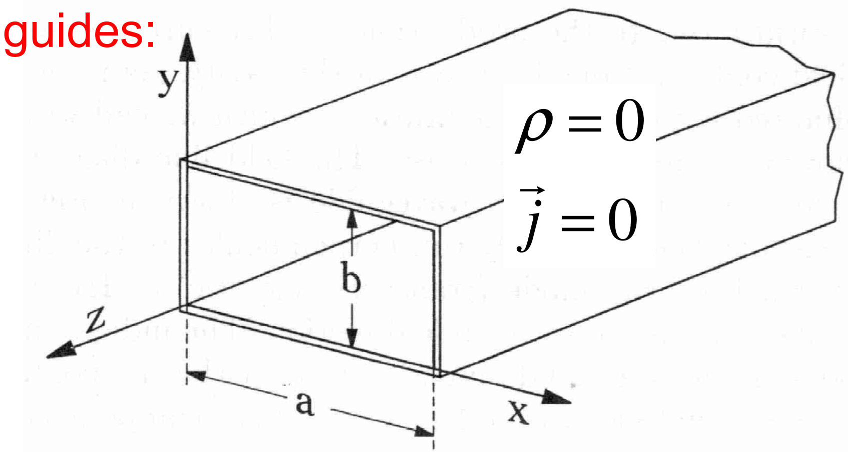
RF in Accelerators

$$\left.\begin{aligned}\vec\nabla\times\vec E&=-\partial_t\vec B\\ \vec\nabla\times\vec B&=\tfrac{1}{c^2}\partial_t\vec E+\mu_0\vec j\end{aligned}\right\}\begin{aligned}\vec\nabla\times(\vec\nabla\times\vec E)&=-\tfrac{1}{c^2}\partial_t^2\vec E-\mu_0\partial_t\vec j\\ \vec\nabla\times(\vec\nabla\times\vec B)&=-\tfrac{1}{c^2}\partial_t^2\vec B-\mu_0\vec\nabla\times\vec j\end{aligned}$$

Wave guides:



$$\rho=0$$
$$\vec j=0$$

Wave equation for all components

$$\vec\nabla^2\vec E=\tfrac{1}{c^2}\partial_t^2\vec E$$
$$\vec\nabla^2\vec B=\tfrac{1}{c^2}\partial_t^2\vec B$$

$$\left.\begin{aligned}\vec\nabla\times\vec E&=-\partial_t\vec B\\ \vec\nabla\times\vec B&=\tfrac{1}{c^2}\partial_t\vec E\end{aligned}\right\}\begin{aligned}\vec\nabla_\perp\times\vec E_\perp&=-\partial_t\vec B_z\\ \vec\nabla_\perp\times\vec B_\perp&=\tfrac{1}{c^2}\partial_t\vec E_z\end{aligned}$$

$$\left.\begin{aligned}\vec\nabla\cdot\vec E&=0\\ \vec\nabla\cdot\vec B&=0\end{aligned}\right\}\begin{aligned}\vec\nabla_\perp\cdot\vec E_\perp+\partial_z E_z&=0\\ \vec\nabla_\perp\cdot\vec B_\perp+\partial_z B_z&=0\end{aligned}$$

Search for simple modes:

Transverse electric and magnetic (TEM) waves cannot exists, since:

$$E_z=0 \text{ and } B_z=0\ \Rightarrow\ \vec E_\perp=const\ \text{ and } \vec B_\perp=const$$

TE and TM Modes

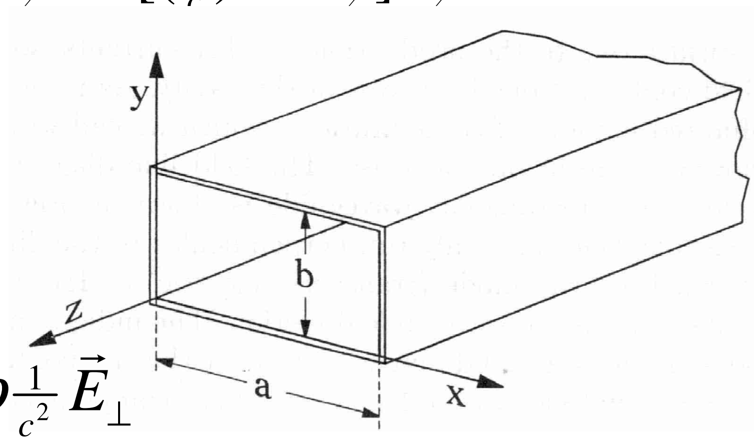
CHESS & LEPP

Fourier expansion of the z-dependence: $\vec{E}(x,y,z,t) = \int \vec{E}_{k_z\omega}(x,y)e^{ik_z z - i\omega t}dk_z d\omega$

$$\vec{\nabla}^2\vec{E} = \tfrac{1}{c^2}\partial_t^2\vec{E} \qquad \Rightarrow \qquad \vec{\nabla}_\perp^2 E_z = -[(\tfrac{\omega}{c})^2 - k_z^2]E_z$$

$$\vec{\nabla}^2\vec{B} = \tfrac{1}{c^2}\partial_t^2\vec{B} \qquad\qquad \vec{\nabla}_\perp^2 B_z = -[(\tfrac{\omega}{c})^2 - k_z^2]B_z$$

Eigenvalue equation with boundary conditions:



Walls:

$$\vec{E}_{//} = 0 \quad \vec{B}_r = 0$$

$$E_z = 0 \quad \partial_r B_z = 0$$

$$\vec{\nabla}\times\vec{B} = \tfrac{1}{c^2}\partial_t\vec{E}$$

$$\vec{\nabla}_\perp \times \vec{B}_z + ik_z\vec{e}_z \times \vec{B}_\perp = -i\omega\tfrac{1}{c^2}\vec{E}_\perp$$

$$\vec{\nabla}_r \times \vec{B}_z + ik_z\vec{e}_z \times \vec{B}_r = -i\omega\tfrac{1}{c^2}\vec{E}_\varphi \Rightarrow \partial_r B_z = 0$$

Solutions for E or B only exist for a discrete set of eigenvalues: $(\tfrac{\omega}{c})^2 - k_z^2 = k_n^{(E)2}$

$$(\tfrac{\omega}{c})^2 - k_z^2 = k_n^{(B)2}$$

Due to different boundary conditions, E_z and B_z cannot simultaneously be nonzero.

TE modes have $E_z = 0$ TM modes have $B_z = 0$

Dispersion relation

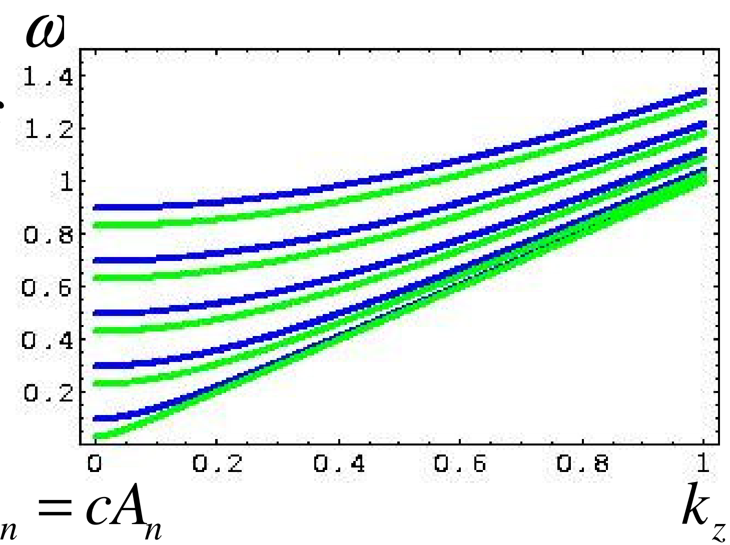
$$\omega(k_z) = c\sqrt{A_n{}^2 + k_z^2}$$

Phase velocity $v_{ph} = \omega / k_z = c\sqrt{1+\left(\frac{A_n}{k_z}\right)^2} > c$

Group velocity $v_{gr} = d\omega / dk_z = c / \sqrt{1+\left(\frac{A_n}{k_z}\right)^2} < c$

For each excitation frequency ω one obtains a propagation in the wave guide of

$$e^{ik_z z}\ , \quad k_z = \sqrt{(\tfrac{\omega}{c})^2 - A_n^2}$$



Transport for ω above the cutoff frequency $\omega > \omega_n = cA_n$

Damping for ω below the cutoff frequency $\omega < \omega_n = cA_n$