $n + m_x\upsilon_x + m_y\upsilon_y \approx 0$

$$m_x\varphi_x + m_y\varphi_y = \vec{m}\cdot\vec{\varphi}$$

Sum and Difference Resonances

$n+m_x\nu_x+m_y\nu_y\approx 0$ means that oscillations in y can drive oscillations in x in

$$x''=-K\,x+\Delta f_x(x,y,s)$$

The resonance term in the Hamiltonian then changes only slowly:

$$H(\vec{\varphi},\vec{J},\vartheta)=\vec{\nu}\cdot\vec{J}+H_{00}(\vec{J})+H_{n\vec{m}}(\vec{J})\cos(n\vartheta+\vec{m}\cdot\vec{\varphi}+\Psi_{n\vec{m}}(\vec{J}))$$

$$\tfrac{d}{d\vartheta}\vec{\varphi}=\vec{\partial}_J H \quad , \quad \tfrac{d}{d\vartheta}\vec{J}=-\vec{\partial}_\varphi H$$

$$J=\vec{m}\cdot\vec{J}$$

$$J_\perp=m_xJ_x-m_yJ_y=\vec{m}\times\vec{J} \quad \Rightarrow \quad \tfrac{d}{d\vartheta}J_\perp=0$$

Difference resonances lead to stable motion since:

$$n+|m_x|\nu_x-|m_y|\nu_y\approx 0\Rightarrow |m_x|J_x+|m_y|J_y=const.$$

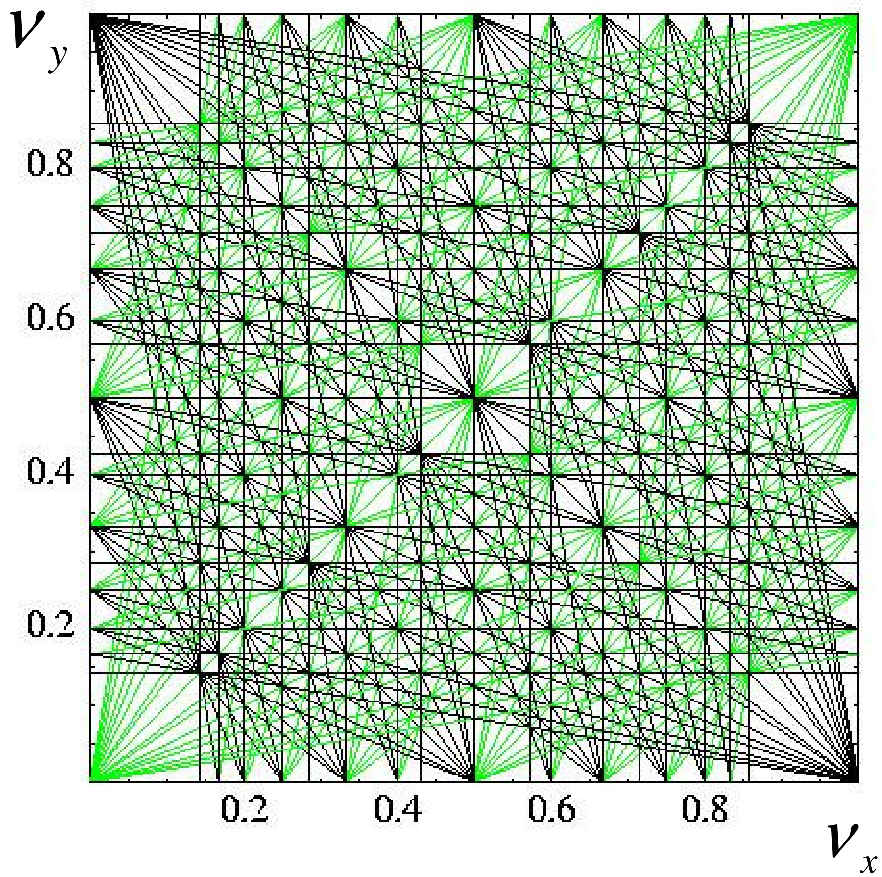
Sum resonances lead to unstable motion since:

$$n+|m_x|\nu_x+|m_y|\nu_y\approx 0\Rightarrow |m_x|J_x-|m_y|J_y=const.$$

Resonances Diagram

$n + m_x \upsilon_x + m_y \upsilon_y \approx 0$ means that oscillations in y can drive oscillations in x in

$$x'' = -K\,x + \Delta f_x(x, y, s)$$



All these resonances have to be avoided by their respective resonance width.

The position of an accelerator in the tune plane s called its Working Point.