Twiss parameters in accelerating cavities

$$\alpha = -\tfrac{1}{2}\beta' , \quad \gamma = \tfrac{1+\alpha^2}{\beta}$$

$$a = r' = \sqrt{2J\tfrac{mc}{p}}[-\tfrac{2\alpha+\beta\frac{p'}{p}}{2\sqrt{\beta}}\sin(\psi+\phi_0) + \tfrac{\beta\psi'}{\sqrt{\beta}}\cos(\psi+\phi_0)]$$

$$a' \approx -\tfrac{1}{p}[r(pK + \tfrac{1}{2}p'') + ap']$$

$$a' = -\sqrt{2J\tfrac{mc}{\beta p}}\begin{pmatrix} \frac{(\beta\psi')^2+\alpha^2}{\beta} + \alpha' - \alpha\frac{p'}{p} + \beta\frac{p''}{2p} - \beta\frac{3p'^2}{4p^2} \\ 2\alpha\psi' + \beta\frac{p'}{p}\psi' - \beta\psi'' \end{pmatrix}\begin{pmatrix}\sin(\psi+\phi_0) \\ \cos(\psi+\phi_0)\end{pmatrix}$$

$$= -\sqrt{2J\tfrac{mc}{\beta p}}\begin{pmatrix} \beta(K + \frac{1}{2}\frac{p''}{p}) - (\alpha + \beta\frac{p'}{2p})\frac{p'}{p} \\ \beta\frac{p'}{p}\psi' \end{pmatrix}\begin{pmatrix}\sin(\psi+\phi_0) \\ \cos(\psi+\phi_0)\end{pmatrix}$$

$$\Rightarrow \quad \psi' = \frac{A}{\beta}\text{, choice} : A = 1$$

$$\alpha' + \gamma = \beta\left[K + \left(\tfrac{p'}{2p}\right)^2\right]$$

$$\begin{pmatrix} r \\ a \end{pmatrix} = \sqrt{2J\tfrac{mc}{p}}\begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha+\beta\frac{p'}{2p}}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix}\begin{pmatrix}\sin(\psi+\phi_0) \\ \cos(\psi+\phi_0)\end{pmatrix}$$

Beta functions in accelerating cavities

$$\begin{pmatrix} r \\ a \end{pmatrix} = \sqrt{2J_n \frac{mc}{p}} \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\tilde{\alpha}}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos(\psi + \phi_0) \\ \sin(\psi + \phi_0) \end{pmatrix} \quad , \quad \tilde{\alpha} = \alpha + \beta \frac{p'}{2p}$$

For systems with changing energy one uses the normalized Courant-Snyder invariant $\mathbf{J_n = J}\ \beta_r \gamma_r$

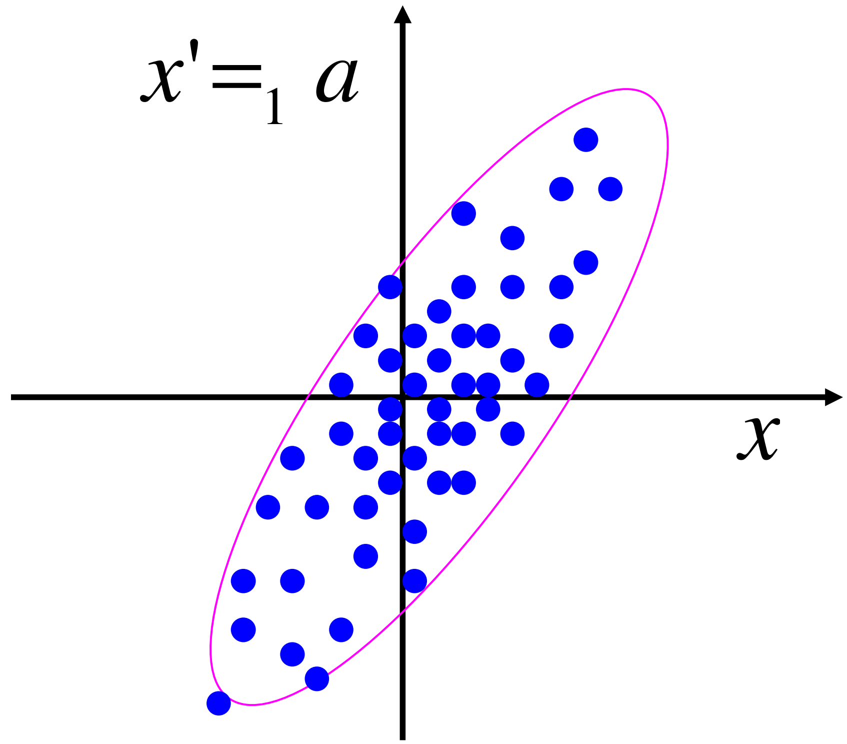
$$(r \quad a) \begin{pmatrix} \frac{1}{\sqrt{\beta}} & \frac{\tilde{\alpha}}{\sqrt{\beta}} \\ 0 & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\tilde{\alpha}}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} r \\ a \end{pmatrix} \frac{p}{2mc} = (r \quad a) \begin{pmatrix} \frac{1+\tilde{\alpha}^2}{\beta} & \tilde{\alpha} \\ \tilde{\alpha} & \beta \end{pmatrix} \begin{pmatrix} r \\ a \end{pmatrix} \frac{p}{2mc} = J_n$$

Reasons:

(1) **J** is the phase space amplitude of a particle in **(x , a)** phase space, which is the area in phase space (over 2p) that its coordinate would circumscribe during many turns in a ring. However, $\mathbf{a=p_x/p_0}$ is not conserved when p0 changes in a cavity. Therefore J is not conserved.

(2) $\mathbf{J_n = J\ p_0/mc}$ is therefore proportional to the corresponding area in $\mathbf{(x , p_x)}$ phase space, and is thus conserved.

The normaized emittance



Remarks:

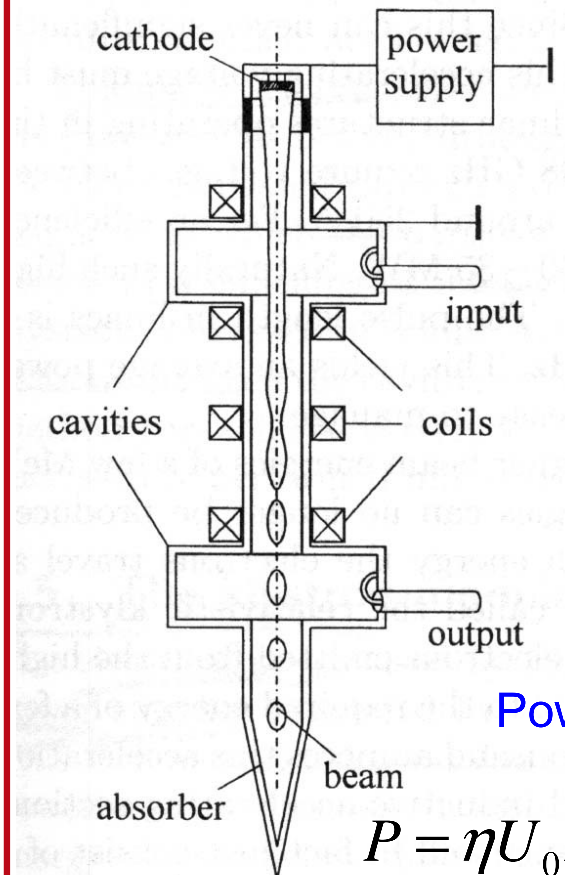
(1) The phase space area that a beam fills in (x , a) phase space shrinks during acceleration by the factor p_i/p. This area is the emittance ε.

(2) The phase space area that a beam fills in (x , p_x) phase space is conserved. This area (divided by mc) is the normalized emittance ε_n.

$$\varepsilon = \varepsilon_n / \beta_r \gamma_r$$

The Klystron as Power Source

CHESS & LEPP



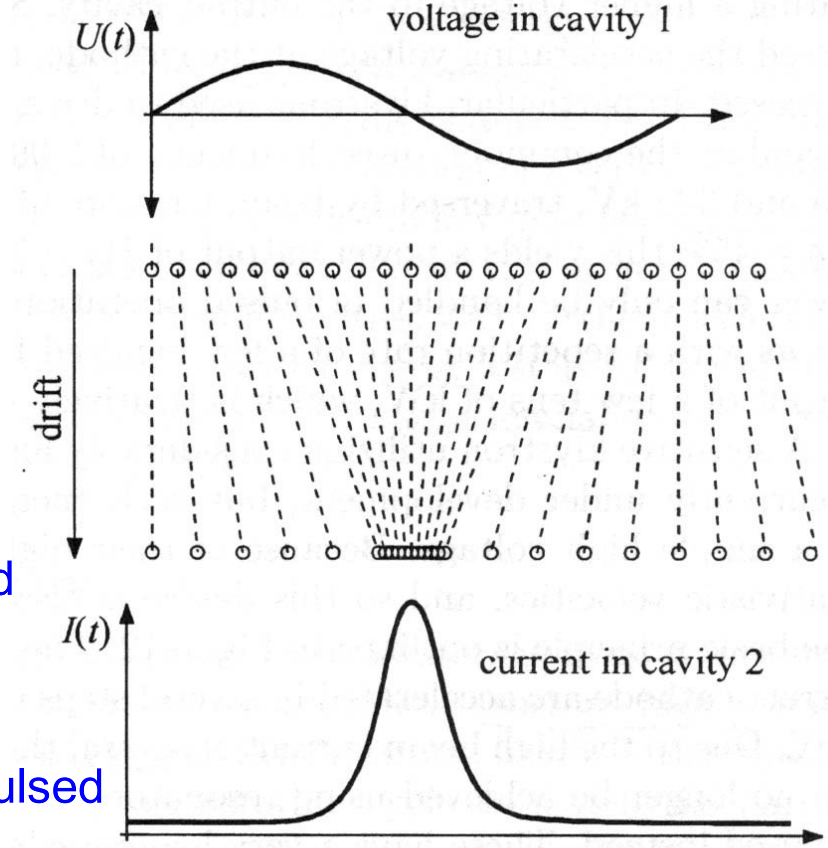
I up to > 10A

Power < 1.5MW

Power < 40MW pulsed

$$P = \eta U_0 I_{\text{beam}} \,, \quad \eta \leq 65\%$$

Time of flight bunching



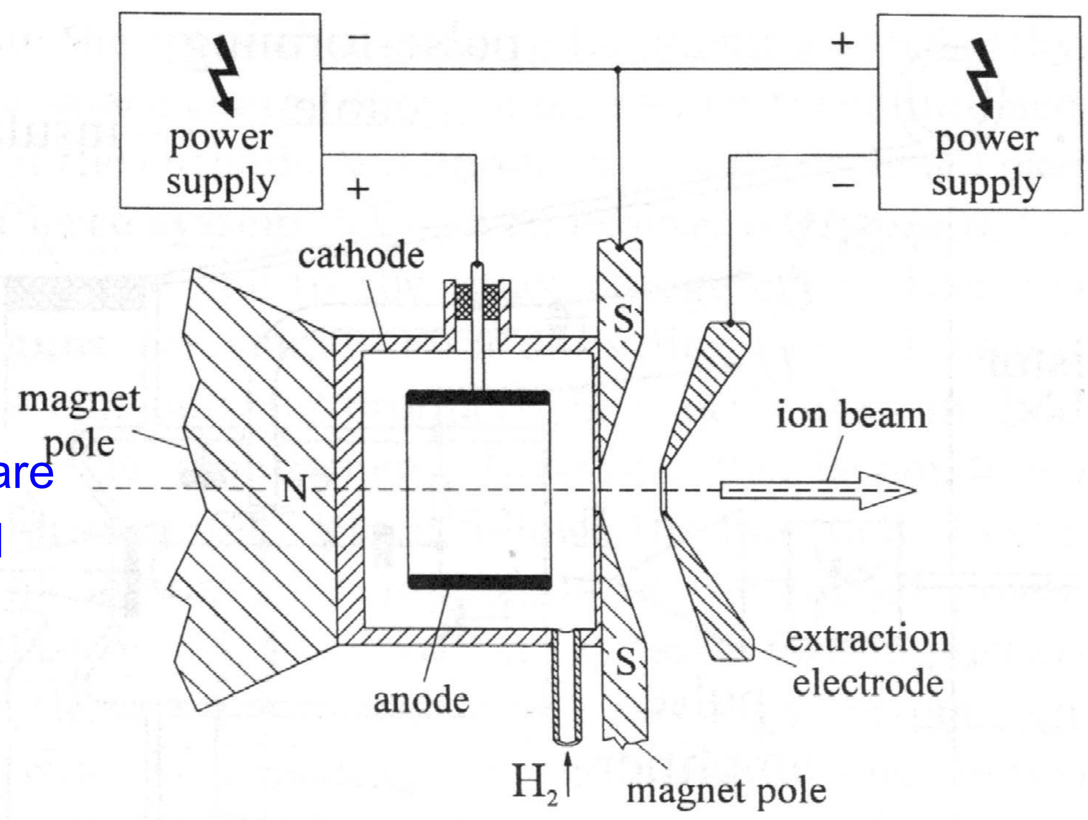
- DC acceleration to several 10kV, 100kV pulsed
- Energy modulation with a cavity
- Time of flight density modulation
- Excitation of a cavity with output coupler

Only works for non-relativistic electrons

The PIG ion source

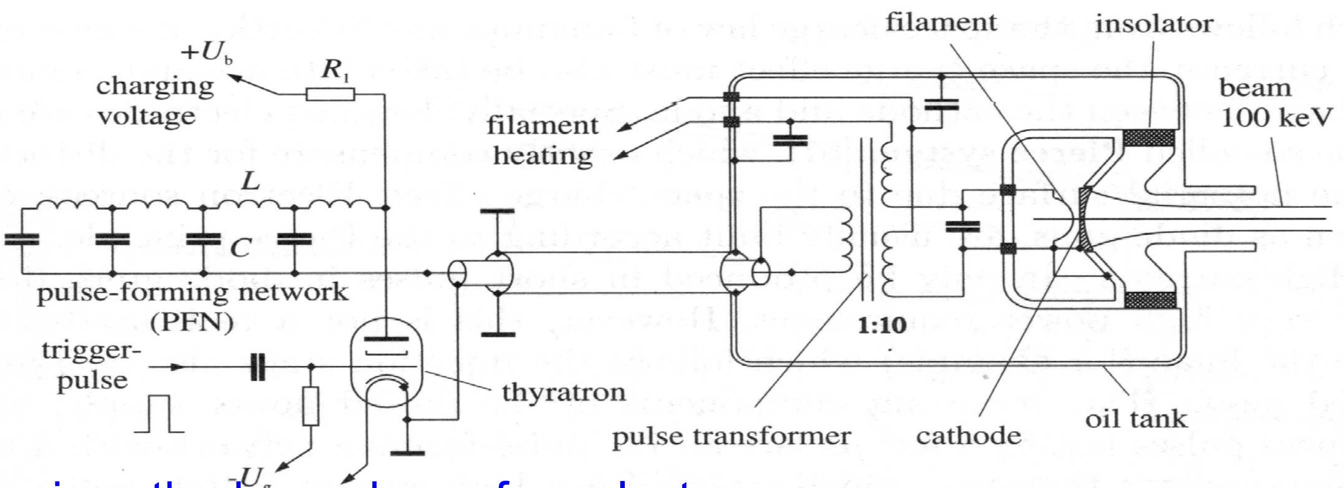
Penning Principle
(of the Philips Ion Gage)

- Magnetic field of about 0.01T.
- Pressurized gas is inserted
  at <100Pa (10-3Atm)
- Gas is ionized and remains
  ionized since electrons are
  accelerated in the E and
circle
  in the B-field.
- Positive ions are accelerated
  through a hole in the
cathode
  to several 100V.



Diode Electron Source

CHESS & LEPP



- A thermionic cathode produces free electrons.
- An earthed anode accelerates them through an aperture into a linac.
- The cathode is not flat but curved (Pierce Cathode) to produce a force that counters Coulomb expulsion (the Space Charge Force)
- Typical voltages are 100-150kV, typical peak currents are a few Ampere.
- Due to power limits, only short pulses can be produced (> a few μs long)
- A thyratron is used as fast high-current switch and capacitors provide the short pulse.
- The pulse from the capacitors is magnified (by about 10) in a transformer to reach the 100-150kV.

Child-Langmuir Law

CHESS & LEPP

$$\left.\begin{array}{l} \partial_z^2\Phi = -\frac{1}{\varepsilon_0}\rho \\ I = \rho\,\dot z \\ \partial_z I = \partial_t \rho = 0 \\ m\gamma c^2 + q\Phi = mc^2 \\ \Phi(0)=0\,,\quad \Phi(d)=V \end{array}\right\}$$

$$\gamma'' = -\frac{q}{mc^2}\partial_z^2\Phi = \frac{q}{mc^2\varepsilon_0}\frac{I}{\dot z} = \frac{\mu_0 qI}{mc}\frac{\gamma}{\sqrt{\gamma^2-1}} = \frac{d\gamma'}{d\gamma}\gamma'$$

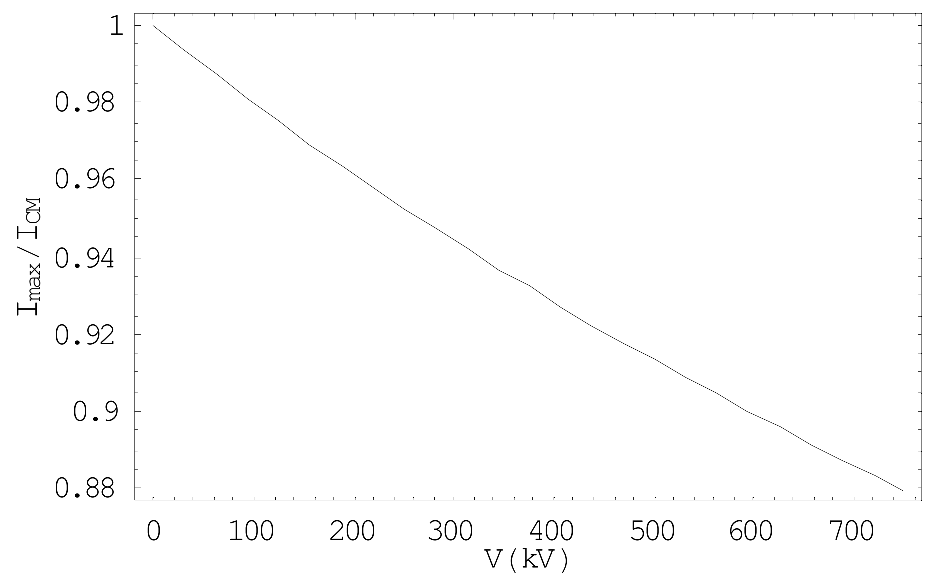
$$\gamma'^2 = \frac{2\mu_0 qI}{mc}\sqrt{\gamma^2-1}$$

$$d\sqrt{\frac{2\mu_0 qI}{mc}} = \int_1^{\gamma_{tot}}\frac{d\gamma}{(\gamma^2-1)^{1/4}} = \int_0^{\Delta\gamma_{tot}}\frac{d\Delta\gamma}{[\Delta\gamma(\Delta\gamma+2)]^{1/4}} \approx \frac{1}{2^{1/4}}\frac{4}{3}\Delta\gamma_{tot}^{\;3/4}$$

$$I_{\max} \approx I_{CM} = \frac{4}{9}\varepsilon_0\sqrt{\frac{2q}{m}}\frac{V^{3/2}}{d^2}$$

$$d\sqrt{\frac{2\mu_0 qI}{mc}} = \frac{1}{2^{1/4}}\frac{4}{3}\Delta\gamma_{tot}^{\;3/4}\,{}_2F_{1(\frac{1}{4},\frac{3}{4},\frac{7}{4})}\left(\frac{-\Delta\gamma_{tot}}{2}\right)$$

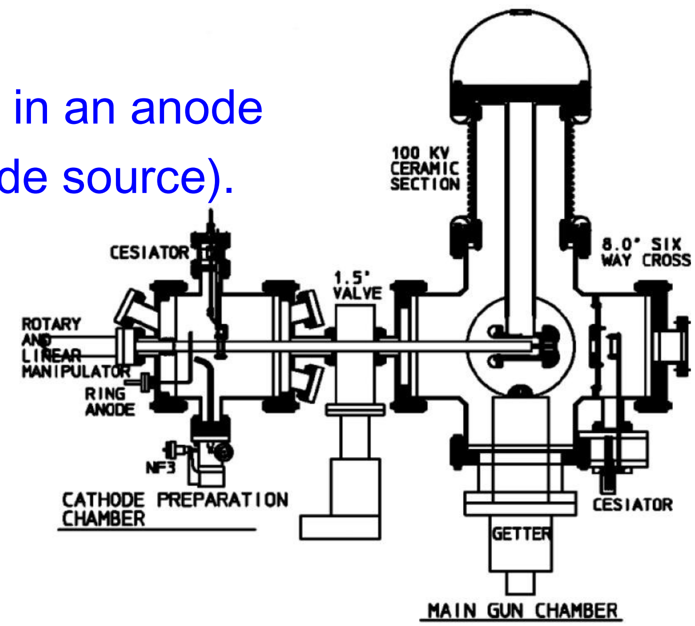
$$I_{\max} = I_{CL}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{\Delta\gamma_{tot}}{2}\right)^2$$



Other Electron and Positron Sources

Photo-Cathode Sources

- A laser shines on a high voltage cathode, which emits photo electrons.
- These are accelerated either through an aperture in an anode (DC source), or in an RF field (RF photo-cathode source).
- With GaAs as cathode and with a polarized laser, polarized electrons are produced.
- Bunches can be as short as a few ps.
- Peak currents of a few 100A can be achieved.



Positron Source

- Electrons are accelerated to about 200MeV in a linac and hit a tungsten target.
- Pair production leads to e+/e- pairs.
- A following linac has the correct phase to accelerate e+ and decelerate e-.
- Due to multiple collisions in the target, the energy spread is up to 30MeV and
- The beam is very wide. A following damping ring is needed to produce narrow beams.