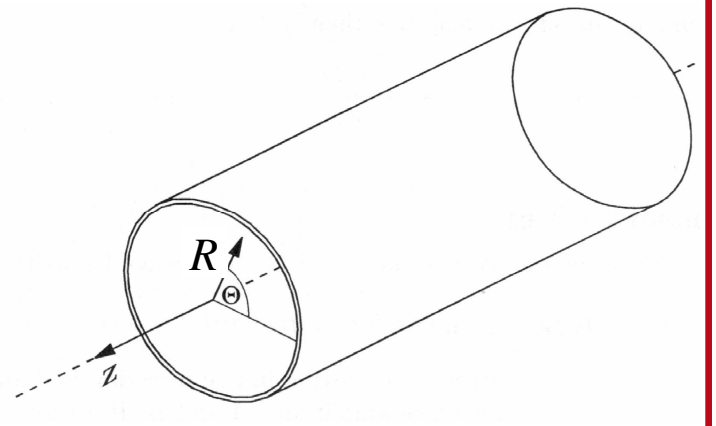
Cylindrical Wave Guides

TM Modes:

$$E_z(\vec{x}_0) = 0 \qquad \vec{\nabla}_\perp^2 E_z = [k_z^2 - (\tfrac{\omega}{c})^2] E_z$$

$$(\partial_r^2 + \tfrac{1}{r}\partial_r + \tfrac{1}{r^2}\partial_\varphi^2) E_z = [k_z^2 - (\tfrac{\omega}{c})^2] E_z$$

$$(\xi^2 \partial_\xi^2 + \xi \partial_\xi + \xi^2 - n^2) E_z = 0\,, \quad \xi = k_{nm}^{(B)}\, r$$



$$E_z(\vec{x}) = E_0 J_n(k_{nm}^{(B)} r) e^{in\varphi} \qquad k_{nm}^{(B)} = \frac{Z_{nm}}{R}$$ with the mth 0 of the nth Bessel function

Notation: TM$_{nm}$ Mode

TE Modes:

$$\partial_r B_z(\vec{x}_0) = 0 \qquad \vec{\nabla}_\perp^2 B_z = [k_z^2 - (\tfrac{\omega}{c})^2] B_z$$

$$B_z(\vec{x}) = B_0 J_n(k_{nm}^{(E)} r) e^{in\varphi} \qquad k_{nm}^{(E)} = \frac{S_{nm}}{R}$$ with the mth extremum of J$_n$

Notation: TE$_{nm}$ Mode

Fundamental Mode

Mode for particle acceleration: TM_{01}

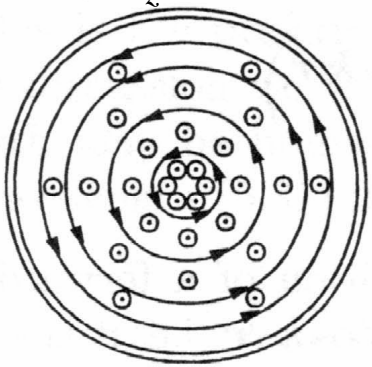
$$E_z(\vec{x}) = E_z J_0(\tfrac{r}{r_0})\cos(k_z z - \omega t)$$

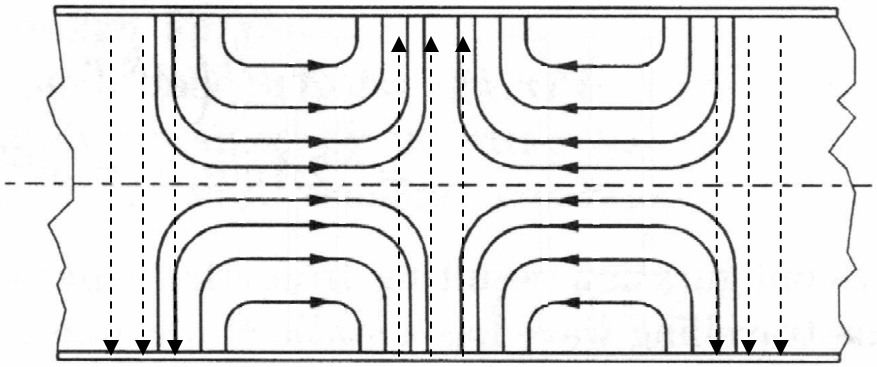
$$E_r(\vec{x}) = -E_z r_1 k_z J_0'(\tfrac{r}{r_1})\sin(k_z z - \omega t)$$

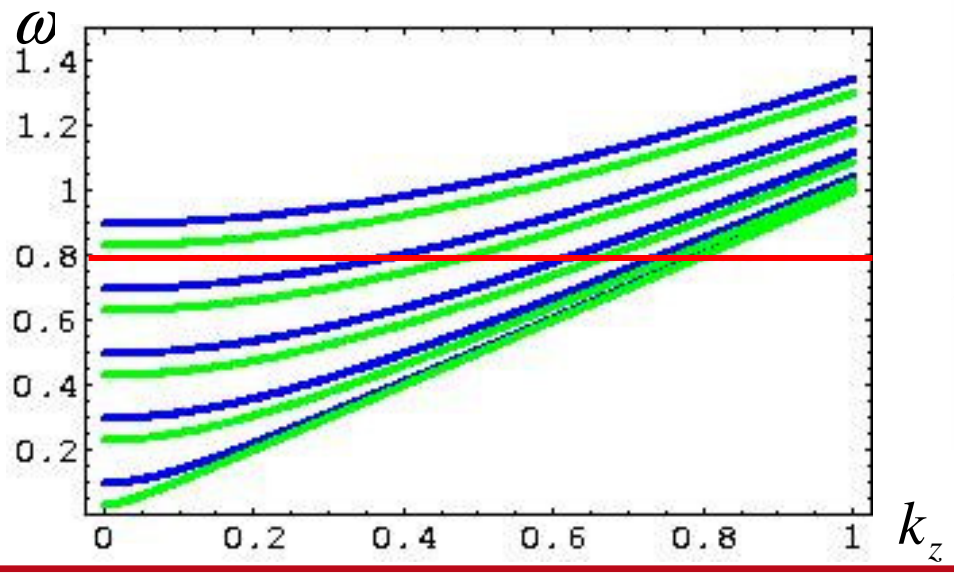
$$E_\varphi(\vec{x}) = 0$$

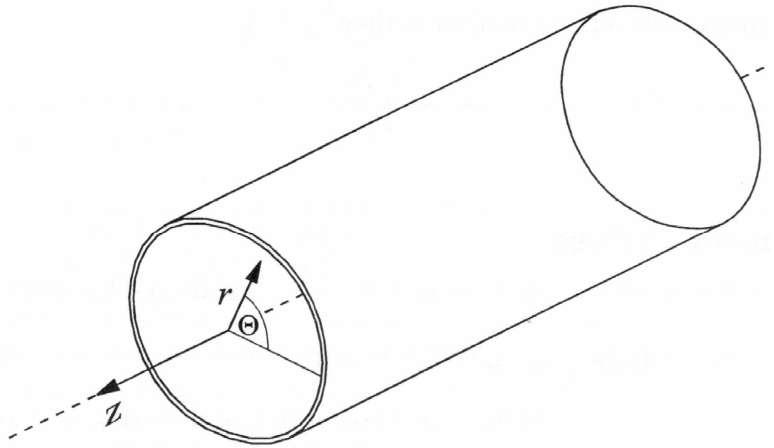
$$B_r(\vec{x}) = 0$$

$$B_\varphi(\vec{x}) = -E_z r_1 \tfrac{\omega}{c^2} J_0'(\tfrac{r}{r_1})\sin(k_z z - \omega t)$$







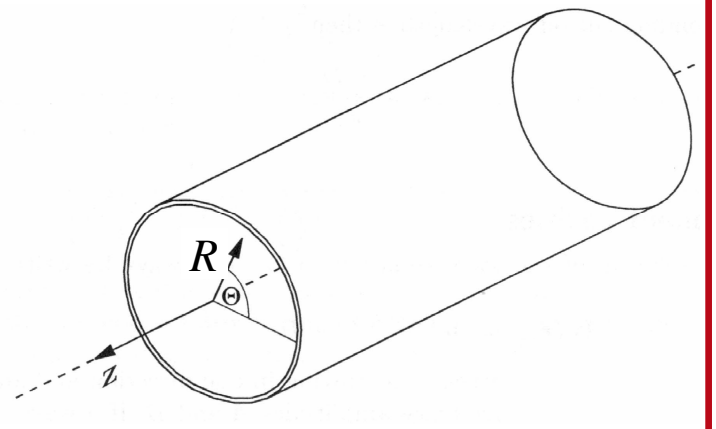


Cylindrical Wave TE Modes

$$E_z(\vec{x}) = 0\,, \quad B_z(\vec{x}) = B_0 J_n(\tfrac{S_{nm}}{R} r) e^{in\varphi}$$

$$\vec{E}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp E_z + \omega \vec{\nabla}_\perp \times \vec{B}_z)$$

$$\vec{B}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp B_z - \frac{\omega}{c^2} \vec{\nabla}_\perp \times E_z)$$



$$E_r = i\omega\left(\frac{R}{S_{nm}}\right)^2 \frac{1}{r}\partial_\varphi B_z = -B_0 n \omega R \frac{1}{S_{nm}^2}\frac{R}{r} J_n(\tfrac{S_{nm}}{R} r)\cos(n\varphi + k_z z - \omega t)$$

$$E_\varphi = -i\omega\left(\frac{R}{S_{nm}}\right)^2 \partial_r B_z = B_0 \omega R \frac{1}{S_{nm}} J'_n(\tfrac{S_{nm}}{R} r)\sin(n\varphi + k_z z - \omega t)$$

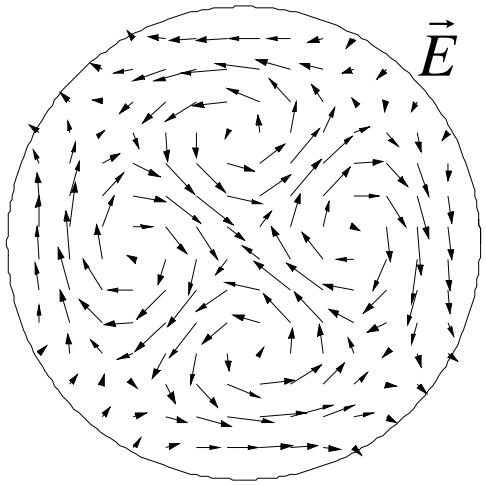
$$E_z = 0$$

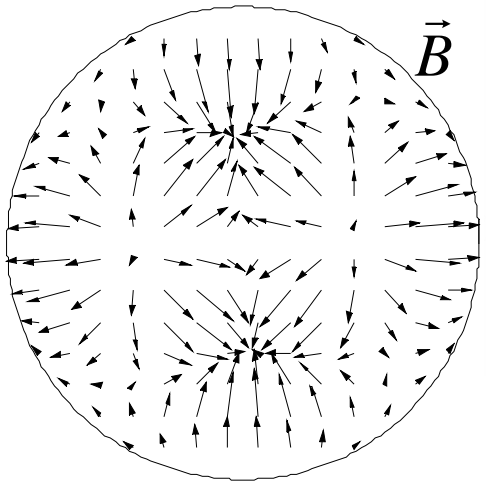
$$B_r = ik_z\left(\frac{R}{S_{nm}}\right)^2 \partial_r B_z = -B_0 k_z R \frac{1}{S_{nm}} J'_n(\tfrac{S_{nm}}{R} r)\sin(n\varphi + k_z z - \omega t)$$

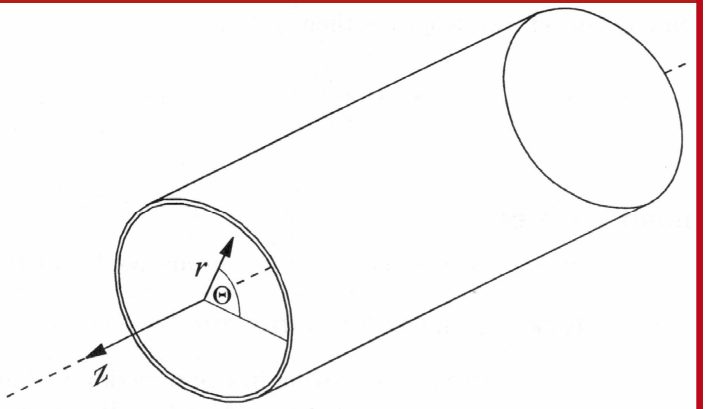
$$B_\varphi = ik_z\left(\frac{R}{S_{nm}}\right)^2 \frac{1}{r}\partial_\varphi B_z = -B_0 n k_z R \frac{1}{S_{nm}^2}\frac{R}{r} J_n(\tfrac{S_{nm}}{R} r)\cos(n\varphi + k_z z - \omega t)$$

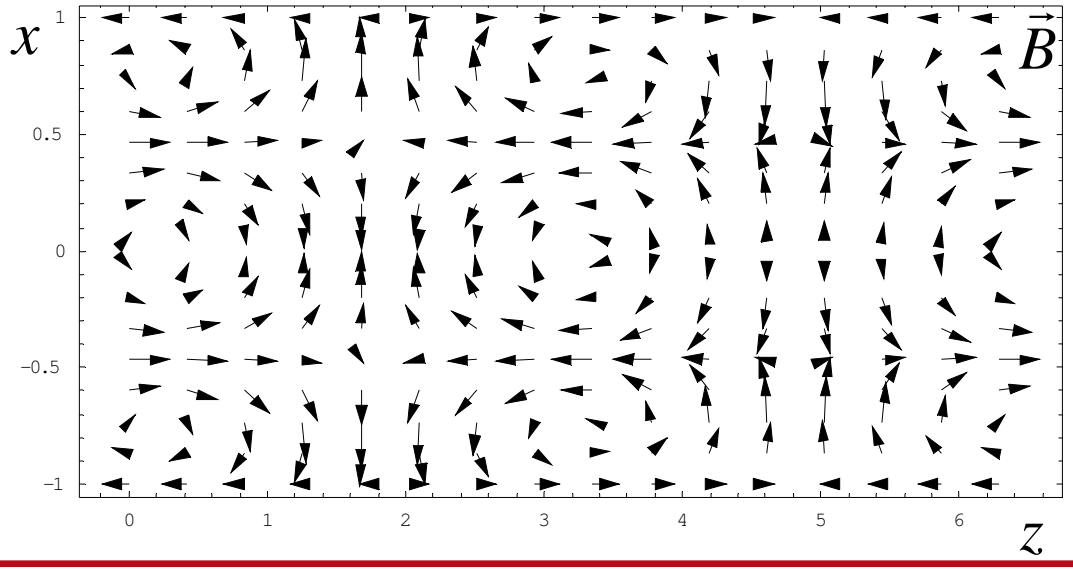
$$B_z = B_0 J_n(\tfrac{S_{nm}}{R} r)\cos(n\varphi + k_z z - \omega t)$$

Rectangular TE_{22} Mode







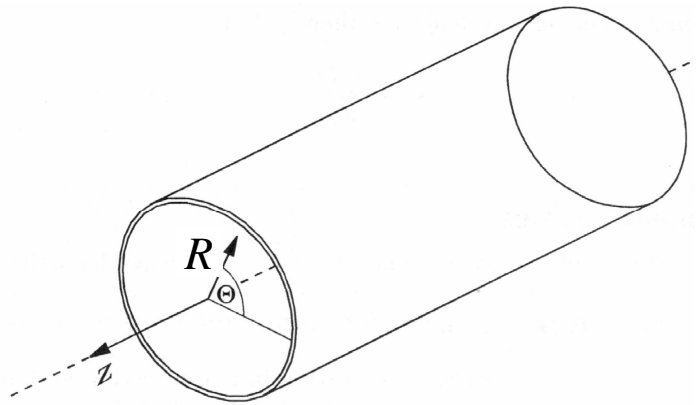


Cylindrical Wave TM Modes

$$E_z(\vec{x}) = E_0 J_n(\tfrac{Z_{nm}}{R} r) e^{in\varphi}\ , \quad B_z(\vec{x}) = 0$$

$$\vec{E}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp E_z + \omega \vec{\nabla}_\perp \times \vec{B}_z)$$

$$\vec{B}_\perp = \frac{i}{\frac{\omega^2}{c^2} - k_z^2}(k_z \vec{\nabla}_\perp B_z - \frac{\omega}{c^2} \vec{\nabla}_\perp \times \vec{E}_z)$$



$$E_r = i\frac{Rk_z}{Z_{nm}} \partial_r E_z = -E_0 k_z J'_n(\tfrac{Z_{nm}}{R} r) \sin(n\varphi + k_z z - \omega t)$$

$$E_\varphi = i\frac{Rk_z}{Z_{nm}} \frac{1}{r} \partial_\varphi E_z = -E_0 n k_z \frac{1}{Z_{nm}} \frac{R}{r} J_n(\tfrac{Z_{nm}}{R} r) \cos(n\varphi + k_z z - \omega t)$$

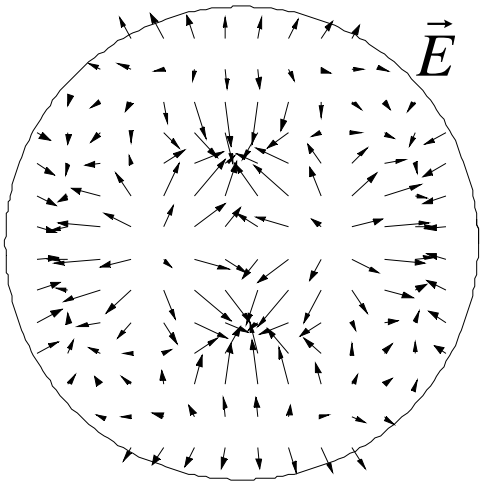
$$E_z = E_0 J_n(\tfrac{Z_{nm}}{R} r) \cos(n\varphi + k_z z - \omega t)$$

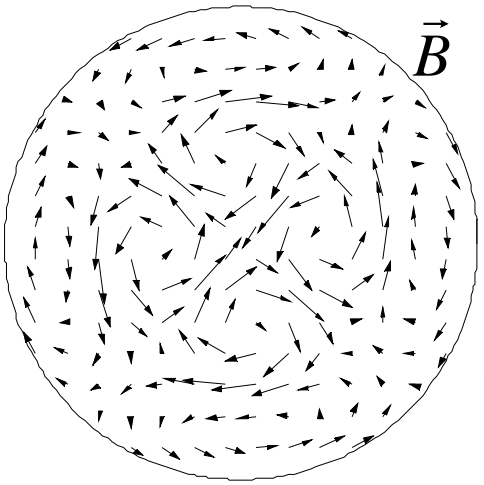
$$B_r = -i\frac{R\omega}{Z_{nm}} \frac{1}{r} \partial_\varphi E_z = E_0 n \frac{\omega}{c^2} \frac{1}{Z_{nm}} \frac{R}{r} J_n(\tfrac{Z_{nm}}{R} r) \cos(n\varphi + k_z z - \omega t)$$

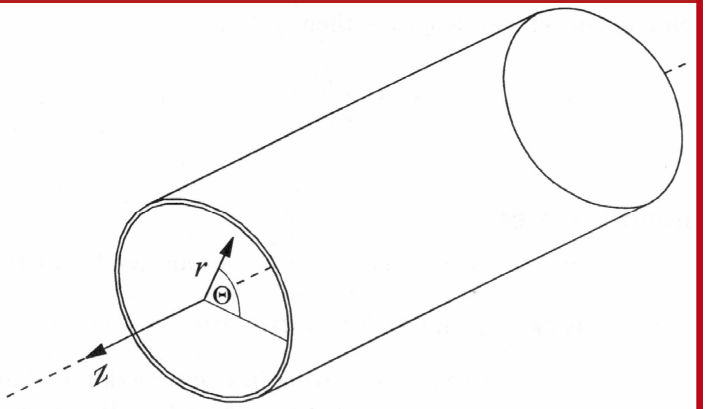
$$B_\varphi = i\frac{R\omega}{Z_{nm}} \partial_r E_z = -E_0 \frac{\omega}{c^2} J'_n(\tfrac{Z_{nm}}{R} r) \sin(n\varphi + k_z z - \omega t)$$

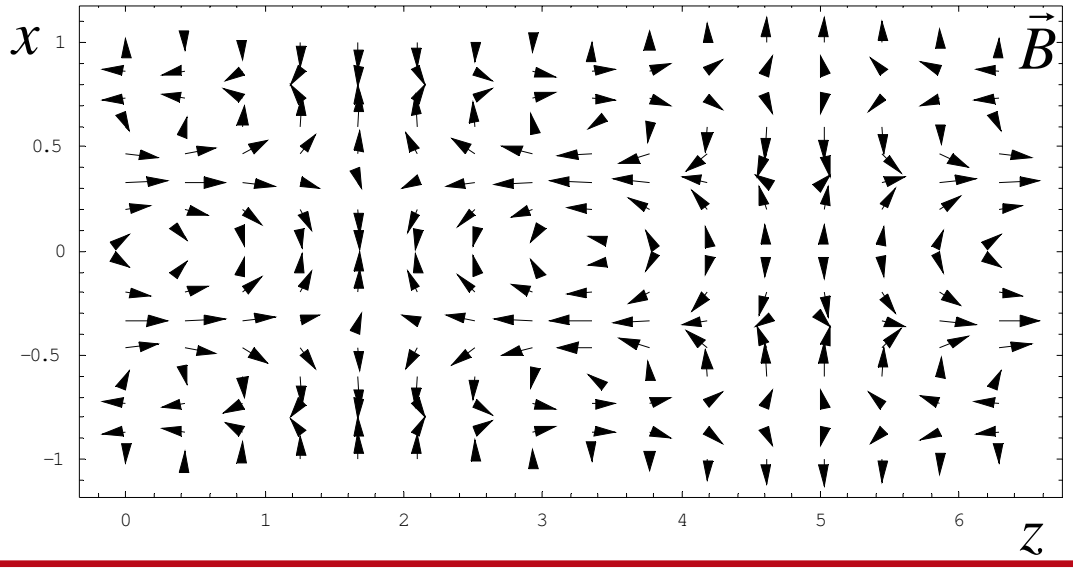
$$B_z = 0$$

Rectangular TM_{22} Mode

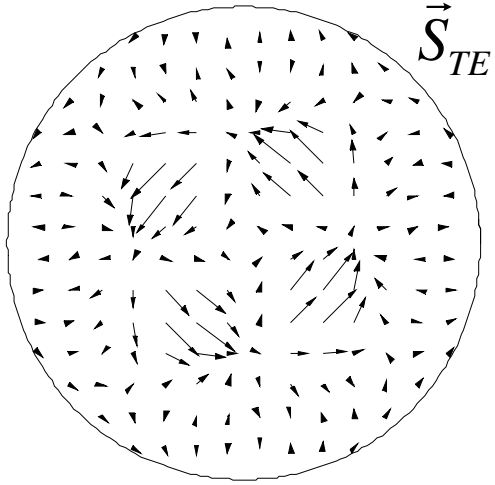


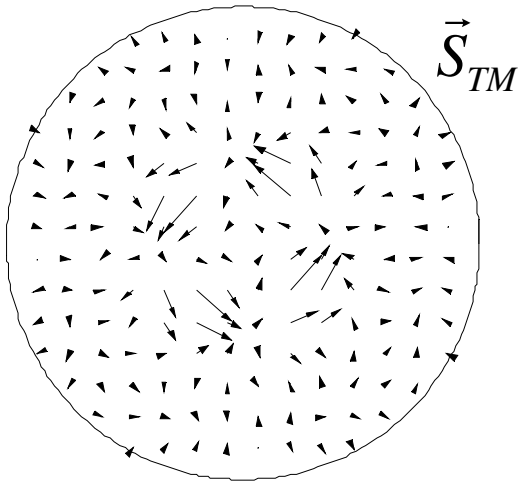


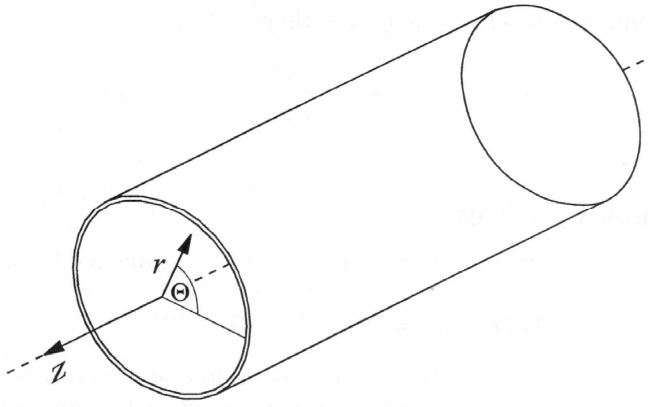


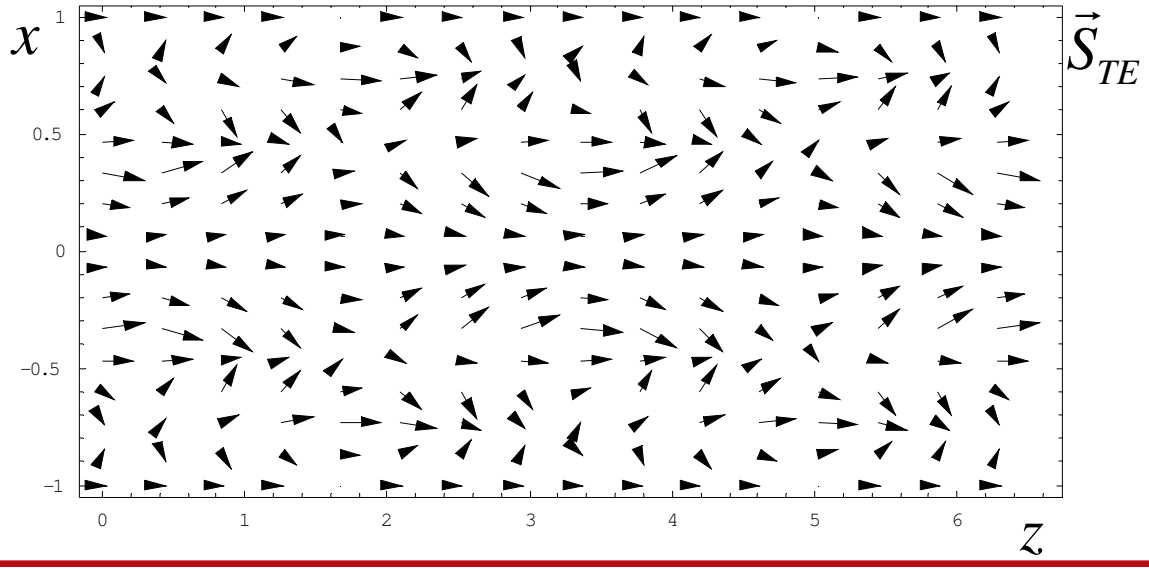


Pointing Vector

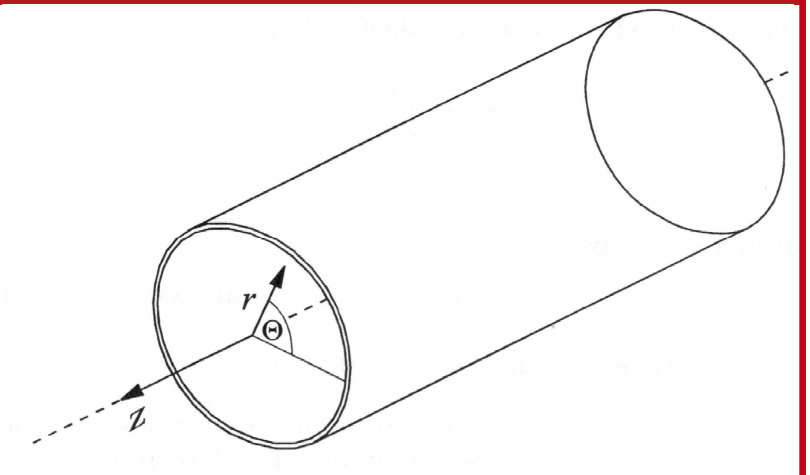




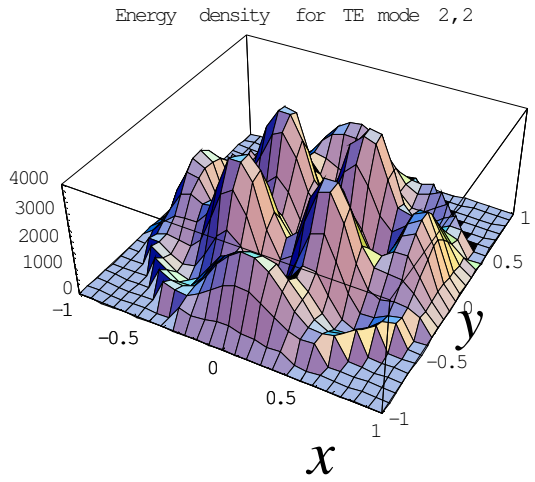




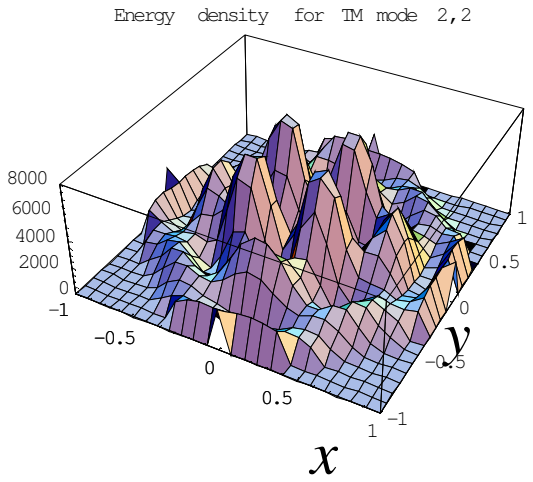
Energy Density



U_{TE}



U_{TM}



U_{TM}