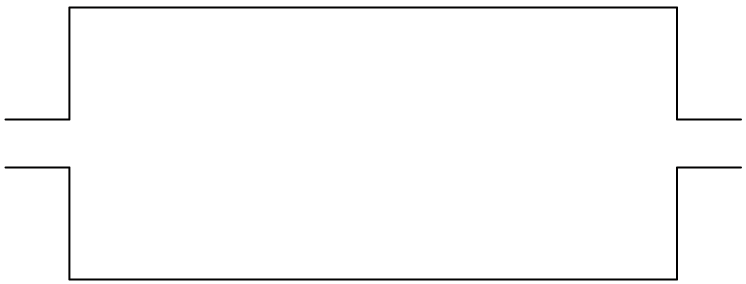
Resonant Cavities



TE Modes: Standing waves with nodes

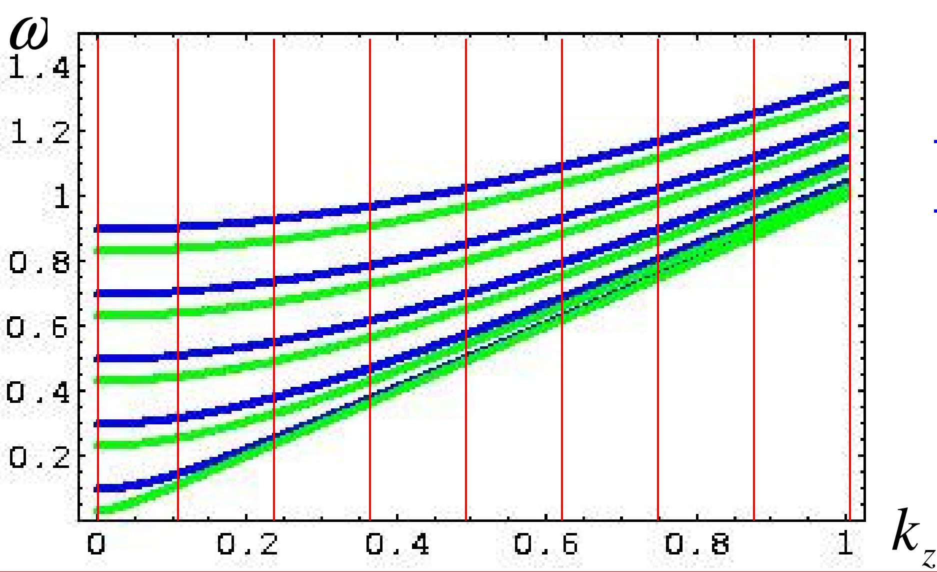
$$B_z(\vec{x}) \propto \sin(k_z z)\sin(\omega t)\,, \quad k_z = \tfrac{l\pi}{L}$$

$$l > 0$$

TM Modes: Standing waves with nodes

$$E_z(\vec{x}) \propto \cos(k_z z)\cos(\omega t)\,, \quad k_z = \tfrac{l\pi}{L}$$

$$l \geq 0$$

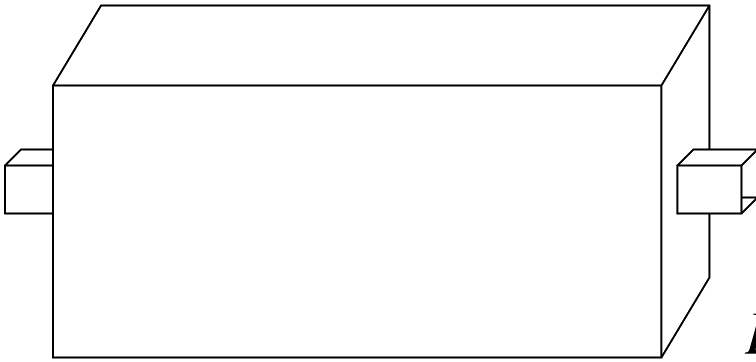


For each mode TE_{nm} or TM_{nm} there is a discrete set of frequencies that can be excited.

$$\omega_{nm}^{(E/B)} = c\sqrt{k_{nm}^{(E/B)2} + \left(\tfrac{l\pi}{L}\right)^2}$$

Resonant Cavities Examples

Rectangular cavity:

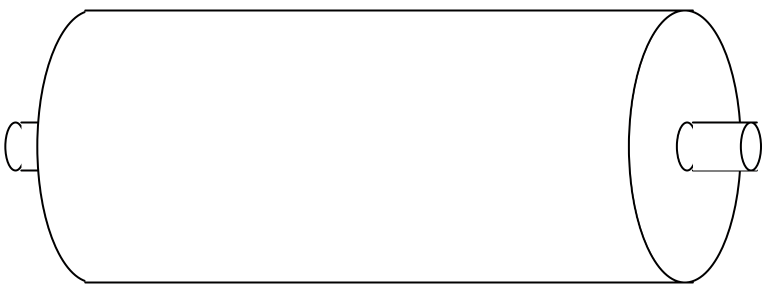


$$\omega_{nml}^{(E/B)} = c\sqrt{\left(\frac{n\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2 + \left(\frac{l\pi}{L_z}\right)^2}$$

Fundamental acceleration mode: $\omega_{110}^{(B)} = c\frac{\pi}{L}\sqrt{2}$

$$L_x = L_y = 21.2cm \quad \Rightarrow \quad f_{110}^{(B)} = 1.0\text{GHz}$$

Pill Box cavity:



$$\omega_{nm}^{(E/B)} = c\sqrt{k_{nm}^{(E/B)2} + \left(\frac{l\pi}{L}\right)^2}$$

$k_{nm}^{(B)}r$ is the m^th 0 of the n^th Bessel function.

$k_{nm}^{(E)}r$ is the m^th extremeum of J_n

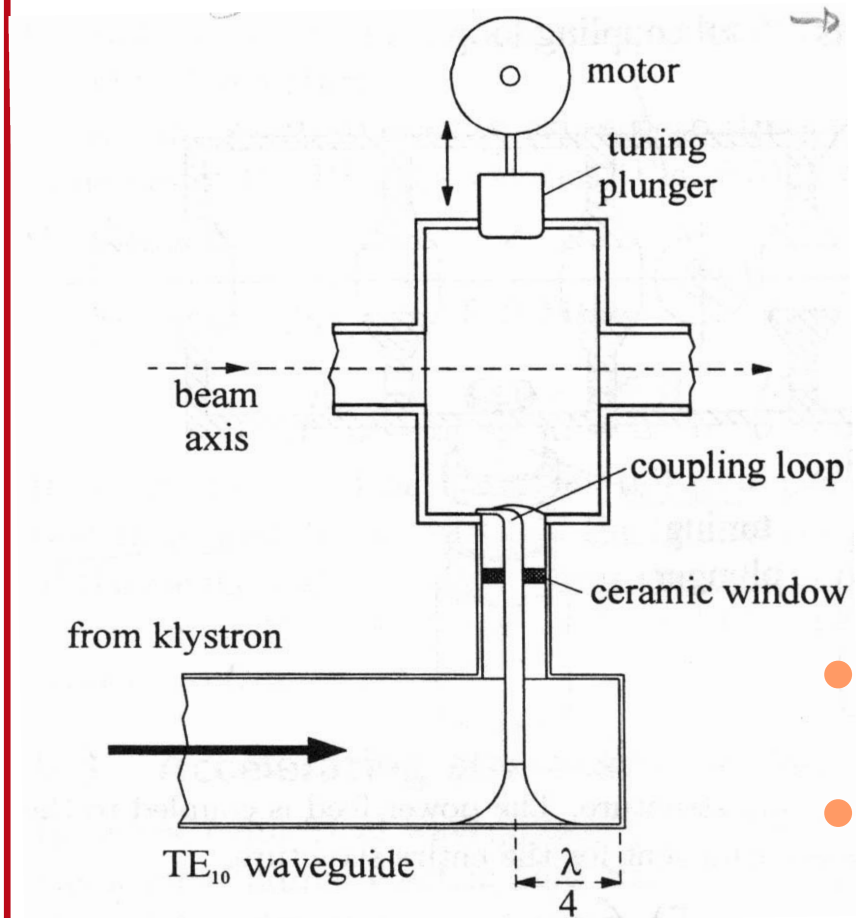
Fundamental acceleration mode: $\omega_{010}^{(B)} = c\frac{2.40}{r}$

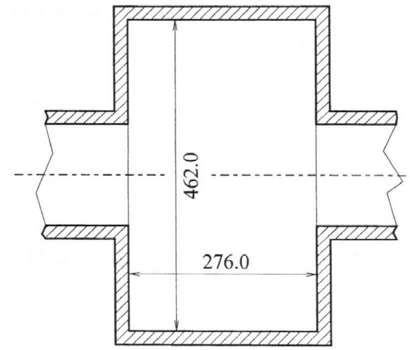
$$2r = 22.9cm \quad \Rightarrow \quad f_{010}^{(B)} = 1.0\text{GHz}$$

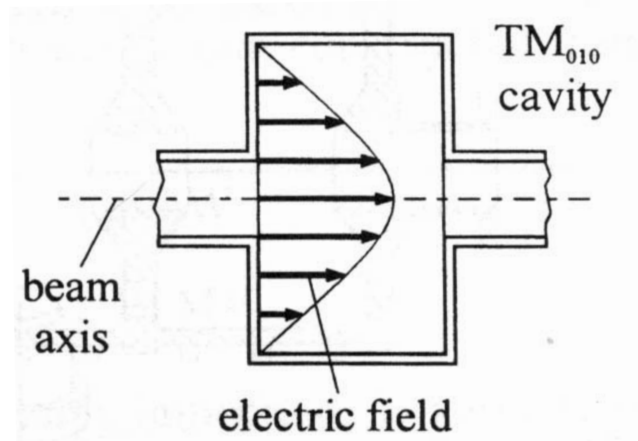
Cavity Operation

500MHz Cavity of DORIS:

$$r = 23.1cm \quad \Rightarrow \quad f_{010}^{(M)} = 0.4967\text{GHz}$$



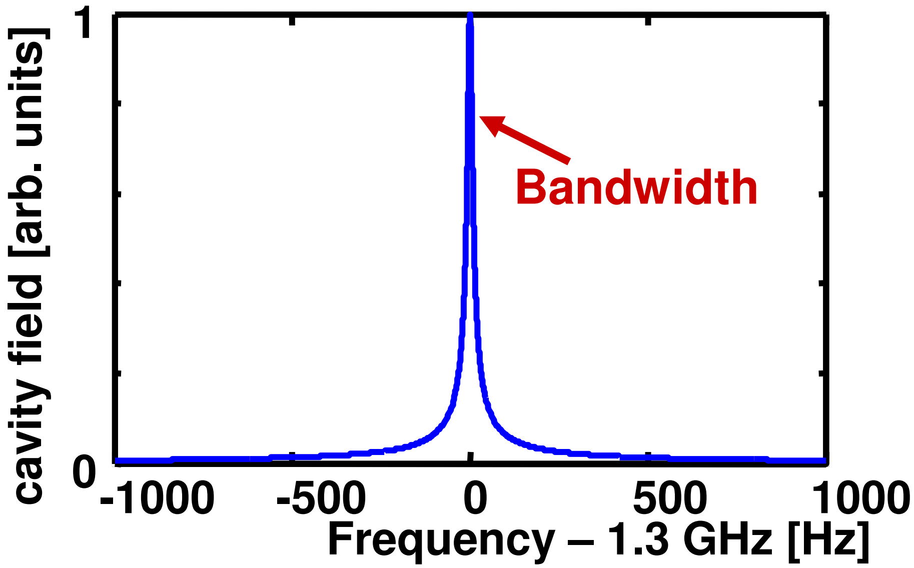




- The frequency is increased and tuned by a tuning plunger.
- An inductive coupling loop excites the magnetic field at the equator of the cavity.

3 dominant features of RF systems

CHESS & LEPP



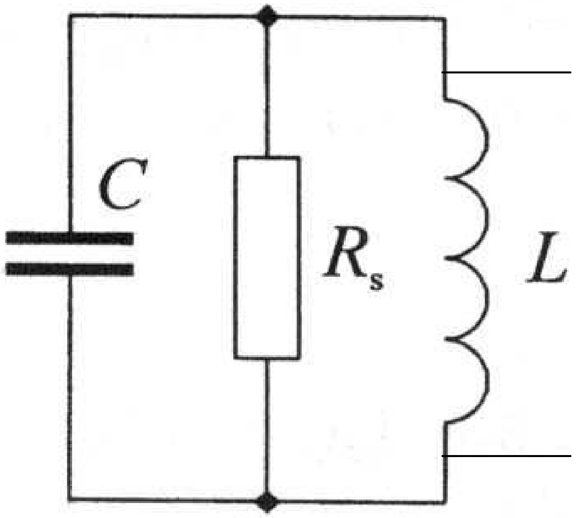
$$I_{in} \Rightarrow V(\omega)$$

(1) The RF system has a resonant frequency ω_0

(2) The resonance curve has a characteristic width $\Delta\omega = \frac{\omega_0}{2Q}$

A resonant L/C/R circuit also has such characteristics

RF systems for accelerators



L and C: determined by the cavity geometry

R_s : shunt impedance, related to surface res. R

$$I_{in} = \left(\frac{1}{R_s} + iC\omega + \frac{1}{iL\omega}\right) U_C$$

$$\hat{U}_C = \frac{1}{\sqrt{\frac{1}{R_s^2} + \left(\frac{1}{L\omega} - C\omega\right)^2}} \hat{I}_{in} \rightarrow \hat{U}_{Cres} = R_s \hat{I}_{in}$$

$$P_{RF} = \langle U_C I_{in} \rangle_t = \frac{1}{T}\int_0^T \mathrm{Re}[\left(\frac{1}{R_s} + iC\omega + \frac{1}{iL\omega}\right) U_C] \mathrm{Re}[U_C] dt = \frac{1}{2}\frac{1}{R_s}\hat{U}_C^2$$

Quallity factor: $$Q = 2\pi \frac{E}{\Delta E} = 2\pi \frac{\frac{1}{2} C U_C^2}{T P_{RF}} = \omega R_s C = R_s \sqrt{\frac{C}{L}}$$

Geometry factor: $$\frac{R_s}{Q} = \sqrt{\frac{L}{C}}$$

Geoetry factor

$$\vec\nabla\cdot\vec E=\tfrac{1}{\varepsilon_0}\rho \qquad \vec E'=\tfrac{1}{\alpha}\vec E(\alpha\vec r',\alpha t') \qquad \vec\nabla'\cdot\vec E'=\tfrac{1}{\varepsilon_0}\rho'$$

$$\vec\nabla\times\vec E=-\partial_t\vec B \quad\Rightarrow\quad \vec B'=\tfrac{1}{\alpha}\vec B(\alpha\vec r',\alpha t') \quad\Rightarrow\quad \vec\nabla'\times\vec E'=-\partial_{t'}\vec B'$$

$$\vec\nabla\cdot\vec B=0 \qquad \rho'=\rho(\alpha\vec r',\alpha t') \qquad \vec\nabla'\cdot\vec B'=0$$

$$\vec\nabla\times\vec B=\tfrac{1}{c^2}[\tfrac{1}{\varepsilon_0}\vec j+\partial_t\vec E] \qquad \vec j'=\vec j(\alpha\vec r',\alpha t') \qquad \vec\nabla'\times\vec B'=\tfrac{1}{c^2}[\tfrac{1}{\varepsilon_0}\vec j'+\partial_{t'}\vec E']$$

Reducing all sizes by α, letting the time pass α times faster, reducing all charges by α^3 and all currents by α^2 leads to fields that are α times smaller !

$$L=\frac{V}{\dot I}=\frac{\alpha^2 V'}{\alpha\dot I'}=\alpha L' \qquad\qquad C=\frac{Q}{V}=\frac{\alpha^3 Q'}{\alpha^2 V'}=\alpha C$$

For any oscillating circuit $\sqrt{\frac{L}{C}}$ is a size independent geometry factor !